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LIGHT SCALAR MESONS DECAY CONSTANTS AND MASSES WITHIN QCD SUM RULES

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Abstract. Understanding the internal structure of the scalar mesons has been a prominent topic in the last decades. Although the scalar mesons have been investigated both theoretically and experimentally, however, many properties of them are not so clear yet. Identifying the scalar mesons is difficult, experimentally; hence the theoretical and phenomenological works can play a crucial role in this respect. In this work, decay constant and mass of all light scalar mesons are evaluated in the context of the two-point QCD sum rules. The results on masses and decay constants are in a good consistency with existing experimental data. Our predictions for those which we have not any experimental results yet can be checked in the future experiments.

The nature and quark structure of the light scalar mesons denoted by $J^{PC} = 0^{++}$ is still an open problem in hadron physics because of their small experimental width. The light scalar meson sector is the most complex one, both experimentally and theoretically. In spite of the success of the quark model, the explanation with the same concept on light scalar mesons, such as the nonet composed of isoscalars $\sigma(600\ MeV)$ and $f_0(980\ MeV)$, isovector $a_0(980\ MeV)$ and isodoublet$\kappa(800\ MeV)$, still has puzzles. It is not known why $a_0(980\ MeV)$ and $f_0(980\ MeV)$ are degenerate in masses and why the widths of σ and κ are broader than those of $a_0(980\ MeV)$ and $f_0(980\ MeV)$ [1]. In the literature, many suggestions are discussed such as conventional $q\bar{q}$ mesons, $q\bar{q}q\bar{q}$ or meson-meson bound states mixed with a scalar glueball [2]. In reality, they can be superpositions of these components, and one depends on models to determine the dominant one. Although it has been seen progress in recent years, this question remains open.

Understanding the nature of light scalar mesons can explain the Chrial symmetry breaking. In the cure of chiral symmetry breaking, the light quark masses are very small ($m_q \leq 10\ GeV$) and therefore the higher order terms are negligible. However, quark masses are not so small in generalized chiral perturbation theory approaches and higher order terms might become dominant. Therefore an exact determination of the condensate is of great importance for clarifying the nature of the mechanism of chiral symmetry breaking [3].

Mesons are simplest systems for investigating the properties of the fundamental theory of the strong interaction, i.e. Quantum ChromoDynamics (QCD). However, understanding the hadronic structure from the viewpoint of the quark and gluon degrees of freedom is one of the most challenging problems due to the nonperturbative nature of the QCD. Because of the quarks confinement inside hadrons, perturbative calculation of quark-gluon Feynman diagrams is not sufficient. Hence, one has to combine the perturbative QCD result with nonperturbative results. For doing this, one needs to know the QCD dynamics at distances of order of the hadron size [4]. In this letter, we calculate the mass and decay constant of the light scalar mesons with quantum numbers $J^{PC} = 0^{++}$ using QCD sum rules approach (QCDSR).

The method of QCD Sum Rules developed by Shifman, Vainshtein and Zakharov [5] has become very powerful and informative tool in hadron phenomenology in the nonperturbative region. In the QCDSR method, hadrons are represented by their interpolating quark currents. The correlation function of these currents is treated within the framework of the Operator Product Expansion (OPE) [6], where the short and long distance quark–gluon interactions are separated. In order to study the QCDSR for light scalar mesons, we consider the two-point correlation function

$$\Pi_S(q) = i\int d^4x\, e^{iq\cdot x}\langle 0 \,|\, T\{ j_S(x) j_S(0)\} \,|\, 0\rangle \qquad (1)$$

where T is the time ordered product, q is the four-momentum, and $j_S(x)$ is the light scalar quark current with a given flavor $\psi = u, d, s.$ A quark-antiquark pair created by the current j_s with the spin-parity

$J^{PC} = 0^{++}$ is called the scalar meson. The correlation function, $\Pi_S(q)$ **satisfies the standard dispersion relation,**

$$\Pi^{QCD}(q^2) = \int_0^{\infty} \frac{ds}{(s-q^2)} \rho(s) + \Pi^{nonpert} \tag{2}$$

here the spectral density function $\rho(s) = \frac{1}{\pi} Im \Pi^{pert}(q)$ includes the contributions of higher resonances and the continuum. In the OPE or theoretical side, each meson interpolating field is written in terms of the quark field operators as $j_S(x) = \bar{\psi}_1(x)\psi_2(x)$. The correlator (Eq.1) receive contributions from all terms in the OPE. The first and dominant contribution comes from the perturbative term and it is represented in Fig. 1 (a). The two-point correlator function, Eq. (1) can also be written in terms of hadron masses and decay constants. This is the so called phenomenological side of the sum rule.

$$\Pi_S(q) = \frac{\langle 0|j|S\rangle\langle S|j|0\rangle}{m_S^2 - q^2} + \ldots, \tag{3}$$

where …, represents the contributions of the higher states and continuum and m_S is mass of the heavy scalar meson. The coupling of the scalar meson S, to the scalar current J_S that is, the vacuum to meson transition amplitude, can be defined in terms of the meson decay constant f_S as:

$$\langle 0|J_S|S(q)\rangle = f_S m_S \tag{4}$$

The value of the decay constant, which is basically the amplitude to find a quark and anti-quark at the origin, can help us to distinguish between different quark content of the meson [7-10]. The final representation for the physical side can be written in terms of the mass and decay constant as:

$$\Pi_S(q) = \frac{f_S^2 m_S^2}{m_S^2 - q^2} + \ldots, \tag{5}$$

By equating both the theoretical and phenomenological sides, we obtain sum rule for decay constant. For removing the unknown subtraction terms from the dispersion relation and suppress the contributions from the excited resonances and continuum states heavier than the scalar meson *S*, the following Borel transformation is applied:

$$\Pi(M^2) \equiv B_{M^2}\Pi(q^2) = \lim_{\substack{-q^2, n \to \infty \\ -q^2/n = M^2}} \frac{(-q^2)^{n+1}}{n!} \left(\frac{d}{dq^2}\right)^n \Pi(q^2) \tag{6}$$

where M^2 is the Borel mass parameter. We should find range of Borel mass parameter M^2 and the continuum threshold s_0 coming from continuum subtraction such that the results of masses and decay constants do not depend on the value of these parameters since they are not physical quantities.

Now, we focus our attention to calculate the theoretical side of the correlation function. Within the framework of the QCDSR, the correlation function of two scalar currents can be treated with generalized Wilson OPE, where the short and long distance quark–gluon interactions are separated. In OPE expansion, the operators are ordered according to their mass dimension d.

$$\begin{aligned}\Pi^{QCD}(q^2) &= i\int d^4x\, e^{(iq\cdot x)}\langle 0|T\{j_S(x)j_S(0)\}|0\rangle \\ &= C_0 I + C_3\langle 0|\bar{\psi}\psi|0\rangle + C_4\langle 0|G^a_{\alpha\beta}G^{a\alpha\beta}|0\rangle + C_5\langle 0|\bar{\psi}\sigma_{\alpha\beta}(\lambda^a/2)G^{a\alpha\beta}\psi|0\rangle \\ &\quad + C_6\langle 0|(\bar{\psi}\Gamma_r\psi)(\bar{\psi}\Gamma_s\psi)|0\rangle + \cdots \end{aligned} \tag{7}$$

where C_i, $i = 0, 3, 4, 6, \ldots$ are Wilson coefficients, $\Gamma_{r,s}$ denote various combination of Lorentz and color matrices, I is the unit operator, $\bar{\psi}\psi$ is the local Fermion field operator of light quarks, $G^a_{\alpha\beta}$ is the gluon strength tensor. In the above expression the lowest-dimension operator $(d = 0)$ is the unit operator associated with the perturbative contribution: $C_0(q^2) = \Pi^{pert}(q^2)$. The other terms of this expansion are the vacuum expectation values of the $d \neq 0$ operators, composed of quark condensates $\langle 0|\bar{\psi}\psi|0\rangle$, gluon condensates $\langle 0|G^a_{\alpha\beta}G^a_{\alpha\beta}|0\rangle$ and operators with higher dimensions. Wilson coefficients receive dominant contributions from the regions of short distance. The current-vacuum interaction is determined by the long-distance dynamics by the condenstaes with $d \neq 0$. The condensates with $d > 6$ usually play a minor role in the most QCDSR applications and we will not consider them. While for the heavy quarks only the interactions with the vacuum gluons are important, in the case of light quarks, quark condensates are dominant [11, 12]. Up to dimension six, all the diagrams contributing to the correlation function are shown in Figure 1:

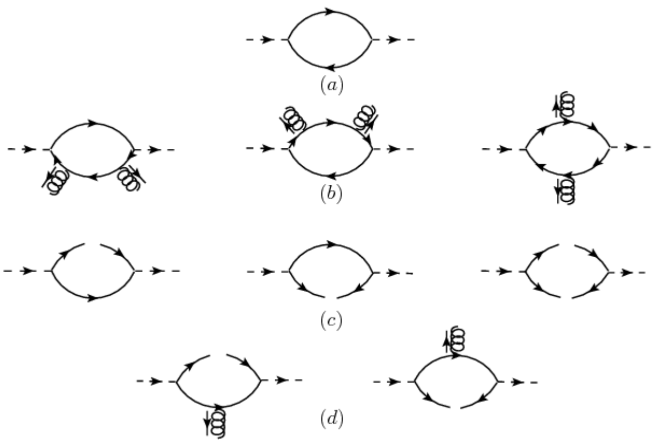


Figure 1: Possible diagrams for two-point correlation function for d ≤ 6. (a) Lowest order bare-loop diagram, (b) gluon condensate diagrams, (c) quark-condensate diagrams, (d) quark-gluon mix condensate diagrams.

After doing the standard calculations and applying the Cutkosky rule which is defined as $1/(k^2 - m^2) \to -2\pi i\delta(k^2 - m^2)$, the spectral density is obtained as:

$$\rho(s) = \frac{N_c}{(4\pi^2 s)}(m_1^2 + m_2^2 - s + 2m_1 m_2)\sqrt{(s + m_1^2 - m_2^2)^2 - 4sm_1^2} \tag{8}$$

where $N_c = 3$ is the color number. In our calculations, we used the fermion propagator as $\mathrm{D = i/(p - m)}$ which is derived from quantization of QCD Lagrangian. Fermion field expanded in Taylor series as

$$\psi_\sigma(x) = \psi_\sigma(0) + x_\alpha \nabla_\alpha|_{x=0} + \cdots \tag{9}$$

where $x_\alpha = -i\frac{\partial}{\partial p_\alpha}$. We also use the Fock-Scwinger gauge, $x^\mu A_\mu^a(x) = 0$. Here, A is the gluon field and it is expressed in momentum space as:

$$A_\tau^c(k) = -\frac{i}{2}(2\pi)^4 G_{\lambda\tau}^c(0)\frac{\partial}{\partial k_\lambda}\delta^4(k) \dots . \tag{10}$$

where k is the gluon momentum. In our calculations quark-gluon-quark vertex function is used as in the following form

$$(\Gamma^c)^{ij} = -ig\left(\frac{\lambda^c}{2}\right)^{ij}\gamma_\tau . \tag{11}$$

After doing calculations and using Feynman parametrization to perform integrals, the contributions coming from different diagrams in the nonperturbative part are obtained as:

$$\begin{aligned}
\Pi_{(b)}^{nonpert} &= \int_0^1 dx \frac{1}{96\pi}\, i < \alpha_s G^2 > x^2((2m_2 x^2(3m_1^5(-1+x)^2 + 3m_1^4 m_2(-1+x)^2 - 2m_1^3(-1+x) \\
&\times (3q^2(-1+x) + m_2^2(1+3x)) + m_2^3 x(2q^2(-1+x) + m_2^2(2+3x)) \\
&+ m_1 m_2 (3q^4(-1+x)^2 + m_2^4\, x(2+3x) + m_2^2 q^2(-1+x)(3+5x)) \\
&- m_1^2(-1+x)(3q^2(-1+x) + m_2^2(2+6x)))) / (m_1^2(-1+x) - (m_2^2 + q^2(-1+x))\, x)^4 \\
&+ (2m_1 x^2(3m_1 m_2^2(m_2 - q)(m_2 + q)(-1+x)^2 + 3m_2(m_2^2 - q^2)^2(-1+x)^2 \\
&+ m_1^5 x(2+3x) + m_1^4 m_2 x(2+3x) + 2m_1^3(-1+x)(q^2 x - m_2^2(1+3x)) \\
&- m_1^2 m_2(-1+x)(-q^2(3+5x) + m_2^2(2+6x)))) / (m_1^2 x + (-1+x)(-m_2^2 + q^2 x))^4 \\
&+ (3(-1+x)^2(-4m_1^5 m_2(-1+x)(-5+3x) - 6m_1^6(-1+x)^2(-5+4x) - 4m_1 m_2(m_2 - q) \\
&\times (m_2 + q)\, x(2q^2(-1+x) + m_2^2(2+3x)) + 4m_1^3 m_2(q^2(-5+x)(-1+x) + m_2^2(-5 \\
&+ 6(-1+x)x)) - 2m_1^2\, x(6q^4(-1+x)^2(-1+2(-1+x)x) + 2m_2^2 q^2(-1+x) \\
&\times (-7 + 18(-1+x)x) + m_2^4(-8 - 33x + 36x^2)) + 3x^2(m_2^6(2+8x) \\
&+ q^6(-1+x)^2(-1+2(-1+x)x) + m_2^4\, q^2 - 5 + 12(-1+x)x) \\
&+ 4m_2^2 q^4) + m_1^4(-1+x)(2m_2^2(-5 - 39x + 36x^2) + 3q^2 \\
&\times (5 + x(7 + 12(-2+x)x)))) / (m_1^2(-1+x) - (m_1^2 + q^2(-1+x))\, x)^4)
\end{aligned} \tag{12}$$

$$\Pi_{(c)}^{nonpert} = \frac{im_1^3\left(\frac{m_0^2}{2} - m_2^2\right)\langle\bar\psi\psi\rangle}{3(m_1^2 - q^2)^3} - \frac{im_1\langle\bar\psi\psi\rangle}{m_1^2 - q^2} - \frac{im_1(2m_1^2 - q^2)\langle\bar\psi\psi\rangle}{6(m_1^2 - q^2)^2}$$

$$+\frac{i(\frac{mo^2}{2}-m_1^2)m_2^3\langle\bar{\psi}\psi\rangle}{3(m_2^2-q^2)^3}-\frac{im_2\langle\bar{\psi}\psi\rangle}{m_2^2-q^2}-\frac{im_1(2m_2^2-q^2)\langle\bar{\psi}\psi\rangle}{6(m_2^2-q^2)^2}-\frac{i\langle\bar{\psi}\psi\rangle^2}{12} \quad (13)$$

$$\Pi_{(d)}^{nonpert}=-\frac{im_0^2(-m_1^3+m_1^2q^2)\langle\bar{\psi}\psi\rangle}{2(m_1^2-q^2)^3}-\frac{im_0^2(-m_2^3+m_2^2q^2)\langle\bar{\psi}\psi\rangle}{2(m_2^2-q^2)^3}$$
$$-\frac{i\alpha_s(m_1^4-m_1^2q^2)\pi\langle\bar{\psi}\psi\rangle^2}{9(m_1^2-q^2)^4}-\frac{i\alpha_s(m_2^4-m_2^2q^2)\pi\langle\bar{\psi}\psi\rangle^2}{9(m_2^2-q^2)^4} \quad (14)$$

here, α_s is the strong coupling constant and $\langle\bar{\psi}\psi\rangle$ is quark condensate. After applying Borel transformation to the above expressions and equating the phenomenological and QCD sides of the correlation function, the sum rules for the mass and decay constant of scalar meson are obtained as

$$m_S^2=\frac{\frac{d}{d(-\frac{1}{M^2})}\int_{(m_1+m_2)^2}^{s_0}ds\,\rho(s)e^{-s/M^2}+\hat{B}\Pi^{\rm nonpert}}{\int_{(m_1+m_2)^2}^{s_0}ds\,\rho(s)e^{-s/M^2}+\hat{B}\Pi^{\rm nonpert}}, \quad (15)$$

$$f_S^2\,e^{-m_S^2/M^2}=\int_{(m_1+m_2)^2}^{s_0}ds\,\rho(s)\,e^{-s/M^2}+\hat{B}\Pi^{\rm nonpert} \quad (16)$$

where $\hat{B}$ represent Borel transformation. Now, we present our numerical results. We choose the light quark masses $m_u=5\ MeV$, $m_d=7\ MeV$ and $m_s=140\ MeV$ [13, 14]. We also take the strong coupling constant $\alpha(M_Z)=0.119\ MeV$ [15], the gluon condensate $\langle\bar{\psi}gG_{\lambda\tau}^c\sigma_{\lambda\tau}\psi\rangle=m_0^2\langle\bar{\psi}\psi\rangle$ with parameter $m_0^2=0.8\ GeV^2$ [16], $\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle=(0.012\pm0.004)\ GeV^4$ and the up, down and strange quark condensates, respectively as $\langle\bar{u}u\rangle=\langle\bar{d}d\rangle=-(0.24)^3\ GeV^3$, $\langle\bar{s}s\rangle=0.8\langle\bar{u}u\rangle$ [17]. The sum rules for the masses and decay constants contain also two auxiliary parameters, namely the continuum threshold s_0 and Borel mass parameter M^2. The physical quantities should be independent of these mathematical objects. Therefore, we look for the regions where the physical quantities do not sensitively depend on these auxiliary parameters. We obtain the working region for the Borel parameter $1GeV^2\leq M^2\leq 4GeV^2$ for all light scalar mesons. Figure 2 shows the dependence of the masses and decay constants of the considered mesons on the Borel mass M^2 for $\sigma(600\ MeV)$, $\kappa(800\ MeV)$, $a_0(980\ MeV)$, $f_0(980\ MeV)$ and $K_0^*(1430\ MeV)$. The region of the continuum threshold has been chosen as $(m_{meson}+0.3)^2\leq s_0\leq (m_{meson}+0.5)^2$, at which dependence on this parameter is weak. Considering the working regions for auxiliary parameters, we obtain the values presented in Tables 1 and 2 for masses and decay constants of light scalar mesons. In these Tables, besides the QCD sum rules results, we also present the existing results from the experiment as well as Lattice QCD. The errors in our values presented in these Tables are due to the uncertainties in the calculation of the working regions for the auxiliary parameters as well as errors in other input parameters. From our analysis, we see that approximately 70% of the total errors presented in the Tables belong to the auxiliary parameters, while the rest come from the errors of the other input parameters. Our results overall have larger uncertainties comparing to the experiment. This is reasonable since QCD sum rules approach contains many input parameters as well as systematic errors. Additionally, the perturbative contribution coming from the bare the loop is large comparing to the nonperturbative contribution. Our calculations show that the nonperturbative part constitutes only (5-20) % of the total values for different considered mesons. This means that the contribution of the higher order operators is small and the sum rules converge and are reliable.

Table 1: Masses of light scalar mesons.

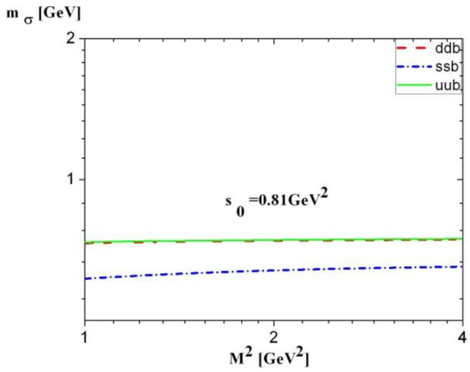
	Our Result (MeV)	Experiment (MeV)	Lattice (MeV)
$m_{\sigma(600\,MeV)}(u\bar{u})$	(743 ± 75)		
$m_{\sigma(600\,MeV)}(d\bar{d})$	(740 ± 70)		
$m_{\sigma(600\,MeV)}(s\bar{s})$	(648 ± 55)		
$m_{\kappa(800\,MeV)}$	(854 ± 90)	(672 ± 40) [18]	(890 ± 21) [19]
$m_{a_0(980\,MeV)}$	(1027 ± 101)	(980 ± 20) [18]	(1010 ± 40) [8]
$m_{f_0(980\,MeV)}(u\bar{u})$	(1034 ± 98)		
$m_{f_0(980\,MeV)}(d\bar{d})$	(1033 ± 105)		
$m_{f_0(980\,MeV)}(s\bar{s})$	(1005 ± 96)		
$m_{K_0^*(1430\,MeV)}$	(1374 ± 120)	(1425 ± 50) [18]	$(1000 - 1200)$ [8], (1420±130) [9]

Table 2: Decay constants of light scalar mesons.

	Our Result (MeV)	QCD Sum Rule [21](MeV)
$f_{\sigma(600\,MeV)}(u\bar{u})$	(140 ± 39)	
$f_{\sigma(600\,MeV)}(d\bar{d})$	(148 ± 37)	
$f_{\sigma(600\,MeV)}(s\bar{s})$	(171 ± 42)	
$f_{\kappa(800\,MeV)}$	(198 ± 55)	(320 ± 20)

$f_{a_0(980\,MeV)}$	(218 ± 60)	(210 ± 50)
$f_{f_0(980\,MeV)}\,(u\bar{u})$	(217 ± 60)	
$f_{f_0(980\,MeV)}(d\bar{d})$	(217 ± 60)	
$f_{f_0(980\,MeV)}(s\bar{s})$	(215 ± 58)	
$f_{K_0^*(1430\,MeV)}$	(298 ± 75)	(445 ± 50)

Conclusion: In the present work, we calculated the decay constants and masses of the light scalar mesons in the context of the two-point QCD sum rules. We only estimated contributions of each quark flavor to the σ and f_0 meson, separately. Our results on the masses and some decay constants are in a good agreement with existing experimental data and lattice QCD predictions specially when the errors in the values are taken into account, but our predictions on some decay constants are slightly different than those of the other approaches. Our predictions for the masses and decay constants which we have no experimental data yet, can be checked in the future experiments. Comparing the theoretical predictions with experimental data would give us valuable information about the nature and structure of these scalar mesons.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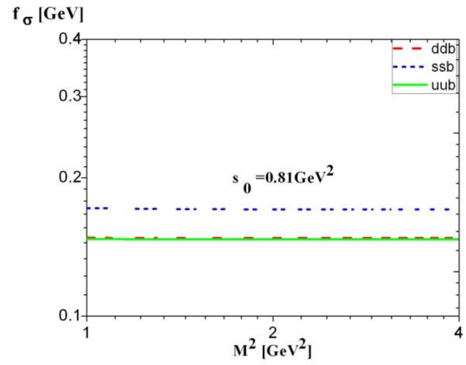
M2 [GeV2]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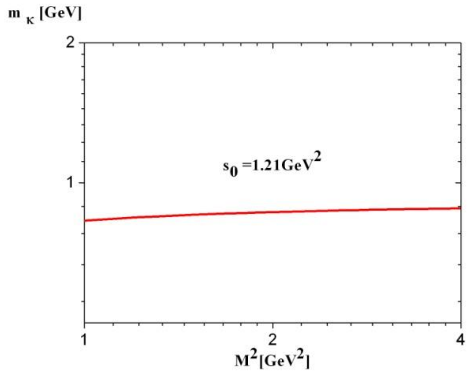
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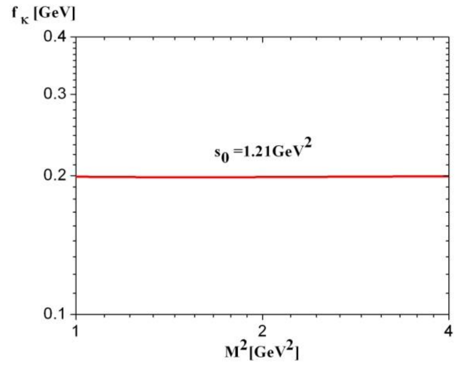
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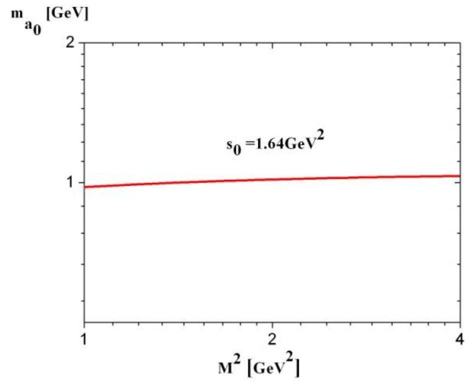
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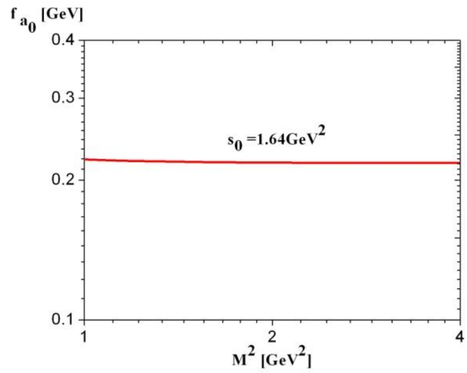
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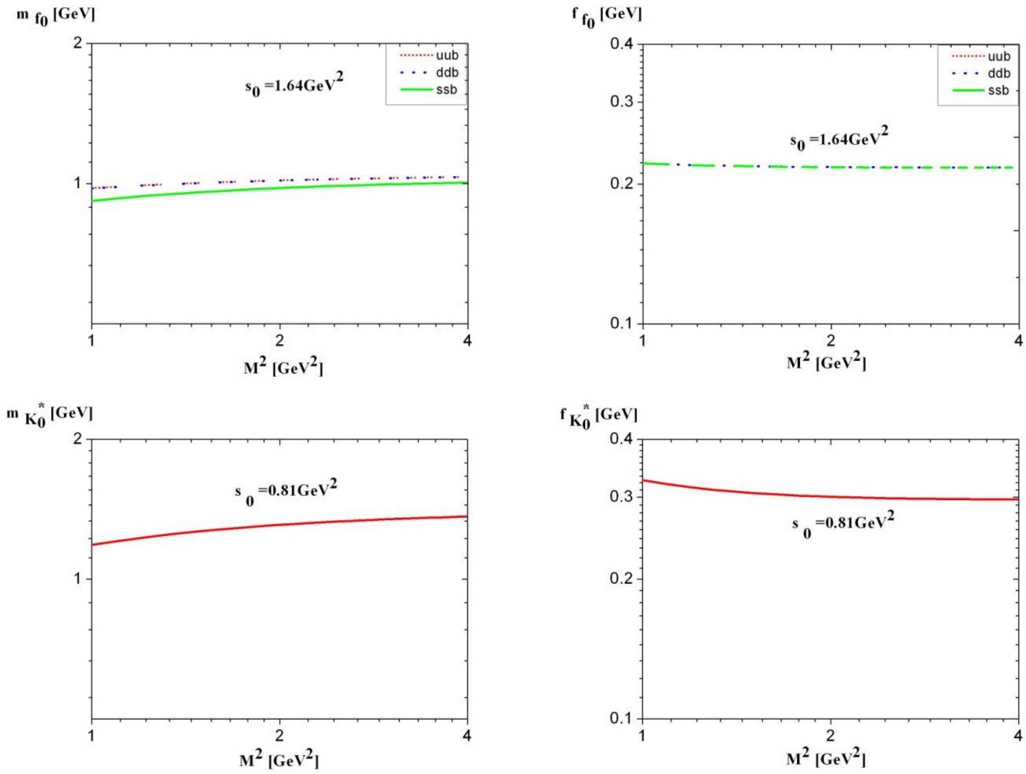

Figure 2: Observables (masses and decay constants) versus $M^2\,(GeV)$: $m_\sigma\,(GeV)$, $f_\sigma\,(GeV)$, $m_\kappa\,(GeV)$, $f_\kappa\,(GeV)$, $m_{a_0}\,(GeV)$, $f_{a_0}\,(GeV)$, $m_{f_0}\,(GeV)$, $f_{f_0}\,(GeV)$ and $m_{K_0^*}\,(GeV)$, $f_{K_0^*}\,(GeV)$ respectively.

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