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Transfer of Quantum Information and Genesis of Superfluid Vacuum in the Pre-Inflationary Universe †

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Abstract

We conjecture that during the time period preceding the inflationary epoch, the background matter was initially a condensate formed from a many-body system of indistinguishable particles whose states were in a quantum superposition. This resulted in the occurrence of a statistical ensemble of spacetimes, thus causing the probabilistic uncertainty in the spacetime geometry of the pre-inflationary multiverse. Then, at a certain moment in time, a measurement event occurred, which broke the linear superposition and reduced the primordial geometrical multiverse to a single state. This process can be described as a quantum Shannon information transfer, which induces logarithmic nonlinearity in the evolution equations of the background system. The latter, therefore, transformed into a logarithmic quantum liquid of a superfluid type and formed the physical vacuum. This measurement also generated the primary mass energy necessary for the Universe's further evolution into the inflationary epoch, followed by the contemporary "dark energy" era.

Keywords: quantum gravity; quantum cosmology; superfluid vacuum; logarithmic fluid; pre-inflationary universe

1. Introduction

The currently popular theory of cosmology deals with the two periods of the Universe's evolution: the contemporary acceleration epoch, often referred to as the "dark energy" era (DE), which was preceded by inflation, the period of space's exponential expansion [1,2]. Most of the observational data known to date are related to these two epochs; these data come from the cosmic microwave background, supernovae, and redshift surveys [3–6].

The time preceding these epochs, the pre-inflationary era, largely remains a mystery, both empirically and theoretically. At those times, even the notion of matter, as we know it, did not yet exist. The currently observed matter and fields are products of the inflationary and post-inflationary epochs. However, one large and easily observable natural phenomenon exists, which is directly related to the pre-inflationary period. This is our Universe itself, with its diverse yet quite orderly and mathematically predictable structure, allowing us to exist and rationalize our reality by assigning laws to the surrounding world. Leaving the anthropic arguments aside, the theoretical describability of our Universe and the experimental repeatability of its laws and processes were not the obvious or mandatory outcome because the pre-inflationary world was evolving under conditions in a strong quantum regime, characterized by high levels of uncertainty in the dynamics and structure

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of whatever kind of matter existed at that time. How (and why) our Universe evolved from the proverbial Schrödinger's cat box, namely the primordial multiverse, remains an open question, probably the biggest one of them all.

On top of this question, one still has to resolve the problems of inflationary cosmology that are likely to be related to the pre-inflationary period: cosmic singularity, the fine-tuning of inflation conditions, and the origin of the primary mass energy necessary for the Universe to begin a rapid exponential expansion.

The relativistic cosmological models are not likely to be able to resolve these problems on their own because their range of applicability ends where quantum effects become significant. As we know, the theory of general relativity (GR) is a classical theory whose quantization is still an open problem. Moreover, the relativistic gravity approach itself probably requires a drastic generalization, similar to Newtonian gravity being absorbed into general relativity when it approached the borders of its applicability. This generalization can be called a theory of quantum gravity or a theory of the physical vacuum because these two notions underlie all other natural phenomena and objects and thus should be strongly related. In any case, a realistic theory of cosmology cannot be a standalone model but must originate from the quantum approach.

The outline of this paper is as follows: In Section 2, we describe one of the candidates for a theory of quantum gravity and physical vacuum: the superfluid vacuum theory. In Section 3, we consider the time period before inflation, when a transfer of quantum Shannon-type information led to the formation of the superfluid vacuum, reduced the primordial multiverse to the Universe, and generated the initial mass energy necessary for further evolution. In Section 4, we present a unified cosmological model originating from the logarithmic superfluid vacuum approach, which describes the inflationary and contemporary "dark energy" epochs. Conclusions are drawn in Section 5.

2. Superfluid Vacuum Theory

Implying standard quantum-mechanical conventions, let us assume that physical vacuum is quantum liquid or condensate whose time evolution is described, for reasons we will justify later, by the logarithmic Schrödinger equation (LogSE)

$$i\hbar\partial_t\Psi = \left[\hat{H} - b\ln(|\Psi|^2/\bar{\rho}) + \ldots\right]\Psi, \tag{1}$$

where $\Psi(\mathbf{x}, t) = \langle \mathbf{x}|\Psi\rangle$ is a condensate wave function representing a state vector $|\Psi\rangle$ in a position space (three-dimensional and Euclidean), $\hat{H} = -\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2 + \mathcal{V}_{\text{ext}}(\mathbf{x}, t)$ is a Hamiltonian operator on a Hilbert space of vectors $|\Psi\rangle$, and $\mathcal{V}_{\text{ext}}(\mathbf{x}, t)$ is an external potential, and b and $\bar{\rho}$ are real-valued parameters. A notation "…" refers to polynomial terms that are powers of fluid density $\rho = |\Psi|^2$; for simplicity such terms will not be considered in this paper.

Logarithmic nonlinearity of this type is instrumental in the theory of physical vacuum known as the theory of superfluid vacuum (SV), or simply SVT, with some landmark works being [7–11]. (There are also extensive mathematical studies of logarithmically nonlinear models, from the classical works [12,13] to the most recent reports [14–53] and latest reviews of applications [54–56].)

According to the superfluid vacuum paradigm, the physical vacuum is a quantum liquid with suppressed dissipative fluctuations, a superfluid (SF). This kind of quantum liquid is formally defined in three-dimensional Euclidean space but makes the latter unobservable for observers who can operate only with small-amplitude, low-momentum fluctuations of this liquid. Instead, such observers "see" four-dimensional Lorentzian spacetime by measuring geodesic trajectories of small excitations of the background superfluid, which can be described according to the irreducible representations of the Poincaré group and

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thus viewed as relativistic particles [57]; for this reason, such observers will be referred to as R(elativistic)-observers in what follows to differentiate them from the F(ull)-observers. A formal mapping, called the superfluid–spacetime correspondence (SF/ST), can be established that relates these two pictures: quantum three-dimensional Euclidean and classical four-dimensional Lorentzian spacetimes; see Appendix A and Ref. [10] for technical details.

Moreover, superfluidity itself implies the inviscid flow of the physical vacuum, thus explaining the negative results of Earth's dragging in the Michelson–Morley-type experiments. In addition, arguments based on the Heisenberg uncertainty principle can be given for the absence of a preferred direction and a reference frame inside quantum background media of a general kind [58]. This, along with the SF/ST correspondence, constitutes an important difference between SVT and the classical ether theories abandoned in favor of the theory of relativity more than a century ago [59,60].

In other words, Lorentz symmetry is not an exact symmetry but an approximation, which is valid in the "infrared" limit (fluctuations with small momentum and amplitude compared to the wave function of the background superfluid). The approach thus has two types of observers: the F(ull)-observer, who can "see" the vacuum as a three-dimensional quantum fluid, and the R-observer, who is unable to observe the underlying quantum Euclidean dynamics and observes the four-dimensional relativistic phenomena instead.

Apart from explaining the occurrence of Lorentz symmetry and relativistic phenomena, superfluid vacuum theory is a well-defined quantum theory with respect to the F-observer. Therefore, it can be regarded as a quantum theory of the gravitational interaction experienced by R-observers. The consistent workflow is to regard Lorentz-covariant gravity as an effective theory for macroscopic measurements by the R-observer but to quantize the underlying Euclidean superfluid and then use the above-mentioned SF/ST correspondence as a "dictionary" to translate the outcomes into the R-observer's language. The direct workflow of superfluid vacuum theory, from three-dimensional Euclidean superfluid dynamics and wave function to the four-dimensional relativistic stress-energy tensor, can be represented as

$$\underbrace{SV\ \ dynamics \xRightarrow{\text{LogSE}} \Psi}_{\text{quantum Euclidean}} \xRightarrow{\text{SF/ST}} \underbrace{g_{\mu\nu} \xRightarrow{\text{'EFE'}} T_{\mu\nu}}_{\text{relativistic gravity}},$$

where "EFE" refers to Einstein field equations in a general sense, i.e., any field-theoretical equations that relate the geometry of induced spacetime $g_{\mu\nu}$ to the distribution of matter within it seen by R-observers, and $T_{\mu\nu}$ is the stress-energy tensor of this distribution. In practice, however, $T_{\mu\nu}$ is known (from R-observers' measurements) but not the background superfluid's dynamics and wave function; therefore, one needs to follow the reversed workflow.

Here, we will adhere to the canonical general relativity, not adding higher powers or derivatives of curvature, Cartan terms, torsion, et cetera. We therefore adopt the conventional Einstein field equations (and, consequently, the Einstein–Hilbert action), which can be introduced via the requirement of the covariant "conservation" or continuity

$$\nabla_\mu T^{\mu\nu} = 0 = \nabla_\mu (R^{\mu\nu} - \tfrac{1}{2} g^{\mu\nu} R), \tag{2}$$

where ∇_μ, $R_{\mu\nu}$, and R are the covariant derivative, Ricci tensor, and scalar curvature, respectively, derived from the metric $g_{\mu\nu}$ induced by the superfluid dynamics via the superfluid–spacetime correspondence. In other words, superfluid vacuum theory also uses the argument of covariant continuity to relate the induced spacetime metric and curvature to the distribution of matter observed by R-observers. The associated stress-energy tensor can now be defined, up to an overall factor, as [8]:

$$T_{\mu\nu} \propto R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \,, \tag{3}$$

where the gravitational constant is merely a proportionality coefficient, which is necessary to make a quantitative agreement with observations of relativistic gravitational phenomena known to date.

Notice that the general relativistic stress-energy tensor is not conserved in the conventional sense: $\partial_\mu T^{\mu\nu} \neq 0$. This poses a closedness question for the theory of general relativity alone because it requires introducing non-Lorentz-covariant notions such as the Landau–Lifshitz pseudotensor of energy-momentum [61], but not for the superfluid vacuum theory, where Lorentz symmetry and spacetime are approximate notions valid only within the limit of small fluctuations, whereas conservation laws are exact in the F-observer picture, cf. Equation (A4).

To summarize, superfluid vacuum theory is a framework for quantum gravity models: In its "infrared" limit (fluctuations with small amplitudes and momenta), it generalizes a theory of general relativity, places it on quantum-mechanical foundations, and thus includes it in a subset of quantum fluid mechanics and condensed matter physics. In the full quantum regime, superfluid vacuum theory is capable of describing phenomena outside an applicability range of relativistic approximation, as we demonstrate below.

3. Before Inflation

In this section, we consider the time period preceding the inflationary epoch. We ask ourselves the following questions: What kind of reality existed before the inflationary epoch's spacetime was generated? Why and how did the superfluid vacuum form in the first place? Why do the logarithmic nonlinearity and associated superfluid model stand out in the SVT framework?

It is natural to begin by assuming that the original medium consisted of $\mathcal{N}$ indistinguishable particles, each described by state vectors $|\aleph_k\rangle$, $k = 1, 2, \ldots, \mathcal{N}$ defined in a single-particle Hilbert space. The dynamics of those particles is defined in three-dimensional Euclidean space and thus governed by a set of $\mathcal{N}$ conventional (linear) Schrödinger equations for $|\aleph_k\rangle$'s. The total system is then described by the product of state vectors $|\aleph_1\rangle \otimes |\aleph_2\rangle \otimes \ldots \otimes |\aleph_{\mathcal{N}}\rangle$ defined, in general, in a Fock space.

As an inevitable outcome of their evolution, at a moment in time, the indistinguishable particles appear to be in one state, $|\aleph_k\rangle \to |\aleph\rangle$. This results in the occurrence of a collective system of exact copies, the Bose–Einstein condensate, which can be described by a vector $|\Psi\rangle$ defined in a single-particle Hilbert space: $|\aleph\rangle \otimes |\aleph\rangle \otimes \ldots \otimes |\aleph\rangle \mapsto |\Psi\rangle$. This state vector's evolution is now governed by a linear Schrödinger equation, which obeys the superposition principle, meaning that the linear combination of its solutions is also a solution.

As a result, the emerged collective object was the proverbial Schrödinger's cat in a closed box: it had a non-zero probability of being in any and all of the states forming that superposition. Since, according to the fluid–spacetime correspondence (more precisely, its special case with kinetic energy and capillary effects but without any wave function–nonlinear parts [10]), each of those wave function solutions generates its own spacetime. As a result, the primordial background was the "geometrical multiverse", i.e., the statistical ensemble of different spacetimes, an actual realization of Everett's interpretation of quantum superposition in physical objects. If a relativistic observer existed at that time, then a repeated measurement would produce, apart from standard measurement errors, different results, with their probability weights assigned by the different multiverse's states.

Since the statistical uncertainty of large-scale spacetime geometry is not reported in the literature, to the best of our knowledge, it is natural to expect that the linear superposition was broken at some point so that the primordial statistical ensemble was reduced to one state. What could possibly have caused the breakdown of this superposition, thus reducing the geometrical multiverse to the Universe we are currently experiencing?

Let us assume that, at a certain moment in time, some kind of measurement was performed by an F-observer on a primordial quantum liquid. In its simplest form, measurement can be viewed as extracting or adding Shannon-type information [62–68], which, in our case, is proportional to the logarithm of the probability that a measurement results in the outcome $|\Psi\rangle$:

$$I_\Psi = -\log_2(|\Psi|^2/|\overline{\Psi}|^2), \tag{4}$$

in bits, where $|\overline{\Psi}|^2$ is a reference value of the probability density (it can be associated with uniform distribution, for example). The change of the system's total energy during such information transfer can be written in the following form (assuming the units $k_B = 1$ in what follows):

$$E_{\text{tot}} = E + \mathcal{E}_\Psi \langle\Psi|I_\Psi|\Psi\rangle = E \pm T_\Psi \mathcal{S}_\Psi, \tag{5}$$

where $\mathcal{E}_\Psi$ is the energy cost per transferred bit, $E = \langle\Psi|\hat{H}|\Psi\rangle$ is the average dynamical energy of the system, $\hat{H}$ is the Hamiltonian operator, $\mathcal{S}_\Psi = -\langle\Psi|\ln(|\Psi|^2/|\overline{\Psi}|^2)|\Psi\rangle$ is quantum-informational entropy, and $T_\Psi = |\mathcal{E}_\Psi|/\ln 2$ is quantum-mechanical temperature forming a thermodynamic conjugate pair with $\mathcal{S}_\Psi$. More details regarding quantum information thermodynamics can be found in Appendix B.

Formula (5) indicates that the Hamiltonian operator $\hat{H}$ gets generalized to the following time evolution operator:

$$\hat{H}|\Psi\rangle \to \hat{H}_{\text{tot}}|\Psi\rangle = \left[\hat{H} - \mathcal{E}_\Psi \log_2(|\Psi|^2/\bar{\rho})\right]|\Psi\rangle, \tag{6}$$

if we set $\bar{\rho} = |\overline{\Psi}|^2$. This means that the corresponding evolution equation acquires a nonlinearity of type (1) where coupling b is associated with quantum-mechanical temperature T_Ψ and energy cost $\mathcal{E}_\Psi$ of measurement and information transfer:

$$b \propto (T_\Psi - T_\Psi^{(0)}), \tag{7}$$

where $T_\Psi^{(0)}$ being a reference value of T_Ψ.

Thus, our condensate's evolution equation is no longer linear, and quantum superposition is broken, inevitably leading to the multiverse's reduction.

Moreover, logarithmic nonlinearity introduces another important effect into the system. Notice first that Equation (1) can be derived as the Euler–Lagrange equation by varying an action functional with the first-order Lagrangian:

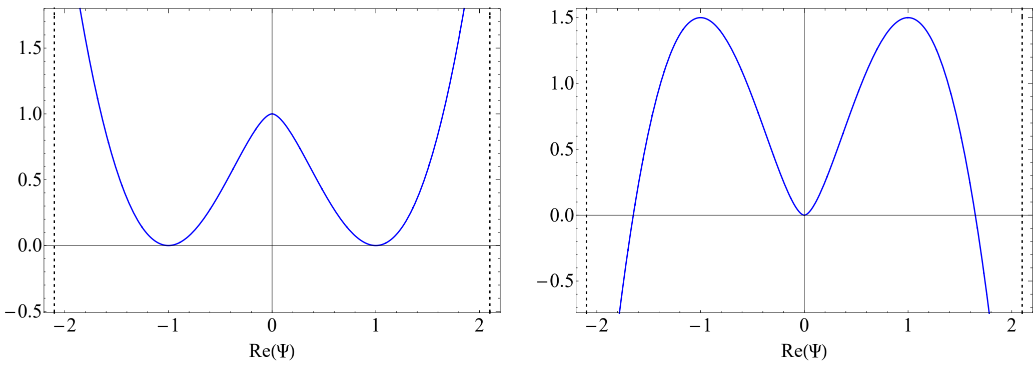
$$\mathcal{L} = \frac{i\hbar}{2}(\Psi^*\partial_t\Psi - \Psi\partial_t\Psi^*) - \frac{\hbar^2}{2m}|\boldsymbol{\nabla}\Psi|^2 - \mathcal{V}_{\text{ext}}(\mathbf{x},t)|\Psi|^2 - \mathcal{V}(|\Psi|^2), \tag{8}$$

where

$$\mathcal{V}(\rho) = -b\rho[\ln(\rho/\bar{\rho}) - 1] + \mathcal{V}_0 \tag{9}$$

is the field-theoretical potential. Here, $\mathcal{V}_0 = \mathcal{V}(0)$ is a shift constant whose value can be ad hoc chosen such that the function (9) always vanishes at its local minima: $\mathcal{V}_0 = \bar{\rho}(|b| - b)/2$. It has nontrivial local extrema at $|\Psi_e| = \sqrt{\bar{\rho}}$: they are minima if coupling b is negative and maxima if it is positive, as illustrated in Figure 1. Thus, this potential is of a Higgs type (also known as the Mexican hat), where the sign of b differentiates between two topologically different sectors of the model, corresponding to the "hat" and "upside hat" versions of the potential (9). These sectors can be regarded as describing different phases if Equation (1) describes matter of some sort [69], as we also expect here.

Figure 1 shows that, for both negative and positive bs, the energetically favored state would be the one with non-zero density. This can be viewed as a mass generation mechanism; the difference between these cases is that mass generation is induced by spontaneous symmetry breaking in the former case and a tunneling-like effect in the latter. It is not yet clear which scenario was realized during the background superfluid formation, but both would have resulted in the superfluid acquiring a non-zero initial mass.



(**a**) $b < 0$ (**b**) $b > 0$

Figure 1. Potential (9) in units of $\bar{\rho}$ versus $\mathrm{Re}(\Psi)$ in units of $\sqrt{\bar{\rho}}$, for $b = \pm 1$ in units of energy. Vertical dotted lines represent the finite density condition $|\Psi|^2 < \infty$ ensured by normalization (A1).

In the first case, topologically nontrivial solutions exist that can be viewed as droplets of lower internal density or "bubbles". Their stability is ensured by a standard topological charge conservation, similarly to solitons in quartic Mexican hat models [70].

In the second case ("upside hat"), one can notice that at least one regular localized solution exists [13], with the energy level below that of the trivial minimum, whereas the wave function's normalization prevents the energy from reaching arbitrarily large negative values. This state corresponds to a Gaussian-like probability distribution because the condensate's wave function normalization also prevents density ρ from taking arbitrarily large values. Gaussian solutions are stable over a wide range of their parameter values, according to both variational and Vakhitov–Kolokolov approaches [71,72]. They can be viewed as droplets of larger internal density than a density background value, which can interact with each other and form the superfluid in a similar way to the one found in laboratory cold gases, such as He-II [73,74].

Jumping ahead, this mechanism also solves the cosmic singularity problem in the spacetime (10) induced by unperturbed laminar flow discussed in the next section: since the minimal-energy density of the newly formed quantum liquid $|\Psi|^2$ cannot be zero or infinite, metric (10) has its singular-value domain excluded because its conformal factor must always be non-zero and finite.

To summarize, the measurement process based on the transfer of quantum information breaks linear superposition, reduces the primordial geometrical multiverse to the Universe, generates its mass, and forms the quantum background of a logarithmic liquid type (which can be proven to be superfluid)—or physical vacuum as we know it. The further evolution of the system is as described in the following section.

4. Inflationary and "Dark Energy" Epochs

In this section, we proceed to the inflationary epoch, assuming the knowledge we have gained from the previous section.

Let us consider a special case where the phase of a superfluid vacuum wave function is a linear function with respect to spatial coordinates and time. This corresponds to a laminar constant-velocity flow in Euclidean space along one of the directions, if viewed by F-observers. The R-observers, however, would see a different picture, according to the SF/ST correspondence. According to the derivation given in Appendix A, they will find themselves in one of the conformally flat four-dimensional spacetimes:

$$g_{\mu\nu} \sim \rho\, \eta_{\mu\nu} \sim |\Psi|^2\, \eta_{\mu\nu}, \tag{10}$$

where $\eta_{\mu\nu}$ is a metric of Minkowski spacetime, and the conformal factor depends on the superfluid wave function squared. This is a rather large class of spacetimes, type **O** in the Petrov(-Pirani-Penrose) classification, which includes de Sitter and others with exponential-type expansion. Such spacetimes can be directly associated with solutions of the condensate Equation (1) for background superfluid (if necessary, this equation can always be extended by including an external potential or polynomial nonlinear corrections with respect to the wave function's amplitude).

From this metric, using the definition (3) and conformal rescaling formulae, one can reverse-engineer the basic Lorentz-covariant action functional [8,11]:

$$S_{\text{Inf}} = \frac{1}{2} \int d^4x \sqrt{-g}\, e^{2\Phi} \left[R + 3!(\partial\Phi)^2 \right] - \int d^4x \sqrt{-g} V_0\,, \tag{11}$$

where $(\partial f)^2 \equiv g^{\mu\nu}\partial_\mu f\, \partial_\nu f$, and $\Phi(\rho) = \ln(\rho/\bar{\rho}) + \Phi_0$ is a function of SV density $\rho = |\Psi|^2$ (for brevity, we assume $\Phi_0 = \Phi(\bar{\rho}) = 0$ in what follows). The topological term with constant V_0 can always be added to a field action. In Equations (10) and (11), we assume Planck units but only for the purpose of brevity of dimensionless notations. In the end, appropriate scale parameters must be introduced to restore dimensionality. This scale freedom comes from the metric tensor (10) being defined up to an overall factor (see Appendix A for details; more detailed comments are in the concluding section).

One can see that, in the R-observer's picture, the background superfluid generates not only the spacetime but also the scalar field Φ, which drives expansion. Therefore, model (11) can be applied to the inflationary epoch in the early Universe, where it can describe not only exponential-type expansion but also the origin of the dilaton scalar field from superfluid vacuum density. In fact, action (11) is a popular type used in dilaton-driven inflationary models, where it delivers the inflationary spacetimes for a large range of conformal factors.

Furthermore, this model can be further extended to describe the transition from the inflation epoch to the contemporary accelerating expansion of the Universe, commonly referred to as the "dark energy" period. From working with laboratory fluids, we know that the laminar flow cannot remain as it is for long due to fluctuations that inevitably occur and cause turbulence. By perturbing the superfluid density in Equation (11) and changing to the Einstein frame, $g^{(\text{Inf})}_{\mu\nu} = e^{-\sqrt{2/3}\,\phi} g^{(\text{DE})}_{\mu\nu}$, one arrives at the model [11]:

$$S_{\text{DE}} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} R - \frac{1}{2}(\partial\phi)^2 + \frac{1}{2} e^{-\lambda\phi} (\partial\sigma)^2 - V_0 e^{-2\lambda\phi} - \Delta V(\phi,\sigma) \right] + S^{(M)}, \tag{12}$$

where ϕ and σ are the quintom, a combination of quintessence and phantom fields, respectively; λ is the quintessence scale parameter (necessary for the right dimensionality once we change to geometrized units); and $\Delta V(\phi,\sigma)$ is the scalar potential's perturbation. The term $S^{(M)} = \int d^4x\sqrt{-g}\mathcal{L}_M$ is added to account for the other matter and radiation content of the Universe, which was generated during and after the inflaton–quintom transition.

Model (12) is a non-minimally-coupled generalization of models that are popular in quintessence and phantom cosmology [75–79]. Note that, unlike conventional phantom models, the scalar–tensor part of the action functional (12) is not postulated but derived from quantum origins.

Field-theoretical actions (11) and (12) thus form the *quinton* system, which comprises the dilaton field that drives inflation and subsequently transforms into the quintom, a combination of the quintessence and tachyonic phantom fields, which plays the role of dark energy. From the F-observer's viewpoint, these three fields are projections of the dynamical evolution of superfluid vacuum density and its fluctuations onto the measuring apparatus of a relativistic observer.

5. Conclusions

Working within the framework of the superfluid vacuum approach to quantum gravity, we studied the time period preceding the inflationary epoch.

We conjectured that the newly formed background was a Bose–Einstein condensate formed from a many-body system of indistinguishable particles. Originally, this pre-vacuum condensate was in a quantum superposition of its states, which created the primordial multiverse with probabilistically uncertain geometry, composed of a statistical ensemble of spacetimes. Then, at a certain moment in time, a measurement event occurred, which broke the linear superposition and reduced this multiverse to one state, the Universe.

We demonstrated that this measurement can be viewed as a transfer of quantum information of the Shannon type, which explains the occurrence of logarithmic nonlinearity in evolution equations. The background condensate thus became the logarithmic quantum liquid and superfluid; it formed what we now call the physical vacuum. In other words, the phenomenon of superfluidity is not only a dynamical one but also a quantum-informational one because it involves interaction with a measuring apparatus of some sort.

The measurement also induced symmetry breaking of either a dynamical or a tunnelling type, depending on the sign of logarithmic coupling. This process generated the primary mass energy necessary for the Universe's further evolution into the inflationary epoch. Superfluid vacuum also induced a four-dimensional Lorentzian spacetime and the relativistic gravity perceived by measuring apparatuses operating with small fluctuations of the background superfluid.

After the inflationary epoch, described by dilaton-type gravity action (11), the Universe entered the contemporary "dark energy" period driven by a combination of the quintessence and phantom fields being non-minimally coupled to each other, cf. model (12). Thus, the superfluid vacuum offers a golden mean in the dispute between corpuscular and geometrical explanations of phenomena attributed to dark energy and matter: the nature of the fundamental background superfluid is ultimately corpuscular if viewed by Euclidean F-observers, but its particles are unobservable by relativistic observers, who perceive the Lorentzian curved spacetime instead and work with a modified theory of gravity [9].

Apart from plentiful predictions of phenomena originating from the inflationary and DE epochs, which could be derived from the modified gravity models (11) and (12), superfluid vacuum cosmology draws a number of predictions from its idea of the pre-inflationary epoch. The main set of predictions is based on the possibility of the breakdown of the "infrared" (small-fluctuation) regime in some domain of space, under yet unknown but highly likely extreme conditions. As a result of such a transition, the four-dimensional Lorentzian description must be replaced with the quantum Euclidean one. This makes various post-relativistic phenomena, such as luminal boom, superluminal propagation, and vacuum Cherenkov radiation (which are a very powerful and fast way of releasing energy), possible [80]. For instance, such phenomena can significantly contribute to the generation and release of energy in blazars/quasars, fast radio bursts, and ultra-relativistic radio objects with continuous optical spectra. Superluminal propagation of astrophysical jets and other objects might also induce optical phenomena somewhat analogous to the acoustic effects caused by shock waves during sonic booms in the atmosphere (some examples can be found in Ref. [81]).

Another set of predicted phenomena, though a rather speculative one, can be made on the assumption that the primordial geometrical multiverse somehow survived somewhere in a large-scale spatial domain. As a result, such a domain would have a geometry that is a statistical ensemble of multiple spacetimes. Observational data coming from such a domain would have additional statistical uncertainty, depending on the probabilistic weight of the spacetime members of the ensemble, although such uncertainty has not yet been reported elsewhere to the best of our knowledge.

Last but not least, remarks must be made about the theory's scale and parameters. The initial model (1) has two free constant parameters, m and $\bar{\rho}$, which are associated with the constituent particles of the background superfluid. It also has one coupling b, which is associated with the quantum-mechanical temperature, cf. Equation (7). In general, this coupling can be a function of position and time, but in the homogeneous and $\mathcal{S}_\Psi$-isentropic limit of superflow, it can be assumed constant: $b \approx b_0$. To this list, one can add integration constants fixed by (hitherto unknown) initial conditions for Equation (1). Leaving these aside, one can ask, what are the characteristic scales or bounds of m, $\bar{\rho}$, and b_0?

While a temptation exists to declare them to be of the Planck scale a priori, this situation is more complex. Equation (1) is the one observed by F-observers but not by R-observers; therefore, the latter are unable to directly observe the superfluid and its constituent particles and measure the above-mentioned three parameters. Instead, relativistic observers must rely on predictions and bounds coming from the "infrared" (small-fluctuations) limit. This limit, however, allows a certain scale of freedom, as discussed after Equation (11). Because the value Φ_0 is not fixed by superfluid dynamics and one can always alter the characteristic scale of the SV density, $\Phi(\rho) = \ln(\rho/\bar{\rho}) + \Phi_0 = \ln(\rho/\bar{\rho}')$, where $\bar{\rho}' = \bar{\rho} \exp(-\Phi_0) = \bar{\rho} \exp[-\Phi(\bar{\rho})]$. Changing the gauge Φ_0 affects the effective gravitational "constant" $\kappa_{\text{eff}} \equiv \exp(-\Phi) = \bar{\rho} \exp(-\Phi_0) |\Psi|^{-2}$ during the inflationary epoch, which in turn affects scale parameters of the dilaton's descendant fields, such as quintom. As a result, even if the density scale $\bar{\rho}'$ is Planckian, it does not yet mean that $\bar{\rho}$ is, and vice versa.

Thus, the only fundamental bound known so far is $c_b^2 + \hbar\omega/2m = c^2$ according to Equation (A20), whereas other bounds are expected to come from empirical arguments and observations based on models (11) and (12) and from local gravitational effects [9,82]. Fortunately, the theory's fundamental predictions, such as the occurrence of the inflation- and DE/DM-associated fields and phenomena, are valid for a large range of values of m, $\bar{\rho}$, and b_0 and thus would generally be protected from fine-tuning problems.

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Appendix A. Superfluid–Spacetime Correspondence

Let us assume that a physical vacuum is a quantum liquid or condensate whose states are described by the state vector $|\Psi\rangle$. In the position representation, the condensate wave function $\Psi(\mathbf{x}, t) = \langle \mathbf{x}|\Psi\rangle$ obeys the normalization condition in the three-dimensional Euclidean space $\mathbb{R}^3$:

$$\int_{\mathbb{R}^3} |\Psi|^2 d\mathbf{x} = \int_{\mathbb{R}^3} \rho d\mathbf{x} = \mathcal{M} = m\mathcal{N} > 0, \tag{A1}$$

where $\mathcal{M}$ and ρ are the total mass and density of the liquid, and m and $\mathcal{N}$ are the mass and number of constituent particles.

We also assume that the time evolution of this liquid is described by the logarithmic Schrödinger equation:

$$i\partial_t \Psi = \left[-\frac{\mathcal{D}}{2} \boldsymbol{\nabla}^2 + \frac{1}{\hbar} \mathcal{V}_{\text{ext}}(\mathbf{x}, t) - \frac{1}{\hbar} b(\mathbf{x}, t) \ln\left(|\Psi|^2 / \bar{\rho} \right) \right] \Psi, \tag{A2}$$

where $b(\mathbf{x}, t)$ is, in general, a real-valued coupling function (for simplicity, assumed to be constant here); $\mathcal{D}$ and $\bar{\rho}$ are constant parameters, positive but otherwise free; and $\mathcal{V}_{\text{ext}}(\mathbf{x}, t)$ is external potential. The constant parameter $\mathcal{D}$ can be written as $\mathcal{D} = \hbar/m$, where m is the mass of a constituent particle.

Furthermore, our wave function can always be assumed to be in the Madelung form $\Psi = \sqrt{\rho} \exp(i\mathcal{S})$, where $\mathcal{S} = \mathcal{S}(\mathbf{x}, t)$ is a phase of the wave function, which is related to the fluid velocity $\mathbf{u} = \mathcal{D}\boldsymbol{\nabla}\mathcal{S}$, and $\rho = |\Psi|^2$ is the density. Then, wave Equation (A2) splits into a set of two equations:

$$\begin{aligned} &\partial_t \rho + \mathcal{D} \boldsymbol{\nabla} \cdot (\rho \boldsymbol{\nabla} \mathcal{S}) = 0, \\ &\partial_t \mathcal{S} - \frac{\mathcal{D}}{2} \left[\boldsymbol{\nabla} \cdot \left(\frac{\boldsymbol{\nabla}\sqrt{\rho}}{\sqrt{\rho}} \right) + \frac{(\boldsymbol{\nabla}\sqrt{\rho})^2}{\rho} - (\boldsymbol{\nabla}\mathcal{S})^2 \right] = \frac{1}{\hbar} [b \ln(\rho/\rho_0) - \mathcal{V}_{\text{ext}}(\mathbf{x}, t)], \end{aligned} \tag{A3}$$

where a dot denotes an inner scalar product. These equations can be rewritten in an explicitly fluid-mechanical form:

$$\begin{aligned} &\partial_t \rho + \boldsymbol{\nabla} \cdot (\rho \mathbf{u}) = 0, \\ &\rho D_t \mathbf{u} = \rho [\partial_t \mathbf{u} + (\mathbf{u} \cdot \boldsymbol{\nabla})\, \mathbf{u}] = \boldsymbol{\nabla} \cdot \mathbb{T} + \mathbf{f}_{\text{ext}}, \end{aligned} \tag{A4}$$

where $D_t = D/Dt$ is a material derivative; $\mathbf{f}_{\text{ext}} = -\frac{\mathcal{D}}{\hbar} \rho \boldsymbol{\nabla} \mathcal{V}_{\text{ext}}$ is the external force per unit volume; and $\mathbb{T} = -\frac{\mathcal{D}^2}{4\rho} \boldsymbol{\nabla}\rho \otimes \boldsymbol{\nabla}\rho - \tilde{p}\, \mathbb{I}$ is the Korteweg-type stress tensor with capillarity, in which $\mathbb{I}$ is the identity matrix, $\tilde{p} = p - \frac{1}{4} \mathcal{D}^2 \boldsymbol{\nabla}^2 \rho$ is the capillary pressure, and $p = p(\mathbf{x}, \rho)$ is the pressure given by the equation of state in a differential form

$$\boldsymbol{\nabla} p = -\frac{\mathcal{D}}{\hbar} \rho \boldsymbol{\nabla} [b \ln(\rho/\bar{\rho})], \tag{A5}$$

which can be easily integrated

$$p = p_0 + c_b^2 \rho, \tag{A6}$$

where we introduced the notation $c_b^2 = -\frac{\mathcal{D}}{\hbar} b = \frac{\mathcal{D}}{\hbar} |b|$. Equation (A6) indicates that the classical limit of our quantum fluid is a barotropic fluid with the ideal equation of state, which implies that collisions inside such fluid are elastic and therefore dissipation-free. The latter is one of the indicators that logarithmic liquid models can be applied in a theory of superfluidity, although the full picture is more complicated [73,74].

For the rest of this section, we will be neglecting quantum capillary effects, which are of a higher order with respect to the Planck constant than the pressure terms. Therefore, we will be working in a linear approximation with respect to $\mathcal{D}$:

$$\mathcal{O}(\mathcal{D}^2) \approx 0, \tag{A7}$$

where $\mathcal{O}(f)$ denotes terms of order f; the complete formulae including capillary effects can be found in [10]. This approximation is equivalent to assuming the kinetic energy term in Equation (A2) to be smaller than the rest, which is a robust assumption for quantum condensates. We also remember that the phase of the wave function is of an order $\mathcal{D}^{-1}$ due to its relation to fluid velocity.

Fluid perturbations and emergent Lorentz symmetry. Let us consider small perturbations of the condensate (A2). We introduce some shorthand notations $\phi \equiv -\mathcal{D}\,\mathcal{S}$, $\mathbf{u} = -\boldsymbol{\nabla}\phi$, and perform the following expansions:

$$\rho = \rho_{(0)} + \rho_{(1)},\ \phi = \phi_{(0)} + \phi_{(1)},\ \mathbf{u} = \mathbf{u}_{(0)} + \mathbf{u}_{(1)}, \tag{A8}$$

where bracketed subscripts label orders of magnitude, such that $|\rho_{(1)}| \ll |\rho_{(0)}|$ and so on.

Then, we linearize Equations (A3)–(A5) by keeping only first powers of the first-order values. We obtain

$$\partial_t\rho_{(0)} - \boldsymbol{\nabla}\cdot\left(\rho_{(0)}\boldsymbol{\nabla}\phi_{(0)}\right) = 0, \tag{A9}$$

$$\partial_t\rho_{(1)} - \boldsymbol{\nabla}\cdot\left(\rho_{(0)}\boldsymbol{\nabla}\phi_{(1)} + \rho_{(1)}\boldsymbol{\nabla}\phi_{(0)}\right) = 0, \tag{A10}$$

$$\partial_t\phi_{(0)} - \frac{1}{2}\left(\boldsymbol{\nabla}\phi_{(0)}\right)^2 = c_b^2 \ln\left(\frac{\rho_{(0)}}{\bar{\rho}}\right) + \frac{\mathcal{D}}{\hbar}\mathcal{V}_{\rm ext}(\mathbf{x},t), \tag{A11}$$

$$\partial_t\phi_{(1)} - \boldsymbol{\nabla}\phi_{(0)}\cdot\boldsymbol{\nabla}\phi_{(1)} = c_b^2\frac{\rho_{(1)}}{\rho_{(0)}}, \tag{A12}$$

also bearing in mind the approximation (A7). From Equations (A10) and (A12), we obtain

$$-\partial_t\left[\frac{\rho_{(0)}}{A_b}\left(\partial_t\phi_{(1)} - \boldsymbol{\nabla}\phi_{(0)}\cdot\boldsymbol{\nabla}\phi_{(1)}\right)\right] + \boldsymbol{\nabla}\cdot\left[\rho_{(0)}\boldsymbol{\nabla}\phi_{(1)} + \frac{\rho_{(0)}}{A_b}\boldsymbol{\nabla}\phi_{(0)}\left(\partial_t\phi_{(1)} - \boldsymbol{\nabla}\phi_{(0)}\cdot\boldsymbol{\nabla}\phi_{(1)}\right)\right] = 0, \tag{A13}$$

where $A_b = c_b^2$. This equation can be rewritten in the explicitly four-dimensional Lorentz-covariant form

$$\partial_\mu(f^{\mu\nu}\partial_\nu\phi_{(1)}) = 0, \tag{A14}$$

where we assume the Einstein summation convention for repeating Greek indices labeling the coordinates $x^\mu = \{t, \mathbf{x}\}$ and introduce the rank-4 matrix

$$f^{\mu\nu} \propto \rho_{(0)}\begin{pmatrix} -1 & \vdots & \partial_j\phi_{(0)} \\ \cdots & \cdot & \cdots \\ \partial_i\phi_{(0)} & \vdots & A_b\delta_{ij} - \partial_i\phi_{(0)}\partial_j\phi_{(0)} \end{pmatrix}, \tag{A15}$$

where Latin indices label spatial coordinates and the proportionality symbol indicates that this matrix is defined up to a multiplicative constant.

Furthermore, we restore the wave function-related notations $\phi_{(0)} = -\mathcal{D}\,\mathcal{S}$, $\phi_{(1)} = -\mathcal{D}\,\delta\mathcal{S}$, $\rho_{(0)} = \rho = |\Psi|^2$, and rewrite Equation (A14) in the form

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\delta\mathcal{S}) = 0, \tag{A16}$$

where we introduce the Lorentzian spacetime metric tensor

$$g^{\mu\nu} \propto \frac{1}{\sqrt{-f}} f^{\mu\nu} \propto \frac{1}{\rho} \begin{pmatrix} -1 & \vdots & -\mathcal{D}\partial_j \mathcal{S} \\ \cdots & \cdot & \cdots \\ -\mathcal{D}\partial_i \mathcal{S} & \vdots & A_b \delta_{ij} - \mathcal{D}^2 \partial_i \mathcal{S}\, \partial_j \mathcal{S} \end{pmatrix}, \quad \text{(A17)}$$

$$g_{\mu\nu} = (g^{\mu\nu})^{-1} \propto \rho \begin{pmatrix} -N^2 & \vdots & -\mathcal{D}\partial_j \mathcal{S} \\ \cdots & \cdot & \cdots \\ -\mathcal{D}\partial_i \mathcal{S} & \vdots & \delta_{ij} \end{pmatrix}, \quad \text{(A18)}$$

where $N^2 \equiv \frac{1}{2}\mathcal{D}\omega + A_b - \mathcal{D}^2 (\boldsymbol{\nabla}\mathcal{S})^2$ is a lapse function. The metric is defined up to an overall factor that plays the role of a gauging function dependent on a choice of units for clocks and measuring rods.

Due to the approximate Lorentz symmetry that emerged, it is necessary to differentiate between observers who can *only* perceive small fluctuations of the background superfluid (as relativistic gravity, particles, and fields), to be referred to as the R(elativistic)-observers, from F(ull)-observers who can directly operate with the quantum Equation (A2) and corresponding superfluid. Since the known physical observers are R-observers, the SF/ST correspondence map forms a basis for the dictionary for translating between the quantum 3D Euclidean and "classical" 4D Lorentzian descriptions of reality.

Special case: linear phase. Let us study a special case which will be instrumental for the purposes of this paper: that the logarithmic liquid is in a stationary state, with the phase:

$$\mathcal{S} = \omega t + \mathcal{D}^{-1} \mathbf{u}_0 \cdot \mathbf{x}, \;\; \mathbf{u}_0 = \text{const}, \quad \text{(A19)}$$

which means that its flow is laminar with a constant velocity $\mathcal{D}\boldsymbol{\nabla}\mathcal{S} = \mathbf{u}_0 = \text{const}$ in Euclidean space. This is the picture observed by F-observers.

The R-observers perceive, according to Equation (A18), spacetime with the conformally Minkowski metric tensor

$$g_{\mu\nu} \propto \rho \begin{pmatrix} -(c_s^2 - \mathbf{u}_0^2) & \vdots & -(\mathbf{u}_0)_j \\ \cdots & \cdot & \cdots \\ -(\mathbf{u}_0)_i & \vdots & \delta_{ij} \end{pmatrix} + \mathcal{O}(\mathcal{D}^2), \quad \text{(A20)}$$

where $c_s^2 = c_b^2 + \mathcal{D}\omega/2$ with ω assumed to be a frequency eigenvalue of the logarithmic background condensate in a given quantum state, and $\mathcal{O}(\mathcal{D}^2)$ reminds us that we neglected quantum capillary effects, cf. Equation (A7). This metric can be further simplified: without loss of generality, we can set $\mathbf{u}_0 = 0$ either by choosing a fluid-comoving frame of reference (in the Euclidean F-observer's picture) or due to the metric tensor's invariance under diffeomorphisms (in the R-observer's picture).

From Equation (A20) one can see that value $c_s = \sqrt{c_b^2 + \hbar\omega/2m}$, plays a role of the maximally attainable velocity of small fluctuations of superfluid vacuum (perceived as relativistic particles by R-observers). Therefore, from the viewpoint of those observers, this velocity can be associated with the fundamental constant of the speed of light. In other words, in superfluid vacuum theory, this constant is no longer fundamental but a derivable value.

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Appendix B. Quantum Information and Entropy

Let us start with the information theory and introduce some concepts. Assume that we have a random variable $\mathbf{R}$ with a set of values $\{R_i\}$ with distribution $P(R_i)$. Let us define the following number:

$$\langle I\rangle_{\mathrm{R}} = -\sum_i P_i \ln P_i, \tag{A21}$$

where $P_i = P(R_i)$; this number thus depends on the probabilities P's but not on the values $\{R_i\}$ themselves. One can notice that the value $\langle I\rangle_{\mathrm{R}}$ vanishes for "singular" distributions, in which the probability is 1 for one of the values of $\{R_i\}$ and null for all others. On the other hand, its value increases to $\ln n$ for distributions over n equal elements of the set $\{R_i\}$. In fact, any changes in the distribution P_i that "flatten" the probabilities increase the value of $\langle I\rangle_{\mathrm{R}}$.

Therefore, this number can be used as a measure of how far a probability distribution is from being "singular", i.e., as a measure of the probability distribution's spread. It is somewhat similar to variance, but it can be defined for distributions over arbitrary sets, while variances are defined only for distributions over real numbers.

Let us extrapolate this concept to quantum mechanics. Assume our system is in the state $|\psi\rangle$ and a basis $\{\psi_i\}$ determines a square-amplitude distribution of $|\psi\rangle$ over the set $\{\psi_i\}$. Therefore, we can assign

$$P_i = |\langle\psi_i|\psi\rangle|^2, \tag{A22}$$

then, the Formula (A21) yields

$$\langle I\rangle_\psi = -\sum_i |\langle\psi_i|\psi\rangle|^2 \ln\Big(|\langle\psi_i|\psi\rangle|^2\Big), \tag{A23}$$

or, in the basis-free notations,

$$\langle I\rangle_\psi = -\langle\psi|\ln(\alpha|\psi|^2)|\psi\rangle, \tag{A24}$$

where α is a scale parameter that ensures a dimensionless value of the logarithm's argument.

The parameter α can be defined as a reference number for the measurement of $\langle I\rangle_\psi$. Recalling the remarks made in the paragraph after Equation (A21), it is natural to connect the value of α to a uniform distribution $|\overline{\psi}|^2$. By imposing

$$\langle\psi|\ln(\alpha|\overline{\psi}|^2)|\psi\rangle = 0, \tag{A25}$$

we obtain

$$\alpha = 1/|\overline{\psi}|^2, \tag{A26}$$

which can be used, along with Equation (A24), to define quantum information entropy

$$\mathcal{S}_\psi = -k_B \int_{\mathbb{R}^{\bar{d}}} |\psi(\mathbf{x})|^2 \ln\Big(|\psi(\mathbf{x})|^2/\bar{n}\Big) d\mathbf{x}, \tag{A27}$$

where $|\psi(\mathbf{x})|^2 = n(\mathbf{x})$ is particle density, $\bar{n} = |\overline{\psi}|^2$, k_B is the Boltzmann constant. This defines the relative entropy of information stored in $|\psi\rangle$ with respect to the entropy of information stored in $|\overline{\psi}\rangle$.

Let us consider now a case of a quantum liquid in $\bar{d}$-dimensional space. Then the state-vector normalization gets slightly more complicated. The condition (A1) generalizes to

$$\int_{\mathbb{R}^{\bar{d}}} |\Psi(\mathbf{x})|^2 d\mathbf{x} = \int_{\mathbb{R}^{\bar{d}}} |\Phi(\mathbf{k})|^2 d\mathbf{k} = \mathcal{M}, \tag{A28}$$

where $\mathbf{k}$ is a wave vector and $\Phi(\mathbf{k})$ is the condensate wave function in the momentum representation.

To preserve the probabilistic interpretation under the normalization (A28), one needs to adjust the definitions of quantum-mechanical averages $\langle \hat{O} \rangle_{\mathcal{M}} \equiv (1/\mathcal{M}) \int_{\mathbb{R}^{\bar{d}}} \Psi^*(\mathbf{s}) \hat{O}\, \Psi(\mathbf{s}) d\mathbf{s}$. Alternatively, one can maintain standard conventions of averages

$$\langle \hat{O} \rangle \equiv \int_{\mathbb{R}^{\bar{d}}} \psi^*(\mathbf{s}) \hat{O}\, \psi(\mathbf{s}) d\mathbf{s}, \tag{A29}$$

but work with rescaled wave functions

$$\psi(\mathbf{x}) = \Psi(\mathbf{x})/\sqrt{\mathcal{M}},\ \phi(\mathbf{k}) = \Phi(\mathbf{k})/\sqrt{\mathcal{M}}, \tag{A30}$$

which are normalized to one:

$$\langle \psi(\mathbf{x}) | \psi(\mathbf{x}) \rangle = \int_{\mathbb{R}^{\bar{d}}} |\psi(\mathbf{x})|^2 d\mathbf{x} = 1,\ \ \langle \phi(\mathbf{k}) | \phi(\mathbf{k}) \rangle = \int_{\mathbb{R}^{\bar{d}}} |\phi(\mathbf{k})|^2 d\mathbf{k} = 1. \tag{A31}$$

Then, we can use the definition (A27) without modifications. Its momentum-space analogue would be, assuming the normalization (A31),

$$\widetilde{\mathcal{S}}_\phi = -k_B \int_{\mathbb{R}^{\bar{d}}} |\phi(\mathbf{k})|^2 \ln\Big(|\phi(\mathbf{k})|^2 / |\overline{\phi}|^2 \Big) d\mathbf{k}, \tag{A32}$$

which defines the relative entropy of information stored in the Fourier image of $|\psi(\mathbf{x})\rangle$ with respect to the entropy of information stored in $|\overline{\phi}\rangle$. It is convenient to assume

$$|\overline{\phi}|^2 = \zeta / \bar{n}, \tag{A33}$$

where ζ is a dimensionless constant measuring the difference between reference values of the position-space and momentum-space entropies (A27) and (A32).

This kind of entropy satisfies a certain inequality that has a profound quantum-mechanical meaning. To derive it, we generalize the method of Ref. [63] for a $\bar{d}$-dimensional space, and use these formulae for $\bar{d}$-dimensional integrals:

$$\begin{aligned}
&\int_{\mathbb{R}^{\bar{d}}} e^{-\frac{1}{2} a\, \mathbf{x}^2 - i \mathbf{k}\cdot\mathbf{x}}\, d\mathbf{x} = \left(\frac{2\pi}{a}\right)^{\bar{d}/2} e^{-\mathbf{k}^2/2a}, \\
&\int_{\mathbb{R}^{\bar{d}}} \mathbf{b}\cdot\mathbf{x}\, e^{-\frac{1}{2} a\, \mathbf{x}^2 - i \mathbf{k}\cdot\mathbf{x}}\, d\mathbf{x} = -\frac{i}{a} \mathbf{b}\cdot\mathbf{k} \left(\frac{2\pi}{a}\right)^{\bar{d}/2} e^{-\mathbf{k}^2/2a}, \\
&\int_{\mathbb{R}^{\bar{d}}} \mathbf{x}^2 e^{-\frac{1}{2} a\, \mathbf{x}^2 - i \mathbf{k}\cdot\mathbf{x}}\, d\mathbf{x} = \frac{\bar{d}}{a} \left(1 - \frac{\mathbf{k}^2}{a}\right) \left(\frac{2\pi}{a}\right)^{\bar{d}/2} e^{-\mathbf{k}^2/2a}, \\
&\int_{\mathbb{R}^{\bar{d}}} \left| C e^{-\frac{1}{4} a x^2} \right|^2 \ln\left(\left| C e^{-\frac{1}{4} a x^2} \right|^2 / b \right) d\mathbf{x} = |C|^2 \left(\frac{2\pi}{a}\right)^{\bar{d}/2} \left[\ln\left(\frac{|C|^2}{b}\right) - \frac{1}{2}\bar{d} \right],
\end{aligned}$$

where $\mathbf{x}$ and $\mathbf{k}$ are $\bar{d}$-dimensional vectors, respectively. Assuming the norm (A31) and units $k_B = 1$, we derive the inequality

$$\mathcal{S}_\psi + \widetilde{\mathcal{S}}_\phi \geqslant \bar{d}\Big[1 + \ln\Big(\pi \zeta^{1/\bar{d}} \Big) \Big], \tag{A34}$$

where $\zeta = \bar{n} |\overline{\phi}|^2 = |\overline{\psi}|^2 |\overline{\phi}|^2 = (|\overline{\Psi}\, \overline{\Phi}| / \mathcal{M})^2$, according to the notations above. This inequality generalizes the ones obtained in the works [63–65], which did not take into account that a multiplicative constant in the logarithm's argument is necessary for dimensionality reasons, and this constant can, in general, be different for the position-space and momentum-space entropies considered above.

One can notice that Equation (A34) becomes saturated on the Gaussian functions

$$\psi_g(\mathbf{x}) = (2\pi\sigma^2)^{-\bar{d}/4}\exp\left(-\mathbf{x}^2/4\sigma^2\right), \;\; \phi_g(\mathbf{k}) = (2\sigma^2/\pi)^{\bar{d}/4}\exp\left(-\sigma^2\mathbf{k}^2\right), \tag{A35}$$

which are both normalized according to Equation (A31).

Let us prove now that inequality (A34) is stronger than the conventional Heisenberg uncertainty relation, in the sense that the latter can be derived from the former, but not vice versa. To do that, let us calculate the maximum value of the entropy $\mathcal{S}_\psi$ under the constraints (A31) and $\langle(\mathbf{x}-\langle\mathbf{x}\rangle)^2\rangle \equiv \langle\psi(\mathbf{x})|(\mathbf{x}-\langle\psi(\mathbf{x})|\mathbf{x}|\psi(\mathbf{x})\rangle)^2|\psi(\mathbf{x})\rangle = \mathbf{x}_0^2$. This is equivalent to finding the extremum of the following functional:

$$\mathcal{S}_\psi[n(\mathbf{x})] \equiv \mathcal{S}_\psi - \lambda_1(\langle\psi(\mathbf{x})|\psi(\mathbf{x})\rangle - 1) - \lambda_2\left(\langle(\mathbf{x}-\langle\mathbf{x}\rangle)^2\rangle - \mathbf{x}_0^2\right), \tag{A36}$$

where λ's being the Lagrange multipliers and $n(\mathbf{x}) = |\psi(\mathbf{x})|^2$ is the particle density normalized to one. The vanishing variation $\delta\mathcal{S}_\psi[n(\mathbf{x})]/\delta n(\mathbf{x})$ yields the equation

$$\ln\left(n_{\max}(\mathbf{x})/\bar{n}\right) + \lambda_1 + 1 + \lambda_2\left(\mathbf{x}^2 - \langle\mathbf{x}\rangle\cdot\mathbf{x}\right) = 0, \tag{A37}$$

where $n_{\max}(\mathbf{x})$ is the particle density corresponding to the extremum (maximum) of entropy. We therefore obtain

$$n_{\max} = \left(2\pi x_0^2/\bar{d}\right)^{-\bar{d}/2}\exp\left[-\bar{d}(\mathbf{x}-\langle\mathbf{x}\rangle)^2/2x_0^2\right], \tag{A38}$$

$$\max[\mathcal{S}_\psi] = \frac{\bar{d}}{2}\ln\left(2\pi e x_0^2\bar{n}^{2/\bar{d}}/\bar{d}\right), \tag{A39}$$

where $x_0 = \sqrt{\mathbf{x}_0^2}$. The last equation suggests the inequality

$$\mathcal{S}_\psi \leqslant \frac{\bar{d}}{2}\ln\left(2\pi e x_0^2\bar{n}^{2/\bar{d}}/\bar{d}\right), \tag{A40}$$

which can be rewritten as

$$e\pi\exp\left(-2\mathcal{S}_\psi/\bar{d}\right) \geqslant \frac{\bar{d}}{2}\bar{n}^{-2/\bar{d}}\left[\langle(\mathbf{x}-\langle\mathbf{x}\rangle)^2\rangle\right]^{-1}, \tag{A41}$$

remembering the above-mentioned constraints (A37). By performing similar calculations in the momentum space, we obtain

$$\frac{2}{\bar{d}}\langle(\mathbf{k}-\langle\mathbf{k}\rangle)^2\rangle \geqslant (e\pi)^{-1}(\bar{n}/\zeta)^{2/\bar{d}}\exp\left(2\widetilde{\mathcal{S}}_\phi/\bar{d}\right). \tag{A42}$$

Combining the last two inequalities with the inequality (A34), we obtain

$$\begin{aligned}\frac{2}{\bar{d}}\langle(\mathbf{k}-\langle\mathbf{k}\rangle)^2\rangle &\geqslant (e\pi)^{-1}(\bar{n}/\zeta)^{2/\bar{d}}\exp\left(2\widetilde{\mathcal{S}}_\phi/\bar{d}\right)\\ &\geqslant e\pi\bar{n}^{2/\bar{d}}\exp\left(-2\mathcal{S}_\psi/\bar{d}\right) \geqslant \frac{\bar{d}}{2}\left[\langle(\mathbf{x}-\langle\mathbf{x}\rangle)^2\rangle\right]^{-1},\end{aligned} \tag{A43}$$

therefore,

$$\sqrt{\langle(\mathbf{k}-\langle\mathbf{k}\rangle)^2\rangle}\sqrt{\langle(\mathbf{x}-\langle\mathbf{x}\rangle)^2\rangle} \geqslant \bar{d}/2, \tag{A44}$$

which thus establishes a limit to the precision of a simultaneous position-momentum measurement in $\bar{d}$ dimensions due to the quantum uncertainty, if one recalls that $\mathbf{k} = \mathbf{p}/\hbar$. For a number of dimensions larger than one (three is our special interest here, obviously), this inequality is actually stronger than the canonical Heisenberg uncertainty relation;

cf. $\sqrt{\langle(\mathbf{k} - \langle\mathbf{k}\rangle)^2\rangle}\sqrt{\langle(\mathbf{x} - \langle\mathbf{x}\rangle)^2\rangle} \geqslant 1/2$. In addition, quantum entropic uncertainty has a larger range of applicability than variance-based uncertainty relations because variance can be divergent in some distributions.

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