Indefinite causality in quantum mechanics and its thermodynamic applications

Kyrylo Simonov

Fakultät für Mathematik, Universität Wien, Oskar-Morgenstern-Platz 1, 1090 Vienna, Austria

E-mail: kyrylo.simonov@univie.ac.at

Abstract. The nature of causality remains one of the key puzzles in science. In quantum theory, the causal structure is not subject to quantum uncertainty and plays rather a background role. One can ask whether the background causal structure can be dropped, for example, by respecting causality only locally. Such scenarios of local validity of quantum theory while relaxing the global definite causal order of operations can be described via the machinery of process matrices. An important example of scenarios of this kind is quantum SWITCH, a process realizing a quantum superposition of causal orders of operations. Looking for the possible applications of quantum SWITCH has been the subject of growing interest in the scientific community as it could provide communication and computational resources not realizable via standard quantum theory. In the last few years, the benefits potentially offered by quantum SWITCH for thermodynamic tasks have appeared in the spotlight. This contribution aims at reviewing the recent proposals of thermodynamic applications of quantum SWITCH and draw the perspectives.

1. Introduction

It is well known that causality plays a fundamental role in our everyday life. In particular, the causal nature of our actions is one of the first things we learn about. Nevertheless, the fundamental status of causality, being famously criticized by Bertrand Russell [1], still remains a subject of debates and one of the key puzzles in physics and philosophy. In particular, one of the difficulties with the unification of quantum theory with general relativity could arise from the very notion of time and the place of causality in quantum theory. Recently, there have appeared theoretical constructions that challenge the fixed global causal structure assumed by standard quantum theory. A significant breakthrough in this direction has been achieved by Hardy that focused on theories that borrow both indeterminism of quantum theory and dynamic treatment of space-time geometry of general relativity and formulated an operational approach to such theories known as the causaloid framework. In such theories, just as dynamic quantities manifest uncertainty in quantum theory, space-time geometry is treated as indefinite, leading thus to indefinite causal structures [2]. Later on, Chiribella et al. have shown that quantum theory allows for scenarios which respect causality only locally by constructing a process with quantum operations executed without fixed causal order, the quantum SWITCH [3]. Description of such processes without pre-assumed fixed causal order of quantum operations has been encapsulated by Oreshkov, Costa, and Brukner within the process matrix formalism [4]: its development has demonstrated that indefinite causal structures can be incorporated to quantum theory without abandoning its linear structure.

Journal of Physics: Conference Series **2533** (2023) 012039 doi:10.1088/1742-6596/2533/1/012039

Quantum SWITCH has provided an important paradigmatic example of a process realizing indefinite causal order of operations since it has been implemented in table-top experiments [5, 6]. These have significantly increased interest in indefinite causal structures and their possible benefits, first of all, for quantum information processing. Nevertheless, despite intimate connection between information and thermodynamics, potential benefits of the quantum SWITCH for thermodynamic tasks remained in shadow and have been put under scrutiny only very recently.

Quantum SWITCH relaxes the implied causal relations between quantum operations by making their application order subject to superposition. This is achieved by coherently controlling the order of operations by another quantum system which gets entangled with the application orders of the operations. For two operations $\mathcal{A}[.]$ and $\mathcal{B}[.]$, the control system is a qubit in state ω, and the resulting evolution of both target system (prepared in a state ρ) and control qubit can be represented as a new quantum operation

$$\mathcal{S}_{\mathcal{A},\mathcal{B}}[\rho \otimes \omega] = \sum_{ij} K_{ij}(\rho \otimes \omega)K_{ij}^{\dagger}, \tag{1}$$

whose Kraus decomposition is given by the operators $K_{ij} = A_i B_j \otimes |0\rangle\langle 0| + B_j A_i \otimes |1\rangle\langle 1|$, where the sets of operators $\{A_i\}$ and $\{B_j\}$ constitute the Kraus decompositions of $\mathcal{A}$ and $\mathcal{B}$, respectively. Hence, if the control qubit is prepared in a computational basis state $|0/1\rangle$, quantum SWITCH places the channels into a specific order in which they act on the system, namely $(\mathcal{A}\circ\mathcal{B})[.]$ or $(\mathcal{B}\circ\mathcal{A})[.]$, respectively. As a consequence of that, any incoherent state of the control qubit (with respect to computational basis), i.e., $\omega = \mathrm{diag}(\phi, 1-\phi)$, realizes a convex combination of the composite channels,

$$\mathcal{S}_{\mathcal{A},\mathcal{B}}\left[\rho \otimes \begin{pmatrix} \phi & 0 \\ 0 & 1-\phi \end{pmatrix}\right] = \phi\Big((\mathcal{A}\circ\mathcal{B})[\rho] \otimes |0\rangle\langle 0|\Big) + (1-\phi)\Big((\mathcal{B}\circ\mathcal{A})[\rho] \otimes |1\rangle\langle 1|\Big), \tag{2}$$

i.e., the channels are applied in a random order chosen, for example, by tossing a coin. However, any coherence in the state of the control qubit inevitably results in a combination of the $\mathcal{A}[.]$ and $\mathcal{B}[.]$ not of the form of Eq. (2) and incompatible with any well-defined causal order between them. In particular, for identical channels $\mathcal{A}[.] = \mathcal{B}[.]$, the action of quantum SWITCH can be given as

$$\mathcal{S}_{\mathcal{A},\mathcal{A}}[\rho \otimes \omega] = \frac{1}{4}\sum_{ij}\Big(\{A_i, A_j\}\rho\{A_i, A_j\}^{\dagger} \otimes \omega + [A_i, A_j]\rho[A_i, A_j]^{\dagger} \otimes Z\omega Z\Big), \tag{3}$$

where $[A_i, A_j] = A_i A_j - A_j A_i$ denotes the commutator, $\{A_i, A_j\} = A_i A_j + A_j A_i$ denotes the anti-commutator, and Z denotes the Pauli Z-operator.

2. Thermodynamic protocols with quantum-controlled thermalization

One of the most striking effects of quantum SWITCH is its ability to reduce noise in communication channels (up to its complete removal for certain channels), so that information can be transmitted even via zero capacity channels [7]. A particular example of such channels is provided by a completely depolarizing channel $\mathcal{T}_0[\rho] = \frac{I}{d}$, where I denotes a d-dimensional unity operator, that outputs a d-dimensional maximally mixed state for any d-dimensional input state ρ. From the thermodynamic point of view, the action of $\mathcal{T}_0[.]$ can be seen as a result of interchanging the states of the system and a bath at infinite temperature. Hence, it can be generalized to a family of channels $\{\mathcal{T}_\beta[.]\}_\beta$ known as thermalizing channels: they model complete thermalization of the system S with a thermal bath B at inverted temperature β swapping their states

$$\mathcal{T}_\beta[\rho] = \mathrm{Tr}_B[U_{SB}(\rho \otimes \tau_\beta)U_{SB}^{\dagger}] = \mathrm{Tr}_B[\tau_\beta \otimes \rho] = \tau_\beta, \tag{4}$$

Journal of Physics: Conference Series **2533** (2023) 012039 doi:10.1088/1742-6596/2533/1/012039

where U_{SB} is the SWAP operator between S and B, and $\tau_\beta = \frac{e^{-\beta H}}{\mathrm{Tr}[e^{-\beta H}]}$ is a thermal state with respect to β and Hamiltonian H.

While several identical thermalizing channels $\mathcal{T}_\beta$ acting consequently on the system leave it eventually in the same thermal state τ_β, putting them into indefinite causal order via quantum SWITCH can result in an output state different from τ_β. In particular, let us consider a qubit system with Hamiltonian $H = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$, which is initially prepared in a thermal state of temperature β_{in}. If the control qubit is initially prepared in the state $|+\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}}$, quantum SWITCH outputs

$$\mathcal{S}_{\mathcal{T}_\beta,\mathcal{T}_\beta}[\rho \otimes |+\rangle\langle+|] = \frac{1}{2}\Big((\tau_\beta + \tau_\beta\tau_{\beta_{in}}\tau_\beta) \otimes |+\rangle\langle+| + (\tau_\beta - \tau_\beta\tau_{\beta_{in}}\tau_\beta) \otimes |-\rangle\langle-|\Big).$$

Therefore, if the control qubit is measured in the Fourier basis $\{|\pm\rangle\} = \{\frac{|0\rangle+|1\rangle}{\sqrt{2}}\}$, the system will be found in a state

$$\begin{aligned} \rho_\pm &= \frac{1}{2p_\pm}(\tau_\beta \pm \tau_\beta\tau_{\beta_{in}}\tau_\beta) \\ &= \frac{1}{2p_\pm(1+e^{-\beta})}\begin{pmatrix} 1 \pm \frac{1}{(1+e^{-\beta})(1+e^{-\beta_{in}})} & 0 \\ 0 & e^{-\beta}\Big(1 \pm \frac{e^{-(\beta+\beta_{in})}}{(1+e^{-\beta})(1+e^{-\beta_{in}})}\Big) \end{pmatrix}, \end{aligned} \tag{5}$$

with probability

$$p_\pm = \frac{1}{2}\,\mathrm{Tr}[\tau_\beta \pm \tau_\beta\tau_{\beta_{in}}\tau_\beta] = \frac{1}{2}\Big(1 \pm \frac{1+e^{-(2\beta+\beta_{in})}}{(1+e^{-\beta})^2(1+e^{-\beta_{in}})}\Big). \tag{6}$$

These output states can have temperature different from β or even be non-thermal. This important observation can be exploited to construct several novel thermodynamic protocols, which we review in the following, assuming the considered qubit system with the Hamiltonian H as working medium.

2.1. Cooling cycle from quantum-controlled thermalization

Among the first thermodynamic protocols that exploit the mentioned property of quantum-controlled thermalizing channels (and quantum control of causal orders of operations in general) is the refrigeration cycle proposed by Felce and Vedral [8] and implemented within several experimental setups [9, 10] as well as on the IBM cloud quantum computer [11]. It starts with working medium in thermal state at temperature $\beta_{in} \equiv \beta_c$ which gets thermalized in indefinite causal order with two identical cold reservoirs at the same temperature $\beta \equiv \beta_c$. Performing a measurement of the control qubit in the Fourier basis, the working medium can be found in one of the states (5) which are thermal of temperatures

$$\beta_+ = \beta_c + \ln\Big(1 + \frac{1-e^{-2\beta_c}}{1+2e^{-\beta_c}+2e^{-2\beta_c}}\Big) \geq \beta_c, \tag{7}$$

$$\beta_- = \ln\Big(1 + \frac{1-e^{-\beta_c}}{1+2e^{-\beta_c}}\Big) \leq \beta_c, \tag{8}$$

respectively. This means that after quantum-controlled thermalization the working medium is cooled or heated with the corresponding probability $p_\pm = \frac{1}{2}\Big(1 \pm \frac{1+e^{-3\beta_c}}{(1+e^{-\beta_c})^3}\Big)$. If it is heated, then heat can be consequently transferred from the working medium by thermalizing it with another (hot) reservoir at a suitable temperature $\beta_- < \beta_h < \beta_c$. The cycle can be closed by the final stroke which consists in thermalization of the working medium with a cold reservoir (regardless of outcome of the control qubit measurement) and thermalization of the cold reveservoirs with each other. The overall heat transfer from the cold reservoirs, working medium, and control qubit is given by the heat transfer to the hot reservoir

$$\begin{aligned} Q &= p_- \operatorname{Tr}[(\rho_- - \rho_h)H] \\ &= p_- \Big(\frac{e^{-\beta_-}}{1+e^{-\beta_-}} - \frac{e^{-\beta_h}}{1+e^{-\beta_h}} \Big) \\ &= p_- \Big(\frac{1+2e^{-\beta_c}}{3(1+e^{-\beta_c})} - \frac{e^{-\beta_h}}{1+e^{-\beta_h}} \Big), \end{aligned} \tag{9}$$

and can be assigned to a heat flow from the cold reservoirs to the hot one. In accordance with Landauer's principle, each run of cycle requires investing work in order to erase the register that contains the measurement outcome of control qubit. Indeed, erasure of the register by thermalization with a resetting reservoir at β_h has the cost

$$\tilde{W}_E = -\frac{p_+ \ln(p_+) + p_- \ln(p_-)}{\beta_h} = \frac{1}{\beta_h} \mathcal{H}\Big(\frac{3e^{-\beta_c}}{2(1+e^{-\beta_c})^2} \Big), \tag{10}$$

where $\mathcal{H}(x) = -x\ln(x) - (1-x)\ln(1-x)$. Therefore, performance of the obtained cycle can be estimated by

$$\eta = \frac{Q}{\tilde{W}_E} = \frac{3\beta_h e^{-\beta_c}}{2(1+e^{-\beta_c})^2 \mathcal{H}\Big(\frac{3e^{-\beta_c}}{2(1+e^{-\beta_c})^2}\Big)} \Big(\frac{1+2e^{-\beta_c}}{3(1+e^{-\beta_c})} - \frac{e^{-\beta_h}}{1+e^{-\beta_h}} \Big), \tag{11}$$

and higher efficiency can be obtained for cold reservoirs at very low temperature. However, it significantly decreases p_- and, hence, makes heat transfer from cold reservoirs to working medium problematic. Nevertheless, the heating probability p_- and the energy of the corresponding state ρ_- (hence, amount of transferred heat Q) can be increased by considering a generalization of quantum SWITCH to $N > 2$ channels and increasing the number of the coherently controlled cold reservoirs [12].

2.2. Activation of passive states via quantum-controlled thermalization

While thermalizing a system with reservoirs at its initial temperature in a quantum-controlled order still produces a thermal state, taking system in a state initially colder than the reservoirs as

$$\beta_{in} > 2\beta \tag{12}$$

and thermalizing it with them via the quantum SWITCH can produce an active state which can be used to perform work [13, 14]. Indeed, the work that can be extracted from a given state $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$ via unitary cycles is estimated by its ergotropy

$$W(\rho) = \operatorname{Tr}[(\rho - U_p \rho U_p^\dagger)H], \tag{13}$$

where U_p is a unitary producing the corresponding passive state $\rho_p = U_p \rho U_p^\dagger = \sum_k p_k |\epsilon_k\rangle\langle\epsilon_k|$ with decreasing populations $p_k \geq p_{k+1}$ and increasing energies $\epsilon_k \leq \epsilon_{k+1}$ (with $|\epsilon_k\rangle$ denoting corresponding eigenstates of the Hamiltonian). Obviously, any convex combination of application orders of $\mathcal{T}_\beta$ produces a state of zero ergotropy since any thermal state τ_β is

Journal of Physics: Conference Series **2533** (2023) 012039 doi:10.1088/1742-6596/2533/1/012039

a completely passive state. On the other hand, if $\mathcal{T}_\beta$ are put into indefinite causal order via quantum SWITCH, the average ergotropy

$$\langle W\rangle = p_+W(\rho_+) + p_-W(\rho_-) = \max\Big\{0, \frac{e^{-2\beta_c} - e^{-\beta_{in}}}{2(1+e^{-2\beta_c})(1+e^{-\beta_{in}})}\Big\}. \tag{14}$$

of states (5) obtained after measurement of control qubit is non-zero if the temperature bound (12) is satisfied. It can be further improved by increasing the number of quantum-controlled reservoirs: indeed, for N reservoirs at β satisfying (12), the average ergotropy of the states obtained after measurement of the control system increases as

$$\langle W_N\rangle = 2\Big(1 - \frac{1}{N}\Big)\langle W\rangle, \tag{15}$$

hence, up to doubling the work that can be extracted in the usual quantum SWITCH scenario [14]. On the other hand, allowing for initial correlations between working medium and control qubit can change temperature bound (12) by getting it shifted towards higher temperatures of the reservoirs in the case of initial classical correlations and even completely reduced in the case of initial quantum correlations [14], hence, allowing for work extraction at the regimes beyond (12).

3. Thermodynamic cycles from quantum-controlled measurements

Recently, it has been observed that performance of a measurement on working medium can play role of a hot reservoir in a thermodynamic cycle [15, 16, 17, 18]. A particular example has been provided by a class of single-temperature cycles consisting of thermalization with a cold reservoir at inverted temperature β and subsequent generalized measurements $\mathcal{M}_a[.]$ and $\mathcal{M}_{1-b}[.]$ of tunable strengths $0 \leq a \leq 1$ and $0 \leq 1-b \leq 1$, where a measurement acts on a system's state as $\mathcal{M}_\lambda[\rho] = \begin{pmatrix} 1-\lambda & 0 \\ 0 & \lambda \end{pmatrix}$ [18]. By adjusting them, it is possible to implement via $\mathcal{M}_a$ and $\mathcal{M}_b$ exchange of energy that can have nature of heat or work. Indeed, each stroke produces a change in internal energy and entropy summarized in Table 1.

Table 1. Change of internal energy and entropy in each stroke.

Stroke 1: $\mathcal{M}_a$	Stroke 2: $\mathcal{M}_{1-b}$	Stroke 3: Thermalization
$\delta U_1 = a - \frac{e^{-\beta}}{1+e^{-\beta}}$	$\delta U_2 = 1-b-a$	$\delta U_3 = \frac{e^{-\beta}}{1+e^{-\beta}} - 1 + b$
$\delta S_1 = \mathcal{H}(a) - \mathcal{H}\Big(\frac{e^{-\beta}}{1+e^{-\beta}}\Big)$	$\delta S_2 = \mathcal{H}(b) - \mathcal{H}(a)$	$\delta S_3 = \mathcal{H}(a) - \mathcal{H}\Big(\frac{e^{-\beta}}{1+e^{-\beta}}\Big)$

Adjusting the measurement strengths in such a way that $\delta S_i = 0$ makes the i-th stroke isentropic, and, hence, investing/extracting work to/from the medium. Otherwise, the stroke can be regarded as one transferring heat. Having this in mind, it is possible to make the cycle work in the regimes of thermal accelerator or heat engine:

- *Thermal accelerator.* In the first stroke, the measurement $\mathcal{M}_a$ realizes heat transfer to the working medium by fixing its strength in the interval $\frac{e^{-\beta}}{1+e^{-\beta}} < a < \frac{1}{2}$, so that $\delta U_1 \equiv Q_{hot} > 0$. The work is absorbed via the second measurement by fixing $b = a$, so that $\delta S_2 = 0$, and $\delta U_2 \equiv W > 0$. The last stroke implements heat transfer to the cold reservoir, so that $\delta U_3 \equiv Q_{cold} < 0$.

Journal of Physics: Conference Series **2533** (2023) 012039 doi:10.1088/1742-6596/2533/1/012039

- *Heat engine.* Similarly to thermal accelerator, the first measurement $\mathcal{M}_a$ implements heat transfer to the working medium, , so that $\delta U_1 \equiv Q_{hot} > 0$, however, its strength is fixed in the interval $\frac{1}{2} \leq a < \frac{1}{1+e^{-\beta}}$. This choice allows one to extract work from the medium via the second measurement by fixing $b = a$, so that $\delta S_2 = 0$, and $\delta U_2 \equiv W < 0$. The last stroke implements heat transfer to the cold reservoir, so that $\delta U_3 \equiv Q_{cold} < 0$.

Dieguez, Lisboa, and Serra have proposed an extension of the considered 3-stroke cycle by increasing the number of generalizing measurements implementing heat transfer and putting them into an indefinite causal order via the quantum SWITCH [19]. Indeed, similarly to the original protocol, the new scheme has three strokes corresponding to measurement-powered heat transfer, measurement-powered work source, and thermalization with a cold reservoir. The difference lies in the heat transfer stage, which is implemented by two measurements with opposite strengths (i.e., $\mathcal{M}_\lambda$ and $\mathcal{M}_{1-\lambda}$ with $0 \leq \lambda \leq 1$) controlled via quantum SWITCH with the control qubit being initially in a certain pure state $\omega = |\theta\rangle\langle\theta|$ with $|\theta\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}|1\rangle$. As the original cycle, it can operate in three regimes which we review below.

3.1. Thermal accelerator regime.

In the thermal accelerator regime, the cycle begins with a pair of coherently controlled measurements $\mathcal{M}_a$ and $\mathcal{M}_{1-a}$ that supports a heat flow from the working medium due to a change in its internal energy $Q^{\pm}_{hot} > 0$, which depends on the outcome of a measurement of the control qubit in the $\pm$-basis. The following measurement $\mathcal{M}_b$ has to realize the isentropic channel required to invest work: crucially, its strength depends on the outcome of the measurement of control qubit in the $|\pm\rangle$-basis. Indeed, accordingly to the obtained outcome, one picks a measurement $\mathcal{M}_{b_\pm}$ with a strength

$$b_\pm = \frac{1}{2}\Big[1 + (2\Omega_\pm - 1)\Big(1 - \frac{2e^{-\beta}}{1+e^{-\beta}}\Big)\Big], \tag{16}$$

where $p_\pm = \frac{1}{2}(1 \pm a(1-a)\sin\theta)$. Finally, the cycle is closed by thermalization with the cold reservoir delivering heat $Q^{\pm}_{cold} < 0$ from it to the working medium.

$$Q^{\pm}_{hot} \equiv \delta U_1 = \Omega_\pm\Big(1 - \frac{2e^{-\beta}}{1+e^{-\beta}}\Big), \tag{17}$$

$$W^{\pm} \equiv \delta U_2 = (1 - 2\Omega_\pm)\Big(1 - \frac{2e^{-\beta}}{1+e^{-\beta}}\Big), \tag{18}$$

$$Q^{\pm}_{cold} \equiv \delta U_3 = (\Omega_\pm - 1)\Big(1 - \frac{2e^{-\beta}}{1+e^{-\beta}}\Big). \tag{19}$$

A thermal accelerator requires heat transfer to the working medium in the first stroke (i.e., $Q^{\pm}_{hot} > 0$), which is obtained by fixing $0 < \Omega_\pm < 1$ in general. Furthermore, in the second stroke, work $W^{\pm} > 0$ has to be invested via $\mathcal{M}_{b_\pm}$, providing the condition $0 < \Omega_\pm < \frac{1}{2}$. Hence, the performance of the cycle depending on the obtained outcome of the control qubit measurement is given by

$$\eta^{\pm}_{acc} = -\frac{Q^{\pm}_{cold}}{W^{\pm}} = 1 - \frac{1}{2 - \frac{1}{\Omega_\pm}}. \tag{20}$$

3.2. Heat engine regime.

The heat engine regime requires a similar scheme of the cycle as the accelerator regime. However, the work supported by the measurement $\mathcal{M}_{b_\pm}$ has to be extracted from the working medium, i.e., $W^{\pm} < 0$. This is achieved by fixing $\frac{1}{2} < \Omega_{\pm} < 1$. Therefore, the performance of the cycle depending on the obtained outcome of the control qubit measurement is given by

$$\eta_{he}^{\pm} = \frac{W^{\pm}}{Q_{hot}^{\pm}} = 2 - \frac{1}{\Omega_{\pm}}. \tag{21}$$

3.3. Performance under incoherent control of the measurements.
Performance of the obtained cycle with a coherent control can be compared with its incoherent counterpart, where the corresponding pair of controlled channels is applied in an incoherent mixture of its orders obtained by tracing out the control qubit instead of measuring it. Performance of the latter for both regimes turns out to be

$$\eta_{acc}^{inc} = 1 - \frac{1}{2 - \frac{1}{\Omega_{inc}}}, \tag{22}$$

$$\eta_{he}^{inc} = 2 - \frac{1}{\Omega_{inc}}, \tag{23}$$

where $\Omega_{inc} = 2p_{\pm}\Omega_{\pm}$, taking into account that the thermal accelerator operates in the intervals of a and θ defined by the condition $0 < \Omega_{inc} < \frac{1}{2}$, whereas heat engine regime requires $\frac{1}{2} < \Omega_{inc} < 1$. Hence, for the values of a and θ that satisfy both the corresponding conditions for $\Omega_{\pm}$ and Ω_{inc}, the coherently-controlled cycle always offers a higher performance than its incoherent counterpart in one of the $|\pm\rangle$-branches. On the other hand, coherently-controlled cycle can operate in a wider interval of values of a and θ that do not necessarily satisfy the corresponding condition of Ω_{inc}, i.e., do not allow its incoherent counterpart to operate.

4. Discussion and outlook

Experimental realization of indefinite causal order of quantum operations via the quantum SWITCH has indicated an important milestone that stimulated research in its applications to information processing and communication. Recent proposals of thermal engines powered by the quantum SWITCH have supplemented this list with quantum thermodynamics. Indeed, there were proposed several protocols that exploit coherently controlled thermalization and generalized measurements in order to construct thermodynamic cycles that do not have classical counterpart or can offer regimes which outperform it. Experimental realizability of these protocols is of high importance here, and a cooling cycle with quantum-controlled thermalization has been already implemented in several table-top experiments. Beyond the reviewed protocols, quantum SWITCH allows one to outperform heat-bath algorithmic cooling technique [20] and boost the charging process of a quantum battery [21].

The development of the field of indefinite causality has just begun, and thermodynamic aspects of indefinite causality are yet to be understood. In particular, it is still debated whether indefinite causal orders can be regarded as a genuine thermodynamic resource [22, 23, 24]. A proper way to study them would be to go beyond quantum SWITCH and question usefulness of causal non-separability of a process in general. In particular, activation of thermal states via coherently controlled thermalization could be a hint for a thermodynamic analogue of the recently developed resource theories of causally non-separable processes [23, 25]: such a resource theory is expected to be richer than its communication counterpart and the existing resource theories of thermodynamics.

References

[1] Russell B 1913 *Proc. Aristot. Soc.* **13** 1
[2] Hardy L 2007 *J. Phys. A: Math. Theor.* **40** 3081

[3] Chiribella G, D'Ariano G M, Perinotti P and Valiron B 2013 *Phys. Rev. A* **88** 022318
[4] Oreshkov O, Costa F and Brukner C 2012 *Nature Commun.* **3** 1092
[5] Rubino G, Rozema L A, Feix A, Araújo M, Procopio L, Brukner Č and Walther P 2017 *Sci. Adv.* **3** e1602589
[6] Goswami K and Romero J 2020 *AVS Quantum Sci.* **2** 037101
[7] Chiribella G, Banik M, Bhattacharya S S, Guha T, Alimuddin M, Roy A, Saha S, Agrawal S and Kar G 2021 *New J. Phys.* **23** 033039
[8] Felce D and Vedral V 2020 *Phys. Rev. Lett.* **125** 070603
[9] Cao H, Wang N N, Jia Z, Zhang C, Guo Y, Liu B H, Huang Y F, Li C F and Guo G C 2022 *Phys. Rev. Research* **4** L032029
[10] Nie X, Zhu X, Huang K, Tang K, Long X, Lin Z, Tian Y, Qiu C, Xi C, Yang X, Li J, Dong Y, Xin T and Lu D 2022 *Phys. Rev. Lett.* **129** 100603
[11] Felce D, Vedral V and Tennie F 2021 Refrigeration with Indefinite Causal Orders on a Cloud Quantum Computer *Preprint* 2107.12413
[12] Nie H, Feng T, Longden S and Vedral V 2022 Quantum cooling activated by coherently-controlled thermalisation *Preprint* 2201.06954
[13] Guha D, Alimuddin M and Parashar P 2020 *Phys. Rev.* A **102** 032215
[14] Simonov K, Francica G, Guarnieri G and Paternostro M 2022 *Phys. Rev.* A **105** 032217
[15] Yi J, Talkner P and Woon Kim Y 2017, *Phys. Rev.* E **96** 022108
[16] Elouard C, Herrera-Mart D, Huard B and Auffèves A 2017 *Phys. Rev. Lett.* **118** 260603
[17] Elouard C and Jordan A N *Phys. Rev. Lett.* **120** 260601
[18] Behzadi N 2020 *J. Phys. A: Math. Theor.* **54** 015304
[19] Dieguez P R, Lisboa V F and Serra R M 2023 *Phys. Rev.* A **107** 012423
[20] Goldberg A Z and Heshami K 2023 *J. Phys. Commun.* **7** 015003
[21] Chen Y and Hasegawa Y 2021 Indefinite causal order in quantum batteries *Preprint* 2105.12466
[22] Guérin P A, Rubino G and Brukner Č 2019 *Phys. Rev.* A **99** 062317
[23] Kristjánsson H, Chiribella G, Salek S, Ebler D and Wilson M 2020 *New J. Phys.* **22** 073014
[24] Capela M, Verma H, Costa F, Céleri L C 2022 Indefinite causal order is not always a resource for thermodynamic processes *Preprint* 2208.03205
[25] Milz S, Bavaresco J and Chiribella G 2022 *Quantum* **6** 788