Time and its arrow from quantum geometrodynamics?

Claus Kiefer[1], **Leonardo Chataignier**[2], **and Mritunjay Tyagi**[3]

[1]Faculty of Mathematics and Natural Sciences, Institute for Theoretical Physics, University of Cologne, Cologne, Germany
[2]Department of Physics and EHU Quantum Center, University of the Basque Country UPV/EHU, Barrio Sarriena s/n, 48940 Leioa, Spain
[3]University of Groningen, University College Groningen, Hoendiepskade 23/24, 9718 BG Groningen, The Netherlands

E-mail: kiefer@thp.uni-koeln.de, leonardo.chataignier@ehu.eus, m.tyagi@rug.nl

Abstract. We discuss how quantum geometrodynamics, a conservative approach to quantum gravity, might explain the emergence of classical spacetime and, with it, the emergence of classical time and its arrow from the universal quantum state. This follows from a particular but reasonable choice of boundary condition motivated by the structure of the Hamiltonian of the theory. This condition can also be seen as defining a quantum version of Penrose's Weyl curvature hypothesis. We comment on the relation of this picture to the 'past hypothesis' and the different observed arrows of time, and we consider how quantum geometrodynamics could serve as a unifying and more fundamental framework to explain these observations.

1. Introduction

Although most fundamental laws of Nature, as we currently know them, are time symmetric, our observations clearly distinguish a "direction" or "arrow" of time. This becomes evident from different types of phenomena: the propagation of radiation (e.g. the distinction between advanced and retarded potentials); the expansion of the Universe and the formation of structure, including black holes (both related to the gravitational field); the general tendency of entropy to increase, as explained by thermodynamics and statistical physics; and the apparent asymmetry and irreversibility of quantum measurements (apparent "collapse" of the wave function). Could there be a "master arrow" of time behind these different phenomena? In other words, could there be a single, well-motivated reason for all these different arrows?

In this article, we entertain the possibility that, once the quantum nature of the gravitational field and the fundamental ontology of quantum mechanics are well understood, then time and its master arrow may follow from the structure of the quantum state itself. In a nutshell, this is due to the fact that, as we currently know of no bounds to the quantum superposition principle, and given the ubiquity of entanglement, it is possible that the whole Universe is described by a single quantum state as a closed quantum system. Furthermore, as the prevailing interaction at large scales is gravitation, the description of the Universe as a single quantum system ought to include a quantum description of gravity, even if we aim to describe only a reduced set of large-scale degrees of freedom instead of literally everything. Finally, due to spacetime diffeomorphism invariance, the universal quantum state must be independent of any time coordinate, so any

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physical notion of time and its arrow must be 'internal' (or 'intrinsic'); that is, it must emerge from the quantum degrees of freedom themselves.

In this way, we are led to the prospect that the Universe could be fundamentally described by a quantum state, from which dynamics and history emerge. The general outline of the ideas reported here have been previously discussed in [1–6]. Although there are other formalisms (see, for instance, the different contributions to the book in [4, 5]), it is important to emphasize how a conservative approach to the canonical quantization of the gravitational field, the theory of quantum geometrodynamics, which only assumes general relativity together with the usual tenets of quantum theory and nothing more, can consistently point us toward an explanation of the emergence of classical time and its arrow.

This contribution to the Symposium proceedings consists of six sections. We first review the various arrows of time and the idea of the *past hypothesis*.[1] We then present a brief section where we focus on the situation in quantum theory. Section 4 and 5 constitute the main part of our contribution. In section 4 we present the main ideas on how the arrows of time can emerge from the fundamental framework of quantum geometrodynamics, and section 5 contains the outline of an example for which technical details and further developments are relegated to a separate publication. We conclude in section 6.

2. Arrows of time and the past hypothesis

We observe many phenomena that frequently occur in a certain order but infrequently in the opposite order. This discrepancy defines an arrow of time as Eddington explained [7] and it brings forth an element of irreversibility into our description of Nature. Historically, as most of the physical laws that we have understood are time symmetric,[2] the origin of irreversibility has been a vexing issue (Loschmidt's *Umkehreinwand* and Zermelo's *Wiederkehreinwand*; see e.g. [3, 8]).

One of the most prevalent arrows of time is the thermodynamical arrow, which is responsible for the time directionality of many events we observe. The second law of thermodynamics establishes the general tendency of entropy to increase:

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \underbrace{\left(\frac{\mathrm{d}S}{\mathrm{d}t}\right)_{\mathrm{ext}}}_{\mathrm{d}S_{\mathrm{ext}}=\delta Q/T} + \underbrace{\left(\frac{\mathrm{d}S}{\mathrm{d}t}\right)_{\mathrm{int}}}_{\geq 0}, \tag{1}$$

and this growth of entropy for an isolated system defines the thermodynamical arrow of time. Its relevance notwithstanding, the question of its fundamental physical origin remains.

While it is clear that the direction of time is related to the second law of thermodynamics, it is not clear whether this is the master arrow (if it exists). Over the years, the precise origin of time's directionality has been a topic of continued debate. For example, Einstein and Ritz represented different opinions regarding the matter in 1909. In a common paper, they state

> " . . . Ritz considers the restriction to the form of the retarded potentials as one of the roots of the Second Law, while Einstein believes that the irreversibility is exclusively based on reasons of probability."[3]

[1] A general reference is the textbook [3], to which we refer for more details and references to original literature.

[2] There are, of course, phenomena associated with the weak interaction that are not time symmetric. Nonetheless, they are still CPT invariant, which can be interpreted as a slight generalization of T-invariance, drastically different from the irreversibility discussed here.

[3] German original: "Ritz betrachtet die Einschränkung auf die Form der retardierten Potentiale als eine der Wurzeln des Zweiten Hauptsatzes, während Einstein glaubt, daß die Nichtumkehrbarkeit ausschließlich auf Wahrscheinlichkeitsgründen beruhe."

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Indeed, in electrodynamics, it is clear that fields interacting with local sources are usually described by retarded solutions. Since a general solution of the wave equation can be decomposed as [3]

$$\begin{aligned} \text{four-potential} = A^\mu &= \text{source term plus boundary term} \\ &= A^\mu_{\text{retarded}} + A^\mu_{\text{in}} \\ &= A^\mu_{\text{advanced}} + A^\mu_{\text{out}} \ , \end{aligned} \tag{2}$$

the restriction to retarded potentials corresponds to the choice of the 'Sommerfeld condition' $A^\mu_{\text{in}} \approx 0$. Why should such a condition hold?

While Ritz essentially expressed the opinion that the electrodynamic (or radiative) arrow could be a candidate for a master arrow of time (which one could perhaps see as a fundamental fact of Nature), Einstein's view is the one favoured today, as it alluded to a statistical explanation of the second law of thermodynamics as the more fundamental concept, with which other arrows, including the electrodynamic one, could conceivably be explained. In fact, the electrodynamic arrow can be understood as a consequence of the second law and the thermodynamic arrow by taking into account thermodynamic properties of absorbers.

The statistical foundations of thermodynamics, which were most notably developed by Boltzmann and Gibbs, attribute the non-decreasing character of the entropy of an isolated system to what Einstein calls "reasons of probability." Low-entropy states are assumed to be less probable than high-entropy states, and thus one expects that states with lower entropy will evolve into states with higher entropy. If this reasoning is extended to the whole Universe, then the observed growth of entropy indicates that entropy was low in the early Universe. This would thus be an initial or boundary condition for the evolution of the Universe. According to Boltzmann (1898):

> That in Nature the transition from a probable to an improbable state does not happen equally often as the opposite transition, should be sufficiently explained by the assumption of a very improbable initial state of the whole Universe surrounding us (...),

or Arthur Eddington [9]:

> Accordingly, we sweep anti-chance [low-entropy state generated by an extremely improbable fluctuation] out of the laws of physics—out of the differential equations. Naturally, therefore, it reappears in the boundary conditions (...).

This boundary condition is often referred to as the "past hypothesis" (see [10, 11] and also the discussion in [12]), which would explain the arrows of time based on a fundamental assumption about the early Universe.[4] But is that really the case?

Similarly to the situation in eletrodynamics, arrows of time arise also in gravitation and cosmology. Indeed, the formation of structure in the Universe (such as stars, galaxies, and clusters), which follows from gravitational instabilities, and the expansion of the Universe can be thought of as defining gravitational and cosmological arrows of time. Can they be connected to the second law of thermodynamics? If so, one must have a notion of entropy of the gravitational degrees of freedom. As gravitational systems have a negative heat capacity, (in)homogeneous states are to have (high) low gravitational entropy, the opposite behaviour with respect to non-gravitational systems.

As gravitational radiation and tidal forces can be captured by the Weyl curvature tensor, it is reasonable to expect that the entropy of gravitational fields may be related to Weyl curvature. Roger Penrose proposed the 'Weyl curvature hypothesis' [13] in order to suggest a possible explanation for the homogeneity and isotropy of the observed Universe at large scales, as well as for the growth of entropy in the Universe. This hypothesis states that the Weyl curvature

[4] Note that the meaning of 'past' is itself defined by this hypothesis.

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tensor must vanish (or at least be non-divergent) at past singularities approached from the future. With this, gravitational waves must be retarded, in a direct analogy to the Sommerfeld condition for electromagnetic waves. In contrast to the electrodynamic case, it is difficult to relate the gravitational and cosmological arrows to the thermodynamic one due to the weakness of gravitational waves.

A vanishing Weyl curvature tensor in the early Universe is thus expected to be related to the Universe's initial low entropy. As the Universe evolved, the growth of entropy, the arrow of time, and the formation of structure would have followed from the increasing effect of Weyl curvature on the dynamics. Although it remains to be seen whether the Weyl curvature hypothesis is correct, the effort to take into account the entropy of gravitational fields is a step in the direction of unifying the arrows of time under a common entropic explanation based on the past hypothesis.

It is also worthwhile to mention that the Bekenstein–Hawking entropy of black holes,

$$S_{\rm BH} = k_{\rm B}\frac{Ac^3}{4\hbar G} \overset{\rm Schwarzschild}{\approx} 1.07\times 10^{77} k_{\rm B}\left(\frac{M}{M_\odot}\right)^2 , \tag{3}$$

indicates the importance of entropy in gravitation, especially at its interface with quantum field theory, and it begs a microscopic explanation (as several approaches to quantum gravity aim to yield [14]). With this analysis, one can also estimate the "probability of our Universe" [4, 15], by considering that the maximal entropy of the observed Universe (more precisely: its region within the particle horizon) is obtained if its mass is concentrated in a single black hole. The fantastical result[5] [4, 15]

$$\frac{\exp\left(\frac{S}{k_{\rm B}}\right)}{\exp\left(\frac{S_{\rm max}}{k_{\rm B}}\right)} \sim \frac{\exp\left(3.1\times 10^{104}\right)}{\exp\left(2.9\times 10^{122}\right)} \approx \exp\left(-2.9\times 10^{122}\right) \tag{4}$$

indicates how special the initial state must be. In general, this might raise issues of fine tuning, although we will see how toy models of quantum gravity could address the problem via a reasonable choice of boundary condition, which implements a version of the past hypothesis.

The number (4) is an unimaginably tiny number. In fact, it is much smaller than the probability of the whole observable Universe including all states of observers emerging in a gigantic fluctuation in the spirit of Boltzmann. This number would correspond to the total entropy of the Universe and thus be of the order 10^{-104} – still tiny but 18 orders of magnitude bigger than 10^{-122}. This discrepancy casts strong doubts on the applicability of the anthropic principle, as the smallness of (4) calls for a physical explanation.

3. Irreversibility and entropy in quantum theory

Even though the Schrödinger equation is invariant under time reversal, the measurement process distinguishes a direction of time via the update (or "collapse") of the wave function, which might be ontic (e.g., a dynamical collapse) or epistemic. In particular, we can take the Everettian view that the quantum state and the Schrödinger equation are the ontological elements of the theory, from which all else is to be derived. In this view, the measurement process entails the branching of Everett paths (the "many worlds"). This is certainly economical from the point of view of formalism, although some (like John Bell) may consider the worlds an ontological extravagance. As we wish to make no further modifications or additions to the formalism here, we adopt the Everett point of view; it is certainly the point of view implicitly assumed in many discussions of quantum cosmology.

A central concept related to the process of measurement and the branching of the quantum state is decoherence, which follows from the increasing entanglement that results

[5] In this estimate, we also include the Gibbons–Hawking entropy [16] associated with Λ [4].

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from interactions with the environment, and it leads to the irreversible emergence of classical properties [17]. Can this quantum irreversibility and the corresponding arrow of time be related to a definition of entropy?

The increasing entanglement can be quantified by the definition of entanglement entropy, for example via the von Neumann or linear entropies, respectively given by

$$\begin{aligned} S_{\mathrm{vN}} &= -k_{\mathrm{B}} \mathrm{tr}\left(\rho \ln \rho\right) \\ S_{\mathrm{lin}} &= k_{\mathrm{B}} \mathrm{tr}\left(\rho - \rho^2\right) \ , \end{aligned} \tag{5}$$

where k_{B} is Boltzmann's constant (needed to express the entropy in thermodynamic units) and ρ is the density matrix representation of the quantum state. This density matrix is reduced in the sense that the environment degrees of freedom are to be traced out, which corresponds to a type of coarse graining, and it leads to a mixed state if the system is entangled with the environment. In general, coarse graining entails the transformation of relevant information (e.g. about macroscopic differences) into irrelevant information (e.g. about microscopic differences without discernible macroscopic consequences),[6] and this may lead to an increase in entropy (from zero for a pure state to nonzero for a mixed state).

Although the precise relation between entanglement entropy and thermodynamical entropy must be spelled out, it is conceivable that, if quantum theory is fundamental and describes the whole Universe, entanglement entropy may play a key role in establishing and explaining a master arrow of time, from which the other arrows (e.g. thermodynamical, gravitational, electrodynamic, cosmological) may emerge. Motivated by this open question, we are led to consider the question of quantum gravity: if the whole Universe is a single quantum system that includes a quantum description of gravitation, what is the role of entanglement entropy and the past hypothesis? Is it possible that quantum gravity accommodates (or ideally explains) the past hypothesis and the observed arrows of time?

In what follows, we would like to emphasize that a conservative approach to quantum gravity, which only assumes the usual tenets of general relativity and (Everettian) quantum theory without further speculative elements, may point towards an explanation of the emergence of classical time and its master arrow from a fundamental theory in which the quantum state and the (time-independent) Schrödinger equation are the basic ontological elements. This envisioned explanation is of course tentative because the theory of canonical quantum gravity is not yet fully developed and, moreover, the connection between entanglement entropy and its other counterparts must still be properly understood, as mentioned above.

4. Quantum gravity, the timeless past hypothesis, and quantum Weyl curvature

In canonical quantum gravity, which was pioneered by Rosenfeld, Bergmann, and Dirac, and developed by Wheeler and DeWitt and others [14], the gravitational field is quantized according to the usual rules of canonical quantization, and the result is a (functional) Schrödinger equation for gravity and matter fields. Due to spacetime diffeomorphism invariance, the quantum states must be invariant under time translations, and this implies that the fundamental Schrödinger equation must be time independent (or timeless):

$$0 = \mathrm{i}\hbar \frac{\mathrm{d}}{\mathrm{d}\tau} \left|\Psi\right\rangle = \hat{H} \left|\Psi\right\rangle \ . \tag{6}$$

This equation is better known as the Wheeler–DeWitt equation. The timelessness of this constraint on the physical states simply signals that no external or absolute time has physical meaning in the theory, and thus a physical notion of evolution, if any, must be internal, that

[6] The classic example is Gibbs's ink drop analogy.

is, derived from the physical degrees of freedom themselves. The intuitive explanation for this timelessness is that there is no time "outside" spacetime in general relativity, and that the classical spacetime is absent in the quantum theory in the same way as a classical trajectory is absent in quantum mechanics; see, for example, [14, 18, 19] for more details.

The quantum state $|\Psi\rangle$ leads to the wave function $\Psi(g_{\mu\nu}, \phi)$, which in general depends on the spacetime metric $g_{\mu\nu}$ and on matter fields ϕ. If we perform a $3+1$ decomposition of these variables (which corresponds at the classical level to a foliation of the four-dimensional spacetime into spacelike leaves Σ), then the requirement of diffeomorphism invariance is equivalent to the validity of the local quantum constraints

$$\hat{\mathcal{H}}_\perp \Psi = 0 \ , \ \hat{\mathcal{H}}_i \Psi = 0 \ , \tag{7}$$

where $\hat{\mathcal{H}}_i$ generates spatial diffeomorphisms, $\hat{\mathcal{H}}_\perp$ is responsible for time reparametrizations, and the Hamiltonian operator is $\hat{H} = \int_\Sigma \mathrm{d}^3x N^\mu \hat{\mathcal{H}}_\mu$ with $\mu = (\perp, i = 1, 2, 3)$, and $N^\mu = (N^\perp \neq 0, N^i)$ being arbitrary functions. After imposing all constraints, it is seen that the wave function only depends on the *spatial* metric, whose components are here called h_{ab}.

If $\Psi(h_{ab}, \phi)$ only depends on combinations of the fields that are invariant under spatial diffeomorphisms, then the constraint $\hat{\mathcal{H}}_i \Psi = 0$ is satisfied. The remaining Hamiltonian constraint reads (for simplicity, the matter content is that of a minimally coupled scalar field) [14, 19]:

$$\begin{aligned} 0 = \sqrt{h}\hat{\mathcal{H}}_\perp \Psi &= \left[-\frac{16\pi G\hbar^2}{c^2}\left(h_{ac}h_{bd} - \frac{1}{2}h_{ab}h_{cd}\right)\frac{\delta^2}{\delta h_{ab}\delta h_{cd}} - \frac{\hbar^2}{2}\frac{\delta^2}{\delta\phi^2} + V\right]\Psi \ , \\ V &= -\frac{c^4}{16\pi G}h(R - 2\Lambda) + hh^{ab}\phi_{,a}\phi_{,b} + h\mathcal{V}(\phi) \ , \end{aligned} \tag{8}$$

where G is Newton's constant, h is the determinant of the spatial metric, R is the Ricci scalar on the spatial leaves, Λ is the cosmological constant, and $\mathcal{V}(\phi)$ is an arbitrary potential for the scalar field. We have ignored factor ordering ambiguities in Eq. (8).

Notice how the potential term V in Eq. (8) vanishes in the limit $h = \det h_{ab} \to 0$, thus yielding an approximate separable equation for gravity and matter, which admits separable solutions [see Eq. (10) below]. This signals a fundamental asymmetry of the Hamiltonian constraint with respect to the "local volume" factor $\sqrt{h}$.[7] The Schrödinger equation of canonical quantum gravity thus distinguishes a direction in configuration space,[8] and we will see how this is related to a distinguished direction of classical time.

Although the definition of the Hamiltonian $\hat{H}$ in the full theory requires regularization, and the solutions to Eqs. (6) and (8) are not readily available, one can already begin to understand the phenomenology of the Wheeler–DeWitt equation in simple toy models which do not aim at describing all possible degrees of freedom, but rather a subset of variables that are relevant to the description of, for instance, degrees of freedom relevant to cosmology. We are thus led to the topic of quantum cosmology, where the quantum state $|\Psi\rangle$ leads to the wave function of the universe,[9] which is in line with the notion that quantum theory is universal.[10]

The Hamiltonian constraint in simple toy models of quantum cosmology (assuming a homogeneous and isotropic universe with small perturbations) can be shown to be [1–6]:

$$\hat{H}\Psi = \left[\frac{\partial^2}{\partial\alpha^2} + \sum_i \left(-\frac{\partial^2}{\partial x_i^2} + \underbrace{V_i(\alpha, x_i)}_{\to 0 \text{ for } \alpha \to -\infty}\right)\right]\Psi = 0 \ , \tag{9}$$

[7] More precisely, the potential term V in Eq. (8) generally changes under the substitution $h \to -h$.
[8] The local volume factor is also distinguished by the signature of the DeWitt metric on the gravitational configuration space, also known as "superspace".
[9] A universe (with lower-case u) is a toy model of the real Universe.
[10] Possible observational tests of quantum cosmology are discussed, for example, in [20]; see also references therein.

where $\alpha = \log(a/a_0)$ is the logarithm of the scale factor of the universe (a_0 is a reference scale factor) and x_i are other degrees of freedom. Also here, the potential terms vanish as $a \to 0$ or $\alpha \to -\infty$ (the analogous condition of the general case $h \to 0$). In this way, we can consider the boundary condition [1–6]

$$\Psi \overset{\alpha\to-\infty}{\longrightarrow} \psi_0(\alpha) \prod_i \psi_i(x_i) \ . \tag{10}$$

As the region of configuration space in which $a \to 0$ corresponds, in particular, to the early universe, such a symmetric boundary condition corresponds to the imposition of a completely unentangled state "in the beginning."[11] This leads to a vanishing "initial" entanglement entropy. If such an entropy could also be connected to other definitions (in particular to thermodynamical entropy [21]), then this condition would correspond to a timeless version of the past hypothesis, as there is no external, absolute nor preferred time in this context. It is interesting that, while this boundary condition is still an assumption, it is a reasonable choice allowed by the very structure of the Hamiltonian, which lends some credibility and motivation to this version of the past hypothesis.

For $a > 0$, the nonvanishing potential will generally lead to an entangled state, as it will no longer be possible to separate matter and gravitational variables. Thus, in regions of configuration space that correspond to the "late-time" universe, the entanglement entropy will no longer be zero, as it increases with increasing scale factor. It is worthwhile to emphasize that this can be seen as a timeless (static) situation, and the variation of entanglement entropy with a concerns variations in the form of the wave function across configuration space. Classical time itself, and our usual classical notions of causality and dynamics, can be defined in the regions of configuration space where decoherence ensures that a classical spacetime background emerges [14,22,23].[12] Here, the system includes the background degrees of freedom, and possibly some of the Weyl scalars that describe scalar and tensor modes, whereas the environment is, for example, composed of small density fluctuations and weak gravitational waves. In contrast to the usual decoherence process in quantum mechanics, which unfolds relative to an external time, such a quantum-cosmological decoherence responsible for the emergence of classical time unfolds relative to the scale factor itself, and thus it also concerns variations across configuration space. With this understanding, it could be that canonical quantum gravity motivates the "initial" low entropy state and the subsequent master arrow of time. In this way, the asymmetry of the Wheeler–DeWitt equation and decoherence (caused by increasing entanglement [17,22,23]) would be the most fundamental sources of irreversibility behind the arrows of (classical) time.

What else can we say about the asymptotic state given in Eq. (10)? Can we fix the form of each factor? As the symmetric boundary condition is a choice, albeit well motivated, we can only fix the form of the factors by a further hypothesis. In analogy to Penrose's classical Weyl curvature hypothesis, we can introduce the following quantum version of this hypothesis [6]:

> *The quantum states for scalar variables that are related to scalar and tensor modes in cosmology assume the form of adiabatic vacuum states in a (quasi-)de Sitter space, as the region of small scale factors is approached from directions of large scale factors.*

Instead of the classical requirement that the Weyl curvature should vanish or that it should be at least non-divergent in the early universe, one requires that (Weyl) scalars be in their adiabatic vacuum states in an inflationary quantum universe, where early times are defined by the region of

[11] Conradi and Zeh compare such a symmetric "initial" condition with the *apeiron* (ἄπειρον) discussed by Anaximander of Milet in about 550 BC [1].

[12] We are tacitly assuming a closed universe. In the presence of an asymptotic structure (e.g. asymptotic flatness), a notion of time is available at the boundary and may persist even in the quantum theory, while the constraints (in particular, the Wheeler–DeWitt equation) are valid in the bulk, where the emergence of a classical background through decoherence can be studied.

configuration space with small scale factors. This has a direct consequence to phenomenology, as gravitational waves can be described by certain Weyl scalars constructed from the Weyl curvature tensor [24], such as $\Psi_4 := -\frac{1}{8c^2}\left(\ddot{h}_+ - \mathrm{i}\ddot{h}_\times\right)$,[13] where h_+ and $h_\times$ indicate the two polarizations of weak gravitational waves (related to the tensor modes). The Newman–Penrose variable Ψ_4 corresponds to the helicity state $s = -2$, whereas its complex conjugate describes $s = +2$. Furthermore, the Mukhanov–Sasaki variable $v_{\vec{k}}$ combines the scalar perturbations of the metric and the inflaton scalar field [25], thus defining invariant scalar modes.

This quantum hypothesis then demands that the x_i variables in Eq. (10), which in this case correspond to the scalar and tensor modes (or to the corresponding Weyl scalars), all now generically denoted by $v_{\vec{k}}$, be in the state

$$\psi_{\vec{k}}(v_{\vec{k}}) = \mathcal{N}_{\vec{k}} \exp\left(-\frac{1}{2}\,\Omega^{(0)}_{\vec{k}} v^2_{\vec{k}}\right) , \tag{11}$$

with $\Omega^{(0)}_{\vec{k}} = k$. This "initial" state at small scale factors will evolve into a two-mode squeezed state, from which the primordial power spectra for density perturbations and gravitational waves can be deduced. Finally, it is important to mention that, once a classical spacetime background has emerged at the onset of inflation [26], the primordial fluctuations also go through the quantum-to-classical transition with a more standard process of decoherence relative to classical time, and they eventually become classical stochastic variables that can be used to describe the seeds for structure formation [27].

Both the symmetric boundary condition (timeless past hypothesis) in Eq. (10) and the quantum Weyl curvature hypothesis are evidently theoretical assumptions, which nevertheless appear to be well motivated and to accommodate well our observations (power spectra in cosmology and the arrow of time) [20]. In order for this formalism to become more robust, one would need not only to achieve a regularization of the quantum Hamiltonian constraint in a full theory of quantum gravity, but also to better understand the connection between entanglement entropy and thermodynamical entropy in this context (this includes the precise connection to all observed arrows of time), perhaps following Chap. 9 of [21]. This task is yet to be completed.

5. Outline of a toy model and associated challenges

Let us outline the calculation of the entanglement entropy from the Wheeler–DeWitt equation in a simplified toy model. Further details, discussions and developments will be the subject of a forthcoming article [28]. The toy model consists of two scalar fields of the same mass m, where one of the fields ϕ_1 acts as the "system" and the other field ϕ_2 acts as the "environment," with the constraint equation

$$\left\{\frac{1}{m_\mathrm{P}^2}\frac{\partial^2}{\partial\alpha^2} - \frac{\partial^2}{\partial\phi_1^2} - \frac{\partial^2}{\partial\phi_2^2} + a_0^6\mathrm{e}^{6\alpha}\left[m^2(\phi_1^2+\phi_2^2) + g\phi_1\phi_2 + m_\mathrm{P}^2\frac{\Lambda}{3}\right]\right\}\Psi(\alpha,\phi_1,\phi_2) = 0\,, \tag{12}$$

and $0 < g \ll m^2$. The parameters have mass dimensions $[\phi_{1,2}] = [m] = [m_\mathrm{P}] = [|\Lambda|^{\frac{1}{2}}] = [g^{\frac{1}{2}}] = [1/a_0] = 1$ and $[\alpha] = 0$.[14] As it is more convenient to work with dimensionless quantities, we can rescale a quantity $f \to m_\mathrm{P}^n f$ if $[f] = n$. Furthermore, we are interested in the early Universe, in which $a = a_0\mathrm{e}^\alpha \to 0$. For convenience, we define our symmetric boundary condition at $\alpha = \alpha_0$, with $\alpha_0 < 0, |\alpha_0| \gg 1$ (the limit $a \to 0$ is thus obtained with $\alpha_0 \to -\infty$). It is then convenient

[13] The variable Ψ_4 is not to be confused with the wave function Ψ that satisfies Eqs. (9) and (10).

[14] The inverse mass dimension of a_0 is the result of absorbing a length scale (with $c = \hbar = 1$) that comes from the integration of the spatial volume in this homogeneous model.

to define the variable $s = \alpha - \alpha_0$, so that the boundary condition is imposed at $s = 0$. Defining $0 < \lambda = a_0^6 e^{6\alpha_0} \ll 1$, the Wheeler–DeWitt equation becomes

$$\left\{ \frac{\partial^2}{\partial s^2} - \frac{\partial^2}{\partial \phi_1^2} - \frac{\partial^2}{\partial \phi_2^2} + \lambda e^{6s} \left[m^2(\phi_1^2 + \phi_2^2) + g\phi_1\phi_2 + \frac{\Lambda}{3} \right] \right\} \Psi = 0 \,, \tag{13}$$

which is now written in terms of the dimensionless quantities. For vanishingly small scale factors ($\lambda = 0$), we can neglect the potential term in Eq. (13) as mentioned before. In this case, we search for a separable solution (at $s = 0$, corresponding to our boundary condition). As the size of the universe increases (with $1 \gg \lambda > 0$ fixed and with increasing s), the potential term becomes relevant. In order to solve the constraint analytically, we consider a perturbative expansion in λ in the form

$$\Psi = e^{\sum_{n=0}^{\infty} (\lambda e^{6s})^n S_n(s,\phi_1,\phi_2)} \equiv \Psi_0 (1 + \lambda e^{6s} \Psi_1) + \mathcal{O}(\lambda^2) \ . \tag{14}$$

It is advantageous to expand only the exponent (instead of the wave function) in powers of λ. Keeping only the lowest orders, we would obtain $\Psi = e^{S_0 + \lambda e^{6s} S_1} e^R$, where R denotes the neglected remaining corrections to the phase and amplitude. Due to $\lambda \ll 1$, we can assume that, at field values that are not too large, the real and imaginary parts of the neglected corrections R are much smaller than the contributions of S_0 and S_1. However, for large field values, the contributions from R could conceivably be larger than those of the lowest orders, invalidating perturbation theory.

It is straightforward to verify that a possible solution is $S_0 = i(k_1\phi_1 + k_2\phi_2 \pm \sqrt{k_1^2 + k_2^2} s)$ (plane wave) and $S_1 = A + B\phi_1 + C\phi_2 - D(\phi_1^2 + \phi_2^2)/2 + E\phi_1\phi_2$, where the coefficients are complex functions of $k_{1,2}$ and the coupling constants. Then, one can either consider a superposition of such solutions for different values of $k_{1,2}$ (e.g., a narrow Gaussian centered at $\bar{k}_{1,2}$, which leads to a separable solution at $s = 0$ and $\lambda = 0$) or one can simply fix the values of $k_{1,2}$. For example, since the constraint is invariant under the exchange of ϕ_1 and ϕ_2, it is convenient to fix $k_1 = k_2$ to obtain a Gaussian solution that is symmetric under this exchange (this solution is also separable at $\lambda = 0$). If this solution is to be normalizable (even non-perturbatively in λ), we assume that $|\Psi|^2$ vanishes as $|\phi_{1,2}| \to \infty$. This then entails that the remainder term e^R cannot diverge or grow faster than the Gaussian decreases as $|\phi_{1,2}| \to \infty$, and thus the higher-order corrections can be neglected also for very large values of $|\phi_{1,2}|$ in this case, and we can simply approximate the state as the Gaussian $\Psi = e^{S_0 + \lambda e^{6s} S_1}$. This solution could then be normalized upon integration over $\phi_{1,2}$ (for fixed s), which would lead to a linear entropy of the form:

$$S_{\text{lin}} \propto g\lambda e^{6s} + \dots \tag{15}$$

with ϕ_2 taken as the environment field, and the entropy would be evaluated at the lowest non-trivial order of the coupling constants and λ, with a positive proportionality constant. This has the desired behavior: the entropy vanishes as $\lambda \to 0$ (low "initial" entropy in the "early Universe") and it increases with respect to s (or with respect to the scale factor, thereby increasing with the expansion of the universe).

However, this result is necessarily tentative. First, it makes use of a perturbative expansion, the validity of which needs to be thoroughly verified. Second, one must ascertain how sensitive the result is to the choice of state Ψ. Third, the entropy is obtained by manually normalizing the state, without considering in detail the definition of the inner product. As other works show [20], the positive-definite inner product may acquire a non-trivial form even in the matter sector when we use techniques from canonical gauge theory. Furthermore, the precise form of the inner product has implications for decoherence, which is another important ingredient to the emergence of a classical background with a notion of time such as s. Thus, this is only a first, but promising attempt at a concrete study of the growth of entropy in the geometrodynamical account of quantum cosmology.

6. Conclusions and outlook

To perform rigorous calculations for more realistic models is a challenge for future works, which might also require the use of sophisticated numerical methods. We believe, however, that the procedure outlined above could point towards a physical explanation of the growth of entropy from the conservative approach of quantum geometrodynamics, a timeless theory that only assumes general relativity and the principles of quantum theory. This is in line with recent work [29] that discusses the "entanglement past hypothesis." As we have seen, this hypothesis can be accommodated by a reasonable choice of boundary condition related to the structure of the Hamiltonian in general relativity. An open issue is, of course, whether the specification of this boundary condition as expressed by the quantum Weyl tensor hypothesis can be derived from an underlying principle instead of being freely chosen. These and other questions motivate further research in this topic.

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