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A Solution to the Quantum Measurement Problem

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Abstract: A novel solution to the quantum measurement problem is presented by using a new asymmetric equation that is complementary to the Schrödinger equation. Solved for the hydrogen atom, the new equation describes the temporal and spatial evolution of the wavefunction, and the latter is used to calculate the radial probability density for different measurements. The obtained results show that Born's position measurement postulates naturally emerge from the theory and its first principles. Experimental verification of the theory and its predictions are also proposed.

Keywords: quantum mechanics; quantum measurement problem; Galilean group of the metrics

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1. Introduction

The measurement problem in nonrelativistic quantum mechanics (NRQM) has been an active area of research for almost 100 years. Numerous attempts to solve it (e.g., [1–29]) have been unsuccessful. The proposed ideas are aligned to different interpretations of NRQM, and they range from objective wavefunction collapse theories, such as the Ghirardi–Rimini–Weber (GRW) model [8,9] and its extension, known as the continuous spontaneous localization (CSL) model [10,11], which are based on the Copenhagen interpretation [2–7], to the deterministic DeBroglie–Bohm interpretation with its non-local hidden variables [7,15] or to quantum decoherence [17–19] and Everett's many-words interpretation [3,15]. Penrose [23] proposed a collapse theory that attributes the wavefunction collapse to the influence of gravity. The role of quantum gravity in solving the measurement problem was further explored in [24]; however, this requires the formulation of quantum gravity as a fundamental theory of modern physics, which has not yet been accomplished.

According to the Copenhagen interpretation, quantum measurements involve the projection of a previously undetermined quantum state into an eigenstate that corresponds to the measurements (e.g., [2,7,15]). This cannot be achieved by the Schrödinger equation (SE) alone. The reason is that the time-evolution of a quantum state $|\Psi >= \hat{U}|\Psi_o >$, where $\hat{U}$ is the time-evolution operator for the equation, is continuous and deterministic; however, the projection of $|\Psi >$ into an eigenstate corresponding to the measurements is discontinuous and indeterministic. Heisenberg [25] argued that an energy transfer between a detector and an observed quantum system disrupts the smooth evolution of the wavefunction through quantum jumps (see [26] for a possible resolution).

On the other hand, von Neumann [2] tried to solve the measurement problem by introducing the reduction postulate, which combines the SE and the collapse of the wavefunction. The reduction postulate remains controversial [27], and some arguments were made to demonstrate that the postulate is incorrect (e.g., [28]) because it leads to ambiguity when a measurement must be switched on and considered as an interaction [29]. A solution to the quantum measurement problem is proposed in this paper.

A quantum system interacting with its surroundings is called an open quantum system, and significant progress in studying such systems has been made during the last few decades, with the Theory of Open Quantum Systems being established (e.g., [30]). Specifically, theoretical and experimental research on open quantum systems by using digital quantum simulators has been performed and general algorithms for such digital simulations have been investigated and developed [31–34]. However, their practical

experimental implementation poses some challenges that still remain unsolved [33,34]. The approach described in this paper is very different from the open system approach, and it has several advantages when compared to the latter, as the presented theory and its results demonstrate.

A solution of the quantum measurement problem proposed in this paper is based on a new asymmetric equation (NAE) [35], whose existence is guaranteed by the irreducible representations (irreps) of the Galilean group of the metric [36–38]. The NAE is complementary to the SE that also originates from the same irreps [39,40]. The main advantage of the NAE, as compared to the SE, is that it allows measurements to be directly accounted for. The effects of the measurements on the evolution of the wavefunction are investigated by solving the new equation for the hydrogen atom. The obtained solutions represent the temporal and spatial evolution of the wavefunction, and they are used to compute the radial probability densities of finding the electron after the measurement process is completed.

To account for the original electron's probability density resulting from the SE, the products of this probability and those resulting from measurements are calculated using the NAE. The obtained products are consistent with Born's position measurement postulates (e.g., [41,42]). Experimental verification of the calculated probability densities is proposed and discussed. The presented results demonstrate that the full description of the wavefunction in NRQM may require both the Schrödinger equation and its complementary new asymmetric equation.

The paper is organized as follows: the basic equations resulting from the Galilean group are given and discussed in Section 2; the governing equation is presented in Section 3; the temporal and spatial solutions to the governing equation are given in Sections 4 and 5, respectively; experimental verification is discussed in Section 6; and conclusions are given in Section 7.

2. Basic Equations from the Galilean Group of the Metrics

Galilean space and time are separated, and their metrics are given by $ds_1^2 = dx^2 + dy^2 + dz^2$ and $ds_2^2 = dt^2$, where x, y, and z are spatial coordinates and t is time. The metrics remain invariant with respect to all the transformations that form the Galilean group of the metric, $\mathcal{G}$, whose structure can be written as $\mathcal{G} = S(4) \otimes_s H(6)$, where $S(4) = T(1) \otimes R(3)$, with $T(1)$ and $R(3)$ being subgroups of translation in time and rotations, respectively. In addition, $H(6) = T(3) \otimes B(3)$, where $T(3)$ and $B(3)$ are subgroups of translations in 3D space and Galilean boosts, respectively [36–38].

The Galilean group of the metrics, $\mathcal{G}$, is a ten-parameter Lie group and $H(6)$ is its invariant Abelian subgroup. The irreducible representations (irreps) of $H(6)$ are one dimensional, they are well known [39,40], and they provide labels for all the irreps of the entire group $\mathcal{G}$. The classification of these irreps was completed by Bargmann [36], who demonstrated that the scalar and spinor irreps are physical, but the vector and tensor irreps are not because they do not allow for the elementary particle localization. As a result, in NRQM, only scalar and spinor wavefunctions are allowed, and their temporal and spatial evolution is described by the Schrödinger [41,42] and Lévy-Leblond [37,38] equations, respectively. In this paper, only the scalar irreps are considered.

The transformations for space translations represented by $T(3)$ and Galilean boosts represented by $B(3)$ may be considered separately and written as follows:

$\hat{T}_{\mathbf{a}}\phi(\mathbf{r},t) \equiv \phi(\mathbf{r}+\mathbf{a},t) = e^{i\mathbf{k}\cdot\mathbf{a}}\phi(\mathbf{r},t)$ and $\hat{B}_{\mathbf{v}}\phi(\mathbf{r},t) \equiv \phi(\mathbf{r}+\mathbf{v},t) = e^{i\mathbf{k}\cdot\mathbf{v}t}\phi(\mathbf{r},t)$, where $\hat{T}_{\mathbf{a}}$ and $\hat{B}_{\mathbf{v}}$ are operators of space translations and boosts, respectively. Moreover, $\phi(\mathbf{r},t)$ is a scalar wavefunction, $\mathbf{a}$ represents a translation in space, $\mathbf{v}$ is the velocity of Galilean boosts, and the real vector $\mathbf{k}$ labels the one-dimensional irreps of $H(6)$.

Expanding $\phi(\mathbf{r}+\mathbf{a},t)$ and $\phi(\mathbf{r}+\mathbf{v}t,t)$ in Taylor series and comparing the results to the above transformations, one obtains the following eigenvalue equation [37,38]:

$$-i\nabla\phi(r,t) = \mathbf{k}\phi(r,t)\,, \tag{1}$$

which has the same form for the boosts and translations in space, and it is also the condition needed so that the wavefunction $\phi(r,t)$ transforms as one of the irreps of $\mathcal{G}$.

However, $S(4)$ is not an invariant subgroup of $\mathcal{G}$, which means that the above procedure cannot be applied to it; the fact that $T(1)$ is an invariant subgroup of $S(4)$ does not help because $T(1)$ is not the 'little group' of $\mathcal{G}$ [39]. The proposed solution [39] requires that $\phi_\omega(\mathbf{r},t) = \eta(\mathbf{r},t)\phi(\mathbf{r},t)$, where $\eta(\mathbf{r},t)$ is a smooth and differentiable function to be determined and $\hat{E} = i\partial_t = i\partial/\partial t$ is the generator of translation; then, $i\partial_t\phi_\omega(\mathbf{r},t) = \omega\phi_\omega(\mathbf{r},t)$. The results presented in [39] demonstrated that $\eta(\mathbf{r},t) = \eta'(\mathbf{r}',t') = 1$ in order to be in agreement with the Galilean Principle of Relativity. Thus, the final temporal eigenvalue equation [39] can be written as

$$i\partial_t\phi(\mathbf{r},t) = \omega\phi(\mathbf{r},t)\,, \tag{2}$$

and this equation supplements Equation (1). The eigenvalue equations given by Equations (1) and (2) guarantee that the wavefunction $\phi(\mathbf{r},t)$ transforms as one of the irreps of $\mathcal{G}$ and that these equations can be used to derive dynamical equations that are consistent with the Galilean group of the metric $\mathcal{G}$.

There is another approach to find dynamical equations by using the Casimir operator of $\mathcal{G}$, which is well known (e.g., [39]). However, this operator does not connect Galilean space and time and therefore cannot be used to obtain any dynamical equation. The problem is resolved by extending $\mathcal{G}$ so that the connection is allowed. This results in the so-called extended Galilean group, whose structure is $\mathcal{G}_e = [R(1)\otimes B(3)]\otimes_s [T(3+1)\otimes U(1)]$, where $T(3+1)$ is an invariant subgroup of $\mathcal{G}_e$ of translations in space and time and $U(1)$ is a one-parameter unitary group (e.g., [37,38]); note that the structure of this group is similar to the Poincaré group (e.g., [43–46]) and that $\mathcal{G}_e$ has three Casimir operators. If a scalar wavefunction is considered, then one of these Casimir operators directly provides the Schrödinger equation [39], but this requires a prior knowledge of $\mathcal{G}_e$, which was introduced only after the Schrödinger equation was already established and used in NRQM. Moreover, it must be also noted that $\mathcal{G}_e$ is not the group of the Galilean metrics. Therefore, the derivation of the Schrödinger equation from this Casimir operator may not be consistent with the Principle of Galilean Relativity.

As a result, the eigenvalue equations given by Equations (1) and (2) are used here to obtain two second-order partial differential equations that are asymmetric in space and time derivatives. The coefficients in these equations are expressed as ratios of the eigenvalues, which label the irreps of $\mathcal{G}_e$. If these labels are identified as wave frequencies and wavenumbers, the resulting asymmetric equations describe the propagation of classical waves [47]. However, the coefficients can be determined by using the de Broglie relationship (e.g., [41,42]); then, one of the resulting asymmetric equations is the Schrödinger equation

$$\left[i\frac{\partial}{\partial t} + \frac{\hbar}{2m}\nabla^2\right]\phi_S(t,\mathbf{x}) = 0\,, \tag{3}$$

and the other can be written as

$$\left[\frac{i}{\omega}\frac{\partial^2}{\partial t^2} + \frac{\hbar}{2m}\mathbf{k}\cdot\nabla\right]\phi_A(t,\mathbf{x}) = 0\,, \tag{4}$$

which is a new asymmetric equation [35], with $\phi_S(t,\mathbf{x})$, and $\phi_A(t,\mathbf{x})$ being the wavefunctions satisfying these equations. The equations can also be derived from the relativistic energy–momentum relation by taking the nonrelativistic limit, which shows tha the expression for the nonrelativistic kinetic energy $E_k = p^2/2m$ underlies both equations. Based on the mathematical structure and physical meaning of the SE and NAE, the equations are complementary to each other and they describe different aspects of the evolution of the wavefunction in NRQM.

There are several important differences between the SE and NAE. The most prominent one is the difference in time derivatives. By being first order in time, the time solutions to

the SE are also the solutions to the continuity equation for probability, which is also first order in time. This means that the SE is invariant with respect to unitary transformations and that NRQM based on the SE is a unitary theory. On the other hand, the NAE being second order in time may have solutions that do not satisfy the continuity equation, and, as a result, the NAE may be used to describe some non-unitary processes in NRQM. Thus, the SE (unitary) $\rightarrow$ the NAE (non-unitary), with the latter describing processes such as the quantum measurement problem investigated in this paper or quantum jumps that represent the absorption and emission of electromagnetic (EM) radiation by atoms, which were both recently studied in [47] by using the NAE.

Another difference between the SE and NAE is the presence of the eigenvalues ω and $\mathbf{k}$ in the latter. To describe any physical event or process by the NAE requires specifying both eigenvalues. By identifying ω and $\mathbf{k}$ as the characteristic frequency and wavevector of dark matter, some attempts were made to formulate a quantum theory of cold dark matter [48] and galactic cold dark matter halos [49]; however, the formulated theories require observational verification of their predictions. In this paper, the eigenvalues are identified with measurements performed on a quantum system, which is a hydrogen atom in its basic 1s state. The changes in the temporal behavior of the wavefunction and in the radial probability densities of the electron resulting from the performed measurements are presented and discussed.

3. Governing Equation and Measurements

The parameters ω and $\mathbf{k}$ that appear explicitly in the NAE (see Equation (4)) are the eigenvalues of the Hermitian operators in the eigenvalue equations given by Equations (1) and (2), which means that they are real and label the irreps of the group. Thus, they can also be associated with a measuring apparatus, specifically with the EM radiation that this apparatus uses to probe a quantum system. For the theory developed in this paper, it is sufficient to prescribe the frequency of EM radiation $\omega = \omega_{ap}$, which gives $E_{ap} = \hbar\omega_{ap}$, and calculate the resulting wavevector $\mathbf{k} = \mathbf{k_{ap}}$, with $\mathbf{k_{ap}} = \hat{k}(1/\lambda_{ap})$, where λ_{ap} is the wavelength of EM used by the apparatus. In general, ω and $\mathbf{k}$ may also be determined by the temporal and spatial scales associated with the measuring process.

Then, the eigenvalue equations for the energy, $\hat{E} = i\hbar\partial_t$, and momentum, $\hat{\mathbf{P}} = -i\hbar\nabla$, operators acting on the wavefunction ϕ_A are given by $\hat{E}\phi_A = \sqrt{E_{ap}E}\ \phi_A$ and $(\hbar\mathbf{k_{ap}} \cdot \hat{\mathbf{P}})\phi_A = (\mathbf{p_{ap}} \cdot \hat{\mathbf{P}})\phi_A = p_{ap}p\ \phi_A$, with $E = \hbar\omega$, $p = \hbar k$, and $p_{ap} = \hbar k_{ap}$ [47–49].

The new asymmetric equation in the spherical variables, with the $r-$dependence only, the potential $V(r)$, and the effects of measurements accounted for by the presence of ω_{ap} and $\mathbf{k_{ap}}$, becomes

$$\left[\frac{i\hbar}{\omega_{ap}}\left(\frac{\partial^2}{\partial t^2}\right) + \frac{\hbar^2}{2m}(\mathbf{k}_{ap} \cdot \nabla) + V(r)\right]\phi_A(t, \mathbf{r}) = 0\,, \tag{5}$$

which is complementary to the SE with the potential to be specified [40,41].

The Schrödinger equation with the Coulomb potential, $V(r) = -e^2/(4\pi\epsilon_o r)$, describes a hydrogen atom by giving the solutions for the wavefunction in terms of Laguerre's polynomials (the radial-dependence) and spherical harmonics (the angular-dependence) and correctly predicts its quantum energy levels given by $E_n = -(m/2n^2\hbar^2)(e^2/4\pi\epsilon_o)^2$. The results given by the SE describe the electron's unitary evolution, and they are valid prior to making any measurement.

As pointed out in the Introduction, all previous attempts to account for measurements by using the SE have failed, including von Neumann's approach with the reduction postulate because of its ambiguity. To remove this ambiguity, it is suggested herein that at the moment when a measuring apparatus is applied to a quantum system, the system is described by the NAE. To verify this suggestion, Equation (5) is now solved for a hydrogen atom in its basic 1s state, and the temporal and spatial behavior of the wavefunction re-

sulting from the measurement process is calculated. Then, the spatial solutions are used to compute changes in the radial probability density caused by measurements.

However, the probability densities resulting from the measurements, and calculated with the NAE, do not account for the previous electron's unitary dynamics. To take this into consideration, the probabilities resulting from the measurements are multiplied by the original probability density obtained for the electron from the SE. The resulting probabilities demonstrate that the original state of the electron is projected into the observable eigenstate, as suggested by Born [50,51]; this is now known as Born's position measurement postulates or Born's rules.

Born's rules suggest that when a quantum particle interacts with a measuring apparatus, then it gets confined into a measuring eigenstate that corresponds to a well-defined position (e.g., [41,42]). In other words, the rules connect NRQM and its theoretical framework to experiments because they allow calculating probabilities from quantum mechanical amplitudes. Thus, they become a crucial link between abstract NRQM and the real world of experience. However, no first principle derivations of the rules exist so far, and they are considered to be Born's intelligent guess (e.g., [52,53]). In this paper, Born's rules are described by the NAE, which makes them to originate from the same basic principles as the NAE does, namely from the irreps of the Galilean group of the metric and the Principles of Galilean Relativity.

4. Time-Dependent Solutions and Their Physical Meaning

To solve Equation (5), the spherical variables are separated into the temporal and spatial (radial only) components, $\phi_A(t, \mathbf{r}) = \chi(t)\ \eta(\mathbf{r})$, and $-\mu^2$ is chosen as the separation constant. To relate this approach directly to the hydrogen atom problem and its description by the SE, it is assumed that $V(r)$ is the Coulomb potential and that $-\mu^2 = E_n$, which gives

$$\frac{d^2\chi}{dt^2} + i\left(\frac{E_{ap}}{2m}\right)\left(\frac{1}{na_o}\right)^2 \chi = 0\,, \tag{6}$$

and

$$\frac{d\eta}{dr} + \frac{1}{(\hat{\mathbf{k}}_{\mathbf{ap}} \cdot \hat{\mathbf{r}})k_{ap}\ a_o}\left(\frac{1}{n^2 a_o} - \frac{2}{r}\right)\eta = 0\,, \tag{7}$$

where $E_{ap} = \hbar\omega_{ap}$, $\mathbf{k_{ap}} = k_{ap}\hat{\mathbf{k}}_{ap}$, $\mathbf{r} = r\hat{\mathbf{r}}$, and the Bohr radius $a_o = (4\pi\epsilon_o\hbar^2)/me^2 = \hbar/(mc\alpha)$, with α being the fine structure constant. It must be noted that both ω_{ap} and k_{ap} are specified by interactions between a measuring apparatus and a quantum system.

Using $i = (1/\sqrt{2} + i/\sqrt{2})^2$, the solutions to Equation (6) are

$$\chi(t) = C_\pm \exp\left[\pm i\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)\sqrt{\frac{E_{ap}}{2m}}\frac{t}{na_o}\right], \tag{8}$$

where $C_\pm$ are the integration constants corresponding to the $\pm$ solutions. The solutions require that a measuring apparatus interacts with a quantum system; that is, $\omega_{ap} \neq 0$ as otherwise $\chi = C_\pm$, and the temporal evolution of the wavefunction must be described by the SE. Both solutions with C_+ and C_- are physical and they correspond to $t \to +\infty$ and $t \to -\infty$, respectively, which shows that the solutions are symmetric with respect to $t = 0$. In the following, only the solution with C_+ is considered because $t = 0$ is the initial time at which the measurement begins and $t > 0$ when it is performed.

The real part of the solution is

$$\mathcal{R}e[\chi(t)] = C_+ \cos\left(\sqrt{\frac{E_{ap}}{m}}\frac{t}{2na_o}\right)\exp\left(-\sqrt{\frac{E_{ap}}{m}}\frac{t}{2na_o}\right). \tag{9}$$

The arguments of the cosine and exponential functions can be estimated when ω_{ap} is given by specifying a measuring apparatus. Consider a quantum microscope designed to

measure the orbital structure of the hydrogen atom (e.g., [54]). Then, ω_{ap} is the frequency of electromagnetic (EM) waves used to perform the measurement. Let this frequency be either the optical, ultraviolet, or X-ray part of the EM spectrum; then, the factor $(\sqrt{E_{ap}/m}/a_o)$ in the arguments of the cosine and exponential functions becomes either $\sim 10^{15}\ s^{-1}$, $\sim 10^{16}\ s^{-1}$, or $\sim 10^{17}\ s^{-1}$, respectively. As a result, the arguments are very large and the exponential function decays very rapidly for any $t > 0$, while the cosine function remains periodic despite its large argument. The time of the decay of the wavefunction decreases with the increasing frequency of the used EM waves.

Since the argument of the exponential function is very large, the temporal part of the wavefunction reaches zero on a very short time scale that ranges from $\sim 10^{-17}\ s$ to $\sim 10^{-15}\ s$. The rapid decay of the wavefunction in time predicted by the NAE and its temporal solutions may be identified as the duration of the measurement, after which the electron resumes its original orbital. If the decay is considered 'instantaneous', then $\mathcal{R}e[\chi(t)]$ may represent the wavefunction that impinges on a screen or photographic film in experiments with double slits or the diffraction of electrons through a narrow aperture, and may be identified with the 'instantaneous collapse' of the wavefunction (e.g., [55]).

In general, it is expected that the time required for the wavefunction collapse is shorter when m increases (e.g., [7,15,41,42]). The arguments of the periodic and exponential parts of the solution $\mathcal{R}e[\chi(t)]$ are the same, and they depend explictly on the mass of electron m. Thus, with a_o being a function of m, the arguments depend on $\sqrt{m}$, whose larger value causes the exponential function to decrease faster and shortens the time of the wavefunction collapse.

5. Time-Independent Solutions and Probability Densities

Having obtained the solutions for the temporal evolution of the wavefunction, the solutions for its spatial behavior are now derived, and the resulting probability densities are calculated. Defining $\beta_{ap} = [(\hat{\mathbf{k}}_{\mathbf{ap}} \cdot \hat{\mathbf{r}}) k_{ap}\ a_o]^{-1}$, the solution to Equation (7) can be written as

$$\eta(r) = \eta_0\ r^{2\beta_{ap}}\ \exp\left[-\left(\frac{\beta_{ap}}{n^2}\right)\frac{r}{a_o}\right], \tag{10}$$

where the integration constant η_0 normalizes the wavefunction and is evaluated from

$$4\pi \int_0^\infty r^2 |\eta(r)|^2 dr = 1\,, \tag{11}$$

showing that η_0 depends on β_{ap} and on the principal quantum numbers n; thus, $\eta_o = \eta_{o,n}(\beta_{ap})$. Moreover, in spherical symmetry, $\hat{\mathbf{r}}$ can always be aligned with $\hat{\mathbf{k}}_{\mathbf{ap}}$, which means that $\hat{\mathbf{k}}_{\mathbf{ap}} \cdot \hat{\mathbf{r}} = 1$ and $\beta_{ap} = (k_{ap}\ a_o)^{-1} = \lambda_{ap}/a_o$, where λ_{ap} is the wavelength of the EM waves used in the measuring process.

The radial probability density in a spherical shell volume element is given by

$$\mathcal{P}_n(r) = r_{max}\frac{dP(r)}{dr} = \eta_o^2(\beta_{ap})\left(\frac{r}{r_{max}}e^{-r/r_{max}}\right)^{4\beta_{ap}+2} \tag{12}$$

where $\eta_o^2(\beta_{ap}) = 1/I(\beta_{ap})$. Defining $x = r/r_{max}$, the integral $I(\beta_{ap})$ becomes

$$I(\beta_{ap}) = \int_0^\infty (x\ e^{-x})^{4\beta_{ap}+2}\ dx\,, \tag{13}$$

and it can be evaluated after β_{ap} is specified. Moreover, r_{max} is the most probable radius, defined as

$$r_{max} = n^2 a_o\left(2 + \frac{1}{\beta_{ap}}\right) = n^2 a_o\left(2 + \frac{a_o}{\lambda_{ap}}\right). \tag{14}$$

The effects of measurements are given by $\beta_{ap} \neq 0$ (or $\lambda_a \neq 0$), which is required for the NAE to represent the radial wavefunction and its evolution. In the absence of measurements, the wavefunction's evolution is described by the SE.

To compare the obtained results directly to the radial probability densities of a hydrogen atom given by the time-independent solutions to the SE, it is convenient to introduce $r_a = r/a_o$. Then, the radial probability density $\mathcal{P}_n(r_a)$ given by Equation (12) can be written as

$$\mathcal{P}_n(r_a) = \frac{dP(r_a)}{dr_a} = \eta_{o,n}^2(\beta_{ap})\, r_a^{4\beta_{ap}+2}\, e^{-2(\beta_{ap}/n^2)r_a}\,, \tag{15}$$

where $\eta_{o,n}^2(\beta_{ap}) = 1/I_n(\beta_{ap})$, and where the integral $I_n(\beta_{ap})$ can be evaluated by using

$$I_n(\beta_{ap}) = \int_0^\infty r_a^{4\beta_{ap}+2}\, e^{-2(\beta_{ap}/n^2)r_a} = \frac{1}{(2\beta_{ap}/n^2)^{4\beta_{ap}+3}}\,\Gamma(4\beta_{ap}+3)\,, \tag{16}$$

which is valid if $\mathcal{R}e(2\beta_{ap}) > 0$ and $\mathcal{R}e(4\beta_{ap}+2) > -1$; note that both conditions are obeyed in the theory presented in this paper.

Using Equations (15) and (16), the radial probability density $\mathcal{P}_1(r_a)$ is calculated for $n = 1$ and for different values of β_{ap}, which varies from 50 to 9000. This range of β_{ap} corresponds to $\lambda_{ap} = 50\ a_o$ and $100\ a_o$ (X-rays), $\lambda_{ap} = 200\ a_o$ and $800\ a_o$ (ultraviolet), and $\lambda_{ap} = 8000\ a_o$ and $9000\ a_o$ (visible); this range can be extended to the other parts of the EM spectrum. Since the selected values of β_{ap} are positive integers, the function $\Gamma(4\beta_{ap}+3) = (4\beta_{ap}+2)!$ in Equation (16). The obtained results are presented in Figure 1, showing the radial probability density, $\mathcal{P}_{1s}(r_a) = 4r_a^2\, e^{-2r_a}$, for the hydrogen 1 s orbital plotted for comparison.

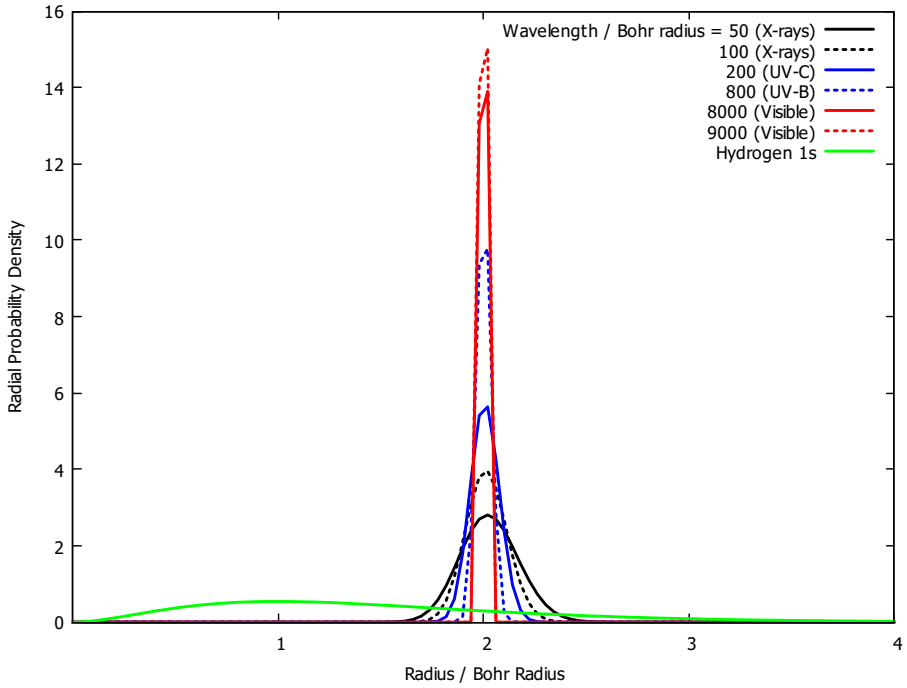

Figure 1. The radial probability density, $\mathcal{P}_1(r_a)$, is plotted against the ratio of the radius to the Bohr radius, r/a_o, for $n = 1$ and $\beta_{ap} = \lambda_{ap}/a_o$, which ranges from X-rays to the ultraviolet and visible parts of the EM spectrum. The theoretically predicted $\mathcal{P}_1(r_a)$ is compared to the radial probability density, $\mathcal{P}_{1s}(r_a)$, for the hydrogen 1 s orbital.

The maxima of $\mathcal{P}_1(r_a)$ plotted in Figure 1 are at $r = r_{max}$, where the latter depends on β_{ap} (see Equation (14)). This dependence on β_{ap} or λ_{ap} is reflected in Figure 1 as small shifts

towards the larger values of r_a for shorter wavelengths, such as X-rays and UV. However, in the limit $\lambda_{ap} \to \infty$, one finds $r = r_{max} = 2a_o$ and

$$\mathcal{P}_1(r_a) = \lim_{\lambda_{ap}\to\infty} \eta^2_{o,1}(\lambda_{ap}) r_a^{4\lambda_{ap}/a_o+2}\, e^{-2(\lambda_{ap}/a_o)r_a} = \delta(r - 2a_o)\,, \tag{17}$$

where $\delta(r - 2a_o)$ is the Dirac delta function. This confirms that for very long wavelengths, $\mathcal{P}_1(r = 2r_a) = 1$, which is the classical limit of the measuring process. The main result is that a quantum particle interacting with a measuring apparatus becomes confined into a measurement eigenstate or in a well-defined position at $r = 2a_o$, as originally suggested by Born's position measurement principles (e.g., [41,42]).

The theory and results presented in this paper describe the process known as quantum decoherence, which occurs when a quantum system interacts with its environment. As a result of this interaction, the information that the system contains is admixed up with its environment. More specifically, the system loses its ability to show superposition and interference effects, and its physical behavior resembles a classical system. This can be seen in Figure 2, which shows that the particle is confined to a very narrow eigenstate after measurements are complete. Moreover, according to Equation (17), the longer the wavelength of EM radiation absorbed by the system, the more likely its classical-like behavior is observed.

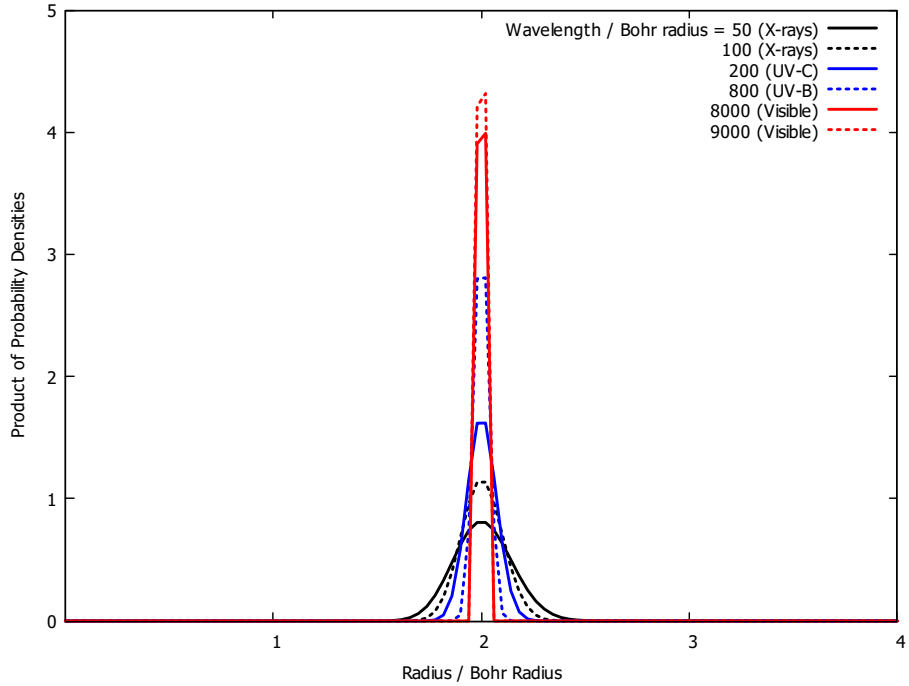

Figure 2. The product of the radial probability densities, $\mathfrak{P}(r_a) = \mathcal{P}_1(r_a) \times \mathcal{P}_{1s}(r_a)$, plotted against the ratio of radius to the Bohr radius, r/a_o, for $n = 1$, and $\beta_{ap} = \lambda_{ap}/a_o$, which ranges from X-rays to the visible part of the EM spectrum.

The results presented in Figure 1 are obtained by solving the spatial part of the NAE for specified wavelengths of EM waves used in the measuring process. However, the results do not take into account the initial state of the electron on its 1 s orbital. The radial probability density for the 1 s orbital, $\mathcal{P}_{1s}(r_a)$, given by the SE is also shown in Figure 1. In order to account for the electron's initial probability density on the 1 s orbital, the product $\mathfrak{P}(r_a) = \mathcal{P}_{1s}(r_a) \times \mathcal{P}_1(r_a)$ is calculated and plotted in Figure 2.

All the computed probability densities in Figure 2 are more narrow and centered at $r = 2\ a_o$, as compared with those in Figure 1. This is an interesting result as it demonstrates how the original electron's probability density on the 1 s orbital has changed because

of the measurement process. Thus, the results of Figure 2 are in better agreement with Born's position measurement postulates than those given in Figure 1, which shows that the electron's initial probability density calculated by the SE must also be accounted for in the procedure that describes the measurements.

Since the presented results depend directly on the principal quantum number n (see Equations (15) and (16)), similar calculations can be performed for $n = 2, 3, 4$; however, they will be presented elsewhere.

6. Experimental Verification

The results presented in Figures 1 and 2 demonstrate that the electron originally located on the 1s orbital, with its most probable radius at $r = a_o$, is shifted to its most probable radius at $r = 2\,a_o$ as a result of the measurements. This means that the electron interacting with a measuring apparatus gets confined into an eigenstate that corresponds to the measurement. The comparison of the original electron's probability density $\mathcal{P}_{1s}(r_a)$ on the orbital 1 s (see Figure 1) to $\mathfrak{P}(r_a)$ plotted in Figure 2 shows that, as a result of the measurement, the electron is confined to a well-defined position at $r = 2a_o$, and that its probability density is very narrow and sharply centered at this position, which is consistent with Born's position measurement principles (e.g., [41,42]). The time scale for the existence of this eigenstate is given by Equation (9), which represents its duration.

The theoretical results obtained in this paper can be verified experimentally by using a quantum microscope similar to that designed by an international team of researchers [54], who used it to measure the orbital structure of Stark states in an excited hydrogen atom (see Figure 3 in their paper) . They reported a similar trend in shifting the maxima and broadening the density distributions for shorter wavelengths, as those shown in Figure 2. There are several other measuring methods developed to study the quantum-classical correspondence [54–59], but some of these methods may not be suitable to observe single orbitals in a hydrogen atom [58].

After using the NAE to solve the quantum measurement problem in this paper and the quantum jumps in [47], the NAE has the potential to develop new quantum-based technologies, which would verify its validity and applicability. As described in a recent comprehensive review [60], cryptographic systems and secure direct communications, as well as emerging fields like the quantum internet, are all based on modern quantum technologies. However, exploring the potential of developing new quantum technologies based on the NAE is out the scope of this paper.

7. Conclusions

In this paper, a new asymmetric equation, which is complementary to the Schrödinger equation, is used to investigate the measurement problem of quantum mechanics. The obtained results demonstrate that while the Schrödinger equation describes the evolution of the wavefunction prior to any measurement, the new asymmetric equation represents the behavior of the wavefunction during the measurement process. The temporal solutions to the new asymmetric equation give a time scale for the duration of the eigenstate that corresponds to the measurement; probability density at this eigenstate is determined from the spatial solutions.

To describe the transition from unitary (reversible) representation of the electron on its 1 s orbital, which is given by the Schrödinger equation, to its non-unitary (irreversible) evolution resulting from the measurement, which is given by the new asymmetric equation, the product of both probability densities is calculated. The main results are that the time scales are of the order of 10^{-17}– 10^{-15} s, depending on the frequency of the EM waves used in the measurement, and that the computed probability densities are sharply centered around $r = 2\,a_o$, which shows that Born's position measurement postulates naturally emerge from the presented theoretical results. In other words, the new asymmetric equation and first principles used to derive it also account for Born's rules of quantum mechanics.

The predicted radial probability densities resulting from the measurement process can be verified experimentally by using a quantum microscope (e.g., [54]). Another way to verify the validity and applicability of the new asymmetric equation would require exploring its potential in contributing to already-known quantum-based technologies (e.g., [60]) or to be used to develop new quantum technologies based on this equation; however, this is out of the scope of this paper.

Based on the presented results, it is proposed that while the unitary evolution of the wavefunction is described by the Schrödinger equation, its non-unitary behavior caused by measurements is represented by the new asymmetric equation; this means that both equations are required in nonrelativistic quantum mechanics to fully describe the spatial and temporal evolution of the wavefunction and the quantum measurement problem.

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