# THE NATURE OF THE TRANSVERSE INSTABILITY IN THE BROOKHAVEN AGS*

E.C. Raka

Brookhaven National Laboratory, Upton, New York

## I. Introduction

The growth of coherent vertical betatron oscillations was first observed in the Brookhaven AGS in June of 1965. It occurred both before and after the transition energy (≅ 7.5 BeV) was reached. Horizontal coherence was also observed early in the acceleration cycle but no significant growth occurred. Some beam loss was produced by the early vertical growth (700 MeV-1 BeV) while that occurring at high energy (≥ 16 BeV) resulted only in enlarging the beam thus making it impossible to obtain a fast extracted beam. The cause of the instability was found to be the presence of a poor vacuum (> $10^{-4}$ mmHg) in a small portion (a fraction of one of the twelve superperiods) of the synchrotron ring.

Subsequent investigation [1] has shown that appreciable vertical amplitudes (> .2 cm p.p. for the center of charge) can be present if the vacuum is larger than $10^{-5}$ mmHg in one or more portions of the ring. Under normal operating conditions (vacuum ≈ 2-5 x $10^{-6}$ or better) the vertical amplitudes are either zero or small (< .2 cm p.p.) and horizontal coherence is not present, even at intensities > 1.8 x $10^{12}$ protons/pulse. Thus except for very brief periods, operation of the AGS has not been limited by the presence of this instability.

Spontaneous growth, with good vacuum, can be obtained, however, if the AGS is flat topped at low energy (600-700 MeV) or growth can be stimulated at low and high energy (16-18 BeV), with or without a flat top, by exciting the beam at one of the frequencies $|n - \nu_y|\omega_o$. This has made it possible to study many of the features of the instability without having to intentionally produce a poor vacuum in some part of the synchrotron.

## II. High Field Characteristics

In order to explain why the vertical instability can occur at high energies in the AGS it is only necessary to examine the time variation on the quantity

$$S_w = \frac{\partial S}{\partial W} = [(n - \nu)\frac{\partial \Omega}{\partial W} - \omega_o \frac{\partial \nu_y}{\partial W}]$$

where $S = (n - \nu_y)\,\Omega$ as defined by Laslett, Neil and Sessler[2]. Here $\Omega$ is the angular frequency of an individual particle, $\nu_y$ its betatron wave number and $\omega_o$ the angular frequency of a phase stable particle with zero betatron amplitude and $\nu$ its betatron wave number. W is $2\pi\,(P_\theta - P_o)$, the deviation of the particle angular momentum from the phase stable value. This can be written as

$$S_{\Delta p} = W\,S_w = [(n - \nu)(\frac{1}{\gamma^2} - \alpha) - s\nu]\,\omega_o \frac{\Delta p}{p}$$

where p is the linear momentum of the phase stable particles, $\alpha$ the momentum compaction and

$$s = \frac{\Delta\nu/\nu}{\Delta p/p} = \frac{\frac{\Delta \nu}{\nu}}{\frac{1}{\alpha}\frac{\Delta r}{r}} = \frac{\Delta\nu}{\Delta r}\frac{\alpha r}{\nu} = \frac{r}{\nu_x^2 \nu}\frac{\Delta\nu}{\Delta r}$$

Here r is the average radius of the phase stable particles and $\nu_x$ the horizontal betatron wave numbers and $\alpha \cong 1/\nu_x^2$.

Now in the AGS the variation of $\nu_y$ with r and hence p is essentially linear over the center of the synchrotron aperture (± 1 cm) and hence at any particular time $\Delta\nu/\Delta r$ = constant. However, the value of the slope changes in time from about -.15 $\nu$ units/inch at low energies to -.024 units/inch at 14 BeV/c (400 msec) and at 21 BeV/c (600 msec) it has the value of ≈ +.005 units/inch ($\nu_y$ ≅ 8.75). At 700 msec (≈ 25 BeV/c) it has increased to +.037 units/inch and is still essentially constant over the central aperture to the accuracy of measurement (± .0014 units).

The slope continues to increase with time but the region of interest is that where it is close to zero. The term $(1/\gamma^2 - \alpha)$ in $S_{\Delta p}$ is small and slowly varying in the AGS once the transition energy has been passed (≈ -.01 between 400 and 700 msec). Thus for a given value of n, the mode number of the coherence, the expression in brackets will pass through zero at a time dependent on this choice. For n=9 the left hand term is negative and cancellation is reached at an earlier time than for n=8 since this makes the first term positive.

This effect has been observed by exciting separately the two modes n=9,8 in the neighborhood of 500 msec during a normal acceleration cycle. Excitation is obtained by driving a pair of 6-ft long coils placed on either side of the orbit in a 10-ft straight section with one or two millisecond bursts of the frequency $|n-\nu|\omega_o/2\pi$.

Figures 1 and 2 show vertical sum and difference signals taken with the different modes excited as indicated. The loss on the sum trace is due to normal targeting while the beats on the difference signal, before the growth predominates, are due to synchrotron oscillation. As can be seen the n=8 growth peaks out at a slightly later time than the n=9 while the overall growth takes place in the region where $S_{\Delta p}$ is near zero. Thus the effective sextupole field present in the AGS provide some stabilization against the growth of the vertical instability in spite of the fact that in general the growth time at high energies is slow compared to the phase oscillation period of ≈ 8 msec. This point was further checked by programming correcting sextupoles so that $S_{\Delta p}$ would pass through zero more rapidly in the neighborhood of 400 msec and then removing the program after 700 msec. If spontaneous growth were present it would be limited to a small value by this procedure. In a like manner if the sextupole program were of the opposite sign $S_{\Delta p}$ would be kept from passing through zero until after 700 msec and again growth would be small since the rate of change of $S_{\Delta p}$ would be more rapid than normal when the program was removed.

In these tests as in the cases shown in Fig. 1 and 2 the fact that the coherent motion of the center of change as seen by the electrodes decreases in amplitude after some peak value is reached does not mean that the individual amplitudes are damped but rather that due to increased spread in S the coherence is washed out.

If the AGS is flat topped in the region where $S_{\Delta p} \cong 0$, and the vacuum is good (< 3-4 x $10^{-6}$), no spontaneous growth of the vertical coherence is observed. Thus there must be another source producing a spread in S. In Ref. 2 a variation of $\nu_y$ with amplitude is considered, and the term

$$S_{a^2} = \frac{\partial \nu_y}{\partial_{a^2}}\,\omega_o\, a^2 ,$$

where a is the amplitude, is introduced as contributing to ΔS. As mentioned above the variation of $\nu_y$ with r is quite linear for the energies considered here and hence there is little if any average octupole field present. Thus it is not evident if there is any contribution to ΔS from a $S_{a^2}$ term present in the AGS. This question is still under investigation. Another possible contributing factor is the coupling between the x and y motion which shows up at a radius of ≈ .-75 cm from the nominal zero used as a reference in measuring $\nu_y$ and $\nu_x$. Here the two $\nu$'s are ≈ equal and deviations $\nu_y$ or $\nu_x$ from a line of constant slope are observed. These are in the directions that one would expect from the simple treatment of weakly coupled oscillators. If the beam is steered to this radius then spontaneous coherence is suppressed or greatly reduced and it is not possible to produce stimulated growth at all.

The growth of large amplitude n=8 and 9 modes has been observed, both with and without a flat top, whether stimulated by rf excitation or occurring spontaneously due to a poor vacuum in some part of the ring. These are essentially pure modes with all twelve bunches in phase, Fig. 1 and 2 for example. In general, however, no matter which mode is stimulated or occurs spontaneously (n=8,9,10) once the coherent amplitude becomes large (≳.5 cm) the bunches are out of phase and it is often not clear what mode or modes are present though evidence of the n=9 mode is most frequently seen.

Just barely above the spontaneous growth threshold the n=9 mode is usually observed and never the n=8. On rare occassions both a pure n=9 and 10, mode of very small amplitude has been seen, superimposed on one another (where the frequencies $|n-\nu|\omega_o/2\pi$ are in the ratio of 5:1 since $\nu_y$ ≅ 8.75). It should be mentioned here that only the n=9 mode is observed when the instability occurs spontaneously at low fields and that almost always the twelve bunches are in phase even at amplitudes large enough to produce beam loss.

Measurements of the growth rates for the modes n=8,9 obtained by rf excitation on a high field flat top (where the time variation of $S_{\Delta p}$ is zero) indicate a definite dependence upon the vacuum as well as a variation with the amount of initial excitation used. The observed growth rates also depend upon the beam intensity but not, as strongly as N the number of protons/pulse. Thus a range of values from 7 $sec^{-1}$ for n=8 with a

*Work performed under the auspices of the U.S. Atomic Energy Commission.

good vacuum and large excitation at N=1.5 x $10^{12}$ protons/pulse to 16 $sec^{-1}$ with poor vacuum in one portion of the ring (> 3 x $10^{-5}$ mmHg) though not enough to produce spontaneous growth, and intermediate excitation at 1.27 x $10^{12}$ protons/pulse or, 35 $sec^{-1}$ with large excitation and these same conditions, has been observed. The growth is not always purely exponential nor do all twelve bunch always participate equally. Some beam loss almost always occurs if the growth time is small compared to the length of the flat top. Growth rates of the same order are obtained at intensities of 1.3 - 1.5 x $10^{12}$ protons/pulse when spontaneous coherence is present. Below intensities of about 6-7 x $10^{11}$ protons/pulse spontaneous growth has not been observed even with a vacuum $\approx 10^{-4}$ in a small portion of the ring. It is still possible to obtain some stimulated growth with a poor vacuum at intensities of 2.5 x $10^{11}$ protons/pulse while the threshold with good vacuum is around 4 x $10^{11}$ protons.

III. Low Field Characteristics

When the vertical instability appears spontaneously below transition it occurs in the energy range 200 MeV - 1.5 BeV (50 MeV injection energy). It is always in the n=9 mode, with the twelve bunches in phase unless there is a large assymmetry in the individual bunch population. The growth is almost always modulated in amplitude at the phase oscillation frequency, ($\approx$ 2 kc to < 1 kc in the above energy range), twice the phase oscillation frequency, the magnet ripple frequency 720 ~, or combinations of these. This effect is shown in Fig. 3 and 4 while Fig. 5 shows a growth without modulation. It can be explained by the variation of $S_{\Delta p}$ that occurs during this period in the accelerating cycle. Although s is large ($\approx$ -1.6) and not changing rapidly, the rotation frequency, phase oscillation frequency, momentum, and momentum spread $\Delta p$ are, and hence $S_{\Delta p}$ is varying at rates that are comparable to or greater than the growth rate of the instability. Since s does not become small until after the transition energy is reached while the instability if it were due only to a resistive effect has a threshold[2] that varies as $1/\beta\gamma^3$, eventual stabilization must again be mostly due to $S_{\Delta p}$. It is also possible to suppress this coherence by programming the correcting sextupoles to increase s and hence $S_{\Delta p}$ during the critical part of the accelerating cycle. Conversely the coherence can often be made to appear by programming the sextupoles in the opposite direction.

The instability is generally more likely to appear if during the critical period the beam radius is steered to the outside of the nominal zero radius defined by the radial control electrodes. The reason for this is not yet understood. At these energies $\nu_x$ and $\nu_y$ are well separated and plots of $\nu_y$ vs r are fairly linear over the central aperture though there is some indication of curvature beyond $\pm$ 1.25 cm from a zero that is slightly inside of that defined by the radial control electrodes. Growth of the instability early in the cycle almost always shows up when the vacuum approaches $10^{-5}$ mm in any portion of the ring which is considerably lower than the 4-6 x $10^{-6}$ required to obtain growth at high fields.

It has been possible to stimulate the modes n=7-10 at low fields with good vacuum. However, subsequent growth does not often occur. Rather the coherence eventually disappears many milliseconds after the excitation is removed, the amplitude never having exceeded the initial stimulated value. On rare occasions a n=8 pattern has appeared some time after low level stimulation with n=9 was employed. The amplitude envelope, after stimulation, always exhibits various modulation effects similar to those described above.

In order to obtain a measure of the early growth rate the AGS was flat topped at about 70 msec ($\approx$ 600 MeV) and spontaneous coherence was obtained at 1.3 x $10^{12}$ protons/pulse. Subsequent measurements showed that growth could occur at about 4 x $10^{11}$ protons and above, with the threshold depending weakly on the vacuum. Again it was found that the growth rate depended quite markedly on the state of the ring vacuum. With the early flat top the growth is quite often not exponential once the amplitude becomes appreciable and many times the growth would limit itself before loss occurred. At other times with ostensibly the same conditions and perhaps less beam intensity exponential growth and some beam loss could occur. A growth rate of 1.57 $sec^{-1}$ at 1.85 x $10^{12}$ protons has been observed while on the same day at .9 x $10^{12}$ rates of 24 and 45 $sec^{-1}$ were obtained (all with good vacuum). At 1.3 x $10^{12}$ protons growth rates varying by almost a factor of two have been observed on different occassions (28-45 $sec^{-1}$).

The reason that spontaneous growth appears so readily on the early flat top was traced to the fact that $d\nu_y/dr$ is $\approx$ 0 here. This change from .15 nu units/inch is due to the absence of eddy current effects on the flat top. Thus again $S_{\Delta p}$ is small and the threshold for instability is considerably reduced. Here also it is possible to control the growth by introducing sextupole fields though complete supression is not always achieved with the modest currents used.

IV. Summary

It seems certain that a resistive wall instability is present in the AGS as evidenced by the vertical coherent growth that occurs when the synchrotron is flat topped at low energy with a good vacuum in the ring. It is also clear that condition of the vacuum can contribute significantly to the growth of the instability. Thus, another mechanism involving a localized disturbance due to a poor vacuum must be considered. It does not seem that the process proposed by H.G. Hereward[4] to explain the horizontal instability observed in the CERN PS can apply to the AGS. He assumes a poor vacuum around the entire synchrotron and arrives at the condition that coherent modes for $n>\nu$ will be damped. Of course the n=8 mode as well as n=9 have been seen to grow in the AGS. The former should be damped according to the resistive wall theory for an unbunched beam[2]. Even the modification of this theory for bunched beams[5] does not predict the occurence of an essentially pure n=8 mode. All these theories, of course, make simplifying assumptions, many of which do not apply to the AGS.

Future investigations of the instability will be centered on determining why the early flat-top growth rates exhibit such a wide variation and why the growth is often limited before beam loss occurs. Also, as previously mentioned, an attempt will be made to determine if there is any amplitude dependent frequency spread present in the y motion.

As for supressing the instability if it becomes prevalent when higher beam intensities are obtained (>2.2 x $10^{12}$ protons/pulse) a narrow band feedback damping system has been developed and tested on the AGS[3]. It has successfully damped the early coherence even on a flat top but was marginally effective on the late coherence. An improved version proposed in Ref. 3 has been built and will be tested soon. The design of a wide band system that would act on the individual bunches is also under consideration. This would provide damping for modes where the bunches are not in phase. Preliminary observations indicate that this is most likely to occur at high fields. Thus, a system that operated over an exceedingly narrow range of $\nu$ and $\omega_o/2\pi$ would be sufficient, eliminating the need for variable delays of any kind.

References


1. E.C. Raka, Observation of Coherent Transverse Oscillations in the Brookhaven AGS, BAPS Series II, 11, 469 (1966).

2. L.J. Laslett, V.K. Neil and A.M. Sessler, Transverse Instabilities of Intense Coasting Beams in Particle Accelerators, RSI 36, 436 (1965).

3. E.C. Raka, Proc. of the Second National Particle Accelerator Conference, Wash. D.C., IEEE Transactions on Nuclr. Sci., March 1967, p. 1091.

4. H.G. Hereward, The Instability of Radial Betatron Oscillations in the CPS, CERN, MPS/Int. DL 64-8.

5. E.D. Courant and A.M. Sessler, Transverse Coherent Resistive Instabilities of Azimuthally Bunched Beams in Particle Accelerators, RSI, 37, 11, 1579-1588, 1966.


DISCUSSION (condensed and reworded)

Kolomenskij (Lebedev): When the oscillations of the center of mass are supressed, oscillations about the center of mass can arise. What is the possibility of such an instability taking place?

Raka: I have not observed any effect of this kind so far. Investigations are in progress on the development of quadrupole coherence.

Hereward (CERN): Do you believe that a localized bad vacuum somewhere in the ring is worse for these coherent phenomena than the same amount of additional pressure distributed all the way around the ring equally?

H. Bruck (Orsay): We observed dependence of instabilities on pressure and attributed to the non-linearity introduced by the fields of the particles of contrary sign. It does not matter whether they are electrons or ions. In this case the effect is the same whether bad vacuum is localized or distributed.

Sessler (LRL): I think this paper and Martin's paper both are very interesting. I would like to emphasize to the theorist that experimentalists in both cases are calling our attention to phenomena which we haven't calculated. As soon as we all rush home we should investigate the accentuation of machine resonances, coupling, sum resonances, ordinary imperfection resonances due to the self-field. I think such calculations are very badly needed because we have seen now concrete evidence in two machines, that this leads to particle loss. It is time that we do the calculations and see how it depends on things and what can be done about it.

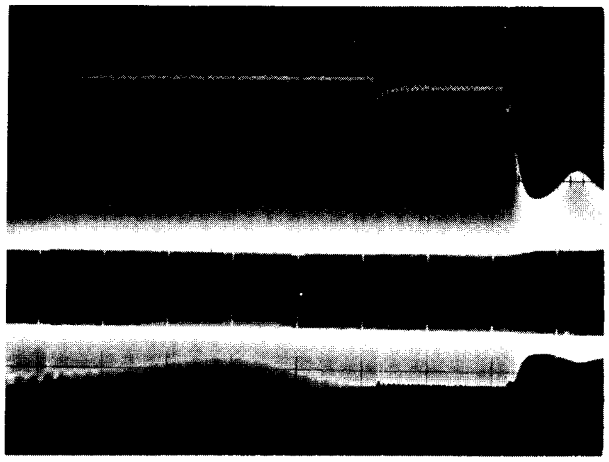

**Fig. 1** Vertical sum and difference 450 msec trigger 50 msec/cm; n = 8 excited (≈280 kc); $\nu_y \approx 8.75$

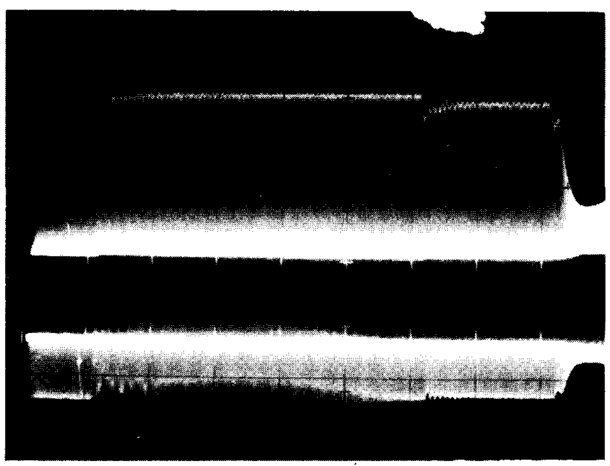

**Fig. 2** Vertical sum and difference 450 msec trigger 50 msec/cm; n = 9 excited (≈90 kc); $\nu_y \simeq 8.75$

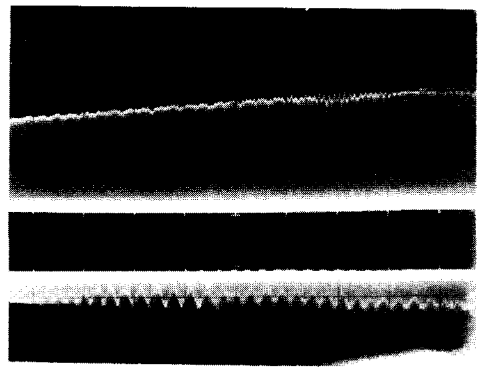

Fig. 3 50 msec trigger
5 msec/cm
Note 720 ~
Modulation of
Coherence Growth

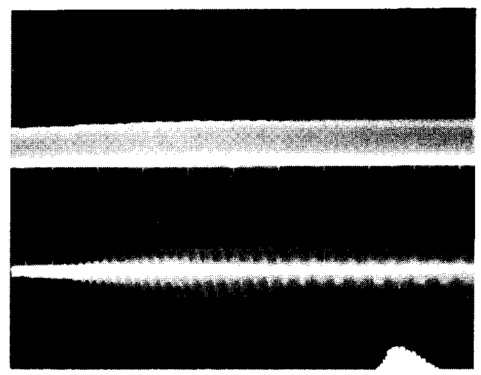

Fig. 4 37 msec trigger
5 msec/cm
Showing varying
frequency modulation
and beam loss
on sum trace

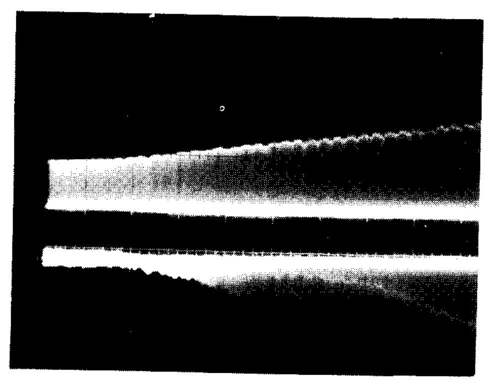

Fig. 5 50 msec trigger
10 msec/cm
Showing a
.2 cm p.p.
amplitude growth
at 1.15 x $10^{12}$ protons