Review Article

The Nature of Massive Neutrinos

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The compelling experimental evidences for oscillations of solar, reactor, atmospheric, and accelerator neutrinos imply the existence of 3-neutrino mixing in the weak charged lepton current. The current data on the 3-neutrino mixing parameters are summarised and the phenomenology of 3-ν mixing is reviewed. The properties of massive Majorana neutrinos and of their various possible couplings are discussed in detail. Two models of neutrino mass generation with massive Majorana neutrinos—the type I see-saw and the Higgs triplet model—are briefly reviewed. The problem of determining the nature, Dirac or Majorana, of massive neutrinos is considered. The predictions for the effective Majorana mass $|\langle m \rangle|$ in neutrinoless double-beta-$((\beta\beta)_{0\nu}$-) decay in the case of 3-neutrino mixing and massive Majorana neutrinos are summarised. The physics potential of the experiments, searching for $(\beta\beta)_{0\nu}$-decay for providing information on the type of the neutrino mass spectrum, on the absolute scale of neutrino masses, and on the Majorana CP-violation phases in the PMNS neutrino mixing matrix, is also briefly discussed. The opened questions and the main goals of future research in the field of neutrino physics are outlined.

1. Introduction: The Three Neutrino Mixing—An Overview

It is a well-established experimental fact that the neutrinos and antineutrinos which take part in the standard charged current (CC) and neutral current (NC) weak interaction are of three varieties (types) or flavours: electron, ν_e and $\bar{\nu}_e$, muon, ν_μ and $\bar{\nu}_\mu$, and tauon, ν_τ and $\bar{\nu}_\tau$. The notion of neutrino type or flavour is dynamical: ν_e is the neutrino which is produced with e^+ or produces an e^- in CC weak interaction processes; ν_μ is the neutrino which is produced with μ^+ or produces μ^-, and so forth. The flavour of a given neutrino is Lorentz invariant. Among the three different flavour neutrinos and antineutrinos, no two are identical. Correspondingly, the states which describe different flavour neutrinos must be orthogonal (within the precision of the current data): $\langle \nu_{l'} \mid \nu_l \rangle = \delta_{l'l}$, $\langle \bar{\nu}_{l'} \mid \bar{\nu}_l \rangle = \delta_{l'l}$, $\langle \bar{\nu}_{l'} \mid \nu_l \rangle = 0$.

It is also well known from the existing data (all neutrino experiments were done so far with relativistic neutrinos or antineutrinos) that the flavour neutrinos ν_l (antineutrinos $\bar{\nu}_l$) are always produced in weak interaction processes in a state that is predominantly left handed (LH) (right handed (RH)). To account for this fact, ν_l and $\bar{\nu}_l$ are described in

the Standard Model (SM) by a chiral LH flavour neutrino field $\nu_{lL}(x)$, $l = e, \mu, \tau$. For massless ν_l, the state of ν_l ($\bar{\nu}_l$), which the field $\nu_{lL}(x)$ annihilates (creates), is with helicity $(-1/2)$ (helicity $+1/2$). If ν_l has a nonzero mass $m(\nu_l)$, the state of ν_l ($\bar{\nu}_l$) is a linear superposition of the helicity $(-1/2)$ and $(+1/2)$ states, but the helicity $+1/2$ state (helicity $(-1/2)$ state) enters into the superposition with a coefficient $\propto m(\nu_l)/E$, E being the neutrino energy, and thus is strongly suppressed. Together with the LH charged lepton field $l_L(x)$, $\nu_{lL}(x)$ forms an $SU(2)_L$ doublet in the Standard Model. In the absence of neutrino mixing and zero neutrino masses, $\nu_{lL}(x)$ and $l_L(x)$ can be assigned one unit of the additive lepton charge L_l and the three charges L_l, $l = e, \mu, \tau$, are conserved by the weak interaction.

At present there is no compelling evidence for the existence of states of relativistic neutrinos (antineutrinos), which are predominantly right handed, ν_R (left handed, $\bar{\nu}_L$). If RH neutrinos and LH antineutrinos exist, their interaction with matter should be much weaker than the weak interaction of the flavour LH neutrinos ν_l and RH antineutrinos $\bar{\nu}_l$; that is, ν_R ($\bar{\nu}_L$) should be "sterile" or "inert" neutrinos (antineutrinos) [1]. In the formalism of the Standard Model, the sterile ν_R and $\bar{\nu}_L$ can be described by $SU(2)_L$ singlet RH

neutrino fields $\nu_R(x)$. In this case, ν_R and $\bar{\nu}_L$ will have no gauge interactions, that is, will not couple to the weak $W^{\pm}$ and Z^0 bosons. If present in an extension of the Standard Model (even in the minimal one), the RH neutrinos can play a crucial role (i) in the generation of neutrino masses and mixing, (ii) in understanding the remarkable disparity between the magnitudes of neutrino masses and the masses of the charged leptons and quarks, and (iii) in the generation of the observed matter-antimatter asymmetry of the Universe (via the leptogenesis mechanism [2, 3]; see also, e.g., [4, 5]). In this scenario which is based on the see-saw theory [6–9], there is a link between the generation of neutrino masses and the generation of the baryon asymmetry of the Universe. The simplest hypothesis (based on symmetry considerations) is that to each LH flavour neutrino field $\nu_{lL}(x)$ there corresponds an RH neutrino field $\nu_{lR}(x)$, $l = e, \mu, \tau$, although schemes with less (more) than three RH neutrinos are also being considered (see, e.g., [10]).

The experiments with solar, atmospheric, reactor, and accelerator neutrinos (see [11] and the references quoted therein) have provided compelling evidences for flavour neutrino oscillations [1, 12–14]—transitions in flight between the different flavour neutrinos ν_e, ν_μ, ν_τ (antineutrinos $\bar{\nu}_e$, $\bar{\nu}_\mu$, $\bar{\nu}_\tau$), caused by nonzero neutrino masses and neutrino mixing. As a consequence of the results of these experiments, the existence of oscillations of the solar ν_e, atmospheric ν_μ and $\bar{\nu}_\mu$, accelerator ν_μ (at $L \sim 250\,\text{km}$, $L \sim 730\,\text{km}$, and $L \sim 295\,\text{km}$, with L being the distance traveled by the neutrinos), and reactor $\bar{\nu}_e$ (at $L \sim 180\,\text{km}$ and $L \sim 1\,\text{km}$), was firmly established. The data imply the presence of neutrino mixing in the weak charged lepton current:

$$\mathscr{L}_{\text{CC}} = -\frac{g}{\sqrt{2}} \sum_{l=e,\mu,\tau} \overline{l}_L(x)\, \gamma_\alpha \nu_{lL}(x)\, W^{\alpha\dagger}(x) + \text{h.c.},$$

$$\nu_{lL}(x) = \sum_{j=1}^{n} U_{lj} \nu_{jL}(x), \tag{1}$$

where $\nu_{lL}(x)$ are the flavour neutrino fields, $\nu_{jL}(x)$ is the left-handed (LH) component of the field of the neutrino ν_j having a mass m_j, and U is a unitary matrix—the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix [1, 12–14], $U \equiv U_{\text{PMNS}}$. All compelling neutrino oscillation data can be described assuming 3-neutrino mixing in vacuum, $n = 3$. The number of massive neutrinos n can, in general, be bigger than 3 if, for example, there exist right-handed (RH) sterile neutrinos [1] and they mix with the LH flavour neutrinos. It follows from the current data that at least 3 of the neutrinos ν_j, say ν_1, ν_2, ν_3, must be light, $m_{1,2,3} \lesssim 1\,\text{eV}$, and must have different masses, $m_1 \neq m_2 \neq m_3$. At present there is no compelling experimental evidence for the existence of more than 3 light neutrinos. Certain neutrino oscillation data exhibit anomalies that could be interpreted as being due to the existence of one or two additional (sterile) neutrinos with mass in the eV range, which have a relatively small mixing ~ 0.1 with the active flavour neutrinos (see, e.g., [15] and the references quoted therein).

In the case of 3 light neutrinos on which we will concentrate on in this review, the neutrino mixing matrix U can be parametrised by 3 angles and, depending on whether the massive neutrinos ν_j are Dirac or Majorana [16] particles, by 1 or 3 CP violation (CPV) phases [17–20]:

$$U = VP, \quad P = \text{diag}\left(1, e^{i(\alpha_{21}/2)}, e^{i(\alpha_{31}/2)}\right), \tag{2}$$

where α_{21} and α_{31} are the two Majorana CPV phases and V is a CKM-like matrix containing the Dirac CPV phase δ:

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}. \tag{3}$$

In (3), $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, the angles θ_{ij} and the Dirac phase δ lie in the intervals $0 \leq \theta_{ij} \leq \pi/2$ and $0 \leq \delta \leq 2\pi$, and, in general, $0 \leq \alpha_{j1}/2 \leq 2\pi$, $j = 2, 3$ [21, 22]. If CP invariance holds, we have $\delta = 0$, π, and [20, 23–25], $\alpha_{21(31)} = k^{(\prime)}\pi$, $k^{(\prime)} = 0, 1, 2, 3, 4$.

Thus, in the case of massive Dirac neutrinos, the neutrino mixing matrix U is similar, in what concerns the number of mixing angles and CPV phases, to the CKM quark mixing matrix. The presence of two additional physical CPV phases in U if ν_j are Majorana particles is a consequence of the special properties of the latter (see, e.g., [17, 26]). On the basis of the existing neutrino data it is impossible to determine whether the massive neutrinos are Dirac or Majorana fermions.

The neutrino oscillation probabilities depend, in general, on the neutrino energy E, the source-detector distance, L, on the elements of U, and, for relativistic neutrinos used in all neutrino experiments performed so far, on the neutrino mass squared differences $\Delta m_{ij}^2 \equiv (m_i^2 - m_j^2)$, $i \neq j$ (see, e.g., [11, 26]). In the case of 3-neutrino mixing there are only two independent neutrino mass squared differences, say $\Delta m_{21}^2 \neq 0$ and $\Delta m_{31}^2 \neq 0$. The numbering of massive neutrinos ν_j is arbitrary. We will employ here the widely used convention of numbering of ν_j which allows to associate θ_{13} with the smallest mixing angle in the PMNS matrix U, and θ_{12}, $\Delta m_{21}^2 > 0$, and θ_{23}, $\Delta m_{31(32)}^2$, with the parameters which drive, respectively, the solar (ν_e), and the dominant atmospheric ν_μ (and $\bar{\nu}_\mu$) (and accelerator ν_μ) oscillations. Under the assumption of CPT invariance, which we will suppose to hold throughout this article, θ_{12} and Δm_{21}^2 drive also the reactor $\bar{\nu}_e$ oscillations at $L \sim 180\,\text{km}$ (see, e.g., [11]). In this convention $m_1 < m_2$, $0 < \Delta m_{21}^2 < |\Delta m_{31}^2|$, and, depending on $\text{sgn}(\Delta m_{31}^2)$, we have either $m_3 < m_1$ or $m_3 > m_2$ (see further). In the case of $m_1 < m_2 < m_3$ ($m_3 < m_1 < m_2$), the neutrino mass squared difference Δm_{21}^2, as it follows from the data to be discussed below, is much smaller than $|\Delta m_{31(32)}^2|$, $\Delta m_{21}^2 \ll |\Delta m_{31(32)}^2|$. This implies that in each of the two cases $m_1 < m_2 < m_3$ and $m_3 < m_1 < m_2$ we have $|\Delta m_{31}^2 - \Delta m_{32}^2| = \Delta m_{21}^2 \ll |\Delta m_{31,32}^2|$. The angles θ_{12} and θ_{23} are sometimes called "solar" and "atmospheric" neutrino mixing angles and are often denoted as $\theta_{12} = \theta_\odot$ and $\theta_{23} = \theta_{\text{atm}}$, while Δm_{21}^2 and $\Delta m_{31(32)}^2$ are sometimes referred to as the "solar" and "atmospheric" neutrino mass squared differences and correspondingly are denoted as $\Delta m_{21}^2 \equiv \Delta m_\odot^2$, $\Delta m_{31(32)}^2 \equiv \Delta m_{\text{atm}}^2$.

TABLE 1: The best-fit values and 3σ allowed ranges of the 3-neutrino oscillation parameters, derived in [33] from a global fit of the current neutrino oscillation data. The values (values in brackets) correspond to $m_1 < m_2 < m_3$ ($m_3 < m_1 < m_2$). The definition of Δm_A^2 used is: $\Delta m_A^2 = m_3^2 - (m_2^2 + m_1^2)/2$.

Parameter	Best fit ($\pm 1\sigma$)	3σ
$\Delta m_\odot^2$ [10^{-5} eV^2]	$7.54^{+0.26}_{-0.22}$	6.99–8.18
$\lvert\Delta m_A^2\rvert$ [10^{-3} eV^2]	$2.43^{+0.06}_{-0.10}$ ($2.42^{+0.07}_{-0.11}$)	2.19 (2.17)–2.62 (2.61)
$\sin^2\theta_{12}$	$0.307^{+0.018}_{-0.016}$	0.259–0.359
$\sin^2\theta_{23}$	$0.386^{+0.024}_{-0.021}$ ($0.392^{+0.039}_{-0.022}$)	0.331 (0.335)–0.637(0.663)
$\sin^2\theta_{13}$	0.0241 ± 0.0025 ($0.0244^{+0.0023}_{-0.0025}$)	0.0169 (0.0171)–0.0313 (0.0315)

Before continuing we would like to note that the preceding discussion is to a large extent based on parts of the text of the review article [11].

The neutrino oscillation data, accumulated over many years, allowed to determine the parameters which drive the solar, reactor, atmospheric, and accelerator neutrino oscillations, Δm_{21}^2, θ_{12}, $\lvert\Delta m_{31(32)}^2\rvert$ and θ_{23}, with a rather high precision. Furthermore, there were spectacular developments in the period since June 2011 in what concerns the angle θ_{13} (see, e.g., [11]). They culminated in March of 2012 in a high precision determination of $\sin^2 2\theta_{13}$ in the Daya Bay experiment with reactor $\bar{\nu}_e$ [27, 28]:

$$\sin^2 2\theta_{13} = 0.089 \pm 0.010 \pm 0.005, \qquad (4)$$

where we have quoted the latest result of the Daya Bay experiment published in [28]. Subsequently the RENO [29], Double Chooz [30], and T2K [31] (see also [32]) experiments reported, respectively, 4.9σ, 2.9σ, and 3.2σ evidences for a nonzero value of θ_{13}, compatible with the Day Bay result.

A global analysis of the latest neutrino oscillation data presented at the Neutrino 2012 International Conference, held in June of 2012 in Kyoto, Japan, was performed in [33]. We give below the best fit values of Δm_{21}^2, $\sin^2\theta_{12}$, $\lvert\Delta m_{31(32)}^2\rvert$, $\sin^2\theta_{23}$, and $\sin^2\theta_{13}$, obtained in [33]:

$$\Delta m_{21}^2 = 7.54 \times 10^{-5} \text{ eV}^2,$$

$$\left\lvert\Delta m_{31(32)}^2\right\rvert = 2.47\,(2.46) \times 10^{-3} \text{ eV}^2, \qquad (5)$$

$$\sin^2\theta_{12} = 0.307, \qquad \sin^2\theta_{23} = 0.39,$$

$$\sin^2\theta_{13} = 0.0241\,(0.0244), \qquad (6)$$

where the values (the values in brackets) correspond to $m_1 < m_2 < m_3$ ($m_3 < m_1 < m_2$). The 1σ uncertainties and the 3σ ranges of the neutrino oscillation parameters found in [33] are given in Table 1 (note that we have quoted the value of $\lvert\Delta m_{31(32)}^2\rvert$ in (5), while the mass squared difference determined in [33] is $\lvert\Delta m_A^2\rvert = \lvert\Delta m_{31}^2 - \Delta m_{21}^2/2\rvert$ ($\lvert\Delta m_A^2\rvert = \lvert\Delta m_{32}^2 + \Delta m_{21}^2/2\rvert$)).

A few comments are in order. We have $\Delta m_{21}^2/\lvert\Delta m_{31(32)}^2\rvert \cong 0.031 \ll 1$, as was indicated earlier. The existing data do not allow to determine the sign of $\Delta m_{31(32)}^2$. As we will discuss further, the two possible signs correspond to two different basic types of neutrino mass spectrum. Maximal solar neutrino mixing, that is, $\theta_{12} = \pi/4$, is ruled out at more than 6σ by the data. Correspondingly, one has $\cos 2\theta_{12} \geq 0.28$ at 3σ. The results quoted in (6) imply that θ_{23} is close to (but can be different from) $\pi/4$, $\theta_{12} \cong \pi/5.4$ and that $\theta_{13} \cong \pi/20$. Thus, the pattern of neutrino mixing is drastically different from the pattern of quark mixing. As we have noticed earlier, the neutrino oscillations experiments are sensitive only to neutrino mass squared differences $\Delta m_{ij}^2 \equiv (m_i^2 - m_j^2)$, $i \neq j$, and cannot give information on the absolute values of the neutrino masses, that is, on the absolute neutrino mass scale. They are insensitive also to the nature-Dirac or Majorana, of massive neutrinos ν_j and, correspondingly, to the Majorana CPV phases present in the PMNS matrix U [17, 34].

After the successful measurement of θ_{13}, the determination of the absolute neutrino mass scale, of the type of the neutrino mass spectrum, of the nature-Dirac or Majorana, of massive neutrinos, and getting information about the status of CP violation in the lepton sector, are the most pressing and challenging problems and the highest priority goals of the research in the field of neutrino physics.

As was already indicated above, the presently available data do not permit to determine the sign of $\Delta m_{31(2)}^2$. In the case of 3-neutrino mixing, the two possible signs of $\Delta m_{31(32)}^2$ correspond to two types of neutrino mass spectrum. In the widely used convention of numbering the neutrinos with definite mass employed by us, the two spectra read:

(i) *spectrum with normal ordering (NO)*: $m_1 < m_2 < m_3$, $\Delta m_{atm}^2 = \Delta m_{31}^2 > 0$, $\Delta m_\odot^2 \equiv \Delta m_{21}^2 > 0$, $m_{2(3)} = (m_1^2 + \Delta m_{21(31)}^2)^{1/2}$;

(ii) *spectrum with inverted ordering (IO)*: $m_3 < m_1 < m_2$, $\Delta m_{atm}^2 = \Delta m_{32}^2 < 0$, $\Delta m_\odot^2 \equiv \Delta m_{21}^2 > 0$, $m_2 = (m_3^2 + \Delta m_{23}^2)^{1/2}$, $m_1 = (m_3^2 + \Delta m_{23}^2 - \Delta m_{21}^2)^{1/2}$.

Depending on the value of the lightest neutrino mass, $\min(m_j)$, the neutrino mass spectrum can be

(a) *normal hierarchical (NH)*: $m_1 \ll m_2 < m_3$, $m_2 \cong (\Delta m_\odot^2)^{1/2} \cong 8.68 \times 10^{-3}$ eV, $m_3 \cong \lvert\Delta m_{atm}^2\rvert^{1/2} \cong 4.97 \times 10^{-2}$ eV; or

(b) *inverted hierarchical (IH)*: $m_3 \ll m_1 < m_2$, with $m_{1,2} \cong \lvert\Delta m_{atm}^2\rvert^{1/2} \cong 4.97 \times 10^{-2}$ eV; or

(c) *quasidegenerate (QD)*: $m_1 \cong m_2 \cong m_3 \cong m_0, m_j^2 \gg |\Delta m_{\text{atm}}^2|, m_0 \gtrsim 0.10 \,\text{eV}$.

The type of neutrino mass spectrum (hierarchy), that is, the sign of $\Delta m_{31(32)}^2$, can be determined (i) using data from neutrino oscillation experiments at accelerators (NOνA, T2K, etc.) (see, e.g., [35]), (ii) in the experiments studying the oscillations of atmospheric neutrinos (see, e.g., [36–39]), and (iii) in experiments with reactor antineutrinos [40–47]. The relatively large value of θ_{13} is a favorable factor for the sgn($\Delta m_{31(32)}^2$) determination in these experiments. If neutrinos with definite mass are Majorana particles, information about the sgn($\Delta m_{31(32)}^2$) can be obtained also by measuring the effective neutrino Majorana mass in neutrinoless double β-decay experiments [48, 49].

More specifically, in the cases (i) and (ii), the sgn($\Delta m_{31(32)}^2$) can be determined by studying oscillations of neutrinos and antineutrinos, say, $\nu_\mu \leftrightarrow \nu_e$ and $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_e$, in which matter effects are sufficiently large. This can be done in long base-line oscillation experiments (see, e.g., [35]). For $\sin^2 2\theta_{13} \gtrsim 0.05$ and $\sin^2 \theta_{23} \gtrsim 0.50$, information on sgn($\Delta m_{31(32)}^2$) might be obtained in atmospheric neutrino experiments by investigating the effects of the subdominant transitions $\nu_{\mu(e)} \to \nu_{e(\mu)}$ and $\bar{\nu}_{\mu(e)} \to \bar{\nu}_{e(\mu)}$ of atmospheric neutrinos which traverse the Earth (for a detailed discussion see, e.g., [36–39]). For $\nu_{\mu(e)}$ (or $\bar{\nu}_{\mu(e)}$) crossing the Earth core, new type of resonance-like enhancement of the indicated transitions takes place due to the (*Earth*) *mantle-core constructive interference effect* (*neutrino oscillation length resonance* (*NOLR*)) [50] (see also [51]). As a consequence of this effect, the corresponding $\nu_{\mu(e)}$ (or $\bar{\nu}_{\mu(e)}$) transition probabilities can be maximal [52–54] (for the precise conditions of the mantle-core (NOLR) enhancement see [50, 52–54]). It should be noted that the Earth mantle-core (NOLR) enhancement of neutrino transitions differs [50] from the MSW one. It also differs [50, 52–54] from the mechanisms of enhancement discussed, for example, in the articles [55, 56]: the conditions of enhancement considered in [55, 56] cannot be realised for the $\nu_{\mu(e)} \to \nu_{e(\mu)}$ or $\bar{\nu}_{\mu(e)} \to \bar{\nu}_{e(\mu)}$ transitions of the Earth core crossing neutrinos. For $\Delta m_{31(32)}^2 > 0$, the neutrino transitions $\nu_{\mu(e)} \to \nu_{e(\mu)}$ are enhanced, while for $\Delta m_{31(32)}^2 < 0$ the enhancement of antineutrino transitions $\bar{\nu}_{\mu(e)} \to \bar{\nu}_{e(\mu)}$ takes place [50] (see also [51–54, 57]), which might allow to determine sgn($\Delta m_{31(32)}^2$). Determining the type of neutrino mass spectrum is crucial for understanding the origin of neutrino masses and mixing as well.

All possible types of neutrino mass spectrum we have discussed above are compatible with the existing constraints on the absolute scale of neutrino masses m_j. Information about the absolute neutrino mass scale can be obtained by measuring the spectrum of electrons near the end point in ^{3}H β-decay experiments [58–60] and from cosmological and astrophysical data (see, e.g., [61]). The most stringent upper bound on the $\bar{\nu}_e$ mass was obtained in the Troitzk [62] experiment (see also [63]):

$$m_{\bar{\nu}_e} < 2.05 \,\text{eV} \quad \text{at 95\% C.L.} \tag{7}$$

We have $m_{\bar{\nu}_e} \cong m_{1,2,3} \gtrsim 0.1 \,\text{eV}$ in the case of quasidegenerate (QD) spectrum. The KATRIN experiment [63], which is under preparation, is planned to reach sensitivity of $m_{\bar{\nu}_e} \sim 0.20 \,\text{eV}$; that is, it will probe the region of the QD spectrum. Information on the type of neutrino mass spectrum can also be obtained in β-decay experiments having a sensitivity to neutrino masses $\sim \sqrt{|\Delta m_{31(32)}^2|} \cong 5 \times 10^{-2} \,\text{eV}$ [64] (i.e., by a factor of $\sim$4 better sensitivity than that of the KATRIN experiment [63]). Reaching the indicated sensitivity in electromagnetic spectrometer β-decay experiments of the type of KATRIN does not seem feasible at present. The cosmic microwave background (CMB) data of the WMAP experiment, combined with supernovae data and data on galaxy clustering can be used to derive an upper limit on the sum of neutrinos masses (see, e.g., [61]). Depending on the model complexity and the input data used one obtains [65] $\sum_j m_j \lesssim (0.3$–$1.3)\,\text{eV}$, 95% C.L. Data on weak lensing of galaxies, combined with data from the WMAP and PLANCK experiments, may allow $\sum_j m_j$ to be determined with an uncertainty of $\sigma(\sum_j m_j) = (0.04$–$0.07)\,\text{eV}$ [66, 67].

Thus, the data on the absolute scale of neutrino masses imply that neutrino masses are much smaller than the masses of the charged leptons and quarks. If we take as an indicative upper limit $m_j \lesssim 0.5 \,\text{eV}$, $j = 1, 2, 3$, we have

$$\frac{m_j}{m_{l,q}} \lesssim 10^{-6}, \quad l = e, \mu, \tau, \; q = d, sb, u, c, t. \tag{8}$$

It is natural to suppose that the remarkable smallness of neutrino masses is related to the existence of a new fundamental mass scale in particle physics, and thus to new physics beyond that predicted by the Standard Model. A comprehensive theory of the neutrino masses and mixing should be able to explain the indicated enormous disparity between the neutrino masses and the masses of the charged leptons and quarks.

At present no experimental information on the Dirac and Majorana CPV phases in the neutrino mixing matrix is available. Therefore the status of the CP symmetry in the lepton sector is unknown. The importance of getting information about the Dirac and Majorana CPV phases in the neutrino mixing matrix stems, in particular, from the possibility that these phases play a fundamental role in the generation of the observed baryon asymmetry of the Universe. More specifically, the CP violation necessary for the generation of the baryon asymmetry within the "flavoured" leptogenesis scenario [68–70] can be due exclusively to the Dirac and/or Majorana CPV phases in the PMNS matrix [71, 72] and thus can be directly related to the low energy CP-violation in the lepton sector. If the requisite CP violation is due to the Dirac phases δ, a necessary condition for a successful (flavoured) leptogenesis is that $\sin \theta_{13} \gtrsim 0.09$ [72], which is comfortably compatible with the Daya Bay result, (4).

With $\theta_{13} \neq 0$, the Dirac phase δ can generate CP violating effects in neutrino oscillations [73] (see also [17, 74]), that is, a difference between the probabilities of $\nu_l \to \nu_{l'}$ and $\bar{\nu}_l \to \bar{\nu}_{l'}$ oscillations in vacuum: $P(\nu_l \to \nu_{l'}) \neq P(\bar{\nu}_l \to \bar{\nu}_{l'})$, $l \neq l' = e, \mu, \tau$. The magnitude of the CP violating effects of

interest is determined [75] by the rephasing invariant J_{CP} associated with the Dirac CPV phase δ in U. It is analogous to the rephasing invariant associated with the Dirac CPV phase in the CKM quark mixing matrix [76, 77]. In the "standard" parametrisation of the PMNS neutrino mixing matrix, (2)-(3), we have

$$
\begin{aligned}
J_{CP} &\equiv \mathrm{Im}\left(U_{\mu 3} U_{e3}^* U_{e2} U_{\mu 2}^*\right) \\
&= \frac{1}{8} \cos\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \sin\delta.
\end{aligned}
\tag{9}
$$

Thus, given the fact that $\sin\theta_{12}$, $\sin\theta_{23}$, and $\sin\theta_{13}$ have been determined experimentally with a relatively good precision, the size of CP violation effects in neutrino oscillations depends essentially only on the magnitude of the currently unknown value of the Dirac phase δ. The current data imply $|J_{CP}| \lesssim 0.039$, where we have used (9) and the 3σ ranges of $\sin^2\theta_{12}$, $\sin^2\theta_{23}$, and $\sin^2\theta_{13}$ given in Table 1. Data on the Dirac phase δ will be obtained in the long baseline neutrino oscillation experiments T2K, NOνA, and other (see, e.g., [78]). Testing the possibility of Dirac CP violation in the lepton sector is one of the major goals of the next generation of neutrino oscillation experiments (see, e.g., [35, 78]). Measuring the magnitude of CP violation effects in neutrino oscillations is at present also the only known feasible method of determining the value of the phase δ (see, e.g., [79]).

If ν_j are Majorana fermions, getting experimental information about the Majorana CPV phases in the neutrino mixing matrix U will be remarkably difficult [80–86]. As we will discuss further, the Majorana phases of the PMNS matrix play important role in the phenomenology of neutrinoless double-beta-$((\beta\beta)_{0\nu}$-$)$ decay—the process whose existence is related to the Majorana nature of massive neutrinos [87]: $(A, Z) \rightarrow (A, Z + 2) + e^- + e^-$. The phases $\alpha_{21,31}$ can affect significantly the predictions for the rates of the (LFV) decays $\mu \rightarrow e + \gamma$, $\tau \rightarrow \mu + \gamma$, and so forth, in a large class of supersymmetric theories incorporating the see-saw mechanism [88, 89]. As was mentioned earlier, the Majorana phase(s) in the PMNS matrix can be the leptogenesis CP violating parameter(s) at the origin of the baryon asymmetry of the Universe [21, 22, 71, 72].

Establishing whether the neutrinos with definite mass ν_j are Dirac fermions possessing distinct antiparticles, or Majorana fermions, that is, spin 1/2 particles that are identical with their antiparticles, is of fundamental importance for understanding the origin of neutrino masses and mixing and the underlying symmetries of particle interactions. Let us recall that the neutrinos ν_j with definite mass m_j will be Dirac fermions if particle interactions conserve some additive lepton number, for example, the total lepton charge $L = L_e + L_\mu + L_\tau$. If no lepton charge is conserved, the neutrinos ν_j will be Majorana fermions (see, e.g., [26]). The massive neutrinos are predicted to be of Majorana nature by the see-saw mechanism of neutrino mass generation [6–9], which also provides an attractive explanation of the smallness of neutrino masses and, through the leptogenesis theory [2, 3], of the observed baryon asymmetry of the Universe. The observed patterns of neutrino mixing and of neutrino mass squared differences driving the solar and the dominant atmospheric neutrino

oscillations can be related to Majorana massive neutrinos and the existence of an *approximate* symmetry in the lepton sector corresponding to the conservation of the *nonstandard* lepton charge $L' = L_e - L_\mu - L_\tau$ [90]. They can also be associated with the existence of *approximate* discrete symmetry (or symmetries) of the particle interactions (see, e.g., [91–94]). Determining the nature (Dirac or Majorana) of massive neutrinos is one of the fundamental and most challenging problems in the future studies of neutrino mixing [11].

2. The Nature of Massive Neutrinos

2.1. Majorana versus Dirac Massive Neutrinos (Particles). The properties of Majorana particles (fields) are very different from those of Dirac particles (fields). A massive Majorana neutrino χ_j (or Majorana spin 1/2 particle) with mass $m_j > 0$ can be described in local quantum field theory which is used to construct, for example, the Standard Model, by 4-component complex spin 1/2 field $\chi_j(x)$ which satisfies the Dirac equation and the Majorana condition:

$$
C\left(\overline{\chi}_j(x)\right)^T = \xi_k \chi_j(x), \qquad \left|\xi_j\right|^2 = 1, \tag{10}
$$

where C is the charge conjugation matrix, $C^{-1}\gamma_\alpha C = -(\gamma_\alpha)^T$ ($C^T = -C$, $C^{-1} = C^\dagger$), and ξ_j is, in general, an unphysical phase. The Majorana condition is invariant under *proper* Lorentz transformations. It reduces by a factor of 2 the number of independent components in $\chi_j(x)$.

The condition (10) is invariant with respect to $U(1)$ global gauge transformations of the field $\chi_j(x)$ carrying a $U(1)$ charge Q, $\chi_j(x) \rightarrow e^{i\alpha Q}\chi_j(x)$, only if $Q = 0$. As a result, (i) χ_j cannot carry nonzero additive quantum numbers (lepton charge, etc.), and (ii) the field $\chi_j(x)$ cannot "absorb" phases. Thus, $\chi_j(x)$ describes 2 spin states of a spin 1/2, *absolutely neutral particle*, which is identical with its antiparticle, $\chi_j \equiv \overline{\chi}_j$. As is well known, spin 1/2 Dirac particles can carry nonzero $U(1)$ charges: the charged leptons and quarks, for instance, carry nonzero electric charges.

Owing to the fact that the Majorana (neutrino) fields cannot absorb phases, the neutrino mixing matrix U contains in the general case of n charged leptons and mixing of n massive Majorana neutrinos $\nu_j \equiv \chi_j$, altogether

$$
n_{CPV}^{(M)} = \frac{n(n-1)}{2}, \qquad \text{Majorana } \nu_j, \tag{11}
$$

CPV phases [17]. In the case of mixing of n massive Dirac neutrinos, the number of CPV phases in U, as is well known, is

$$
n_{CPV}^{(D)} = \frac{(n-1)(n-2)}{2}, \qquad \text{Dirac } \nu_j. \tag{12}
$$

Thus, if ν_j are Majorana particles, U contains the following number of additional Majorana CP violation phases: $n_{MCPV}^{(M)} \equiv n_{CPV}^{(M)} - n_{CPV}^{(D)} = (n-1)$. In the case of n charged leptons and n massive Majorana neutrinos, the PMNS matrix U can be cast in the form [17]

$$
U = VP, \tag{13}
$$

where the matrix V contains the $(n-1)(n-2)/2$ Dirac CP violation phases, while P is a diagonal matrix with the additional $(n-1)$ Majorana CP violation phases $\alpha_{21}, \alpha_{31}, \ldots, \alpha_{n1}$,

$$P = \text{diag}\left(1, e^{i(\alpha_{21}/2)}, e^{i(\alpha_{31}/2)}, \ldots, e^{i(\alpha_{n1}/2)}\right). \quad (14)$$

As will be discussed further, the Majorana phases will conserve CP if [23–25] $\alpha_{j1} = \pi q_j$, $q_j = 0, 1, 2$, $j = 2, 3, \ldots, n$. In this case $\exp(i\alpha_{j1}) = \pm 1$ and $\exp[i(\alpha_{j1} - \alpha_{k1})] = \pm 1$ have a simple physical interpretation: these are the relative CP-parities of the Majorana neutrinos ν_j and ν_1 and of ν_j and ν_k, respectively.

It follows from the preceding discussion that the mixing of massive Majorana neutrinos differs, in what concerns the number of CPV phases, from the mixing of massive Dirac neutrinos. For $n = 3$ of interest, we have one Dirac and two Majorana CPV phases in U, which is consistent with the expression of U given in (2). If $n = 2$, there is one Majorana CPV phase and no Dirac CPV phases in U. Correspondingly, in contrast to the Dirac case, there can exist CP violating effects even in the system of two mixed massive Majorana neutrinos (particles).

The Majorana phases do not enter into the expressions of the probabilities of oscillations involving the flavour neutrinos and antineutrinos [17, 34], $\nu_l \rightarrow \nu_{l'}$ and $\bar\nu_l \rightarrow \bar\nu_{l'}$. Indeed, the probability to find neutrino $\nu_{l'}$ (antineutrino $\bar\nu_{l'}$) at time t if a neutrino ν_l (antineutrino $\bar\nu_l$) has been produced at time t_0 and it had traveled a distance $L \cong t$ in vacuum is given by (see, e.g., [11, 26])

$$P\left(\nu_l \longrightarrow \nu_{l'}\right) = \left|\sum_j U_{l'j} e^{-i(E_j t - p_j L)} U^\dagger_{jl}\right|^2,$$

$$P\left(\bar\nu_l \longrightarrow \bar\nu_{l'}\right) = \left|\sum_j U_{lj} e^{-i(E_j t - p_j L)} U^\dagger_{jl'}\right|^2, \quad (15)$$

where E_j and p_j are the energy and momentum of the neutrino ν_j. It is easy to show, using the expression for U in (13), that $P(\nu_l \rightarrow \nu_{l'})$ and $P(\bar\nu_l \rightarrow \bar\nu_{l'})$ do not depend on the Majorana phases present in U since

$$\sum_j (VP)_{l'j} e^{-i(E_j t - p_j L)} (VP)^\dagger_{jl} = \sum_j V_{l'j} e^{-i(E_j t - p_j L)} V^\dagger_{jl}. \quad (16)$$

The same result holds when the neutrino oscillations take place in matter [34].

If CP-invariance holds, Majorana neutrinos (particles) have definite CP-parity $\eta_{\text{CP}}(\chi_j) = \pm i$:

$$U_{\text{CP}}\chi_j(x) U^{-1}_{\text{CP}} = \eta_{\text{CP}}\left(\chi_j\right)\gamma_0\chi_j\left(x_p\right),$$

$$\eta_{\text{CP}}\left(\chi_j\right) \equiv i\rho_j = \pm i, \quad (17)$$

where $x = (x_0, \mathbf{x})$, $x_p = (x_0, -\mathbf{x})$ and U_{CP} is the unitary CP-transformation operator. In contrast, Dirac particles do not have a definite CP-parity—a Dirac field $f(x)$ transforms as follows under the CP-symmetry operation:

$$U_{\text{CP}} f(x) U^{-1}_{\text{CP}} = \eta_f \gamma_0 C\left(\overline{f}\left(x_p\right)\right)^T, \quad |\eta_f|^2 = 1, \quad (18)$$

η_f being an unphysical phase factor. In the case of CP invariance, the CP-parities of massive Majorana fermions (neutrinos) can play important role in processes involving real of virtual Majorana particles (see, e.g., [26, 95]).

Using (18) and (17) and the transformation of the $W^\pm$ boson field under the CP-symmetry operation,

$$U_{\text{CP}} W_\alpha(x) U^\dagger_{\text{CP}} = \eta_W \kappa_\alpha\left(W_\alpha\left(x_p\right)\right)^\dagger, \quad |\eta_W|^2 = 1,$$

$$\kappa_0 = -1, \ \kappa_{1,2,3} = +1, \quad (19)$$

where η_W is an unphysical phase, one can derive the constraints on the neutrino mixing matrix U following from the requirement of CP-invariance of the leptonic CC weak interaction Lagrangian, (1). In the case of massive Dirac neutrinos we obtain $\eta^*_{\nu_j}\eta_l\eta_W U^*_{lj} = U_{lj}$, $l = e, \mu, \tau$, $j = 1, 2, 3$. Setting the product of unphysical phases $\eta^*_{\nu_j}\eta_l\eta_W = 1$, one obtains the well-known result:

$$\text{CP invariance:} \ U^*_{lj} = U_{lj}, \quad l = e, \mu, \tau, \ j = 1, 2, 3,$$

$$\text{Dirac } \nu_j. \quad (20)$$

In the case of massive Majorana neutrinos we obtain using (10), (17), (18), and (19): $\xi^*_j(i\rho_j)\eta^*_l\eta_W U_{lj} = U^*_{lj}$. It is convenient now to set $\xi_j = 1$, $\eta_l = i$, and $\eta_W = 1$. In this (commonly used by us) convention we get [26]

$$\text{CP invariance:} \ U^*_{lj} = \rho_j U_{lj},$$

$$\rho_j = +1 \text{ or } (-1), \ l = e, \mu, \tau, \ j = 1, 2, 3, \quad (21)$$

$$\text{Majorana } \nu_j.$$

Thus, in the convention used the elements of the PMNS matrix can be either real or purely imaginary if ν_j are Majorana fermions. Applying the above conditions to, for example, U_{e2}, $U_{\tau3}$, and U_{e3} elements of the PMNS matrix (2) we obtain the CP conserving values of the phases α_{21}, α_{31}, and δ, respectively: $\alpha_{21} = k\pi$, $k = 0, 1, 2, \ldots$, $\alpha_{31} = k'\pi$, $k' = 0, 1, 2, \ldots$, $\delta = 0, \pi, 2\pi$.

One can obtain in a similar way the CP-invariance constraint on the matrix of neutrino Yukawa couplings, λ_{kl}, which plays a fundamental role in the leptogenesis scenario of baryon asymmetry generation, based on the (type I) see-saw mechanism of generation of neutrino masses [2–5, 79]:

$$\mathscr{L}_Y(x) = -\lambda_{kl}\overline{N_{kR}}(x) H^\dagger(x)\psi_{lL}(x) + \text{h.c.},$$

$$\mathscr{L}^N_M(x) = -\frac{1}{2} M_k \overline{N_k}(x) N_k(x). \quad (22)$$

Here $N_{kR}(x)$ is the field of a heavy right-handed (RH) sterile Majorana neutrino with mass $M_k > 0$, ψ_{lL} denotes the Standard Model left-handed (LH) lepton doublet field of flavour $l = e, \mu, \tau$, $\psi^T_{lL} = (\nu^T_{lL} \ l^T_L)$, and H is the Standard Model Higgs doublet field whose neutral component has a vacuum expectation value $v = 174\,\text{GeV}$. The term $\mathscr{L}_Y(x) + \mathscr{L}^N_M(x)$ includes all the necessary ingredients of the see-saw mechanism. Assuming the existence of two heavy Majorana

neutrinos, that is, taking $k = 1, 2$ in (22), and adding the term $\mathscr{L}_Y(x) + \mathscr{L}_M^N(x)$ to the Standard Model Lagrangian, we obtain the minimal extension of the Standard Model in which the neutrinos have masses and mix and the leptogenesis can be realised. In the leptogenesis formalism it is often convenient to use the so-called orthogonal parametrisation of the matrix of neutrino Yukawa couplings [96]:

$$\lambda_{kl} = \frac{1}{v} \sqrt{M_k} R_{kj} \sqrt{m_j} \left(U^\dagger \right)_{jl}, \tag{23}$$

where R is, in general, a complex orthogonal matrix, $RR^T = R^T R = 1$. The CP violation necessary for the generation of the baryon asymmetry of the Universe is provided in the leptogenesis scenario of interest by the matrix of neutrino Yukawa couplings λ (see, e.g., [4, 5, 79]). It follows from (23) that it can be provided either by the neutrino mixing matrix U, or by the matrix R, or else by both the matrices U and R. It is therefore important to derive the conditions under which λ, R, and U respect the CP symmetry. For the PMNS matrix U these conditions are given in (21). For the matrices λ and R in the convention in which (i) $N_k(x)$ satisfy the Majorana condition with a phase equal to 1 (i.e., $\xi_k = 1$), (ii) $\eta^l = i$ and $\eta^H = 1$, η^l and η^H being the unphysical phase factors which appear in the CP-transformations of the LH lepton doublet and Higgs doublet fields $\psi_{lL}(x)$ and $H(x)$, respectively (this convention is similar to, and consistent with, the convention about the unphysical phases we have used to derive the CP-invariance constraints on the elements of the PMNS matrix U), they read [72]

$$\lambda_{kl}^* = \lambda_{kl} \rho_k^N, \quad \rho_k^N = \pm 1, \ j = 1, 2, 3, \ l = e, \mu, \tau,$$
$$R_{kj}^* = R_{kj} \rho_k^N \rho_j, \quad j, k = 1, 2, 3, \tag{24}$$

where $i \rho_k^N \equiv \eta_{CP}(N_k) = \pm i$ is the CP-parity of N_k. Thus, in the case of CP invariance also the elements of λ and R can be real or purely imaginary. Note that, as it follows from (21) and (24), given which elements are real and which are purely imaginary of any two of the three matrices U, λ and R, determines (in the convention we are using and if CP invariance holds), which elements are real or purely imaginary in the third matrix. If, for instance, U_{e2} is purely imaginary ($\rho_2 = -1$) and $\lambda_{1\mu}$ is real ($\rho_1^N = 1$), then R_{12} must be purely imaginary. Thus, in the example we are considering, a real R_{12} would signal that the CP symmetry is broken [72].

The currents formed by Majorana fields have special properties, which make them also quite different from the currents formed by Dirac fields. In particular, it follows from the Majorana condition that the following currents of the Majorana field $\chi_k(x)$ are identically equal to zero (see, e.g., [26]):

$$\overline{\chi}_k(x) \gamma_\alpha \chi_k(x) \equiv 0, \tag{25}$$

$$\overline{\chi}_k(x) \sigma_{\alpha\beta} \chi_k(x) \equiv 0,$$
$$\overline{\chi}_k(x) \sigma_{\alpha\beta} \gamma_5 \chi_k(x) \equiv 0. \tag{26}$$

Equations (25) and (26) imply that Majorana fermions (neutrinos) cannot have nonzero $U(1)$ charges and intrinsic

magnetic and electric dipole moments, respectively. A Dirac spin 1/2 particle can have nontrivial $U(1)$ charges, as we have already discussed, and nonzero intrinsic magnetic moment (the electron and the muon, e.g., have it). If CP invariance does not hold, Dirac fermions can have also nonzero electric dipole moments. Equations (26) imply also that the Majorana particles (neutrinos) cannot couple to a real photon. The axial current of a Majorana fermion, $\overline{\chi}_k(x) \gamma_\alpha \gamma_5 \chi_k(x) \neq 0$. Correspondingly, $\chi_k(x)$ can have an effective coupling to a *virtual* photon via the *anapole momentum* term, which has the following form in momentum space:

$$\left(g_{\alpha\beta} q^2 - q_\alpha q_\beta \right) \gamma_\beta \gamma_5 F_a^{(k)} \left(q^2 \right), \tag{27}$$

where q is the momentum of the virtual photon and $F_a^{(k)}(q^2)$ is the anapole form factor of χ_k. The fact that the vector current of χ_k is zero while the axial current is nonzero has important implications in the calculations of the relic density of the lightest and stable neutralino, which is a Majorana particle and the dark matter candidate in many supersymmetric (SUSY) extensions of the Standard Model [97].

In certain cases (e.g., in theories with a keV mass Majorana neutrino (see, e.g., [15]), in the TeV scale type I see-saw model (see, e.g., [98]), in SUSY extensions of the Standard Model) one can have effective interactions involving two different massive Majorana fermions (neutrinos), say χ_1 and χ_2. We will consider two examples. The first is an effective interaction with the photon field, which can be written as

$$\mathscr{L}_{\text{eff}}^{(A)}(x) = \overline{\chi}_1(x) \sigma_{\alpha\beta} \left(\mu_{12} - d_{12} \gamma_5 \right) \chi_2(x) F^{\alpha\beta}(x) + \text{h.c.,} \tag{28}$$

where μ_{12} and d_{12} are, in general, complex constants, $F^{\alpha\beta}(x) = \partial^\alpha A^\beta(x) - \partial^\beta A^\alpha(x)$, $A^\mu(x)$ being the 4-vector potential of the photon field. Using the Majorana conditions for $\chi_1(x)$ and $\chi_2(x)$ in the convention in which the phases $\xi_1 = \xi_2 = 1$, it is not difficult to show that the constants μ_{12} and d_{12} enter into the expression for $\mathscr{L}_{\text{eff}}^{(A)}(x)$ in the form: $(\mu_{12} - \mu_{12}^*) = 2i \, \text{Im}(\mu_{12}) \equiv \widetilde{\mu}_{12}$, $(d_{12} + d_{12}^*) = 2 \, \text{Re}(d_{12}) \equiv \widetilde{d}_{12}$, that is, $\widetilde{\mu}_{12}$ is purely imaginary and $\widetilde{d}_{12}$ is real. In the case of $\chi_1(x) \equiv \chi_2(x) = \chi(x)$, the current $\overline{\chi}(x) \sigma_{\alpha\beta}(\mu_{12} - d_{12}\gamma_5)\chi(x)$ has to be Hermitian, which implies that $\widetilde{\mu}_{12}$ should be real while $\widetilde{d}_{12}$ should be purely imaginary. Combined with constraints on $\widetilde{\mu}_{12}$ and $\widetilde{d}_{12}$ we have just obtained, this leads to $\widetilde{\mu}_{12} = \widetilde{d}_{12} = 0$, which is consistent with (26). In the case of CP invariance of $\mathscr{L}_{\text{eff}}^{(A)}(x)$, the constants μ_{12} ($\widetilde{\mu}_{12}$) and d_{12} ($\widetilde{d}_{12}$) should satisfy

$$\text{CP invariance: } \mu_{12} = -\rho_1 \rho_2 \mu_{12}, \qquad d_{12} = +\rho_1 \rho_2 d_{12}. \tag{29}$$

Thus, if $\rho_1 = \rho_2$, that is, if $\chi_1(x)$ and $\chi_2(x)$ possess the same CP-parity, $\mu_{12} = 0$ and d_{12} (and $\widetilde{d}_{12}$) can be different from zero. If $\rho_1 = -\rho_2$, that is, if $\chi_1(x)$ and $\chi_2(x)$ have opposite CP-parities, $d_{12} = 0$ and μ_{12} (and $\widetilde{\mu}_{12}$) can be different from zero. If CP invariance does not hold, we can have both $\mu_{12} \neq 0$ and $d_{12} \neq 0$ ($\widetilde{\mu}_{12} \neq 0$ and $\widetilde{d}_{12} \neq 0$).

As a second example we will consider effective interaction of χ_1 and χ_2 with a vector field (current), which for concreteness will be assumed to be the Z^0-boson field of the Standard Model:

$$\mathscr{L}_{\text{eff}}^{(Z)}(x) = \overline{\chi}_1(x)\gamma_\alpha(v_{12} - a_{12}\gamma_5)\chi_2(x) Z^\alpha(x) + \text{h.c.} \quad (30)$$

Here v_{12} and a_{12} are, in general, complex constants. Using the Majorana conditions for $\chi_1(x)$ and $\chi_2(x)$ with $\xi_1 = \xi_2 = 1$, one can easily show that v_{12} has to be purely imaginary, while a_{12} has to be real. In the case of $\chi_1(x) \equiv \chi_2(x) = \chi(x)$, the hermiticity of the current $\overline{\chi}(x)\gamma_\alpha(v_{12} - a_{12}\gamma_5)\chi(x)$ implies that both v_{12} and a_{12} have to be real. This, together with constraints on v_{12} and a_{12} just derived, leads to $v_{12} = 0$, which is consistent with the result given in (25). The requirement of CP invariance of $\mathscr{L}_{\text{eff}}^{(Z)}(x)$, as can be shown, leads to ($\xi_1 = \xi_2 = 1$):

CP invariance: $v_{12} = -\rho_1\rho_2 v_{12}, \qquad a_{12} = +\rho_1\rho_2 a_{12}.$ (31)

Thus, we find, similarly to the case considered above, that if $\chi_1(x)$ and $\chi_2(x)$ posses the same CP-parity ($\rho_1 = \rho_2$), $v_{12} = 0$ and a_{12} can be different from zero; if $\chi_1(x)$ and $\chi_2(x)$ have opposite CP-parities ($\rho_1 = -\rho_2$), $a_{12} = 0$ while v_{12} can be different from zero. If CP invariance does not hold, we can have both $v_{12} \neq 0$ and $a_{12} \neq 0$.

These results have important implications, in particular, for the phenomenology of the heavy Majorana neutrinos N_k in the TeV scale (type I) see-saw models, for the neutralino phenomenology in SUSY extensions of the Standard Model, in which the neutralinos are Majorana particles, and more specifically for the processes $e^- + e^+ \rightarrow \chi_1 + \chi_2$, $\chi_2 \rightarrow \chi_1 + l^+ + l^-$ ($m(\chi_2) > m(\chi_1)$), $l = e, \mu, \tau$, where χ_1 and χ_2 are, for instance, two neutralinos of, for example, the minimal SUSY extension of the Standard Model (see, e.g., [95, 99]).

Finally, if $\Psi(x)$ is a Dirac field and we define the standard propagator of $\Psi(x)$ as

$$\langle 0 | T(\Psi_\alpha(x)\overline{\Psi}_\beta(y)) | 0 \rangle = S_{\alpha\beta}^F(x - y), \quad (32)$$

one has

$$\begin{aligned} \langle 0 | T(\Psi_\alpha(x)\Psi_\beta(y)) | 0 \rangle = 0, \\ \langle 0 | T(\overline{\Psi}_\alpha(x)\overline{\Psi}_\beta(y)) | 0 \rangle = 0. \end{aligned} \quad (33)$$

In contrast, a Majorana neutrino field $\chi_k(x)$ has, in addition to the standard propagator

$$\langle 0 | T(\chi_{k\alpha}(x)\overline{\chi}_{k\beta}(y)) | 0 \rangle = S_{\alpha\beta}^{Fk}(x - y), \quad (34)$$

two nontrivial *nonstandard* (*Majorana*) propagators

$$\begin{aligned} \langle 0 | T(\chi_{k\alpha}(x)\chi_{k\beta}(y)) | 0 \rangle = -\xi_k^* S_{\alpha\delta}^{Fk}(x - y) C_{\delta\beta}, \\ \langle 0 | T(\overline{\chi}_{k\alpha}(x)\overline{\chi}_{k\beta}(y)) | 0 \rangle = \xi_k C_{\alpha\delta}^{-1} S_{\delta\beta}^{Fk}(x - y). \end{aligned} \quad (35)$$

This result implies that if $v_j(x)$ in (1) are massive Majorana neutrinos, $(\beta\beta)_{0\nu}$-decay can proceed by exchange of virtual neutrinos v_j since $\langle 0|T(v_{j\alpha}(x)v_{j\beta}(y))|0\rangle \neq 0$. The Majorana propagators play a crucial role in the calculation of the baryon asymmetry of the Universe in the leptogenesis scenario of the asymmetry generation (see, e.g., [4, 5, 79]).

2.2. Generating Dirac and Majorana Massive Neutrinos.

The type of massive neutrinos in a given theory is determined by the type of the (effective) mass term $\mathscr{L}_m^\nu(x)$ neutrinos have, more precisely, by the symmetries of $\mathscr{L}_m^\nu(x)$ and of the total Lagrangian $\mathscr{L}(x)$ of the theory. A fermion mass term is bilinear in the fermion fields which is invariant under the proper Lorentz transformations.

Massive Dirac neutrinos arise in theories in which the neutrino mass term conserves some additive quantum number that could be, for example, the (total) lepton charge $L = L_e + L_\mu + L_\tau$, which is conserved also by the total Lagrangian $\mathscr{L}(x)$ of the theory. A well-known example is the Dirac mass term, which can arise in the minimally extended Standard Model to include three RH neutrino fields v_{lR}, $l = e, \mu, \tau$, as $SU(2)_L$ singlets:

$$\mathscr{L}_D^\nu(x) = -\overline{v_{l'R}}(x) M_{Dl'l} v_{lL}(x) + \text{h.c.}, \quad (36)$$

where M_D is a 3×3, in general complex, matrix. The term $\mathscr{L}_D^\nu(x)$ can be generated after the spontaneous breaking of the Standard Model gauge symmetry by an $SU(2)_L \times U(1)_{Y_w}$ invariant Yukawa coupling of the lepton doublet, Higgs doublet, and the RH neutrino fields [100]:

$$\mathscr{L}_Y(x) = -Y_{l'l}^\nu \overline{v_{l'R}}(x) H^\dagger(x) \psi_{lL}(x) + \text{h.c.}, \quad (37)$$

$$M_D = vY^\nu. \quad (38)$$

If the nondiagonal elements of M_D are different from zero, $M_{Dl'l} \neq 0$, $l \neq l' = e, \mu, \tau$, the individual lepton charges L_l, $l = e, \mu, \tau$, will not be conserved. Nevertheless, the total lepton charge L is conserved by $\mathscr{L}_D^\nu(x)$. As in the case of the charged lepton and quark mass matrices generated via the spontaneous electroweak symmetry breaking by Yukawa type terms in the SM Lagrangian, M_D is diagonalised by a biunitary transformation: $M_D = U_R^{\text{lep}} M_D^{\text{diag}}(U_L^{\text{lep}})^\dagger$, where U_R^{lep} and U_L^{lep} are 3×3 unitary matrices. If the mass term in (36) is written in the basis in which the charged lepton mass matrix is diagonal, U_L^{lep} coincides with the PMNS matrix, $U_L^{\text{lep}} \equiv U_{\text{PMNS}}$. The neutrinos v_j with definite mass $m_j > 0$ are Dirac particles: their fields $v_j(x) = (U_L^{\text{lep}})_{jl}^\dagger v_{lL}(x) + (U_R^{\text{lep}})_{jl'}^\dagger v_{l'R}(x)$ do not satisfy the Majorana condition, $C(\overline{v}_j(x))^T \neq \xi_j \chi_j(x)$. Although the scheme we are considering is phenomenologically viable (it does not contain a candidate for a dark matter particle though), it does not provide an insight of why the neutrino masses are much smaller than the charged fermion masses. The only observable "new physics" is that related to the neutrino masses and mixing: apart from the neutrino masses and mixing themselves, this is the phenomenon of neutrino oscillations [100].

Indeed, given the fact that the lepton charges L_l, $l = e, \mu, \tau$, are not conserved, processes like $\mu^+ \rightarrow e^+ + \gamma$ decay, $\mu^- \rightarrow e^- + e^+ + e^-$ decay, $\tau^- \rightarrow e^- + \gamma$ decay, and so forth are allowed. However, the rates of these processes are suppressed by the factor [100] $|U_{l'j}U_{lj}^* m_j^2/M_W^2|^2$, $l' \neq l$, $M_W \cong 80$ GeV being the $W^\pm$-mass and $l = \mu$, $l' = e$ for the $\mu^\pm \rightarrow e^\pm + \gamma$

decay, and so forth, and are unobservably small. For instance, for the $\mu \rightarrow e + \gamma$ decay branching ratio we have [100]

$$\text{BR}\,(\mu \longrightarrow e + \gamma) = \frac{3\alpha}{32\pi} \left| U_{ej} U_{\mu j}^* \, \frac{m_j^2}{M_W^2} \right|^2 \tag{39}$$

$$\cong (2.5 - 3.9) \times 10^{-55},$$

where we have used the best fit values of the neutrino oscillation parameters given in (5) and (6) and the two values correspond to $\delta = \pi$ and 0. The current experimental upper limit reads [101] $\text{BR}(\mu^+ \rightarrow e^+ + \gamma) < 2.4 \times 10^{-12}$. Thus, although the predicted branching ratio $\text{BR}(\mu^+ \rightarrow e^+ + \gamma) \neq 0$, its value is approximately by 43 orders of magnitude smaller than the sensitivity reached in the experiments searching for the $\mu \rightarrow e + \gamma$ decay, which renders it unobservable in practice.

As was emphasised already, massive Majorana neutrinos appear in theories with no conserved additive quantum number, and more specifically, in which the total lepton charge L is not conserved and changes by two units. In the absence of RH singlet neutrino fields in the theory, the flavour neutrinos and antineutrinos ν_l and $\overline{\nu}_l$, $l = e, \mu, \tau$, can have a mass term of the so-called Majorana type:

$$\mathscr{L}_M^\nu (x) = -\frac{1}{2} \overline{\nu_{l'R}^c} (x) \, M_{l'l} \nu_{lL} (x) + \text{h.c.,}$$
$$\nu_{l'R}^c \equiv C(\overline{\nu_{l'L}} (x))^T, \tag{40}$$

where M is a 3×3, in general complex matrix. In the case when all elements of M are nonzero, $M_{l'l} \neq 0$, $l, l' = e, \mu, \tau$, neither the individual lepton charges L_l nor the total lepton charge L is conserved: $L_l \neq$ const., $L \neq$ const. As it is possible to show, owing to the fact that $\nu_{lL}(x)$ are fermion (anticommuting) fields, the matrix M has to be symmetric (see, e.g., [26]): $M = M^T$. A complex symmetric matrix is diagonalised by the *congruent transformation*:

$$M^{\text{diag}} = U^T M U, \quad U\text{-unitary,} \tag{41}$$

where U is a 3×3 unitary matrix. If $\mathscr{L}_M^\nu(x)$ is written in the basis in which the charged lepton mass matrix is diagonal, U coincides with the PMNS matrix: $U \equiv U_{\text{PMNS}}$. The fields of neutrinos ν_j with definite mass m_j are expressed in terms of $\nu_{lL}(x)$ and ν_{lR}^c:

$$\mathscr{L}_M^\nu (x) = -\frac{1}{2} \, \overline{\nu}_j (x) \, m_j \nu_j (x), \tag{42}$$

$$\nu_j (x) = U_{jl}^\dagger \nu_{lL} (x) + U_{jl}^T \nu_{lR}^c = C\left(\overline{\nu}_j (x)\right)^T, \quad j = 1, 2, 3. \tag{43}$$

They satisfy the Majorana condition with $\xi_j = 1$, as (43) shows.

The Majorana mass term (40) for the LH flavour neutrino fields ν_{lL} can be generated

(i) effectively after the electroweak symmetry (EWS) breaking in the type I see-saw models [6–9],

(ii) effectively after the EWS breaking in the type III see-saw models [102],

(iii) directly as a result of the EWS breaking by an $\text{SU}(2)_L$ triplet Higgs field which carries two units of the weak hypercharge Y_W and couples in an $\text{SU}(2)_L \times U_{Y_W}$ invariant manner to two lepton doublets [18, 103, 104] (the Higgs triplet model (HTM) sometimes called also "type II see-saw model"),

(iv) as a one-loop correction to a Lagrangian which does not contain a neutrino mass term [105, 106] (see also [107]),

(v) as a two-loop correction in a theory where the neutrino masses are zero at tree and one-loop levels [108, 109] (see also [107]),

(vi) as a three-loop correction in a theory in which the neutrino masses are zero at tree, one-loop and two-loop levels [107].

In all three types of see-saw models, for instance, the neutrino masses can be generated at the EWS breaking scale and in this case the models predict rich beyond the Standard Model physics at the TeV scale, some of which can be probed at the LHC (see, e.g., [110] and further). We will consider briefly below the neutrino mass generation in the type I see-saw and the Higgs triplet models.

In a theory in which the $\text{SU}(2)_L$ singlet RH neutrino fields ν_{lR}, $l = e, \mu, \tau$, are present (we consider in the present paper the case of three RH sterile neutrinos, but schemes with less than 3 and more than 3 sterile neutrinos are also discussed in the literature, see, e.g., [10, 15]), the most general neutrino mass Lagrangian contains the Dirac mass term (36), the Majorana mass term for the LH flavour neutrino fields (40), and a Majorana mass term for the RH neutrino fields $\nu_{lR}(x)$ [111]:

$$\mathscr{L}_{D+M}^\nu (x) = -\overline{\nu_{l'R}} (x) \, M_{Dl'l} \nu_{lL} (x)$$
$$- \frac{1}{2} \overline{\nu_{l'R}^c} (x) \, M_{Ll'l} \nu_{lL} (x) \tag{44}$$
$$- \frac{1}{2} \overline{\nu_{l'L}^c} (x) \, M_{Rl'l} \nu_{lR} (x) + \text{h.c.,}$$

where $\nu_{l'L}^c \equiv C(\overline{\nu_{l'R}}(x))^T$ and M_D, M_L, and M_R are 3×3, in general complex matrices. By a simple rearrangement of the neutrino fields this mass term can be cast in the form of a Majorana mass term which is then diagonalised with the help of the congruent transformation [26]. In this case there are six Majorana mass eigenstate neutrinos; that is, the flavour neutrino fields $\nu_{lL}(x)$ are linear combinations of the fields of six Majorana neutrinos with definite mass. The neutrino mixing matrix in (1) is a 3×6 block of a 6×6 unitary matrix.

The Dirac-Majorana mass term is at the basis of the type I see-saw mechanism of generation of the neutrino masses and appears in many grand unified theories (GUTs) (see, e.g., [26] for further details). In the see-saw models, some of the six massive Majorana neutrinos typically are too heavy to be produced in the weak processes in which the initial states of the flavour neutrinos and antineutrinos

ν_ℓ and $\bar{\nu}_\ell$, used in the neutrino oscillation experiments, are being formed. As a consequence, the states of ν_ℓ and $\bar{\nu}_\ell$ will be coherent superpositions only of the states of the light massive neutrinos ν_j, and the elements of the neutrino mixing matrix U_{PMNS}, which are determined in experiments studying the oscillations of ν_ℓ and $\bar{\nu}_\ell$, will exhibit deviations from unitarity. These deviations can be relatively large and can have observable effects in the TeV scale see-saw models, in which the heavy Majorana neutrinos have masses in the $\sim (100\text{--}1000)$ GeV range (see, e.g., [112]).

If after the diagonalisation of $\mathcal{L}^\nu_{D+M}(x)$ more than three neutrinos will turn out to be light, that is, to have masses ~ 1 eV or smaller, active-sterile neutrino oscillations can take place (see, e.g., [15, 26]): an LH (RH) flavour neutrino ν_{lL} (antineutrino $\bar{\nu}_{lR}$) can undergo transitions into LH sterile antineutrino(s) $\bar{\nu}_{l'L} \equiv \overline{\nu^s}_{l'L}$ (RH sterile neutrino(s) $\nu_{l'R} \equiv \nu^s_{l'R}$). As a consequence of this type of oscillations, one would observe a "disappearance" of, for example, ν_e and/or ν_μ ($\bar{\nu}_e$ and/or $\bar{\nu}_\mu$) on the way from the source to the detector.

We would like to discuss next the implications of CP invariance for the neutrino Majorana mass matrix, (40). In the convention we have used to derive (24), in which the unphysical phase factor in the CP transformation of the lepton doublet field $\Psi_{lL}(x)$, and thus of $\nu_{lL}(x)$, $\eta^l = i$, the requirement of CP invariance leads to the reality condition for M:

$$\text{CP-invariance: } M^* = M. \qquad (45)$$

Thus, M is real and symmetric and therefore is diagonalised by an orthogonal transformation; that is, if CP invariance holds, the matrix U in (41) is an orthogonal matrix. The nonzero eigenvalues of a real symmetric matrix can be positive or negative (the absolute value of the difference between the number of positive and number of negative eigenvalues of a real symmetric matrix A is an invariant of the matrix with respect to transformations $A' = PAP^T$, where P is a real matrix which has an inverse). Consequently, M^{diag} in (41) in general has the form

$$M^{\text{diag}} = \left(m'_1, m'_2, m'_3\right), \qquad m'_j = \rho_j m_j, \quad m_j > 0, \ \rho_j = \pm 1. \qquad (46)$$

Let us denote the neutrino field which has a mass $m'_j \neq 0$ by $\nu'_j(x)$. According to (43), the field $\nu'_j(x)$ satisfies the Majorana condition: $\nu'_j(x) = C(\overline{\nu'_j}(x))^T$. One can work with the fields $\nu'_j(x)$ remembering that some of them have a negative mass. It is not difficult to show that the CP-parity of the fields $\nu'_j(x)$ is $\eta_{\text{CP}}(\nu'_j) = i$, $j = 1, 2, 3$. The physical meaning of the signs of the masses $m'_j \neq 0$ of the Majorana neutrinos becomes clear if we change to a "basis" of neutrino fields $\nu_j(x)$ which have positive masses $m_j > 0$. This can be done, for example, by introducing the fields [26]:

$$\nu'_j(x) = (-\gamma_5)^{(1/2)(1-\rho_j)} \nu_j(x) : \nu'_j(x) = \nu_j(x) \quad \text{if } \rho_j = 1;$$

$$\nu'_j(x) = -\gamma_5 \nu_j(x) \quad \text{if } \rho_j = -1. \qquad (47)$$

As it is not difficult to show, if $\nu'_j(x)$ has a mass $m'_j < 0$, CP-parity $\eta_{\text{CP}}(\nu'_j) = i$ and satisfies the Majorana condition $C(\overline{\nu'_j}(x))^T = \nu'_j(x)$, the field $\nu_j(x)$ possesses a mass $m_j > 0$, CP-parity $\eta_{\text{CP}}(\nu_j) = i\rho_j$ and satisfies the Majorana condition $C(\overline{\nu_j}(x))^T = \rho_j \nu_j(x)$:

$$\nu_j : m_j > 0, \qquad \eta_{\text{CP}}\left(\nu_j\right) = i\rho_j, \qquad C\left(\overline{\nu_j}(x)\right)^T = \rho_j \nu_j(x). \qquad (48)$$

Thus, in the case of CP invariance, the signs of the nonzero eigenvalues of the neutrino Majorana mass matrix determine the CP-parities of the corresponding positive mass Majorana (mass eigenstate) neutrinos (for further discussion of the properties of massive Majorana neutrinos (fermions) and their couplings, see, e.g., [26]).

2.3. A Brief Historical Detour. It is interesting to note that Pontecorvo in his seminal article on neutrino oscillations [13], which was published in 1958 when only one type of neutrino and antineutrino was known, assumed that the state of the neutrino ν, emitted in weak interaction processes, is a linear superposition of the states of two Majorana neutrinos ν^M_1 and ν^M_2 which have different masses, $m_1 \neq m_2$, opposite CP-parities, $\eta_{\text{CP}}(\nu^M_1) = -\eta_{\text{CP}}(\nu^M_2)$ and are maximally mixed, while the state of the corresponding antineutrino $\bar{\nu}$ is just the orthogonal superposition of the states of ν^M_1 and ν^M_2:

$$|\nu\rangle = \frac{\left|\nu^M_1\right\rangle + \left|\nu^M_2\right\rangle}{\sqrt{2}},$$

$$|\bar{\nu}\rangle = \frac{\left|\nu^M_1\right\rangle - \left|\nu^M_2\right\rangle}{\sqrt{2}}. \qquad (49)$$

Thus, the oscillations are between the neutrino ν and the antineutrino $\bar{\nu}$, in full analogy with the $K^0 - \overline{K}^0$ oscillations. From contemporary point of view, Pontecorvo proposed active-sterile neutrino oscillations with maximal mixing and massive Majorana neutrinos. To our knowledge, the article [13] was also the first in which fermion mixing in the weak interaction Lagrangian was introduced.

The article of Maki et al. [14] was inspired, in part, by the discovery of the second type of neutrino—the muon neutrino, in 1962 at Brookhaven. These authors considered a composite model of elementary particles in which the electron and muon neutrino states are superpositions of the states of *composite Dirac neutrinos* ν^D_1 and ν^D_2 which have different masses, $m^D_1 \neq m^D_2$:

$$|\nu_e\rangle = \left|\nu^D_1\right\rangle \cos\theta_c + \left|\nu^D_2\right\rangle \sin\theta_c,$$

$$|\nu_\mu\rangle = -\left|\nu^D_1\right\rangle \sin\theta_c + \left|\nu^D_2\right\rangle \cos\theta_c, \qquad (50)$$

where θ_c is the neutrino mixing angle. The model proposed in [14] has lepton-hadron symmetry built in and as consequence of this symmetry the neutrino mixing angle coincides with what we call today the Cabibbo angle $\theta_c \cong 0.22$ (the article by

Maki et al. [14] appeared before the article by Cabibbo [113] in which the "Cabibbo angle" θ_c was introduced and the hadron phenomenology related to this angle was discussed, but after the article by Gell-Mann and Lévy [114] in which θ_c was also introduced (by the way, in a footnote)). The authors of [14] discuss the possibility of $\nu_\mu - \nu_e$ oscillations, which they called "virtual transmutations."

In an article [115] by Katayama et al., published in 1962 somewhat earlier than [14], the authors also introduce two-neutrino mixing. However, this is done purely for model construction purposes and does not have any physical consequences since the neutrinos in the model constructed in [115] are massless particles.

In 1967 Pontecorvo independently considered the possibility of $\nu_e \leftrightarrow \nu_\mu$ oscillations in the article [1], in which the notion of a "sterile" or "inert" neutrino was introduced. Later in 1969, Gribov and Pontecorvo [116] introduced for the first time a Majorana mass term for the LH flavour neutrinos ν_e and ν_μ, the diagonalisation of which leads to two Majorana neutrinos $\nu_{1,2}^M$ with definite but different masses $m_{1,2}$, $m_1 \neq m_2$, and two-neutrino mixing with an arbitrary mixing angle θ:

$$
\begin{aligned}
\left| \nu_e \right\rangle &= \left| \nu_1^M \right\rangle \cos\theta + \left| \nu_2^M \right\rangle \sin\theta, \\
\left| \nu_\mu \right\rangle &= -\left| \nu_1^M \right\rangle \sin\theta + \left| \nu_2^M \right\rangle \cos\theta.
\end{aligned}
\tag{51}
$$

This was the first modern treatment of the problem of neutrino mixing which anticipated the way this problem is addressed in gauge theories of electroweak interactions and in grand unified theories (GUTs). In the same article for the first time the analytic expression for the probability of $\nu_e \leftrightarrow \nu_\mu$ oscillations was also derived.

2.4. Models of Neutrino Mass Generation: Two Examples

Type I See-Saw Model. A natural explanation of the smallness of neutrino masses is provided by the type I see-saw mechanism of neutrino mass generation [6–9]. Integral part of this rather simple mechanism are the RH neutrinos ν_{lR} (RH neutrino fields $\nu_{lR}(x)$). The latter are assumed to possess a Majorana mass term $\mathscr{L}_M^N(x)$ as well as Yukawa type coupling $\mathscr{L}_Y(x)$ with the Standard Model lepton and Higgs doublets, $\psi_{lL}(x)$ and $H(x)$, given in (37). In the basis in which the Majorana mass matrix of RH neutrinos is diagonal, we have

$$
\begin{aligned}
\mathscr{L}_{Y,M}(x) &\equiv \mathscr{L}_Y(x) + \mathscr{L}_M^N(x) \\
&= -\left(\lambda_{kl} \overline{N_{kR}}(x) H^\dagger(x) \psi_{lL}(x) + \text{h.c.} \right) \\
&\quad - \frac{1}{2} M_k \overline{N_k}(x) N_k(x),
\end{aligned}
\tag{52}
$$

where we have combined the expressions given in (22). When the electroweak symmetry is broken spontaneously, the neutrino Yukawa coupling generates a Dirac mass term: $m_{kl}^D \overline{N_{kR}}(x) \nu_{lL}(x) + \text{h.c.}$, with $m^D = v\lambda$, $v = 174$ GeV being the Higgs doublet v.e.v. In the case when the elements of m^D are much smaller than M_k, $|m_{il}^D| \ll M_k$, $i, k = 1, 2, 3$, $l = e, \mu, \tau$,

the interplay between the Dirac mass term and the mass term of the heavy (RH) Majorana neutrinos N_k generates an effective Majorana mass (term) for the LH flavour neutrinos [6–9]:

$$
M_{l'l} \cong -\left(m^D \right)_{l'k}^T M_k^{-1} m_{kl}^D = -v^2 (\lambda)_{l'k}^T M_k^{-1} \lambda_{kl}.
\tag{53}
$$

In grand unified theories, m^D is typically of the order of the charged fermion masses. In SO(10) theories, for instance, m^D coincides with the up-quark mass matrix. Taking indicatively $M \sim 0.1$ eV, $m^D \sim 100$ GeV, one obtains $M_k \sim M_N \sim 10^{14}$ GeV, which is close to the scale of unification of the electroweak and strong interactions, $M_{\text{GUT}} \cong 2 \times 10^{16}$ GeV. In GUT theories with RH neutrinos one finds that indeed the heavy Majorana neutrinos N_k naturally obtain masses which are by few to several orders of magnitude smaller than M_{GUT} (see, e.g., [7, 8]). Thus, the enormous disparity between the neutrino and charged fermion masses is explained effectively in this approach by the huge difference between the electroweak symmetry breaking scale and M_{GUT}.

An additional attractive feature of the see-saw scenario under discussion is that the generation and smallness of neutrino masses are related via the leptogenesis mechanism [2, 3] (see also, e.g., [4, 5, 68–70, 79]) to the generation of the baryon asymmetry of the Universe. Indeed, the Yukawa coupling in (52), in general, is not CP conserving. Due to this CP-nonconserving coupling, the heavy Majorana neutrinos undergo, for example, the decays $N_j \rightarrow l^+ + H^{(-)}$, $N_j \rightarrow l^- + H^{(+)}$, which have different rates: $\Gamma(N_j \rightarrow l^+ + H^{(-)}) \neq \Gamma(N_j \rightarrow l^- + H^{(+)})$. When these decays occur in the Early Universe at temperatures somewhat below the mass of, say, N_1, so that the latter are out of equilibrium with the rest of the particles present at that epoch, CP violating asymmetries in the individual lepton charges L_l and in the total lepton charge L of the Universe are generated. These lepton asymmetries are converted into a baryon asymmetry by $(B - L)$ conserving, but $(B + L)$ violating, sphaleron processes, which exist in the Standard Model and are effective at temperatures $T \sim (100–10^{12})$ GeV [117]. If the heavy neutrinos N_j have hierarchical spectrum, $M_1 \ll M_2 \ll M_3$, the observed baryon asymmetry can be reproduced provided the mass of the lightest one satisfies $M_1 \gtrsim 10^9$ GeV [118] (in specific type I see-saw models this bound can be lower by a few orders of magnitude, see, e.g., [119]). Thus, in this scenario, the neutrino masses and mixing and the baryon asymmetry have the same origin—the neutrino Yukawa couplings and the existence of (at least two) heavy Majorana neutrinos. Moreover, quantitative studies based on advances in leptogenesis theory [68–70], in which the importance of the flavour effects in the generation of the baryon asymmetry was understood, have shown that the Dirac and/or Majorana phases in the neutrino mixing matrix U can provide the CP violation, necessary in leptogenesis for the generation of the observed baryon asymmetry of the Universe [71, 72]. This implies, in particular, that if the CP symmetry is established not to hold in the lepton sector due to the PMNS matrix U, at least some fraction (if not all) of the observed baryon asymmetry might be due to the Dirac and/or Majorana CP violation present in the neutrino mixing.

In the see-saw scenario considered, the scale at which the new physics manifests itself, which is set by the scale of masses of the RH neutrinos, can, in principle, have an arbitrary large value, up to the GUT scale of 2×10^{16} GeV and even beyond, up to the Planck mass. An interesting possibility, which can also be theoretically well motivated (see, e.g., [120, 121]), is to have the new physics at the TeV scale, that is, $M_k \sim (100-1000)$ GeV. Low scale see-saw scenarios usually predict a rich phenomenology at the TeV scale and are constrained by different sets of data, such as, the data on neutrino oscillations, from EW precision tests and on the lepton flavour violating (LFV) processes $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$, $\mu^- - e^-$ conversion in nuclei. In the case of the TeV scale type I see-saw scenario of interest, the flavour structure of the couplings of the heavy Majorana neutrinos N_k to the charged leptons and the $W^{\pm}$ bosons, and to the LH flavour neutrinos ν_{lL} and the Z^0 boson, are essentially determined by the requirement of reproducing the data on the neutrino oscillation parameters [98]. All present experimental constraints on this scenario still allow (i) for the predicted rates of the $\mu \rightarrow e + \gamma$ decay, $\mu \rightarrow 3e$ decay, and $\mu - e$ conversion in the nuclei to be [122] within the sensitivity range of the currently running MEG experiment on $\mu \rightarrow e + \gamma$ decay [101] planned to probe values of BR($\mu^+ \rightarrow e^+ + \gamma$) $\gtrsim 10^{-13}$, and of the future planned experiments on $\mu \rightarrow 3e$ decay and $\mu - e$ conversion [123–127], (ii) for an enhancement of the rate of neutrinoless double-beta-$((\beta\beta)_{0\nu}$-) decay [98], which thus can be in the range of sensitivity of the $(\beta\beta)_{0\nu}$-decay experiments which are taking data or are under preparation (see, e.g., [128]) even when the light Majorana neutrinos possess a normal hierarchical mass spectrum (see further), and (iii) for the possibility of an exotic Higgs decay channel into a light neutrino and a heavy Majorana neutrino with a sizable branching ratio, which can lead to observables effects at the LHC [110] (for further details concerning the low energy phenomenology of the TeV scale type I see-saw model, see, e.g., [98, 120–122]).

Let us add that the role of the experiments searching for lepton flavour violation to test and possibly constrain low scale see-saw models, and more generally, extensions of the Standard Model predicting "new" (lepton flavour violating) physics at the TeV scale, will be significantly strengthened in the next years. Searches for $\mu - e$ conversion at the planned COMET experiment at KEK [124] and Mu2e experiment at Fermilab [125] aim to reach sensitivity to conversion rates CR(μAl $\rightarrow e$Al) $\approx 10^{-16}$, while, in the longer run, the PRISM/PRIME experiment in KEK [126] and the project-X experiment in Fermilab [127] are being designed to probe values of the $\mu - e$ conversion rate on Ti, which are smaller by 2 orders of magnitude, CR(μTi $\rightarrow e$Ti) $\approx 10^{-18}$ [126]. The current upper limit on the $\mu - e$ conversion rate is CR(μAl $\rightarrow e$Al) $< 4.3 \times 10^{-12}$ [129]. There are also plans to perform a new search for the $\mu^+ \rightarrow e^+ e^- e^+$ decay [123], which will probe values of the corresponding branching ratio down to BR($\mu^+ \rightarrow e^+ e^- e^+$) $\approx 10^{-15}$, that is, by 3 orders of magnitude smaller than the best current upper limit [130]. Furthermore, searches for tau lepton flavour violation at superB factories aim to reach a sensitivity to BR($\tau \rightarrow (\mu, e)\gamma$) $\approx 10^{-9}$ (see, e.g., [131]).

The Higgs Triplet Model (HTM). In its minimal formulation this model includes one additional $SU(2)_L$ triplet Higgs field Δ, which has weak hypercharge $Y_W = 2$ [18, 103, 104]:

$$\Delta = \begin{pmatrix} \dfrac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & -\dfrac{\Delta^+}{\sqrt{2}} \end{pmatrix}. \qquad (54)$$

The Lagrangian of the Higgs triplet model which is sometimes called also the "type II see-saw model," reads (we do not give here, for simplicity, all the quadratic and quartic terms present in the scalar potential (see, e.g., [132])):

$$\begin{aligned} \mathscr{L}_{\text{HTM}} = & -M_\Delta^2 \operatorname{Tr}\left(\Delta^\dagger \Delta\right) \\ & -\left(h_{\ell\ell'}\overline{\psi^C}_{\ell L} i\tau_2 \Delta \psi_{\ell' L} + \mu_\Delta H^\dagger \Delta^\dagger i\tau_2 H^* + \text{h.c.}\right), \end{aligned}$$
$$(55)$$

where $\overline{\psi^C}_{\ell L} \equiv (-\nu_{\ell L}^T C^{-1} - \ell_L^T C^{-1})$, C being the charge conjugation matrix, H is the SM Higgs doublet, and μ_Δ is a real parameter characterising the soft explicit breaking of the total lepton charge conservation. We will discuss briefly the low energy version of HTM, where the new physics scale M_Δ associated with the mass of Δ takes values 100 GeV $\lesssim M_\Delta \lesssim 1$ TeV, which, in principle, can be probed by LHC (see [132, 133] and references quoted therein).

The flavour structure of the Yukawa coupling matrix h and the size of the lepton charge soft breaking parameter μ_Δ are related to the light neutrino Majorana mass matrix M^ν, which is generated when the neutral component of Δ develops a "small" vev $\nu_\Delta \propto \mu_\Delta$. Indeed, setting $\Delta^0 = \nu_\Delta$ and $H^T = (\nu \quad 0)^T$ with $\nu \simeq 174$ GeV, from Lagrangian (55) one obtains

$$M_{\ell\ell'}^\nu \simeq 2h_{\ell\ell'}\nu_\Delta. \qquad (56)$$

The matrix of Yukawa couplings $h_{\ell\ell'}$ is directly related to the PMNS neutrino mixing matrix $U_{\text{PMNS}} \equiv U$, which is unitary in this case:

$$h_{\ell\ell'} \equiv \frac{1}{2\nu_\Delta}\left(U^* \operatorname{diag}(m_1, m_2, m_3) U^\dagger\right)_{\ell\ell'}, \quad m_j \geq 0. \quad (57)$$

An upper limit on ν_Δ can be obtained from considering its effect on the parameter $\rho = M_W^2/M_Z^2\cos^2\theta_W$. In the SM, $\rho = 1$ at tree-level, while in the HTM one has

$$\rho \equiv 1 + \delta\rho = \frac{1 + 2x^2}{1 + 4x^2}, \quad x \equiv \frac{\nu_\Delta}{\nu}. \qquad (58)$$

The measurement $\rho \approx 1$ leads to the bound $\nu_\Delta/\nu \lesssim 0.03$, or $\nu_\Delta < 5$ GeV (see, e.g., [132]).

For $M_\Delta \sim (100-1000)$ GeV, the model predicts a plethora of beyond the SM physics phenomena (see, e.g., [132, 134–139]), most of which can be probed at the LHC and in the experiments on charged lepton flavour violation, if the Higgs triplet vacuum expectation value ν_Δ is relatively small, roughly $\nu_\Delta \sim (1-100)$ eV. As can be shown (see, e.g., [132]),

the parameters v_Δ and μ_Δ are related: for $M_\Delta \sim v = 174\,\text{GeV}$ we have $v_\Delta \cong \mu_\Delta$, while if $M_\Delta^2 \gg v^2$, then $v_\Delta \cong \mu_\Delta v^2/(2M_\Delta^2)$. Thus, a relatively small value of v_Δ in the TeV scale HTM implies that μ_Δ has also to be small, and vice versa. A nonzero but relatively small value of μ_Δ can be generated, for example, at higher orders in perturbation theory [140]. The smallness of the neutrino masses is therefore related to the smallness of the vacuum expectation value v_Δ, which in turn is related to the smallness of the parameter μ_Δ.

Under the conditions specified above one can have testable predictions of the model in low energy experiments, and in particular, in the ongoing MEG and the planned future experiments on the lepton flavour violating processes $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$ and $\mu + \mathcal{N} \rightarrow e + \mathcal{N}$ (see, e.g., [122]). The HTM has also an extended Higgs sector including neutral, singly charged and doubly charged Higgs particles. The physical singly charged Higgs scalar field (particle) practically coincides with the triplet scalar field Δ^+, the admixture of the doublet charged scalar field being suppressed by the factor v_Δ/v. The singly and doubly charged Higgs scalars Δ^+ and Δ^{++} have, in general, different masses [140]: $m_{\Delta^+} \neq m_{\Delta^{++}}$. Both cases $m_{\Delta^+} > m_{\Delta^{++}}$ and $m_{\Delta^+} < m_{\Delta^{++}}$ are possible. The TeV scale HTM predicts the existence of rich new physics at LHC as well, associated with the presence of the singly and doubly charged Higgs particles Δ^+ and Δ^{++} in the theory (see, e.g., [132, 137–139]).

3. Determining the Nature of Massive Neutrinos

The Majorana nature of massive neutrinos typically manifests itself in the existence of processes in which the total lepton charge L changes by two units: $K^+ \rightarrow \pi^- + \mu^+ + \mu^+$, $\mu^- + (A, Z) \rightarrow \mu^+ + (A, Z-2)$, and so forth. Extensive studies have shown that the only feasible experiments having the potential of establishing the Majorana nature of massive neutrinos at present are the $(\beta\beta)_{0\nu}$-decay experiments searching for the process $(A, Z) \rightarrow (A, Z+2) + e^- + e^-$ (for reviews see, e.g., [26, 128, 141, 142]). The observation of $(\beta\beta)_{0\nu}$-decay and the measurement of the corresponding half-life with sufficient accuracy not only would be a proof that the total lepton charge is not conserved, but might provide also information (i) on the type of neutrino mass spectrum [48, 49], and (ii) on the absolute scale of neutrino masses (see, e.g., [81]).

The observation of $(\beta\beta)_{0\nu}$-decay and the measurement of the corresponding half-life with sufficient accuracy, combined with data on the absolute neutrino mass scale, might provide also information on the Majorana phases in U [80, 82, 83, 143, 144]. If the neutrino mass spectrum is inverted hierarchical or quasidegenerate, for instance, it would be possible to get information about the phase α_{21}. However, establishing even in this case that α_{21} has a CP violating value would be a remarkably challenging problem [83] (see also [84]). Determining experimentally the values of both the Majorana phases α_{21} and α_{31} is an exceptionally difficult problem. It requires the knowledge of the type of neutrino mass spectrum and high precision determination of both the absolute neutrino mass scale and of the $(\beta\beta)_{0\nu}$-decay effective Majorana mass, $|\langle m \rangle|$ (see, e.g., [80, 83]).

3.1. Majorana Neutrinos and $(\beta\beta)_{0\nu}$-Decay.

Under the assumptions of 3-ν mixing, for which we have compelling evidence, of massive neutrinos ν_j being Majorana particles and of $(\beta\beta)_{0\nu}$-decay generated *only by the* (V-A) *charged current weak interaction via the exchange of the three Majorana neutrinos* ν_j having masses $m_j \lesssim$ few MeV, the $(\beta\beta)_{0\nu}$-decay amplitude of interest has the form (see, e.g., [80, 141, 142]): $A(\beta\beta)_{0\nu} \cong \langle m \rangle M$, where M is the corresponding nuclear matrix element (NME) which does not depend on the neutrino mixing parameters, and

$$|\langle m \rangle| = \left| m_1 |U_{e1}|^2 + m_2 |U_{e2}|^2 e^{i\alpha_{21}} + m_3 |U_{e3}|^2 e^{i(\alpha_{31}-2\delta)} \right|, \tag{59}$$

is the effective Majorana mass in $(\beta\beta)_{0\nu}$-decay, $|U_{e1}| = c_{12}c_{13}$, $|U_{e2}| = s_{12}c_{13}$, $|U_{e3}| = s_{13}$. In the case of CP-invariance one has $2\delta = 0$ or 2π and,

$$\eta_{21} \equiv e^{i\alpha_{21}} = \pm 1, \qquad \eta_{31} \equiv e^{i\alpha_{31}} = \pm 1, \tag{60}$$

$\eta_{21(31)}$ being the relative CP-parity of the Majorana neutrinos $\nu_{2(3)}$ and ν_1.

It proves convenient to express [145] the three neutrino masses in terms of Δm_{21}^2 and $\Delta m_{31(32)}^2$, measured in neutrino oscillation experiments, and the absolute neutrino mass scale determined by $\min(m_j)$ (for a detailed discussion of the relevant formalism, see, e.g., [26, 80, 141, 142]). In both cases of neutrino mass spectrum with normal and inverted ordering one has (in the convention we use): $\Delta m_{21}^2 > 0$, $m_2 = (m_1^2 + \Delta m_{21}^2)^{1/2}$. For normal ordering, $\Delta m_{31}^2 > 0$, and $m_3 = (m_1^2 + \Delta m_{31}^2)^{1/2}$, while if the spectrum is with inverted ordering, $\min(m_j) = m_3$, $\Delta m_{32}^2 < 0$ and $m_1 = (m_3^2 + \Delta m_{23}^2 - \Delta m_{21}^2)^{1/2}$. Thus, given Δm_{21}^2, $\Delta m_{31(32)}^2$, θ_{12}, and θ_{13}, $|\langle m \rangle|$ depends on $\min(m_j)$, the Majorana phases α_{21}, α_{31}, and the type of neutrino mass spectrum.

The problem of obtaining the allowed values of $|\langle m \rangle|$ given the constraints on the parameters following from ν-oscillation data, and more generally of the physics potential of $(\beta\beta)_{0\nu}$-decay experiments, was first studied in [145] and subsequently in a large number of papers (see, e.g., [80, 83, 146–153]; extensive list of references on the subject is given in [141, 142]). The results of this analysis are illustrated in Figure 1. The main features of the predictions for $|\langle m \rangle|$ in the cases of the NH, IH, and QD spectra are summarised below.

(i) NH spectrum:

$$|\langle m \rangle| \cong \left| \left(\Delta m_{21}^2 \right)^{1/2} s_{12}^2 + \left(\Delta m_{31}^2 \right)^{1/2} s_{13}^2 e^{-i(\alpha_{21}-\alpha_{31}+2\delta)} \right|. \tag{61}$$

Using the 3σ allowed ranges of the relevant neutrino oscillation parameters we get

$$4.7 \times 10^{-4}\,\text{eV} \lesssim |\langle m \rangle| \lesssim 4.8 \times 10^{-3}\,\text{eV}, \quad \text{NH.} \tag{62}$$

(ii) IH spectrum:

$$|\langle m \rangle| \cong \left(|\Delta m_{32}^2| \right)^{1/2} \left(1 - \sin^2 2\theta_{21} \sin^2 \frac{\alpha_{21}}{2} \right)^{1/2},$$

$$\left(|\Delta m_{32}^2| \right)^{1/2} \cos 2\theta_{12} \lesssim |\langle m \rangle| \lesssim \left(|\Delta m_{32}^2| \right)^{1/2}. \tag{63}$$
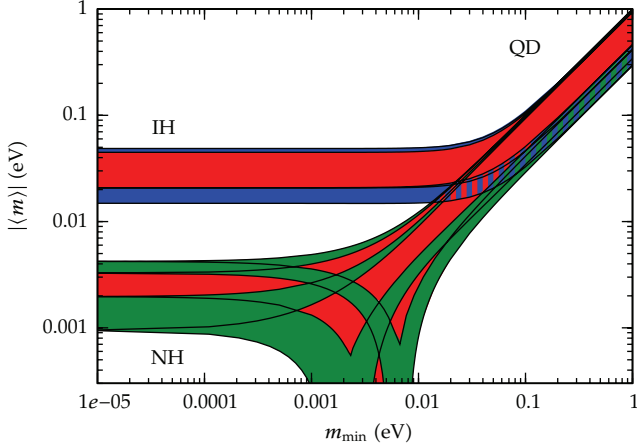
FIGURE 1: The effective Majorana mass $|\langle m \rangle|$ (including a 2σ uncertainty), as a function of $\min(m_j)$ for $\sin^2\theta_{13} = 0.0236 \pm 0.0042$ [27, 28] and $\delta = 0$. The figure is obtained using also the best fit values and 1σ errors of Δm^2_{21}, $\sin^2\theta_{12}$, and $|\Delta m^2_{31(32)}|$ given in Table 1.7 in [11]. The phases $\alpha_{21,31}$ are varied in the interval $[0, \pi]$. The predictions for the NH, IH, and QD spectra are indicated. The red regions correspond to at least one of the phases $\alpha_{21,31}$ and $(\alpha_{31} - \alpha_{21})$ having a CP violating value, while the blue and green areas correspond to $\alpha_{21,31}$ possessing CP conserving values (from [11]).

Numerically one finds

$$0.014\,\text{eV} \lesssim |\langle m \rangle| \lesssim 0.050\,\text{eV}, \quad \text{IH}, \quad (64)$$

the upper and the lower bounds corresponding to the CP-conserving values of $\alpha_{21} = 0; \pi$.

(iii) QD spectrum:

$$|\langle m \rangle| \cong m_0 \left(1 - \sin^2 2\theta_{12} \sin^2 \frac{\alpha_{21}}{2} \right)^{1/2}, \quad (65)$$

$$m_0 \gtrsim |\langle m \rangle| \gtrsim m_0 \cos 2\theta_{12} \gtrsim 0.028\,\text{eV}$$

with $m_0 \gtrsim 0.1\,\text{eV}$, $m_0 < 2.05\,\text{eV}$ [62] (see also [63]), or $m_0 \lesssim (0.3–1.3)\,\text{eV}$ [65] (see (7) and the discussion following after it).

For the IH (QD) spectrum we have also [80, 143]

$$\sin^2 \left(\frac{\alpha_{21}}{2} \right) \cong \left(1 - \frac{|\langle m \rangle|^2}{\widetilde{m}^2} \right) \frac{1}{\sin^2 2\theta_{12}}, \quad (66)$$

$$\widetilde{m}^2 \equiv \left| \Delta m^2_{32} \right| (m_0^2), \quad \text{IH (QD)}.$$

Thus, a measurement of $|\langle m \rangle|$ and m_0 ($|\Delta m^2_{32}|$) for QD (IH) spectrum can allow to determine α_{21}.

The experimental searches for $(\beta\beta)_{0\nu}$-decay have a long history (see, e.g., [154, 155]). The most stringent upper limits on $|\langle m \rangle|$ were set by the IGEX [156] and Heidelberg-Moscow [157], CUORICINO [158], NEMO3 [159], and EXO-200 [160] experiments with ^{76}Ge, ^{130}Te, ^{100}Mo, and ^{136}Xe, respectively (the NEMO3 collaboration has searched for $(\beta\beta)_{0\nu}$-decay of ^{82}Se and other isotopes as well). The IGEX collaboration has obtained for the half-life of ^{76}Ge that $T^{0\nu}_{1/2} > 1.57 \times 10^{25}$ yr

(90% C.L.), from which the limit $|\langle m \rangle| < (0.33–1.35)\,\text{eV}$ was derived [156]. Using the recent more advanced calculations of the corresponding nuclear matrix elements (including the relevant uncertainties) [161] one finds $|\langle m \rangle| < (0.22–0.35)\,\text{eV}$. The NEMO3 and CUORICINO experiments, designed to reach a sensitivity to $|\langle m \rangle| \sim (0.2–0.3)\,\text{eV}$, set the limits: $|\langle m \rangle| < (0.61–1.26)\,\text{eV}$ [159] and $|\langle m \rangle| < (0.19–0.68)\,\text{eV}$ [158] (90% C.L.), where estimated uncertainties in the NME are accounted for. The two upper limits were derived from the experimental lower limits on the half-lives of ^{100}Mo and ^{130}Te, $T^{0\nu}_{1/2} > 5.8 \times 10^{23}$ yr (90% C.L.) [159] and $T^{0\nu}_{1/2} > 3.0 \times 10^{24}$ yr (90% C.L.) [158]. With the NMEs and their uncertainties calculated in [161], the NEMO3 and CUORICINO upper limits read, respectively, $|\langle m \rangle| < (0.50–0.96)\,\text{eV}$ and $|\langle m \rangle| < (0.25–0.43)\,\text{eV}$. A best lower limit on the half-life of ^{136}Xe was obtained recently in the EXO-200 experiment [160]: $T^{0\nu}_{1/2}(^{136}\text{Xe}) > 1.6 \times 10^{25}$ yr (90% C.L.).

The best lower limit on the half-life of ^{76}Ge, $T^{0\nu}_{1/2} > 1.9 \times 10^{25}$ yr (90% C.L.), was found in the Heidelberg-Moscow ^{76}Ge experiment [157]. It corresponds to the upper limit [161] $|\langle m \rangle| < (0.20–0.35)\,\text{eV}$. A positive $(\beta\beta)_{0\nu}$-decay signal at $>3\sigma$, corresponding to $T^{0\nu}_{1/2} = (0.69–4.18) \times 10^{25}$ yr (99.73% C.L.) and implying $|\langle m \rangle| = (0.1–0.9)\,\text{eV}$, is claimed to have been observed in [162], while a later analysis reports evidence for $(\beta\beta)_{0\nu}$-decay at 6σ corresponding to $|\langle m \rangle| = 0.32 \pm 0.03\,\text{eV}$ [163].

Most importantly, a large number of projects aim at a sensitivity to $|\langle m \rangle| \sim (0.01–0.05)\,\text{eV}$ [128]: CUORE (^{130}Te), GERDA (^{76}Ge), SuperNEMO, EXO (^{136}Xe), MAJORANA (^{76}Ge), MOON (^{100}Mo), COBRA (^{116}Cd), XMASS (^{136}Xe), CANDLES (^{48}Ca), KamLAND-Zen (^{136}Xe), SNO+ (^{150}Nd), and so forth. These experiments, in particular, will test the positive result claimed in [163].

The existence of significant lower bounds on $|\langle m \rangle|$ in the cases of IH and QD spectra [48, 49], which lie either partially (IH spectrum) or completely (QD spectrum) within the range of sensitivity of the next generation of $(\beta\beta)_{0\nu}$-decay experiments, is one of the most important features of the predictions of $|\langle m \rangle|$. These minimal values are given, up to small corrections, by $|\Delta m^2_{32}| \cos 2\theta_{12}$ and $m_0 \cos 2\theta_{12}$. According to the combined analysis of the solar and reactor neutrino data [33], (i) the possibility of $\cos 2\theta_{12} = 0$ is excluded at $\sim 6\sigma$, (ii) the best fit value of $\cos 2\theta_{12}$ is $\cos 2\theta_{12} \cong 0.39$, and (iii) at 99.73% C.L. one has $\cos 2\theta_{12} \gtrsim 0.28$. The quoted results on $\cos 2\theta_{12}$ together with the range of possible values of $|\Delta m^2_{32}|$ and m_0 lead to the conclusion about the existence of significant and robust lower bounds on $|\langle m \rangle|$ in the cases of IH and QD spectrum. At the same time one can *always* have $|\langle m \rangle| \ll 10^{-3}$ eV in the case of spectrum with normal ordering [81]. As Figure 1 indicates, $|\langle m \rangle|$ cannot exceed ~ 5 meV for NH neutrino mass spectrum. This implies that $\max(|\langle m \rangle|)$ in the case of NH spectrum is considerably smaller than $\min(|\langle m \rangle|)$ for the IH and QD spectrum. This opens the possibility of obtaining information about the type of ν-mass spectrum from a measurement of $|\langle m \rangle| \neq 0$ [48, 49]. In particular, a positive result in the future $(\beta\beta)_{0\nu}$-decay experiments with $|\langle m \rangle| > 0.01$ eV would imply that

the NH spectrum is strongly disfavored (if not excluded). For $\Delta m^2_{31(32)} > 0$, such a result would mean that the neutrino mass spectrum is with normal ordering, but is not hierarchical. If $\Delta m^2_{31(32)} < 0$, the neutrino mass spectrum would be either IH or QD. Prospective experimental errors in the values of oscillation parameters in $|\langle m \rangle|$ and the sum of neutrino masses, and the uncertainty in the relevant NME, can weaken but do not invalidate these results [82, 83, 86, 164]. Let us note that encouraging results, in what regards the problem of calculation of the NME, were reported at the MEDEX'11 Workshop on Matrix Elements for the Double-beta-decay Experiments [165] (for the bounds on $|\langle m \rangle|$ obtained using the current results on the NME, see, e.g., [86]).

As Figure 1 indicates, a measurement of $|\langle m \rangle| \gtrsim 0.01$ eV would either [81] (i) determine a relatively narrow interval of possible values of the lightest neutrino mass $\min(m_j)$ or (ii) would establish an upper limit on $\min(m_j)$. If an upper limit on $|\langle m \rangle|$ is experimentally obtained below 0.01 eV, this would lead to a significant upper limit on $\min(m_j)$.

The possibility of establishing CP-violation in the lepton sector due to Majorana CPV phases has been studied in [81, 84] and in much greater detail in [82, 83]. It was found that it is very challenging: it requires quite accurate measurements of $|\langle m \rangle|$ (and of m_0 for QD spectrum) and holds only for a limited range of values of the relevant parameters. More specifically [82, 83], establishing at 2σ CP-violation associated with Majorana neutrinos in the case of QD spectrum requires for $\sin^2\theta_\odot = 0.31$, in particular, a relative experimental error on the measured value of $|\langle m \rangle|$ and m_0 smaller than 15%, a "theoretical uncertainty" $F \lesssim 1.5$ in the value of $|\langle m \rangle|$ due to an imprecise knowledge of the corresponding NME, and value of the relevant Majorana CPV phase α_{21} typically within the ranges of $\sim (\pi/4 - 3\pi/4)$ and $\sim (5\pi/4 - 7\pi/4)$.

The knowledge of NME with sufficiently small uncertainty is crucial for obtaining quantitative information on the ν-mixing parameters from a measurement of $(\beta\beta)_{0\nu}$-decay half-life. The observation of a $(\beta\beta)_{0\nu}$-decay of one nucleus is likely to lead to the searches and eventually to observation of the decay of other nuclei. One can expect that such a progress, in particular, will help to solve completely the problem of the sufficiently precise calculation of the nuclear matrix elements for the $(\beta\beta)_{0\nu}$-decay [81] (a possible test of the NME calculations is suggested in [81] and is discussed in greater detail in [166]).

If the future $(\beta\beta)_{0\nu}$-decay experiments show that $|\langle m \rangle| < 0.01$ eV, both the IH and the QD spectrum will be ruled out for massive Majorana neutrinos. If in addition it is established in neutrino oscillation experiments that the neutrino mass spectrum is with *inverted ordering*, that is, that $\Delta m^2_{31(32)} < 0$, one would be led to conclude that either the massive neutrinos ν_j are Dirac fermions or that ν_j are Majorana particles but there are additional contributions to the $(\beta\beta)_{0\nu}$-decay amplitude which interfere destructively with that due to the exchange of light massive Majorana neutrinos. The case of more than one mechanism generating the $(\beta\beta)_{0\nu}$-decay was discussed recently in, for example, [167, 168], where the possibility to identify the mechanisms inducing the decay was also analysed. If, however, $\Delta m^2_{31(32)}$ is determined to be

positive in neutrino oscillation experiments, the upper limit $|\langle m \rangle| < 0.01$ eV would be perfectly compatible with massive Majorana neutrinos possessing NH mass spectrum, or mass spectrum with normal ordering but partial hierarchy, and the quest for $|\langle m \rangle|$ would still be open.

If indeed in the next generation of $(\beta\beta)_{0\nu}$-decay experiments it is found that $|\langle m \rangle| < 0.01$ eV, while the neutrino oscillation experiments show that $\Delta m^2_{31(32)} > 0$, the next frontier in the searches for $(\beta\beta)_{0\nu}$-decay would most probably correspond to values of $|\langle m \rangle| \sim 0.001$ eV. Taking $|\langle m \rangle| = 0.001$ eV as a reference value, the conditions under which $|\langle m \rangle|$ in the case of neutrino mass spectrum with normal ordering would be guaranteed to satisfy $|\langle m \rangle| \gtrsim 0.001$ eV, were investigated in [146]. In the analysis performed in [146], the specific case of normal hierarchical neutrino mass spectrum and the general case of spectrum with normal ordering, partial hierarchy, and values of θ_{13}, including the value measured in the Daya-Bay, RENO, Double Chooz, and T2K experiments, (4), were considered. The ranges of the lightest neutrino mass m_1 and/or of $\sin^2\theta_{13}$, for which $|\langle m \rangle| \gtrsim 0.001$ eV were derived as well, and the phenomenological implications of such scenarios were discussed.

4. Outlook

The last 14 years or so witnessed a spectacular experimental progress in the studies of the properties of neutrinos. In this period the existence of neutrino oscillations, caused by nonzero neutrino masses and neutrino mixing, was established and the parameters which drive the oscillations were determined with a relatively high precision. In spite of these remarkable achievements one has to admit that we are still completely ignorant about some of the fundamental aspects of neutrino mixing: the nature, Dirac or Majorana, of massive neutrinos, the type of spectrum the neutrino masses obey, the absolute scale of neutrino masses, and the status of CP symmetry in the lepton sector. Finding out these aspects and understanding the origins of the neutrino masses and mixing and the patterns they and possibly leptonic CP violation exhibit require an extensive and challenging program of research. The main goals of such a research program include the following.

(i) Determining the nature, Dirac or Majorana, of massive neutrinos ν_j. This is of fundamental importance for making progress in our understanding of the origin of neutrino masses and mixing and of the symmetries governing the lepton sector of particle interactions.

(ii) Determination of the sign of $\Delta m^2_{31(32)}(\Delta m^2_{31})$ and of the type of neutrino mass spectrum.

(iii) Determining or obtaining significant constraints on the absolute neutrino mass scale.

(iv) Determining the status of CP symmetry in the lepton sector.

(v) Understanding at a fundamental level the mechanism giving rise to neutrino masses and mixing and to L_l-non-conservation. This includes understanding

the origin of the patterns of neutrino mixing and neutrino masses, suggested by the data. Are the observed patterns of ν-mixing and of $\Delta m^2_{21,31}$ related to the existence of a new fundamental symmetry of particle interactions? Is there any relation between quark mixing and neutrino (lepton) mixing? What is the physical origin of CP violation phases in the neutrino mixing matrix U? Is there any relation (correlation) between the (values of) CP violation phases and mixing angles in U? Progress in the theory of neutrino mixing might also lead to a better understanding of the mechanism of generation of baryon asymmetry of the Universe.

The successful realization of this research program would be a formidable task and would require many years. It already began with the high precision measurement of θ_{13} in the Daya Bay and RENO experiments, which showed that $\sin^2 2\theta_{13}$ has a relatively large value (4). The Double Chooz and T2K experiments also found values of $\sin^2 2\theta_{13}$, which are different from zero, respectively, at 2.9σ and 3.2σ and are compatible with those obtained in the Daya Bay and RENO experiments. These results on θ_{13} have far reaching implications. As we have already mentioned or discussed, the measured relatively large value of θ_{13} opens up the possibilities, in particular,

(i) for searching for CP violation effects in neutrino oscillation experiments with high intensity accelerator neutrino beams, like T2K, NOνA [169] (the sensitivities of T2K and NOνA on CP violation in neutrino oscillations are discussed, e.g., in [78]),

(ii) for determining the sign of Δm^2_{32}, and thus the type of neutrino mass spectrum, in neutrino oscillation experiments with sufficiently long baselines (see, e.g., [35, 40–47]).

A value of $\sin \theta_{13} \gtrsim 0.09$ is a necessary condition for a successful "flavoured" leptogenesis with hierarchical heavy Majorana neutrinos when the CP violation required for the generation of the matter-antimatter asymmetry of the Universe is provided entirely by the Dirac CP violating phase in the neutrino mixing matrix [72].

With the measurement of θ_{13}, the first steps on the long "road" leading to a comprehensive understanding of the patterns of neutrino masses and mixing, of their origin and implications, were made. The future of neutrino physics is bright.

Acknowledgments

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