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Study of the Hadron Energy Resolution of a Liquid Argon Calorimeter by Applying the H1 Weighting Technique

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Abstract

The H1 weighting technique, which allows to improve the e/h ratio of intrinsically non-compensating calorimeters, is applied to a full liquid argon calorimeter of the ATLAS detector. The dependence of the energy resolution on the transverse and longitudinal granularity of the calorimeter is studied. From this study requirements on these granularities can be derived. The energy resolution of jets is investigated as a function of the jet size.

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1 Introduction

As one of the possible options of the ATLAS calorimeter a full liquid argon (LAr) calorimeter has been considered. It is a sampling calorimeter with lead and stainless steel as absorber materials. Such a calorimeter is intrinsically non-compensating, i.e. it has a different response to the hadron and to the electromagnetic component of a hadron shower.

To improve the e/h ratio as well as the energy resolution a weighting technique has been proposed and is used now for the LAr calorimeter of the H1 detector at HERA.

The aim of this work is to study the possible application of the H1 weighting technique to the ATLAS full LAr calorimeter. The main questions, which have been addressed, are the jet energy resolution and the requirements on the transverse and longitudinal granularity of the calorimeter.

2 H1 Weighting Method

A hadron shower consists of an electromagnetic component due to π^0 production and a hadron component. To correct for the different calorimeter response to these components they should be weighted using different coefficients. Actually the hadron and electromagnetic components of hadron showers overlap and can be separated only statistically. The idea of the H1 weighting technique is to use for the separation the amount of energy deposited in an individual calorimeter cell: locally an electromagnetic shower deposits more energy than a hadron shower.

According to the H1 method the total energy in a calorimeter is defined as

$$E = \sum_i [a_1 + a_2 \exp(-a_3 \frac{E_i^{had}}{V_i^{had}})] E_i^{had} + \sum_i [b_1 + b_2 \exp(-b_3 \frac{E_i^{em}}{V_i^{em}})] E_i^{em}, \qquad (1)$$

where the summing is done over all cells of the calorimeter (separately for the hadron and electromagnetic parts of it), a_k and b_k are calibration parameters, $E_i^{had}(E_i^{em})$ is the visible energy deposited in the i^{th} cell of a hadron (electromagnetic) calorimeter, V_i is the volume of the corresponding cell. The exponential terms in this formula allow to diminish the relative contribution of large energy depositions in small volumes, which are connected mostly with the π^0-component of hadron showers.

Using real or simulated calorimeter data it is possible to determine the calibration parameters by minimizing the following functional

$$F = \sum_{j=1}^{N} (E_j - E_0)^2, \qquad (2)$$

where the summing is done over all events (from 1 to N), E_j is the energy in the calorimeter, defined by equation 1, and E_0 is the initial energy. Data of the whole calorimeter should be used to determine the calibration parameters. In the case of leakage the obtained parameters are distorted and the energy resolution and the response are degradated.

As it was shown by the H1 calorimeter group (see Ref. [1, 2]), such a technique gives an effective e/h ratio of one and a good hadron energy resolution.

3 Simulation Procedure

Studies of the application of the H1 weighting technique for the ATLAS LAr calorimeter are based on detailed Monte Carlo simulations. The electromagnetic and hadron calorimeters are described in the SLUG/GEANT framework together with the inner tracker, coil and cryostat as it is proposed in the Letter of Intent of ATLAS [3].

The electromagnetic calorimeter is a lead/iron LAr sampling calorimeter with an internal structure given in the TGT proposal [4]. Its thickness is 27 radiation lengths or 1.9 interaction lengths (λ). It consists of 22 longitudinal layers. For the present studies they are combined into 5 segments in a ratio of 2:2:8:8:2. A transverse size of a calorimeter cell is $\Delta\eta \times \Delta\varphi = 0.02 \times 0.02$. The hadron calorimeter is assumed to be an iron LAr sampling calorimeter with a total thickness of 12 λ. However for the present studies only an 8 λ calorimeter is used with 8 longitudinal segments. The thickness of each segment is one interaction length. A transverse granularity of the hadron calorimeter is $\Delta\eta \times \Delta\varphi = 0.04 \times 0.04$. Both calorimeters cover a pseudorapidity interval of $|\eta| < 1.2$. This configuration is refered to in the following as the standard one.

The PYTHIA program [5] is used to generate events with two light quark jets at central pseudorapidity ($\eta \simeq 0$). Four different samples of events (500 events in each) are generated with a single jet energy of 50, 100, 500 and 1000 GeV, corresponding to a total energy in the event of $E_0 = 100$, 200, 1000 and 2000 GeV respectively.

The detector response to the final state particles from jets is simulated using the program GEANT 3.14 [6] with the GHEISHA code for shower development.

According to the procedure, described in Sec. 2, six calibration parameters are determined for each sample of events separately. It should be stressed that the energy depositions in all cells of the electromagnetic and hadron calorimeters are used to determine these parameters. The total energy distributions after applying the H1 weighting method are shown in Fig. 1.

Gaussian curves are fitted to these energy distributions in an interval of $\pm 3\sigma$ around the peak value. The mean value E and the standard deviation σ from this fit are used to calculate the resolution. The energy dependence of the resolution is fitted by two functions:

$$\frac{\sigma}{E} = \frac{A}{\sqrt{E_0}} + B \qquad \text{(linear sum)}, \tag{3}$$

$$\frac{\sigma}{E} = \frac{A}{\sqrt{E_0}} \oplus B \qquad \text{(square sum)} \tag{4}$$

with a sampling term A and a constant term B. The energy dependence of the resolution together with the results of the fit by these functions are shown in Fig. 2. The results of the fit are also presented in Tab. 1.

Fitting function	Sampling term A, %	Constant term B, %	χ^2/NDF
$A/\sqrt{E_0} + B$	36.0 ± 2.2	1.16 ± 0.10	2.4
$A/\sqrt{E_0} \oplus B$	44.9 ± 1.7	1.71 ± 0.09	4.8

Table 1: The results of the fit of the energy dependence of the resolution.

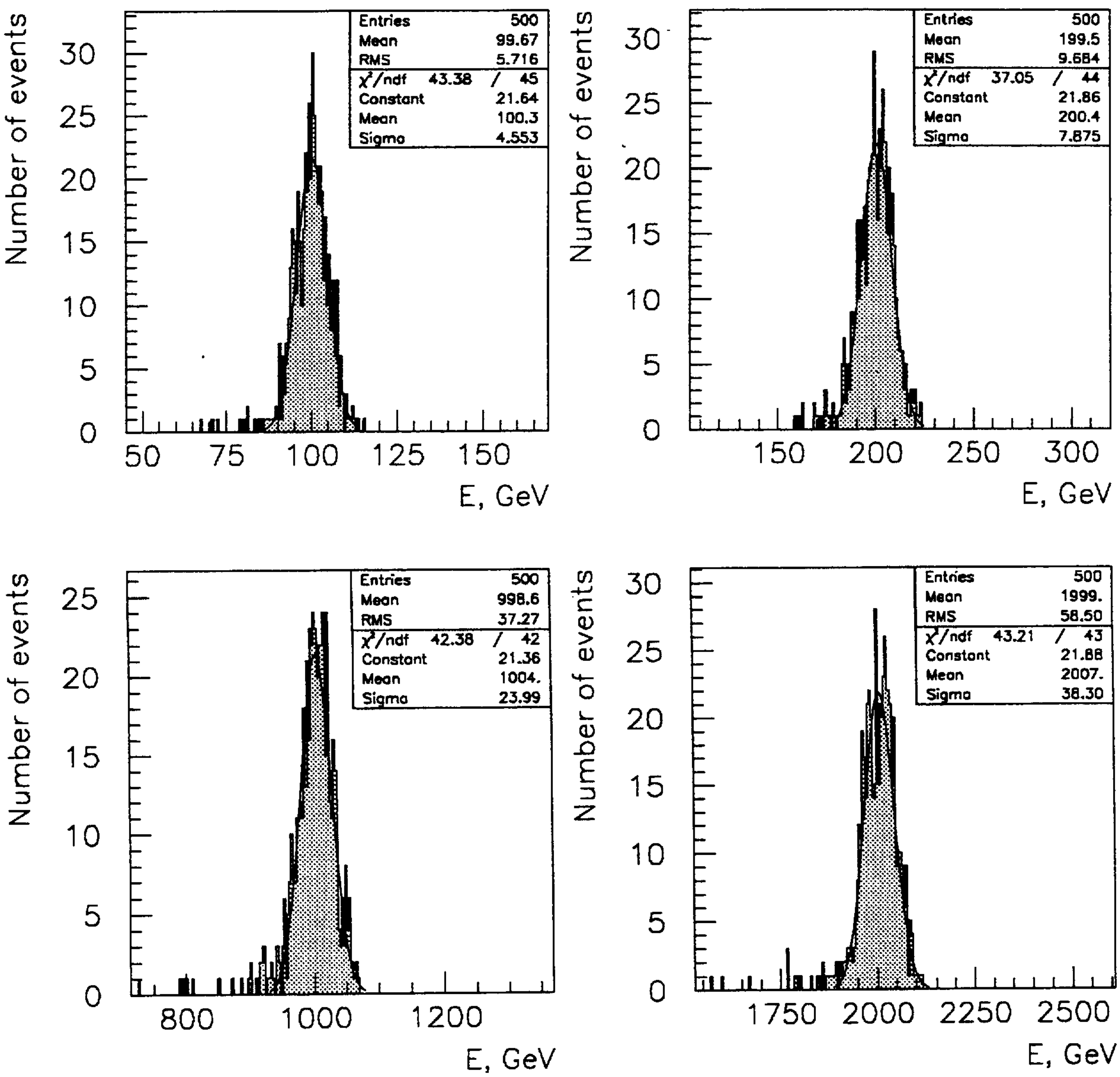

Figure 1: The energy distributions after the H1 weighting for four energy samples.

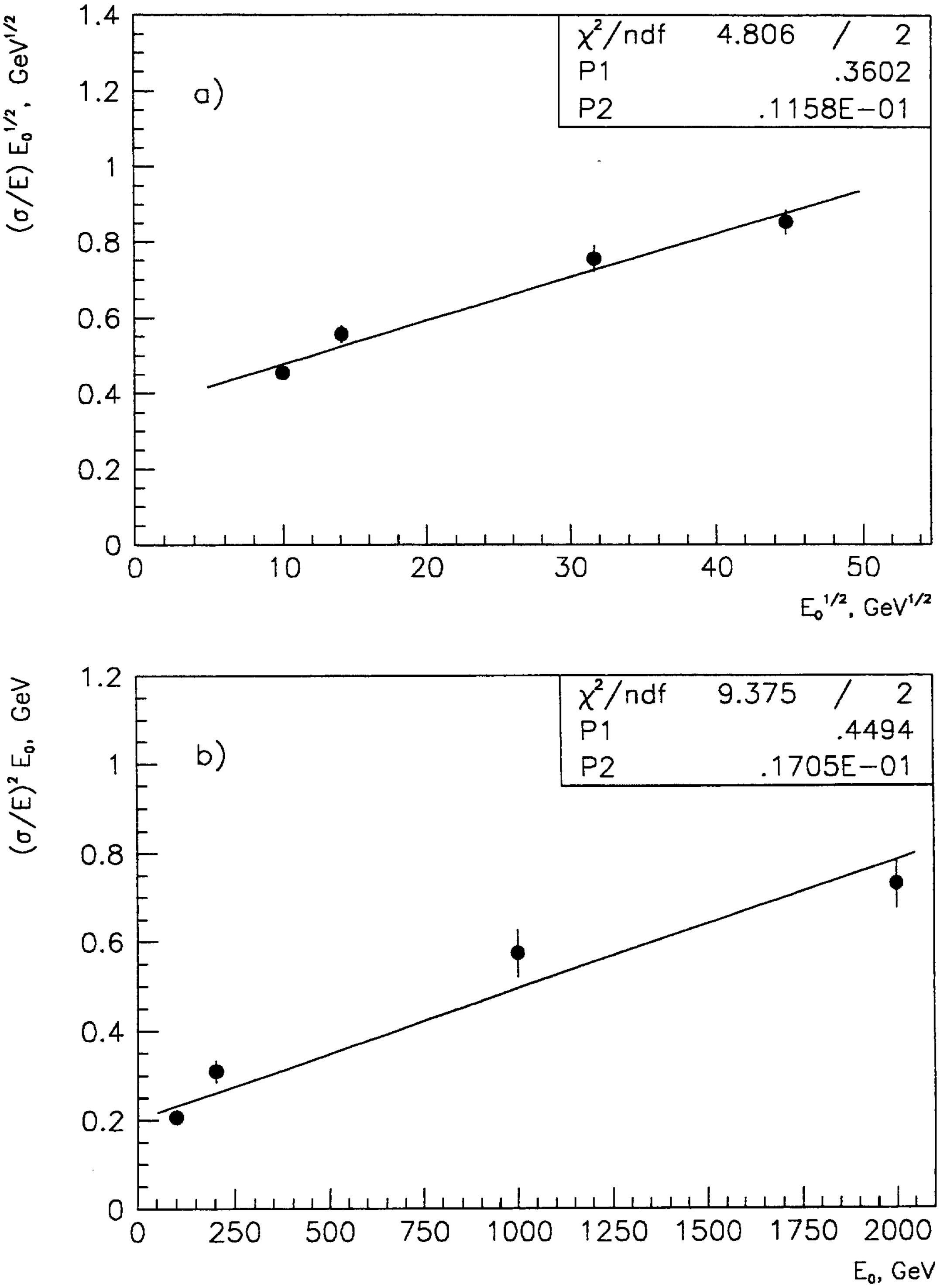

Figure 2: The energy dependence of the resolution with the linear sum fit (a) and with the square sum fit (b).

4 Requirements on the Calorimeter Structure

The results, presented in the previous section, are obtained for the standard configuration of the calorimeter. Requirements, which the H1 weighting technique can impose on the transverse and longitudinal granularity of a calorimeter as well as on its total depth, are analysed in this section. The criterion for these requirements is the optimization of the energy resolution.

4.1 Total Depth of the Calorimeter

The procedure, described in Sec. 3, is repeated for the calorimeter configuration with different number of longitudinal segments in the hadron part of the calorimeter. The thickness of each segment is one interaction length. The number of segments varies from 4 to 12. The total depth of the calorimeter (electromagnetic and hadron) varies from 3.9 to 13.9 λ.

In Fig. 3 the energy resolution of the calorimeter as a function of its total depth is shown. In the interval studied the resolution does not depend on the total depth of the calorimeter for events with 50 and 100 GeV jets. For events with higher energy jets the resolution degradates significantly with decreasing depth. The depth dependences of the sampling and constant terms are presented in Fig. 4. The obtained data show that it is sufficient to have a calorimeter with the depth of $\sim$ 10 λ. The resolution does not improve for thicker calorimeters.

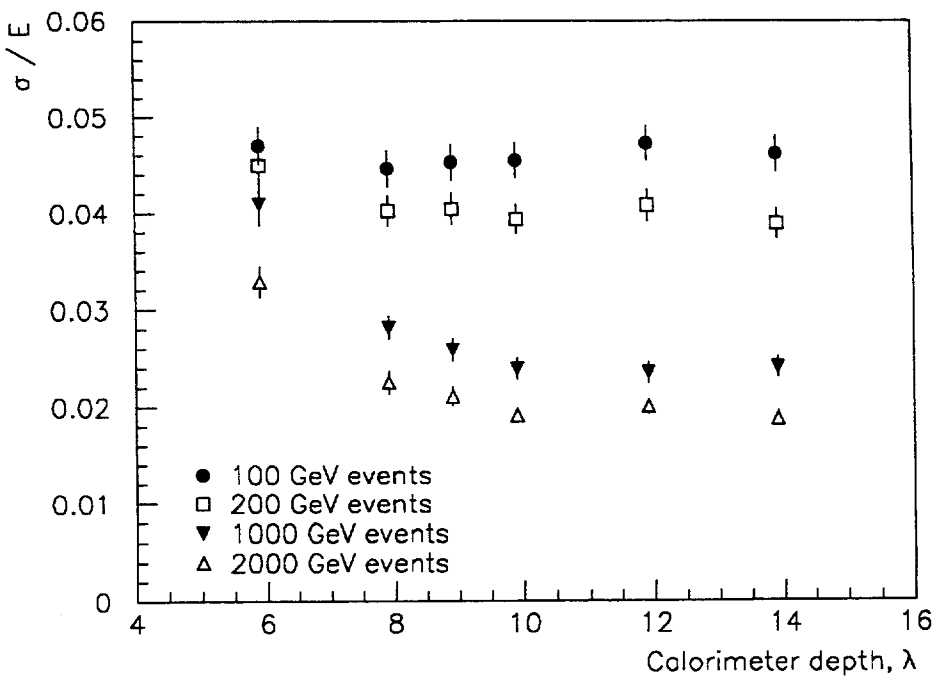

Figure 3: The energy resolution as a function of the total depth of the calorimeter for four energy samples.

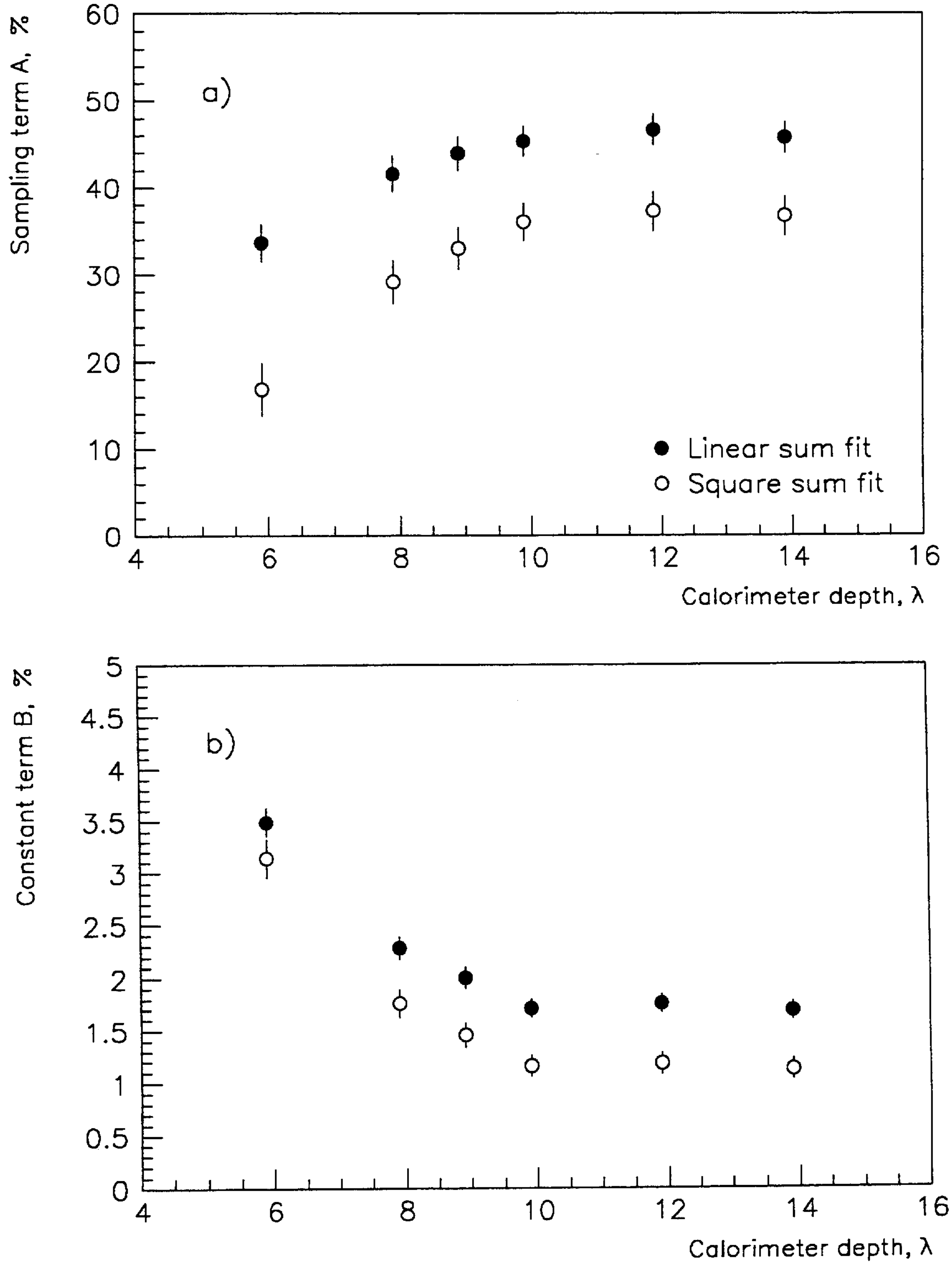

Figure 4: The sampling (a) and constant terms (b) as functions of the total depth of the calorimeter for two fitting functions.

4.2 Granularity of the Hadron Calorimeter

In order to study the sensitivity on the granularity of the hadron calorimeter the following configuration is used. The electromagnetic calorimeter has the standard structure, as it is described in Sec. 3. The total depth of the hadron calorimeter is 8 λ.

At first, the dependence of the resolution on the longitudinal segmentation is studied. The transverse size of a hadron cell is standard: $\Delta\eta \times \Delta\varphi = 0.04 \times 0.04$. The thickness of a segment varies from 0.5 to 8 λ (the corresponding number of longitudinal segments in the hadron calorimeter varies from 16 to 1). The energy resolution as a function of the thickness of a hadron segment is shown in Fig. 5. For all energies the resolution is found to be independent on the longitudinal segmentation of the hadron calorimeter.

The analogous studies are done for the transverse granularity of the the hadron calorimeter. Longitudinally it consists of 8 segments. The transverse size of a cell is varied between 0.02 and 0.20. The corresponding results are shown in Fig. 6. Also in this case no dependence of the resolution on the transverse granularity is found.

In order to check a boundary case, the whole procedure is repeated with the calibration parameters a_2 and a_3 in equation 1 set to zero. Effectively this means that the hadron calorimeter is treated as one block without any segmentation: neither longitudinal nor transverse. The obtained values and error intervals of the resolution are shown in Fig. 5 and 6 as lines for the corresponding energy samples. The results for the different granularities are consistent with the bounadary case. This means, that the H1 weighting technique is not very sensitive to the detailed segmentation of the hadron calorimeter.

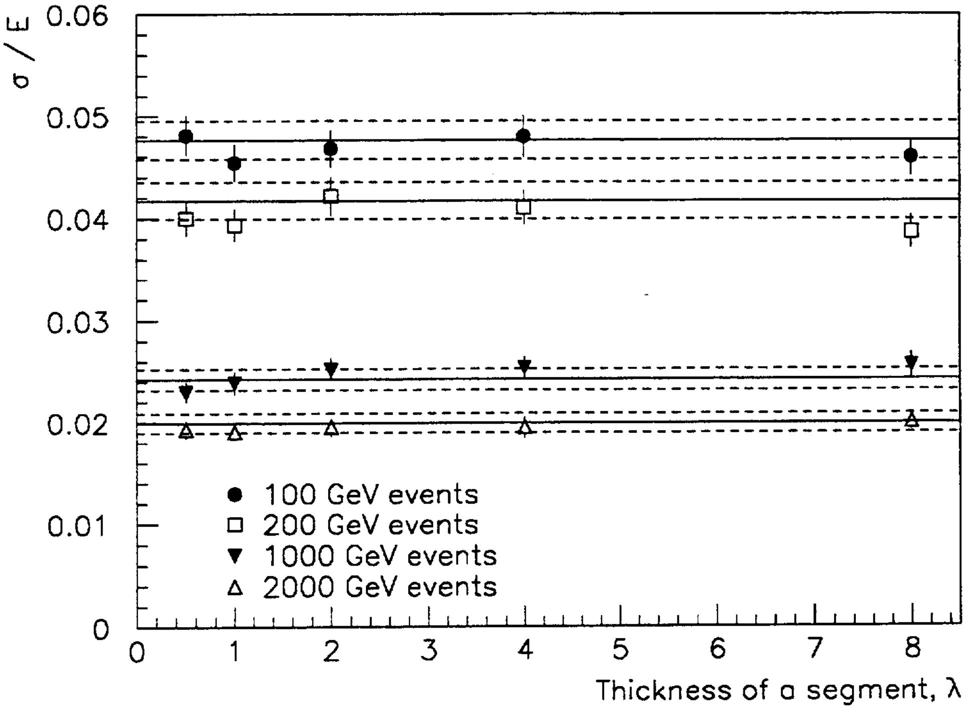

Figure 5: The energy resolution as a function of the thickness of a longitudinal segment of the hadron calorimeter for four energy samples. The lines mark the values and error intervals for the corresponding samples in the case of the hadron calorimeter without any segmentation.

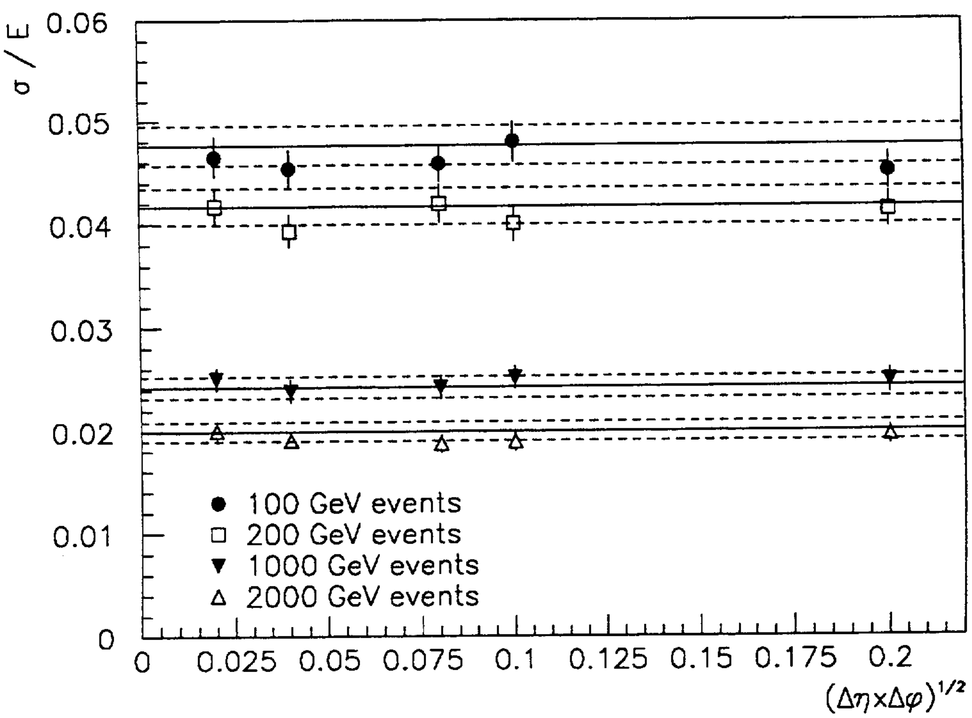

Figure 6: The energy resolution as a function of the transverse size of a cell of the hadron calorimeter for four energy samples. The lines mark the values and error intervals for the corresponding samples in the case of the hadron calorimeter without any segmentation.

4.3 Granularity of the Electromagnetic Calorimeter

The result of the previous subsection can be understood, if one takes into account the characteristic behaviour of longitudinal profiles of hadron showers. They have a very sharp peak around the first interaction length because of local depositions of the electromagnetic energy due to π^0 production in the first interaction. For the proposed calorimeter such a peak is located in the middle of the electromagnetic calorimeter. Therefore the H1 weighting technique should be more sensitive to the granularity of the electromagnetic calorimeter.

To study this, the whole procedure is repeated for different transverse and longitudinal granularities of the electromagnetic calorimeter. The hadron calorimeter in these studies is considered as a block without any segmentation (the calibration parameters $a_2 = a_3 = 0$).

In Fig. 7 the energy resolution is shown for different longitudinal structures of the electromagnetic calorimeter, when the number of longitudinal segments varies from 9 to 1. The transverse size of an electromagnetic cell in this case is always 0.02×0.02. For all energy samples the resolution does not depend on the longitudinal structure of the electromagnetic calorimeter.

The dependence of the resolution on the transverse size of an electromagnetic cell for the case of the standard longitudinal segmentation (5 segments) are shown in Fig. 8. The resolution degrades slightly with increasing cell size. It becomes approximately 10 % worse if the cell size changes from 0.02 to 0.10 for all energy samples.

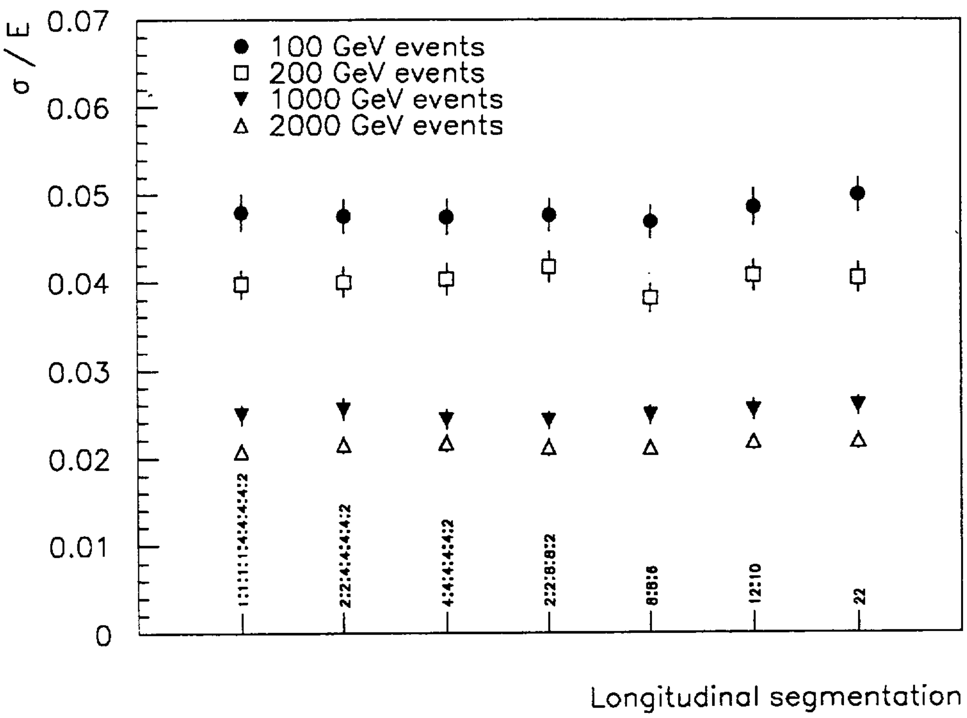

Figure 7: The energy resolution for different longitudinal structures of the electromagnetic calorimeter for four energy samples.

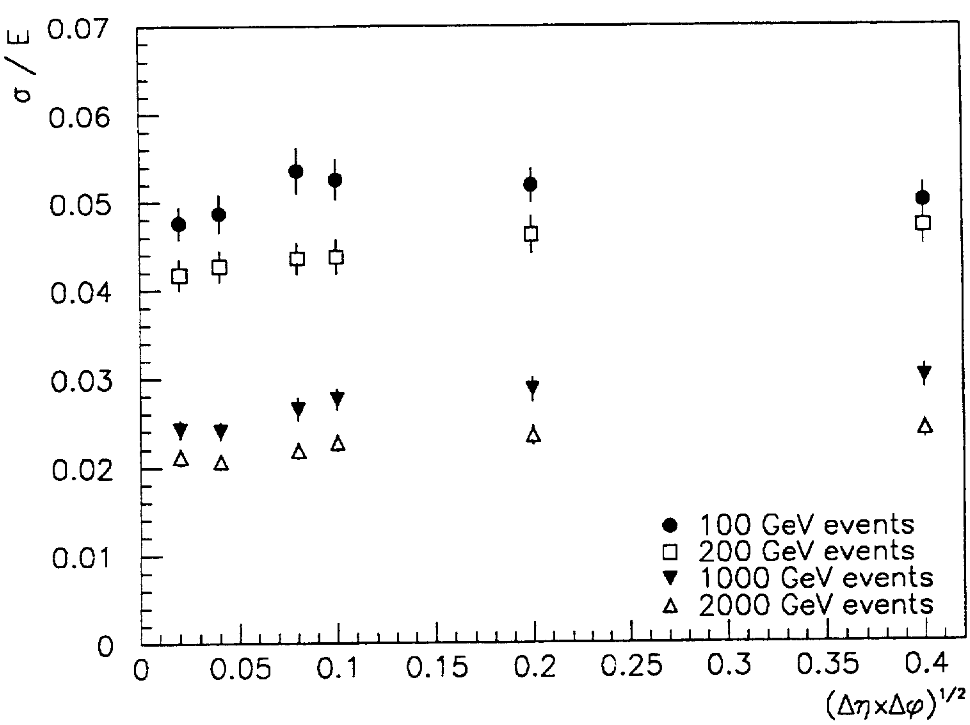

Figure 8: The energy resolution as a function of the transverse size of a cell of the electromagnetic calorimeter for four energy samples.

4.4 Comparison of Weighting Methods

Before making final conclusions about the possible granularity of the calorimeter the comparison of the results, obtained with different weighting procedures, is done. The following procedures are used.

- The H1 weighting technique, applied to the calorimeter with the standard segmentation (as it is described in Sec. 3).
- The H1 weighting technique, applied to the calorimeter with the hadron part without any segmentation (the calibration parameters $a_2 = a_3 = 0$).
- The H1 weighting technique, applied to the calorimeter with the electromagnetic part without any segmentation (the calibration parameters $b_2 = b_3 = 0$).
- The method of linear weighting of the energy depositions in the longitudinal layers. According to this method the total energy in a calorimeter in the j^{th} event is defined as

  $$E_j = \sum_{i=1}^{13} c_i E_{ij},$$

  where c_i are weighting parameters, E_{ij} is the energy, deposited in the i^{th} longitudinal segment. The number of such segments is 13 (5 in the electromagnetic and 8 in the hadron parts of the calorimeter). The weighting parameters are determined by minimization of the corresponding functional (see formula 2), which can be done analytically by solving the system of 13 linear equations:

  $$\sum_{i=1}^{13} c_i \sum_{j=1}^{N} (E_{ij} E_{kj}) = E_0 \sum_{j=1}^{N} E_{kj}.$$

- The method of the linear weighting of the total energies, deposited separately in the electromagnetic and in the hadron calorimeter. This method is equivalent to the H1 weighting with calibration parameters a_2, a_3, b_2 and b_3 set to zero.

In Fig. 9 the energy dependences of the resolution are shown for different weighting procedures. Here also the values of the degradation of the resolution in comparison with the standard H1 technique are presented as functions of the initial energy. The standard H1 method gives the best resolution. The linear weighting with only two parameters gives a resolution which is more than 1.5 times worse. The application of the H1 weighting technique for two boundary configurations of the calorimeter: 1) finely segmented electromagnetic calorimeter plus hadron calorimeter without segmentation and 2) electromagnetic calorimeter without segmentation plus finely segmented hadron calorimeter — gives very different results. In the first case the resolution is close to the best one (only ~ 5 % worse) for all energies. In the second case the H1 weighting does not work for low energies (the resolution is very close to that one obtained with the two parameter linear weighting). For high energies the H1 weighting technique allows to improve the resolution, using a segmented hadron calorimeter. The linear weighting, which takes into account information about the longitudinal development of showers, provides for events with jet energy above 100 GeV a

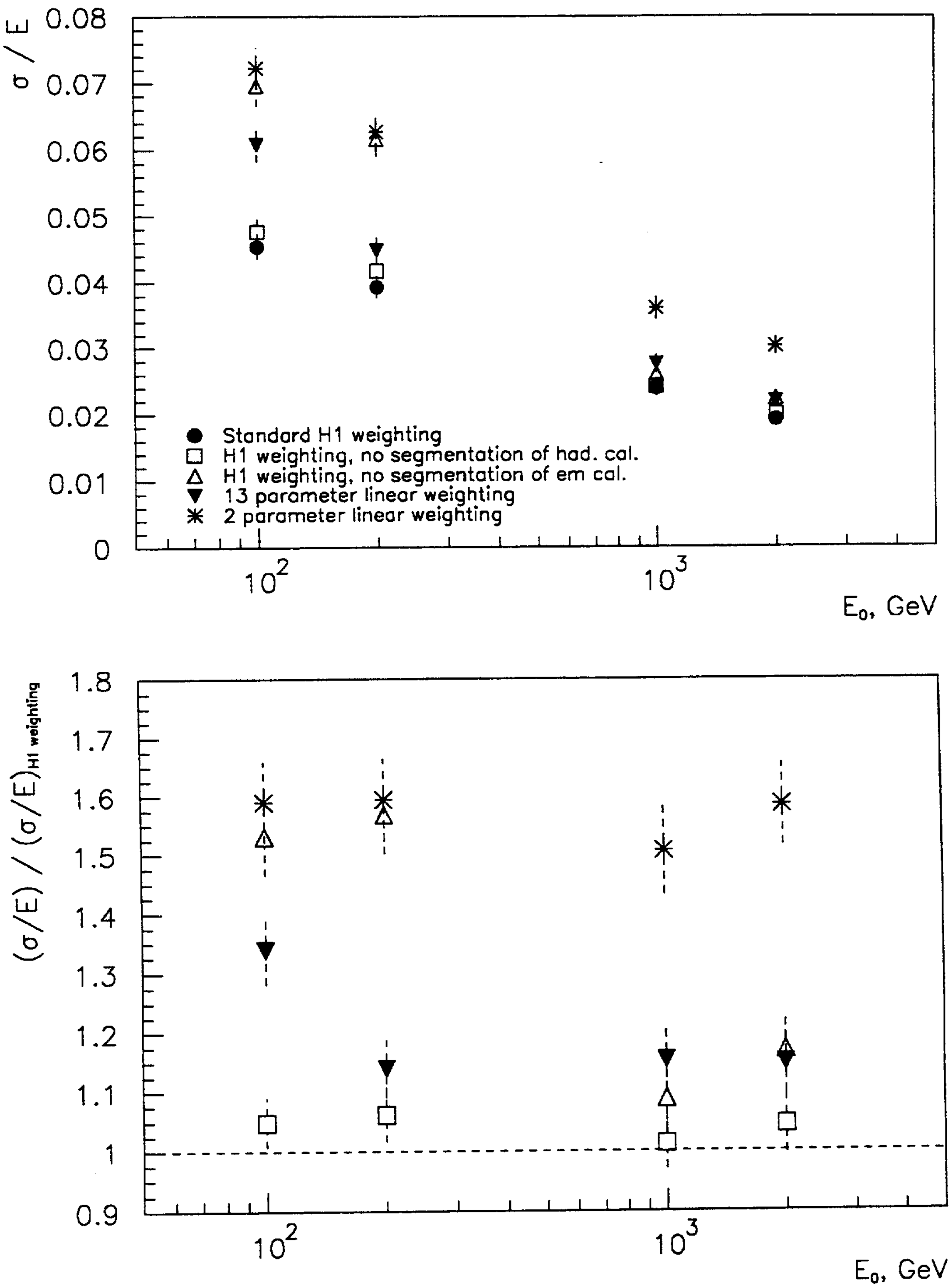

Figure 9: The energy depenedence of the resolution (a) and of the relative degradation of the resolution in comparison with the H1 method (b) for the different weighting techniques.

resolution $\sim$ 15 % worse, than the H1 weighting method. For events with jets of smaller energies this linear weighting method works rather badly.

The results, obtained in this section, allow the following conclusion. The application of the H1 weighting technique does not impose special requirements on the granularity of the calorimeter. The energy resolution is practically independent on both the longitudinal and transverse granularity of the hadron calorimeter as well as on the longitudinal segmentation of the electromagnetic calorimeter. Using the H1 weighting technique any reasonable transverse granularity of the electromagnetic calorimeter ($\sqrt{\Delta\eta \times \Delta\varphi}$ < 0.10) is accepatble.

5 Energy Resolution of Jets

The matter of interest is not the energy resolution of a total event, but the resolution of a single object: particle or jet. So the next step in these studies is to apply the obtained calibration parameters to the energy depositions in the electromagnetic and hadron cells, to reconstruct jets and to study their energy resolution.

The standard configuration of the calorimeter, described in Sec. 3, is used. For each event the energy depositions in cells of the calorimeter are weighted according to the formula 1. The obtained energies are summed up to form towers with a transverse size of a cell of the hadron calorimeter $\Delta\eta \times \Delta\varphi = 0.04 \times 0.04$. In each event this energy distribution in the (η, φ)-plane is used for jet finding. With the help of a clustering algorithm the jet axis (η_0, φ_0) is determined. To obtain the energy of a jet all energies in the towers within a circle with a centre (η_0, φ_0) and radius $R = \sqrt{(\eta - \eta_0)^2 + (\varphi - \varphi_0)^2}$ are summed. The studies are made for different values of the radius R (varying from 0.4 to 1.2). The following procedure is the same as it is described in Sec. 3: a Gaussian curve is fitted to the jet energy distribution, the energy resolution is determined and the energy dependence of the resolution is fitted using the linear sum and the square sum according to formulas 3 and 4.

The energy dependence of the jet resolution for the different radii of a jet as well as the energy dependence of the resolution for a total event are shown in Fig. 10. With decreasing jet size the resolution degradates in a way, that the slope of the energy dependence changes only slightly. The results of the fits of the energy dependence by formulas 3 and 4 are presented in Tab. 2 and 3 respectively. With decreasing jet radius from 1.2 to 0.4 the sampling term increases significantly from 34 to 58 % (from 47 to 71 %), whereas the constant term changes very little 2.2–2.5 % (3.3–3.5 %) for the linear sum fit (square sum fit). The ATLAS requirement to have a jet energy resolution of 50 $\%/\sqrt{E} \oplus 3$ % [3] can be achieved with the H1 weighting method for jets with radius $R > 0.8$.

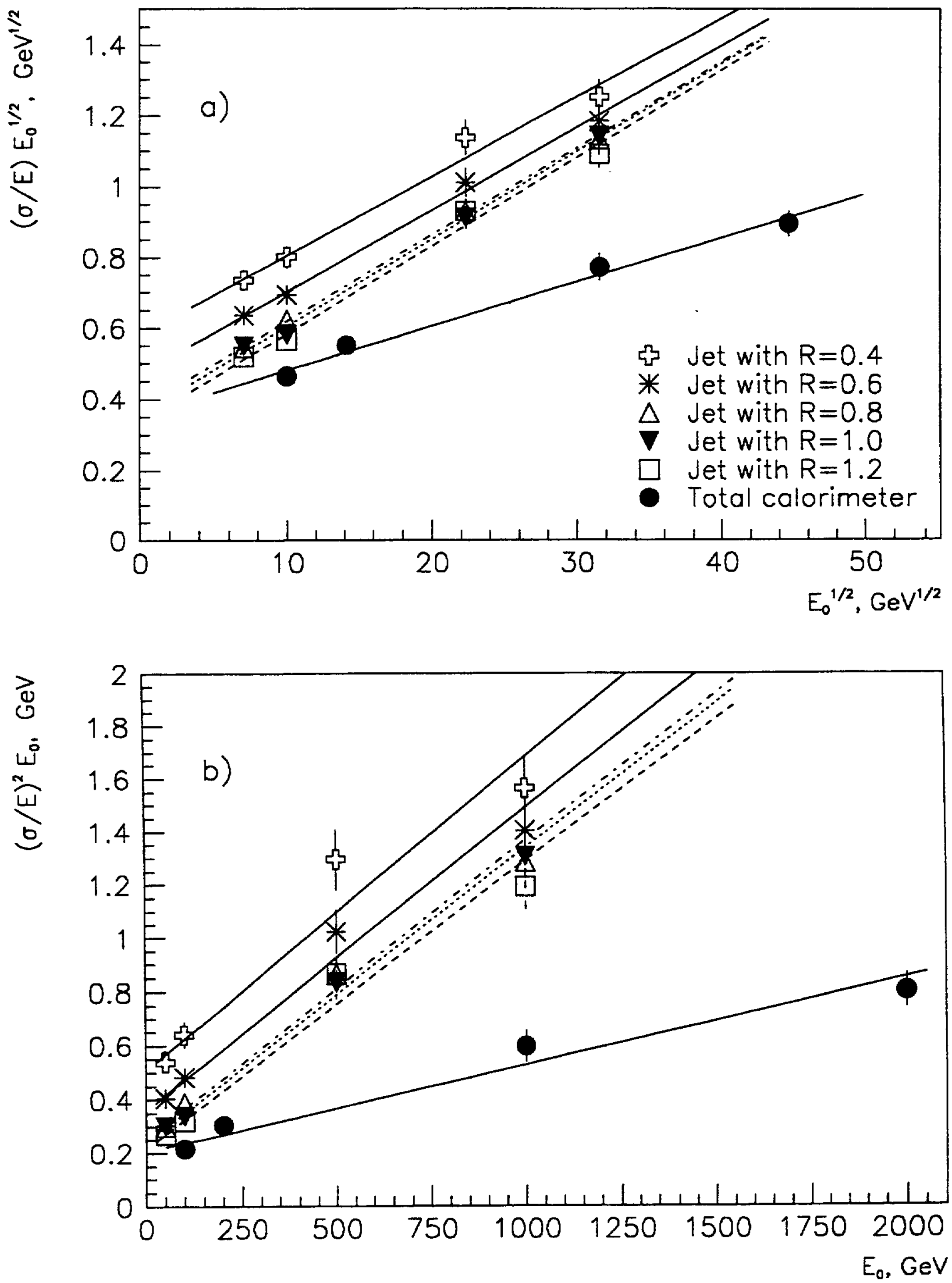

Figure 10: The energy dependence of the resolution for jets and for a total event with the linear sum fit (a) and with the square sum fit (b). E_0 is the jet energy and the initial energy of an event respectively.

Jet radius R	Sampling term A, %	Constant term B, %	χ^2/NDF
1.2	33.6 ± 2.2	2.47 ± 0.15	1.6
1.0	35.8 ± 2.3	2.46 ± 0.15	1.0
0.8	37.6 ± 2.3	2.43 ± 0.16	0.1
0.6	47.0 ± 2.7	2.31 ± 0.18	0.3
0.4	58.2 ± 3.3	2.22 ± 0.21	0.9

Table 2: The results of the fit of the energy dependence of the resolution by the linear sum formula 3.

Jet radius R	Sampling term A, %	Constant term B, %	χ^2/NDF
1.2	46.6 ± 1.8	3.29 ± 0.12	2.6
1.0	49.1 ± 1.9	3.31 ± 0.11	0.5
0.8	50.6 ± 1.9	3.34 ± 0.13	1.4
0.6	59.9 ± 2.1	3.37 ± 0.16	1.2
0.4	71.2 ± 2.5	3.45 ± 0.19	2.3

Table 3: The results of the fit of the energy dependence of the resolution by the square sum formula 4.

6 Conclusions

The possibilities of the H1 weighting technique, which is a software tool for improving the e/h ratio for intrinsically non-compensating calorimeters, have been studied in its application for the ATLAS full LAr option of the calorimeter. It has been shown, that this technique allows to achieve an energy resolution of 36 $\%/\sqrt{E}$ + 2.5 % (linear sum), 49 $\%/\sqrt{E} \oplus 3.3$ % (square sum) for jets with a radius of R=1.0. The application of the H1 weighting technique does not impose serious requirements on a granularity of the electromacnetic or the hadron calorimeter. Using the H1 weighting method any reasonable longitudinal and transverse segmentation, proposed for the ATLAS full LAr calorimeter, is acceptable.

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