# Modified dark matter

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Modified dark matter (MDM, formerly known as MoNDian dark matter) is a phenomenological model of dark matter, inspired by quantum gravity. We review the construction of MDM by generalizing entropic gravity to de-Sitter space as is appropriate for an accelerating universe (in accordance with the ΛCDM model). Unlike cold dark matter models, the MDM mass profile depends on the baryonic mass. We successfully fit the rotation curves to a sample of 30 local spiral galaxies with a single free parameter (viz., the mass-to-light ratio for each galaxy). We show that dynamical and observed masses agree in a sample of 93 galactic clusters. We also comment on strong gravitational lensing in the context of MDM.

*Keywords*: Modified dark matter model; observational tests; flat galactic rotation curves; dynamical masses of clusters.

## 1. Introduction and Summary

The cold dark matter (CDM) model successfully explains several astrophysical phenomena. These include flat galactic rotation curves, gravitational lensing, elemental abundances from big bang nucleosynthesis, and the power spectrum of cosmic microwave background anisotropies. This consistency has led to the widespread acceptance of the ΛCDM paradigm, in which the Universe also exhibits a cosmological constant Λ. CDM does, however, have remaining tensions with observations, especially on $\lesssim$ Mpc scales. These include inconsistency with the observed asymptotic velocity-mass ($v^4 \propto M$) scaling in the Tully-Fisher relation[1].

Efforts have been made to construct theories that better match observations on galactic length scales than CDM. The most prominent of these is modified Newtonian dynamics (MOND)[2] proposed by Milgrom. In MOND the equation of motion is $F = ma\mu(x)$, such that the smooth interpolating function $\mu(x) = 1, x$ for $x \gg 1$

and $x \ll 1$ respectively, where $x \equiv a/a_c$ with the critical acceleration $a_c$ being a tunable parameter, found to be numerically related to the speed of light $c$ and the Hubble parameter $H$ as $a_c \approx c\,H/(2\,\pi) \sim 10^{-8}\mathrm{cm/s}^2$. A favorite interpolating function is given by $\mu(x) = \dfrac{x}{(1+x^2)^{1/2}}$ which we will use in section 4. For a given source mass $M$, we have $a = a_N \equiv GM/r^2$ when $a \gg a_c$, while $a = \sqrt{a_c a_N}$ when $a \ll a_c$, where $a_N$ is the usual Newtonian acceleration without dark matter. It is readily shown that on the outer edges of a galaxy where gravity is weak, MOND yields asymptotically flat rotation curves, and $v^4 \propto M$, the Tully-Fisher relation as observed. MOND however struggles to reproduce observations at cluster and cosmological length scales[3]. Hence, CDM is usually preferred over MOND, with efforts ongoing to reconcile CDM with observations on $\lesssim$ Mpc scales.

Modified dark matter (MDM)[4] is a new form of dark matter quantum that theoretically behaves like cold dark matter (CDM) at cluster and cosmic scales, and naturally accounts for the scaling usually associated with MOND at the galactic scales. The latter feature explains why MDM was originally known as MONDian dark matter[4]; but in view of the confusing connotation to which this nomenclature has probably given rise and in order not to distract from the fact that the model *is* a model of dark matter, we have settled on the name "modified dark matter" (to distinguish it from cold dark matter.) MDM is a phenomenlogical model inspired by quantum gravity. We recall its construction in section 2. In section 3 we provide the first observational test of MDM by fitting rotation curves to a sample of 30 local spiral galaxies ($z \approx 0.003$)[5] and we show that MDM is a more economical model than CDM. We test the MDM model at the cluster scale[5] in section 4 where we show that MDM is superior to MOND. In both the galactic and cluster scales, MDM fares well. Future work will include: 1. Gravitational lensing (can it distinguish MDM from CDM?) 2. Interactions of MDM (unusual particle phenomenology? The Bullet Cluster; how strongly coupled is MDM to baryonic matter and how does MDM self-interact?) 3. Tests at cosmic scales (acoustic oscillations measured in the CMB, simulations of structure formation?)

## 2. Constructing MDM

MDM is a phenomenological model of dark matter inspired by quantum gravity, based on a simple generalization of E. Verlinde's recent proposal of entropic gravity[6], which happens to provide a convenient framework for its construction[4]. Consider a particle with mass $m$ approaching a holographic screen at temperature $T$. Using the first law of thermodynamics to introduce the concept of entropic force $F = T\frac{\Delta S}{\Delta x}$, and invoking Bekenstein's original arguments[7] concerning the entropy $S$ of black holes, $\Delta S = 2\pi k_B \frac{mc}{\hbar}\Delta x$, we get $F = 2\pi k_B \frac{mc}{\hbar} T$. In a deSitter space with cosmological constant $\Lambda$, the net Unruh-Hawking temperature,[8] as measured by a non-inertial observer with acceleration $a$ relative to an inertial observer, is $T = \frac{\hbar}{2\pi k_B c}[\sqrt{a^2+a_0^2} - a_0]$,[9] where $a_0 \equiv \sqrt{\Lambda/3}$. Hence the entropic force (in deSitter space) is given by $F = m[\sqrt{a^2 + a_0^2} - a_0]$. For $a \gg a_0$, we have $F/m \approx a$ which gives $a = a_N$, the familiar Newtonian value for the acceleration due to the source $M$. But for $a \ll a_0$, $F \approx m\frac{a^2}{2\,a_0}$, so the terminal velocity $v$ of the test mass $m$ in a circular motion with radius $r$ should be determined from $ma^2/(2a_0) = mv^2/r$. In this small acceleration regime, in order to fit the galactic rotation curves as Milgrom did, we require $F \approx m\sqrt{a_N a_c}$, which, in turn, requires $a \approx \left(4a_N\, a_0^2 a_c\right)^{\frac{1}{4}} \approx \left(2a_N\, a_0^3/\pi\right)^{\frac{1}{4}}$, where we have noted that numerically $a_0 \approx 2\pi a_c$. From our perspective, MoND is a *classical* phenomenological consequence of *quantum* gravity (with the $\hbar$ dependence in $T \propto \hbar$ and $S \propto 1/\hbar$ cancelled out).[4]

Having generalized Newton's 2nd law, we[4] can now follow the second half of Verlinde's argument[6] to generalize Newton's law of gravity $a = GM/r^2$, by considering an imaginary quasi-local (spherical) holographic screen of area $A = 4\pi r^2$ at temperature $T$. Invoking the equipartition of energy $E = \frac{1}{2}Nk_BT$ with $N = Ac^3/(G\hbar)$ being the total number of degrees of freedom (bits) on the screen, as well as the Unruh temperature formula and the fact that $E = M_{total}c^2$, we get $2\pi k_BT = G\,M_{total}/r^2$, where $M_{total} = M + M'$ represents the *total* mass enclosed within the volume $V = 4\pi r^3/3$, with $M'$ being some unknown mass, i.e., dark matter. For $a \gg a_0$, consistency with the Newtonian force law $a \approx a_N$ implies $M' \approx 0$. But for $a \ll a_0$, consistency with the condition $a \approx \left(2a_N\, a_0^3/\pi\right)^{\frac{1}{4}}$ requires $M' \approx \frac{1}{\pi}\left(\frac{a_0}{a}\right)^2 M$ whence it follows that $F = ma_N\left[1 + \frac{1}{\pi}\left(\frac{a_0}{a}\right)^2\right]$. Thus dark matter indeed exists! And the MOND *force law* derived above, at the galactic scale, *is simply a manifestation of dark matter*!

## 3. Fitting galactic rotation curves with MDM mass profiles

In order to test MDM with galactic rotation curves, we fit computed rotation curves to a selected sample of Ursa Major galaxies given in Ref. 10. The sample contains both high surface brightness (HSB) and low surface brightness (LSB) galaxies. The rotation curves, predicted by MDM as given above by $F = m[\sqrt{a^2 + a_0^2} - a_0] = ma_N\left[1 + \frac{1}{\pi}\left(\frac{a_0}{a}\right)^2\right]$ along with $F = mv^2/r$ for circular orbits, can be solved for $a(r)$ and $v(r)$. We[5] fit these to the observed rotation curves as determined in Ref. 10, using a least-squares fitting routine. As in Ref. 10, the mass-to-light ratio $M/L$, which is our *only* fitting parameter for MDM, is assumed constant for a given galaxy but allowed to vary between galaxies. Once we have $a(r)$, we can find the MDM density profile by using $M' \approx \frac{1}{\pi}\left(\frac{a_0}{a}\right)^2 M$ to give $\rho'(r) = \left(\frac{a_c}{r}\right)^2 \frac{d}{dr}\left(\frac{M}{a^2}\right)$.

Rotation curves predicted by MDM for NGC 4217, a typical HSB galaxy, and NGC 3917, a typical LSB galaxy in the sample are shown in Fig. 1. (See Ref. 5 for the rotation curves for the other 28 galaxies.)

In these figures, observed rotation curves are depicted as filled circles with error bars, and for the two curves at the bottom, the dotted and dash-dotted lines show the stellar and interstellar gas rotation curves, respectively. The solid lines and dashed lines are rotation curves predicted by MDM and the standard cold dark

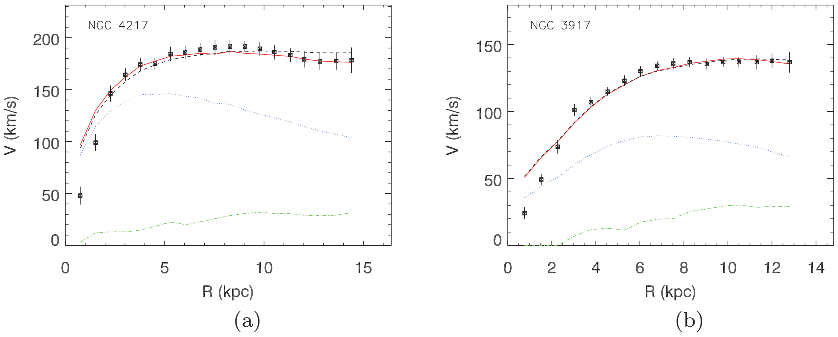



Fig. 1. Galactic rotation curves: (a) NGC 4217 (HSB); (b) NGC 3917 (LSB).

matter (CDM) paradigm respectively. For the CDM fits, we use the Navarro, Frenk & White (NFW)[11] density profile, employing *three* free parameters (one of which is the mass-to-light ratio.) It is fair to say that both models fit the data well; [a] but we remind the readers that while the MDM fits use only 1 free parameter, for the CDM fits one needs to use 3 free parameters. Thus the MDM model is a more economical model than CDM in fitting data at the galactic scale.[b]

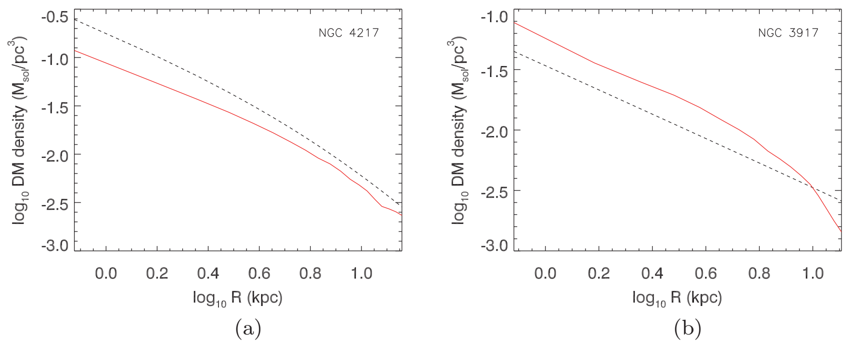



Fig. 2. Dark matter density profiles: (a) NGC 4217 (HSB); (b) NGC 3917 (LSB).

Shown in Fig. 2 are the dark matter density profiles predicted by MDM (solid lines) and CDM (dashed lines) for the HSB galaxy NGC 4217 and the LSB galaxy NGC 3917 in the sample respectively.

[a]We should point out that the rotation curves predicted by MDM and MOND have been found[5] to be virtually indistinguishable over the range of observed radii and both employ only 1 free parameter.

[b]Since MDM employs only the minimal number of free parameters, the speaker (YJN) called it the "minimal dark matter" model in his talk at MG14. But later he found out that the name had been used before for another model. His collaborators and he decided to change the name to "modified dark matter".

## 4. Testing MDM with Galactic Clusters

To test MDM with astronomical observations at a larger scale, we[5] compare dynamical and observed masses in a large sample of galactic clusters. First, let us recall that the MDM profile $M' = \frac{1}{\pi}\left(\frac{a_0}{a}\right)^2 M$ reproduces the flat rotation curves. But we expect that a more general profile should be of the form $M' = \left[\xi\left(\frac{a_0}{a}\right) + \frac{1}{\pi}\left(\frac{a_0}{a}\right)^2\right] M$, with $\xi > 0$ which ensures that $M' > 0$ when $a \gg a_0$.[c] For the more general profile, the entropic force expression is replaced by $F = ma_N\left[1 + \xi\left(\frac{a_0}{a}\right) + \frac{1}{\pi}\left(\frac{a_0}{a}\right)^2\right]$.

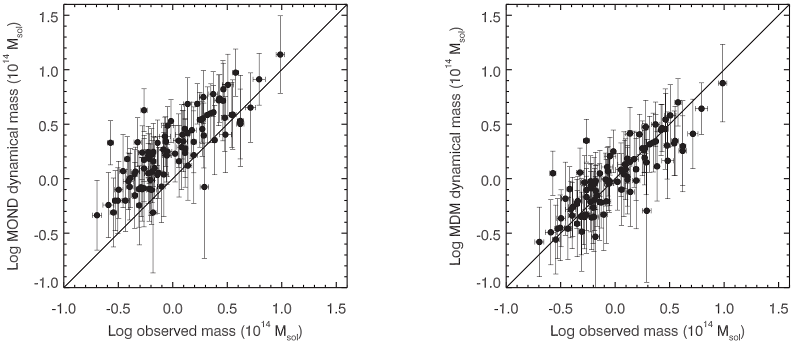
Sanders[12] studied the virial discrepancy (i.e., the discrepancy between the observed mass and the dynamical mass) in the contexts of Newtonian dynamics and MOND. We[5] have adapted his approach to the case of MDM. For his work, Sanders considered 93 X-ray-emitting clusters from the compilation by White, Jones, and Forman (WJF)[13]. He found the well-known discrepancy between the Newtonian dynamical mass ($M_{\rm N}$) and the observed mass ($M_{\rm obs}$): $\left\langle \frac{M_{\rm N}}{M_{\rm obs}} \right\rangle \approx 4.4$. Viewing MOND as a modification of inertia Sanders identified the MOND dynamical mass $M_{\rm MOND}$ as $\frac{G\,M_{\rm MOND}}{r_{\rm out}^2} = a\,\mu\left(\frac{a}{a_c}\right)$. With $\mu(x) = x/(1+x^2)^{1/2}$ as the interpolating function, it can be easily shown that $M_{\rm MOND} = M_{\rm N}/\sqrt{1+\left(\frac{a_c}{a}\right)^2}$. For the sample clusters, Sanders found $\langle M_{\rm MOND}/M_{\rm obs}\rangle \approx 2.1$.

For MDM, the observed (effective) acceleration is given by $a_{obs} = \sqrt{a^2 + a_0^2} - a_0$. Using the more general expression for the MDM profile, we have $a_{obs} = \frac{GM_{MDM}}{r^2}\{1 + \xi\left(\frac{a_0}{a}\right) + \frac{1}{\pi}\left(\frac{a_0}{a}\right)^2\}$. Recalling that $a_{obs} = GM_N/r^2$ for Newtonian dyanmics, we get $M_{\rm MDM} = \frac{M_{\rm N}}{1+\xi\left(\frac{a_0}{a}\right)+\frac{1}{\pi}\left(\frac{a_0}{a}\right)^2}$, for the dynamical mass for MDM.

In Fig. 3a and Fig. 3b, we show the MOND and MDM dynamical masses respectively against the total observed mass for the 93 sample clusters compiled by WJF. For MDM $\xi$ is used as a universal fitting parameter which we find to be $\xi \approx 0.5$. (For completeness we mention that we have used $\xi = 0$ when fitting galactic rotation curves in the previous section. But since now the galaxy cluster sample in our current study implies $\xi \approx 0.5$, we refit the galaxy rotation curves using $\xi = 0.5$ and found the fits are nearly identical with a reduction in mass-to-light ratios of about 35%.) To recapture, while Sanders found $\langle M_{\rm MOND}/M_{\rm obs}\rangle \approx 2.1$, we get $\left\langle \frac{M_{\rm MDM}}{M_{\rm obs}} \right\rangle \approx 1.0$. Thus the virial discrepancy is eliminated in the context of MDM! At the cluster scale, MDM is superior to MOND, as expected.

Finally we comment on strong gravitational lensing in the context of MDM and MOND. (Recall that strong lensing refers to the formation of multiple images of background sources by the central regions of some clusters.) It is known that

[c]We have neglected terms like $(a_0/a)^3$, $(a_0/a)^4$, ...., because they clearly do not lead to flat rotation curves in the regime $a \ll a_0$ and are thus excluded. Therefore, this MDM profile represents the most general profile. Note added: This discussion has since been superseded by more recent work by the authors. See arXiv:1601.00662 [astro-ph.CO].




(a) (b)

Fig. 3. Fit to galactic cluster data using (a) MONDian dynamics; (b) MDM dynamics.

the critical surface density required for strong lensing is $\Sigma_c = \frac{1}{4\pi}\frac{cH_0}{G}F(z_l, z_s)$, with $F \approx 10$ for typical clusters and background sources at cosmological distances. Sanders[12] argued that, in the deep MOND limit, $\Sigma_{MOND} \approx a_c/G$. Recalling that numerically $a_c \approx cH_0/6$, Sanders concluded that MOND cannot produce strong lensing on its own: $\Sigma_c \approx 5\Sigma_{MOND}$. On the other hand, MDM mass distribution appears to be *sufficient* for strong lensing since the natural scale for the critical acceleration for MDM is $a_0 = cH_0 = 2\pi a_c \approx 6a_c$, five to six times that for MOND.

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