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REVIEW

Thermodynamics as a framework for understanding gravitational dynamics and quantum gravity

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Abstract

We present a review of concepts of thermodynamic of spacetime that allows for an understanding of the gravitational dynamics encoding in it, discussing also the recovery of Weyl transverse gravity instead of General Relativity. We also discuss how these tools can provide some hints in the search of quantum gravity phenomenology, by introducing a formalism to analyze low-energy quantum gravity modifications in a completely general framework based on the thermodynamics of spacetime. For that purpose, we consider quantum gravity effects via a parametrized modification of entropy by an extra logarithmic term in the area, predicted in most of the different approaches to quantum gravity. These results provide a general expression for quantum phenomenological equations of gravitational dynamics.

Keywords Thermodynamics of spacetime · Phenomenological quantum gravity · Weyl transverse gravity

Contents

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1 Introduction

In recent decades, the intersection between black hole thermodynamics and quantum physics has provided significant insights into the nature of gravity. The relationship between thermodynamics and black hole physics was first proposed in the 1970s with the discovery that black holes have entropy [1, 2] and radiate at finite temperatures [3]. These very first proposals already lied at the intersection of gravitational dynamics and quantum physics, since the introduction of black hole temperature relies on quantum field theory. It was in the 1990s when a reverse approach started to be considered, where one derives the equations governing gravitational dynamics by thermodynamic considerations [4]. This shift in perspective allows for novel insights into the classical gravitational dynamics, but also might offer a window into quantum gravity.

Here, we review a conceptual perspective on our current research path [5–7], showing how thermodynamic principles can be applied to derive gravitational dynamics, focusing on the minimal requirements for that connection and their implications. It is worth remarking that this thermodynamic framework recovers Weyl transverse gravity, an alternative theory to general relativity. The recovery of general relativity requires extra assumptions. As we will explain, this result allows to follow new directions to explore the role of the cosmological constant in this framework. The understanding of this intersection and its connection with semiclassical gravity lead us to explore a new further step on the interface with quantum gravitational dynamics in a general framework to which the different approaches for quantum gravity must converge in a low-energy limit [7–9].

More specifically, the derivation of gravitational equations from thermodynamic principles involves local equilibrium conditions imposed on local, observer-dependent causal horizons, in which the Einstein equations are encoded. These horizons are observer-dependent and constructed at every point in spacetime. Let us highlight here that this approach requires quantum physics to define the temperature of local causal horizons, thus inherently linking to semiclassical gravitational dynamics instead of pure classical Einstein equations.

The paper is organized as follows. In Sect. 2 we briefly review the main ingredients in the derivation of Einstein equations from thermodynamic considerations, specifically from equilibrium condition on local horizon. In Sect. 3 we describe which are the minimal requirement for this approach and what do they imply. We then delve into the results of the approach, describing first in Sect. 4 the insights into the classical gravitational dynamics and in Sect. 5 the insights it provides on the quantum gravitational dynamics. We end up with a short discussion and future research paths in Sect. 6.

2 Einstein equations from equilibrium conditions on local horizons

Here we summarize the main steps and definitions involved in the derivation of Einstein equations using a concrete construction for the local horizon: the conformal Killing horizon associated with geodesic local causal diamonds (GLCD). These objects make reference to the causality domain associated with a geodesic ball, defining a Killing horizon as the boundary of the domain, and providing then a local horizon for a timelike observer.

A GLCD is constructed at any arbitrary point, P, of spacetime just from a unit time-like vector n^μ and a family of geodesics orthogonal to it and with parameter length l, that define a three-dimensional geodesic ball. The GLCD is then the spacetime region corresponding to the domain of dependence of this ball. By expanding the metric around the point P, one can find an expression for the area of the boundary $\mathcal{B}$ of the geodesic ball (an approximate two-sphere) [10, 11]

$$\mathcal{A} = 4\pi l^2 - \frac{4\pi}{9} l^4 G_{00}(P) + O(l^5), \tag{1}$$

where the 0 subindex implies a contraction with the time-like vector n^μ. The null boundary of the GLCD is a conformal Killing horizon that we will use to define the equilibrium conditions, and whose bifurcate spatial cross-section $\mathcal{B}$.

In order to define the equilibrium condition on that local horizon, $\delta S = 0$, one needs to define the different entropy terms composing that total entropy variation. On the one hand, there is an entropy contribution accounting for the entropy of the fields crossing the horizon, δS_m. This variation of the entropy has been computed either as an entanglement entropy for small perturbations from the approximate Minkowski vacuum in the diamond [11], or as a Clausius entropy flux across the horizon [12]. Although there are subtleties in the different definitions, in both cases one gets a entropy variation proportional to the energy-momentum tensor [5, 6]. Let us write here the expression found from the entanglement entropy for conformal fields resulting

$$\delta S_m = \frac{8\pi^2 k_B l^4}{15\hbar c} \delta \langle T_{00} \rangle. \tag{2}$$

On the other hand, there is another entropy term contribution coming from the vacuum fluctuations across the horizon, this defines an entanglement entropy, δS_e, associated with the entropy of the horizon and that is proportional to the area, $\delta S_e = \eta \delta \mathcal{A}$. This expression has been found independently of the dynamics of general relavitity [1, 13, 14], and later on, it was also found consistent with derivations from the dynamics [15–17]. In order to recover the Einstein equations, this proportionality constant has been interpreted in thermodynamics of spacetime to fit to $\eta = 1/4\hbar G$, adjusting the value of the entropy to the Bekenstein entropy for black hole horizons. The area of the horizon of the GLCD is determined by the area of the boundary of geodesic ball, so the variation of the area in this expression for the entropy is performed on the expression (1).

Finally, the equilibrium condition broken down into the two aforementioned contributions reads as $\delta S_m + \delta S_e = 0$. Expanded, this expression relates the values of T_{00} and G_{00} around P. Then, one needs to take into account that considering the validity of the Einstein Equivalence Principle [18], a GLCD can be constructed at every point of spacetime and for any unit time-like vector n^μ. So then, one can recover the full tensorial expression for the whole spacetime, yielding the traceless equations of motion [5, 6, 10]

$$R_{\mu\nu} - \frac{1}{4}Rg_{\mu\nu} = \frac{8\pi G}{c^4}\left(\langle T_{\mu\nu}\rangle - \frac{1}{4}\langle T\rangle g_{\mu\nu}\right). \tag{3}$$

Only by imposing the local conservation of the energy-momentum tensor $\nabla_\mu T_{\mu\nu} = 0$ one recovers the (semiclassical) Einstein equations, with a cosmological term that appears as a integration constant

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}\langle T_{\mu\nu}\rangle. \tag{4}$$

3 Minimal requirements for local equilibrium approaches

It is of interest to focus here in breaking down the different assumptions that has been introduced in the derivation of the semiclassical derivation of Einstein equations from equilibrium conditions, in order to understand the implications of this approach and which are the requirements to encode a gravitational dynamics into it. We can sum up the necessary requirements into two main assumptions:

1 Einstein Equivalence Principle: In order to define the temperature of local causal horizons we make use of the Unruh effect, which states that uniformly accelerated observers in flat spacetime perceive a thermal bath of particles. To extend this effect to a generic curved spacetime we need to assume the validity of the Einstein equivalence principle, allowing us to define, for a GLCD constructed in every regular point of spacetime, a local Minkowski vacuum. Compared to this vacuum, the quantum fields in the GLCD are perceived by uniformly accelerating observers to be in a thermal state corresponding to the Unruh temperature at the horizon. The Einstein equivalence principle also that spacetime curvature effects are negligible at small scales.

2 Entropy of horizons: We demand that the entropy of local causal observer-dependent horizons must match the Bekenstein expression for black hole entropy, i.e., that entropy is proportional to the area of the horizon and that the proportionality constant is universal throughout the spacetime. This assumption can be interpreted as generalizing Bekenstein entropy to arbitrary horizons, including causal horizons. Since a horizon prevents observers to access the causally separated region of spacetime, this lack of information accounts for Bekenstein entropy. It is widely accepted that this interpretation connects the entropy of the horizon with entanglement entropy [4, 10], although this is not the only possible conclusion [7].

These assumptions seem to imply: (1) We are already providing a notion of a metric in our construction, and in this way we impose a kinematic description of gravity, but without involving any dynamic. So, gravity, if emergent, comes from a description that requires a geometric formulation [19]. (2) Entropy, and by extension thermodynamics, is inherently observer-dependent in gravitational contexts. This seems to be consistent with the Unruh effect, where accelerating observers detect a thermal bath, and with the fact that quantum field theory density operators are observer-dependent [20].

The statement and uniqueness of these assumptions seems to be clear. However, they do not directly address the possible emergence of gravity and the interpretation of the entropy remains an open problem. Even so, these tools already offer insights on the classical structure of gravity and the phenomenological dynamics in the presence of quantum gravitational effects, without the need to introduce any emergence scenario or to have a microscopic understanding of the horizon entropy.

4 New insights into classical gravity

When one derives the gravitational dynamics from the local equilibrium approach without involving any desired input in the theory of gravity, we see that one does not fully recover general relativity but instead the traceless equations suggesting Weyl transverse gravity [21, 22] (WTG).

To understand this claim, let us briefly remind here some of the main features of this theory for our purposes. Weyl transverse gravity is a alternative theory of gravity that differs from general relativity in its symmetry group. It maintains invariance under volume-preserving diffeomorphisms (tranverse diffeomorphisims), restricting the full diffeomorphism invariance of general relativity to the diffeomorphisms that keep fixed the metric determinant $\sqrt{-g} = \omega$, where ω is a positive function translated into a spacetime volume element. But it also incorporates Weyl invariance, meaning that the theory is invariant under Weyl transformations (so it remains unchanged under local rescaling of lengths) $g'_{\mu\nu} = e^{2\sigma} g_{\mu\nu}$ [6].

One of the most appealing features of this theory is that while it is physically indistinguishable from general relativity, it provides a new status for the cosmological constant. Unlike in general relativity, where the cosmological constant is a fixed parameter in the Lagrangian, in WTG, it emerges as an integration constant. This difference has significant implications for the cosmological constant problem, providing a potential explanation for its small observed value by making Λ radiatively stable,[1] meaning it does not receive large quantum corrections [23].

Another aspect to highlight here is that the restriction of the diffeomorphism group to just the transverse diffeomorphisms in not restrictive enough to demand the divergence of the energy-momentum tensor is strictly zero. Instead, it can be proportional to the gradient of an arbitrary scalar function [24].

Now, we can see from the derivation of the gravitational dynamics from local equilibrium conditions that derived equations resemble Einstein's field equations but differ in key aspects. For instance, the energy-momentum tensor's divergence is not constrained to zero, as in general relativity, but only restricted to be proportional to the gradient of an arbitrary scalar function. Additionally, the cosmological constant appears as an integration constant rather than a fixed parameter, and the equations are then conformally invariant. These differences align better the derived equations with Weyl transverse gravity. We have seen also the consistency of this symmetry by

[1] Due to the fact that the vacuum energy does not couple to the gravitational field.

obtaining the expression of the entropy from Weyl invariant theories of gravity [25, 26]. We provide a complete picture of the connection of Weyl transverse gravity with thermodynamics in an upcoming work [27].

5 New insights into low-energy quantum gravity phenomenology

Upon understanding the encoding of the semiclassical gravitational dynamics, one can consider how a modification in the thermodynamics side that incorporates quantum-gravity effects in a general model-independent manner could offer some insight in a then modified gravitational dynamics which will encode also that modification.

Logarithmic corrections to the entropy of black holes and the entropy of causal horizons have emerged as a robust feature across many approaches to quantum gravity [28–39]. The form of this modified entropy looks then

$$S_e = k_B \mathcal{A}/(4l_P^2) + k_B C \ln \mathcal{A}/\mathcal{A}_0 + O(\mathcal{A}_0)/\mathcal{A}), \tag{5}$$

where $C \in \mathbb{R}$ is a dimensionless constant and A_0 is an arbitrary constant with the dimensions of the area.

This form of entropy correction is supported by string theory, loop quantum gravity, AdS/CFT correspondance, entanglement entropy computations and other frameworks [8]. The characterization of the different theories come from the different prediction of the value C, although being in all the approaches sharing the interesting feature of being an universal dimensionless parameter. This agreement suggests that the modified entropy is a universal feature of quantum gravity, reflecting fundamental aspects of spacetime quantization (i.e., the parameter C is often related to the microscopic degrees of freedom in a given theory) [7].

The introduction of a new modified entropy in the equilibrium condition suggests that low energy quantum gravitational effects can be captured by local equilibrium conditions, and that we may be able to derive a phenomenological gravitational dynamic equations.

When performing an analogous derivation from the new modified equilibrium conditions we can arrive, after some manipulation, to [8]

$$\begin{aligned} S_{\mu\nu} - \frac{Cl_{Pl}^2}{30\pi} S_{\mu\lambda} S_\nu^\lambda + \frac{Cl_{Pl}^2}{30\pi}\left(R_{\kappa\lambda}R^{\kappa\lambda} - \frac{1}{4}R^2\right) \\ g_{\mu\nu} = \frac{8\pi G}{c^4}\left(\delta\langle T_{\mu\nu}\rangle - \frac{1}{4}\delta\langle T\rangle g_{\mu\nu}\right), \end{aligned} \tag{6}$$

where $S_{\mu\nu} = R_{\mu\nu} - Rg_{\mu\nu}/4$ denotes the traceless part of the Ricci tensor. In this derivation we required the same assumptions to implement the approach than in the semiclassical case. Let us also remark, that this derivation also accounts for possible modifications of the Unruh temperature showing that they would not have any effect in the gravitational equations. This implies a modified theory of gravity that combines elements of quadratic gravity and scalar-tensor Einstein-Gauss-Bonnet gravity.

In addition, one can also realized that we have again obtained a traceless version of the modified gravitational equations, requiring an extra imposing on a restriction on the divergence of the energy-momentum tensor.

We can also check for the sake of consistency than we can also derive the linearized equations for gravitational dynamics, finding that they align with the corresponding linearized equation of quadratic gravity [9].

These corrections are suppressed by factors related to the Planck length, indicating their relevance at scales where quantum effects are non-negligible but not dominant, providing then a low-energy phenomenology of quantum gravity. Let us remark that these equations provide a modified dynamics that is it is also general, not attached to a particular solution but either to a possible approach to include quantum-gravity effects.

This result thus, on the one side, establish a dynamics that the various approaches need to recover. On the other side, it establishes a starting point to study phenomenological effects associated to particular solutions of the gravity theory [40–42] and to use them to possibly impose constraints on the allowed values of the parameter C.

6 Discussion

We have shown here, how the thermodynamic approach to gravity via the local equilibrium condition on causal horizons provide a powerful framework for understanding gravitational dynamics, offering insights into both classical and quantum regimes. It shows, on the one hand, that the gravitational dynamics recovered in this way correspond to Weyl transverse gravity, which has a different symmetry group. On the other hand, it provides phenomenological gravitational dynamics equations that encode the low-energy effects that need to be recovered from a quantum gravity approach and result in a modified picture of the different classical solutions.

The derivation of Weyl transverse gravity instead of general relativity also connects the spacetime thermodynamics approach to the treatment of the cosmological constant, providing a new framework to understand its current issues.

Notably, this approach does not need to rely in any framework of emergent gravity nor in any specific theory of quantum gravity. This feature not only improves the solidity of the approach, but also paves the road to explore the possibility of setting new constraints on these directions.

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Declarations

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