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Electromagnetohydrodynamics and wave transformation

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Updating of the classic MHD equations by using of full set of Maxwell equation with displacement current gave a possibility to describe hydrodynamic phenomena, together with electromagnetic ones by the same set of equations. These equations applied for study of propagation of different waves and of transformation between different kind of waves in the nonuniform media.

Dispersion equation is derived describing a propagation of a linear wave in a uniform magnetized plasma. It contains MHD, HD, Alfven and EM waves in the limiting cases, and some new types of behaviour in a general situation. It is shown, that transformation of MHD into EM wave may happen only in a highly magnetized plasma. The regions of parameters are found, where no waves exist, and only damping static perturbations may be present.

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1. Introduction

Powerful transient objects in different wave bands have been discovered in recent years. To explain these short (from ms to s), and very powerful events different models, galactic and extra-galactic, have been considered. One of popular model is based on the suggestion of transformation of the magnetized plasma blob, presumably a MHD shock wave, moving with relativistic speed, into electromagnetic pulse after collision with some obstacle, or at entering in media with different properties. Such transformation cannot be described in frame of classical MHD equations, without electromagnetic waves, which don't contain the displacement current, and don't include such transformation [1]. After simple updating of the classic MHD equations, namely, using of full set of Maxwell equation with displacement current [2, 3], we obtain a possibility to describe hydrodynamic phenomena, as well as electromagnetic ones in a conducting medium, like plasma, or liquid metals, separately, or in their interaction and transformations between their propagating waves. In the paper [4] the MHD equations with account of displacement current had been used for investigation of the wave turbulence in astrophysical phenomena.

Analysis of linear waves in the magnetized two-fluid, two-temperature ideal plasma, in 2-D MHD, with account of displacement current, had been done by [5] for non-relativistic motion. We consider equation of classic MHD with displacement current, with finite value of electrical conductivity. Its variability qualitatively change the behavior of waves, their propagation and transformation. Similar to [5] approach was used earlier in [6] for extended study of small-scale alfvenic structure in the Aurora. Numerical calculations in MHD with finite conductivity in presence of displacement current had been performed by [7] in application to dense Z-pinches.

The most famous jets from AGN (M 87), and QSO (3C 273) are given in Figs.1 and 2. The presence of magnetic field in these jets is shown in Fig. 3, where the polarization picture of the jet in M 87 is presented. In the matter outflowing with near-relativistic speed from the accretion disk around the supermassive black hole in the AGN, blobs are formed, and it may be accompanied by formation of a low-frequency electromagnetic wave.

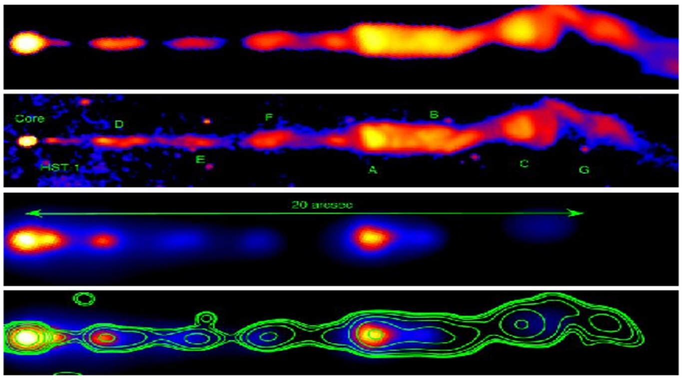

Figure 1: Observational data for the jet from the galaxy $M87$ in different wavelengths (adopted from [8]). The pictures correspond to observations by VLA ($14GHz$, 0.2", first), HST(F814W) (second) and Chandra (0.2", $0.2 - 8keV$, third and forth (adaptively smoothed) ones) telescopes.

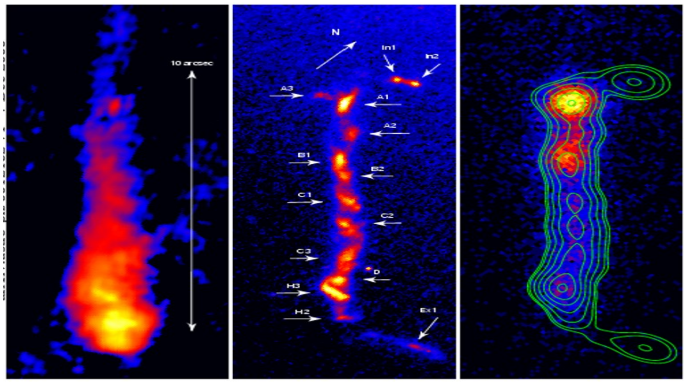

Figure 2: Observational data for the jet in the quasar $3C$ 273 in different wavelengths (adopted from [9]). The pictures correspond to observations by MERLIN ($1.647GHz$, left), HST(F622W) ($6170A$, central) and Chandra (0.1", right) telescopes.

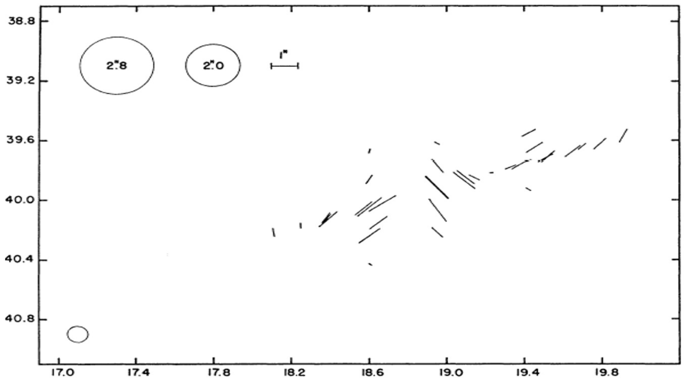

Figure 3: Polarization observations of $M87$ (adopted from [10]).

2. General picture

Consider a plasma flow with velocity field $\mathbf{u}$, which is moving in a region with a variable magnetic field $\mathbf{H}$ and density ρ. It is supposed, that plasma has a scalar conductivity σ. At neglecting the Hall effect and thermo-diffusion, the Maxwell equations and the Ohm's law are written as

$$\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t} = -\mathrm{rot}\mathbf{E}, \quad \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{j} = \mathrm{rot}\mathbf{H}, \quad \mathrm{div}\mathbf{H} = 0,$$
$$\mathrm{div}\mathbf{E} = 4\pi\rho_e, \quad \mathbf{j} = \sigma(\mathbf{E} + \frac{1}{c}[\mathbf{u}\times\mathbf{H}]), \tag{1}$$

where $\mathbf{E}$ is an electric field, ρ_e is a charge density, and $\mathbf{j}$ is an electric current density. Expressing $\mathbf{E}$ from the Ohm's law, inserting it into the first equation in (1), and expressing $\mathbf{j}$ from the second equation in (1), we obtain

$$\frac{\partial \mathbf{H}}{\partial t} = \mathrm{rot}[\mathbf{u}\times\mathbf{H}] - \mathrm{rot}\left(\frac{c\,\mathbf{j}}{\sigma}\right), \quad \mathbf{j} = \frac{c}{4\pi}\mathrm{rot}\mathbf{H} - \frac{1}{4\pi}\frac{\partial \mathbf{E}}{\partial t}. \tag{2}$$

By eliminating $\mathbf{j}$, we get

$$\frac{\partial \mathbf{H}}{\partial t} = \mathrm{rot}[\mathbf{u}\times\mathbf{H}] - \mathrm{rot}(\nu_m\,\mathrm{rot}\mathbf{H}) + \frac{c}{4\pi}\frac{\partial}{\partial t}\left(\mathrm{rot}\frac{\mathbf{E}}{\sigma}\right),$$
$$\text{where}\qquad \nu_m = \frac{c^2}{4\pi\sigma},\qquad \mathrm{rot}\frac{\mathbf{E}}{\sigma} = \frac{\mathrm{rot}\mathbf{E}}{\sigma} + \mathrm{grad}\left(\frac{1}{\sigma}\right)\times\mathbf{E}. \tag{3}$$

Assuming σ to be independent on time, we obtain the equation for the magnetic field in the form

$$\frac{\partial \mathbf{H}}{\partial t} = \mathrm{rot}[\mathbf{u}\times\mathbf{H}] - \mathrm{rot}(\nu_m\,\mathrm{rot}\mathbf{H}) - \frac{\nu_m}{c^2}\frac{\partial^2 \mathbf{H}}{\partial t^2} + \frac{\mathrm{grad}(\nu_m)}{c}\times\frac{\partial \mathbf{E}}{\partial t},$$
$$\text{where}\qquad \mathrm{rot}(\nu_m\,\mathrm{rot}\mathbf{H}) = -\nu_m\Delta\mathbf{H} + \mathrm{grad}(\nu_m)\times\mathrm{rot}\mathbf{H}. \tag{4}$$

The Maxwell equation for $\mathrm{rot}\mathbf{E}$ together with equations for the velocity $\mathbf{u}$, matter density ρ, entropy s, and equation of state for the pressure $P = P(\rho, T)$, should be added. In absence of viscosity and thermal conductivity these equations are written as

$$\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho\mathbf{u}) = 0, \tag{5}$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\nabla)\mathbf{u} = -\frac{1}{\rho}\nabla P + \frac{1}{c\rho}[\mathbf{j}\times\mathbf{H}], \tag{6}$$

$$\rho T\left[\frac{\partial s}{\partial t} + (\mathbf{u}\nabla)s\right] = \frac{\mathbf{j}^2}{\sigma}. \tag{7}$$

Using the expression for $\mathbf{j}$ from the Maxwell equation with a displacement current (2), we obtain together with a previous equation for $\mathbf{H}$, a set of equations for the fluid motion in fields $\mathbf{H}$ and $\mathbf{E}$

$$\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho\mathbf{u}) = 0, \tag{8}$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\nabla)\mathbf{u} = -\frac{1}{\rho}\nabla\left(P + \frac{H^2}{8\pi}\right) + \frac{(\mathbf{H}\nabla)\mathbf{H}}{4\pi\rho} - \frac{1}{4\pi c\rho}\left[\frac{\partial \mathbf{E}}{\partial t}\times\mathbf{H}\right], \tag{9}$$

$$\rho T\left[\frac{\partial s}{\partial t} + (\mathbf{u}\nabla)s\right] = \frac{\nu_m}{4\pi}(\mathrm{rot}\mathbf{H})^2 - \frac{2\nu_m}{4\pi c}\left(\mathrm{rot}\mathbf{H}\cdot\frac{\partial \mathbf{E}}{\partial t}\right) + \frac{\nu_m}{4\pi c^2}\left(\frac{\partial \mathbf{E}}{\partial t}\right)^2, \tag{10}$$

$$\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t} = -\mathrm{rot}\mathbf{E}. \tag{11}$$

For constant values of σ and ν_m in space the equation for $\mathbf{H}$ is written as

$$\frac{\partial \mathbf{H}}{\partial t} = \mathrm{rot}(\mathbf{u}\times\mathbf{H}) + \nu_m\Delta\mathbf{H} - \frac{\nu_m}{c^2}\frac{\partial^2 \mathbf{H}}{\partial t^2}, \tag{12}$$

In limiting cases of very small ($\nu_m \ll 1$) , and very large ($\nu_m \gg 1$) magnetic viscosity this equation is transferring into ideal MHD equation with $\mathbf{H}$ freezing for high conductivity (plasma, metals), and the equation for electromagnetic waves for very low conductivity (dielectric). It describes also an intermediate case with finite non-zero conductivity.

We have called this set of equation as MHDE [3], because of account of displacement current, having in mind to describe transformation of MHD shock wave into electromagnetic one, suggested by Yu. Lyubarskiy for explanation of strong short transient bursts. For constant matter parameters similar set of equations, under the name FMHD (F-Full), had been used earlier in the paper [**?**]. They included the displacement current for construction of the universal numerical scheme of MHD flow with arbitrary strong Alfven waves. Account of the displacement current was done also in few investigations, published during 2000-2021 years, where plasmic and electromagnetic waves in the non-collisional plasma have been considered, using kinetic approach.

3. Linear wave propagation in the uniform plasma with a constant magnetic field

Equations for linear waves, formed in the uniform plasma at rest, with a constant magnetic field, are obtained using a linearized form of previous equations. Arbitrary angle between the direction of wave propagation, and uniform magnetic field direction is used. Consider a uniform plasma at rest, at constant values of σ and ν_m, with parameters

$$\mathbf{u} = 0, \quad \rho = \rho_0, \quad P = P_0, \quad H_{x0} = H_0 = \text{const}, \quad H_{y0} = H_{z0} = E_{z0} = 0, \tag{13}$$

where coordinates "x" and "y" correspond to directions, which are parallel and perpendicular to the field H_0, respectively. Consider waves in this static model, which, by appropriate choice of the coordinate system, always propagate in the direction, perpendicular to "z" axis. In this situation the perturbed values $u_z = H_z = E_x = E_y = 0$.

Other perturbed values depend on time t, and two space coordinates x, y. For small perturbation variables we use the following notations

$$u_x,\ u_y,\ q = \rho - \rho_0,\ p = P - P_0,\ h_x = H - H_0,\ h_y,\ E \equiv E_z. \tag{14}$$

For linear perturbations, the entropy is of a second order of smallness, so, in the linear approximation perturbations are adiabatic, and the entropy remains constant. Other MHDE equations linearized around the uniform medium with constant magnetic field H_0 along x axis, for 2-D perturbations are written as

$$\frac{1}{\rho_0}\frac{\partial q}{\partial t} + \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = 0, \tag{15}$$

$$\frac{\partial h_x}{\partial t} = -H_0\frac{\partial u_y}{\partial y} + \nu_m\left(\frac{\partial^2 h_x}{\partial x^2} + \frac{\partial^2 h_x}{\partial y^2} - \frac{1}{c^2}\frac{\partial^2 h_x}{\partial t^2}\right), \tag{16}$$

$$\frac{\partial h_y}{\partial t} = H_0\frac{\partial u_y}{\partial x} + \nu_m\left(\frac{\partial^2 h_y}{\partial x^2} + \frac{\partial^2 h_y}{\partial y^2} - \frac{1}{c^2}\frac{\partial^2 h_y}{\partial t^2}\right), \tag{17}$$

$$\frac{\partial u_x}{\partial t} + \frac{c_s^2}{\rho_0}\frac{\partial q}{\partial x} = 0, \tag{18}$$

$$\frac{\partial u_y}{\partial t} + \frac{c_s^2}{\rho_0}\frac{\partial q}{\partial y} + \frac{u_A^2}{H_0}\left(\frac{\partial h_x}{\partial y} - \frac{\partial h_y}{\partial x} + \frac{1}{c}\frac{\partial E}{\partial t}\right) = 0, \tag{19}$$

$$\frac{1}{c}\frac{\partial h_x}{\partial t} + \frac{\partial E}{\partial y} = 0. \tag{20}$$

In the momentum equations we use adiabatic perturbations of pressure $p = c_s^2 q$, with c_s^2 as a squared sound speed. The standard MHD Alfven speed is $u_A = H_0/\sqrt{4\pi\rho_0}$. The solution of this system is looked for in the exponential presentation $\sim \exp(ik_x x + ik_y y - i\omega t)$. Substituting into the system with constant coefficients q, h_x, h_y, u_x, u_y, E, we obtain an algebraic system for constant coefficients:

$$\begin{aligned}
\omega q &= \rho_0(k_x u_x + k_y u_y),\\
\omega h_x &= H_0 k_y u_y - i\nu_m(k_x^2 + k_y^2 - \frac{\omega^2}{c^2})h_x,\\
\omega h_y &= -H_0 k_x u_y - i\nu_m(k_x^2 + k_y^2 - \frac{\omega^2}{c^2})h_y,\\
\omega u_x &= k_x \frac{c_s^2}{\rho_0} q,\\
\omega u_y &= k_y \frac{c_s^2}{\rho_0} q + \frac{u_A^2}{H_0}(k_y h_x - k_x h_y - \frac{\omega}{c}E),\\
\omega h_x &= c k_y E.
\end{aligned} \tag{21}$$

The dispersion equations for linear waves $\omega(k_x, k_y)$ defined by equating to zero the determinant of this homogeneous linear algebraic system, are written [3] as ($k^2 = k_x^2 + k_y^2$):

$$(k^2c^2 - \omega^2)(k^2c_s^2 - \omega^2) - i\frac{\omega c^2}{\nu_m}\left[k^2(c_s^2 + u_A^2) - \omega^2\left(1 + \frac{u_A^2}{c^2}\right) + k_x^2 c_s^2 \frac{u_A^2}{c^2}\left(1 - \frac{k^2c^2}{\omega^2}\right)\right] = 0, \tag{22}$$

$$\omega + i\nu_m\left(k^2 - \frac{\omega^2}{c^2}\right) = 0. \tag{23}$$

4. Wave propagating across the magnetic field

For waves, in the direction perpendicular to the magnetic field have $k_x = 0$, $k_y^2 = k^2$, we obtain the dispersion equation in the form

$$(k^2c^2 - \omega^2)(k^2c_s^2 - \omega^2) - i\frac{\omega c^2}{\nu_m}\left[k^2(c_s^2 + u_A^2) - \omega^2\left(1 + \frac{u_A^2}{c^2}\right)\right] = 0. \tag{24}$$

For almost ideal plasma ($\nu_m \ll c^2/\omega$) we obtain in zero approximation ($\nu_m = 0$)

$$\omega_{f0}^2 = k^2\frac{u_A^2 + c_s^2}{1 + \frac{u_A^2}{c^2}}, \quad \omega_s = 0, \tag{25}$$

for fast (f) and slow (s) MHD waves. Here a zero frequency (a static perturbation) corresponds to the limit of slow MHD wave for propagation across the magnetic field. The frequency of the fast MHD wave here differs from the usual MHD because of the effects due to electric field. In the vacuum case with $u_A \gg c$, this wave transforms into electromagnetic one, while in the opposite case we have a usual MHD, without a displacement current. In presence of low dissipation, the static perturbation and fast MHD wave are damping slowly.

In the opposite case of plasma with very low conductivity ($\nu_m \gg c^2/\omega$) we obtain a solution in zero approximation ($\nu_m = \infty$), corresponding to electromagnetic (em) and sound waves (s)

$$\omega_{em}^2 = k^2c^2, \qquad \omega_s^2 = k^2c_s^2. \tag{26}$$

These waves are damping weakly at $\nu_m < \infty$.

4.1 Dispersion curves for different input parameters

For making numerical estimations we introduce following dimensionless parameters

$$x = \frac{\omega}{kc}, \quad \varkappa = \frac{k}{k_0}, \quad s = \frac{c_s}{c}, \quad a = \frac{u_A}{c}, \quad \nu = \frac{c}{k_0 \nu_m}. \tag{27}$$

Here k_0 is an arbitrary scale factor for the absolute value of the wave vector k. In these dimensionless parameters the dispersion equation (24) reads

$$\begin{gathered}(x^2-1)(x^2-s^2) - i\frac{\nu}{\varkappa}x[(s^2-x^2)+a^2(1-x^2)] = 0,\\ x(\nu/\varkappa) = x_r(\nu/\varkappa) + ix_i(\nu/\varkappa),\\ s = \frac{c_s}{c}, \quad a = \frac{u_A}{c}, \quad \nu = \frac{c}{k_0\nu_m}.\end{gathered} \tag{28}$$

At varying value of $\nu/\varkappa$ this dispersion equation may be considered as equation, which describes propagation of the wave at a fixed $\varkappa$ through the medium with a variable ν. In the medium with smoothly changing parameters the short wave propagation is approximately described by this equation. It may be interpreted as WKB approximation [11] describing propagation of waves with a fixed $\varkappa$, which length λ is much less than the characteristic length of varying of parameters, $\lambda \ll \lambda_\rho = \rho/|\nabla\rho|$ and $\lambda \ll \lambda_H = H/|\nabla H|$ in a non-uniform medium. Depending on parameters a and s, different wave modes correspond to various types of waves, and conversion of waves happens when the wave is propagating through the region with varying ν.

We deal here with 4 types of waves:

1. Sound waves (SW).
2. Electromagnetic waves (EM).
3. Fast MHD waves.
4. Static perturbations.

The waves here differ from classical EM waves in vacuum, or waves in the ideal MHD, which do not have imaginary parts. Here EM wave is damping in the media without magnetic field and the speed of the fast MHD wave differs from its classical value. Usually in the experiments and devices on the Earth the non-dimensional parameters are: $a \ll 1, s \ll 1$, but their ratio may be arbitrary. In astrophysical objects, like pulsars, X-ray sources, AGNs relativistic effects could be important and we may expect $a, s \lesssim 1$.

We start from the solution with moderate input parameters $s = 0.03, a = 0.06$, and find for these parameters numerical solutions $x_r(\nu/\varkappa), x_i(\nu/\varkappa)$, for four different modes presented in Figs. 4, 5. In this case, rapid MHD waves are smoothly connected with the sound ones. Additional calculations had shown, that the electromagnetic waves exist only at $\nu/\varkappa \le 2$ for any $a, s \ll 1$.

Similar topology of wave modes takes place for $a < a_b \simeq 2.82s$ for $s \ll 1$, and changes with increase of s to $a_b \simeq 2.75s$ at $s = 0.1$. For increasing values of a and s, the qualitative picture is similar to Figs. 4, 5 for a below the threshold a_{thr}, depending on s (see Table 1).

With increasing of a we see appearance of a region, where all four wave modes have zero real parts $x_r = 0$. All perturbations are damped in this region. In the Figs. 6, 7 the real and imaginary parts of x for the second case are plotted for $s = 0.03, a = 0.15$. The regions with "pure" static perturbations exist, if $a > a_b \simeq 2.82s$ at $a \ll 1$ for low s values, i.e. in highly magnetized

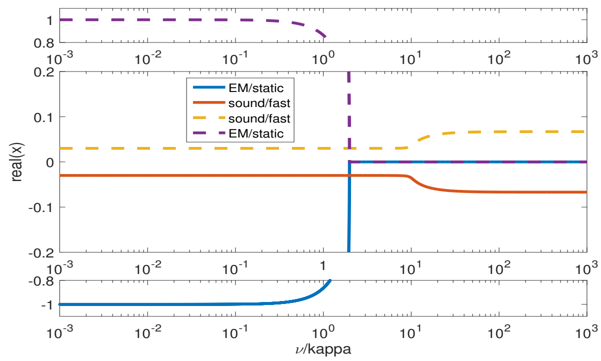

Figure 4: Real parts for parameters $s = 0.03, a = 0.06$ (from [3]). Blue and purple curves correspond to electromagnetic waves and static perturbations. Yellow and red ones are sound and fast MHD waves. Considering waves with a fixed $\varkappa$, we describe qualitatively transformation of types of waves, propagating in medium with varying ν.

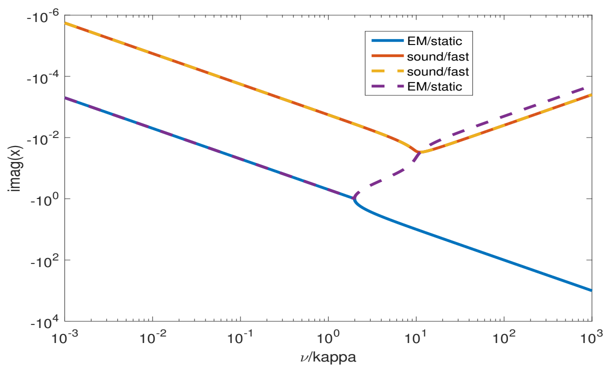

Figure 5: Imaginary parts for parameters $s = 0.03, a = 0.06$ (from [3]). Blue and purple curves correspond to electromagnetic waves and static perturbations. Yellow and red ones correspond to sound and fast MHD waves.

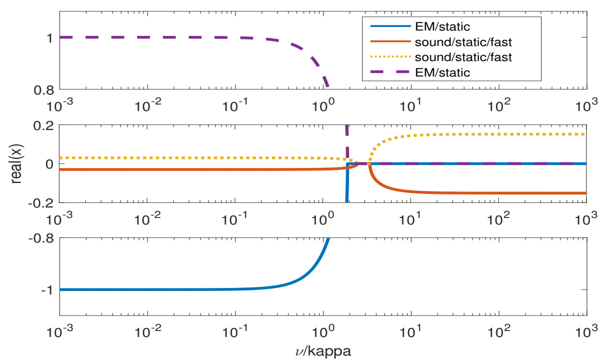

Figure 6: Real parts for parameters $s = 0.03, a = 0.15$ (from [3]). Blue and purple curves correspond to electromagnetic and standing waves. Yellow and red ones correspond to the sound and fast MHD waves. There is an interval in values of $\nu/\varkappa$, where only damping static perturbations are present.

Table 1: Threshold values of normalized Alfven speed for given sound speeds. At larger values of a the electromagnetic wave is forming directly from the fast MHD wave. It can be seen, that for s values lower than 0.1, a_{thr} is almost constant, and for larger values, the ratio a_{thr}/s decreases (from [3]).

$s = c_s/c$	$a_{thr} = u_A/c$
0.01	0.35
0.10	0.36
0.20	0.45
0.30	0.54
0.40	0.62
0.50	0.70
0.60	0.77

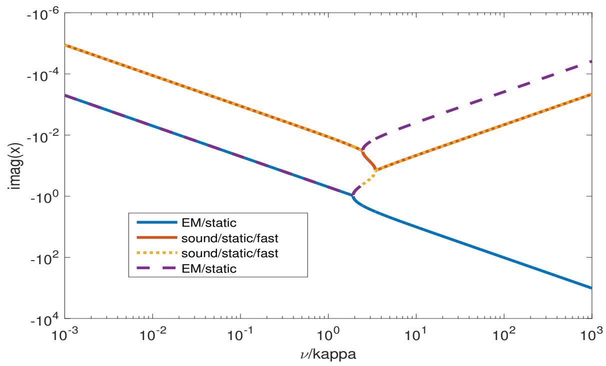

Figure 7: Imaginary parts for $s = 0.03, a = 0.15$ (from [3]). Blue and purple curves correspond to electromagnetic waves and static perturbations. Yellow and red ones correspond to the sound and fast MHD waves.

plasma. The value of a_b decreases slowly with increasing of s. At small $\nu/\varkappa$ we have sound and electromagnetic waves. In this case the connection between asymptotic type solutions is absent, all waves damp after reaching the static region at $\nu/\varkappa \sim 1$.

The picture changes qualitatively for large values of a. At $a > 0.35$, the electromagnetic waves become directly connected with the fast MHD wave if $a > s$, see Figs. 8, 9, and Table 1. The pure sound wave exist only at small $\nu/\varkappa$. Such conditions may appear during accretion processes onto highly magnetized neutron star and during accretion of the magnetized matter into a black hole in galactic sources and AGN's.

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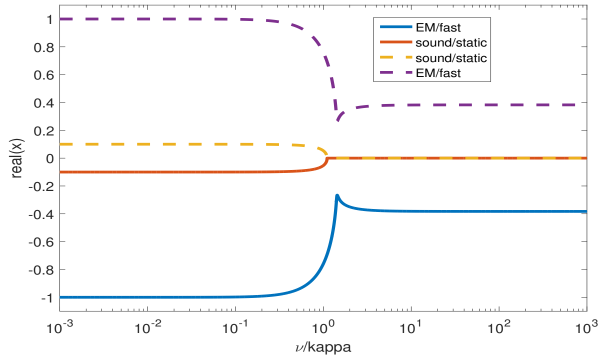

Figure 8: Real parts of solutions for $s = 0.1, a = 0.4$ (from [3]). Blue and purple curves correspond to electromagnetic waves and fast MHD. Yellow and red ones correspond to sound and slow MHD waves.

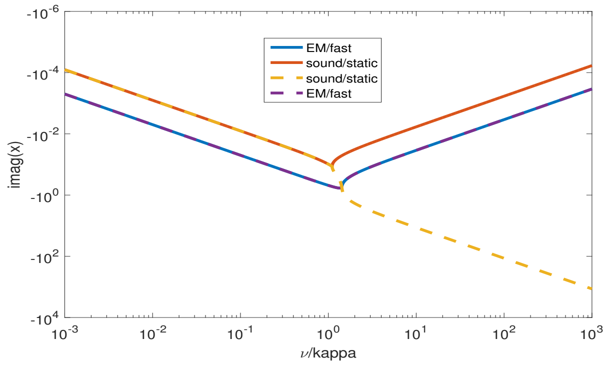

Figure 9: Imaginary parts of solutions for $s = 0.1, a = 0.4$ (from [3]). Blue and purple curves correspond to electromagnetic waves and fast MHD. Yellow and red ones correspond to sound and slow MHD waves.

5. Waves propagation along the magnetic field

For waves, propagating in the direction along the magnetic field have $k_y = 0, k_x^2 = k^2$, we obtain the dispersion equation in the form

$$(k^2c^2 - \omega^2)(k^2c_s^2 - \omega^2) -$$
$$-i\frac{\omega c^2}{\nu_m}\left[k^2(c_s^2 + u_A^2) - \omega^2\left(1 + \frac{u_A^2}{c^2}\right) + k^2c_s^2\frac{u_A^2}{c^2}\left(1 - \frac{k^2c^2}{\omega^2}\right)\right] = 0. \quad (29)$$

After some algebraic transformations this equation is written in the form

$$(k^2c_s^2 - \omega^2)\left\{k^2c^2 - \omega^2 - i\frac{c^2}{\omega\nu_m}\left[\omega^2\left(1 + \frac{u_A^2}{c^2}\right) - k^2u_A^2\right]\right\} = 0, \quad (30)$$

For this configuration the sound waves don't depend on waves, connected with magnetic field. The last ones consist of the electromagnetic waves propagating with the light speed $|\omega_e| = k_x c$ for dielectric conditions, with very large magnetic viscosity $\nu_m >> c^2/\omega$; and Alfven wave, propagating with Alfven speed, where energy of the wave is taken into account, so that $\omega = \frac{k_x u_A}{\sqrt{1+u_A^2/c^2}}$, for ideal

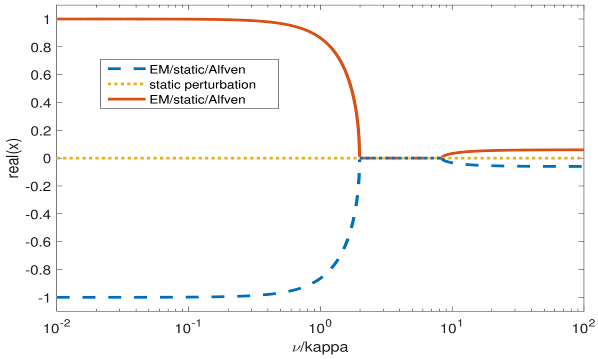

Figure 10: Real parts (without sound waves) for $a = 0.06$ (from [3]). Yellow and red curves on the left side of the plot correspond to EM waves, and the blue one corresponds to static perturbation. On the right side of the picture, red and blue curves correspond to the Alfven waves, the yellow line corresponds to static perturbation. The transformation of waves is absent because of the region with only damping static perturbations.

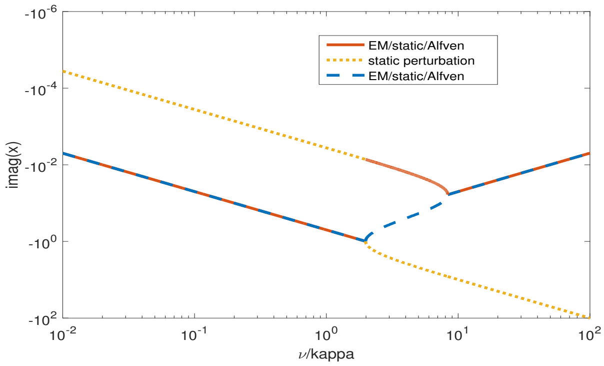

Figure 11: Imaginary parts (without sound waves) for $a = 0.06$ (from [3]).

plasma with $\nu_m = 0$. The mixture of these two waves for intermediate ν_m is shown in the Figs. 10 - 13.

Depending on the parameter a, different modes along the field correspond to various waves, a conversion of the waves may happen when the they propagate through the region with varying ν. We deal here with 4 types of waves:

1. Non-damping sound waves.
2. Electromagnetic waves with damping.
3. Alfven waves.
4. Static perturbations.

Here we don't consider sound waves, which properties are not connected with a magnetic field. A dispersion equation in this case has a form:

$$x(x^2-1) - i\frac{\nu}{\varkappa}(1+a^2)\left(\frac{a^2}{1+a^2} - x^2\right) = 0. \tag{31}$$

In the Figs. 10, 11 the solution of dispersion equation is plotted for $a = 0.06$. Like in the transverse

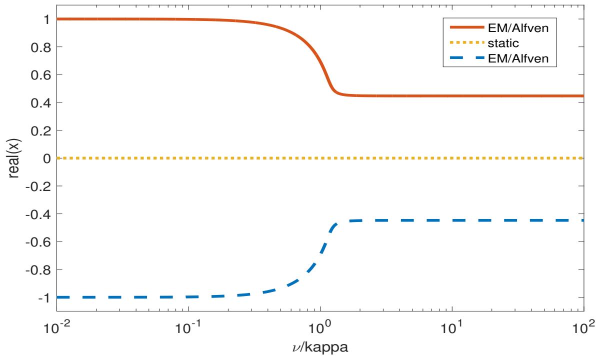

Figure 12: Real parts (without sound waves) for $a = 0.5$ (from [3]). Blue and red curves correspond to EM and Alfven waves. Yellow line corresponds to the static perturbations.

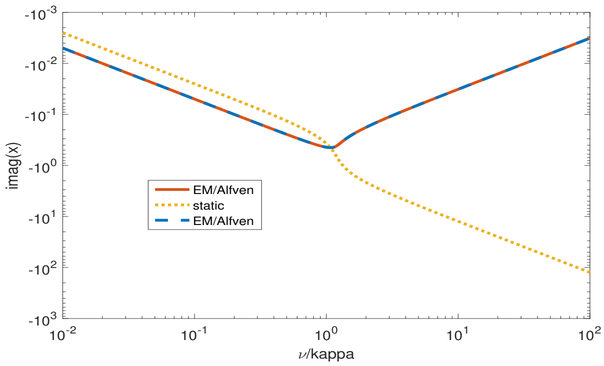

Figure 13: Imaginary parts (without sound waves) for $a = 0.5$ (from [3]). Blue and red curves correspond to EM and Alfven waves. Yellow line corresponds to the static perturbations.

case, there is a region, where the Alfven and EM waves do not exist, and only static perturbations and sound waves are present. Calculations show that EM waves are forming directly from the Alfven ones at $a \gtrsim 0.35$, see Figs. 12, 13 for $a = 0.5$.

6. Discussion and conclusions

The parameters, characterizing the wave vector $\varkappa$, and plasma magnetic viscosity ν are present in the dispersion equation in the form of their ratio $\nu/\varkappa$. Applying WKB approximation we have approximately interpreted the solutions of this equation for describing the change of a speed for different modes with constant $\varkappa$, propagating in the medium with smoothly changing conductivity. In Figs. 6, 10, where there are regions with only static perturbations, the wave transformation does not occur. The evident cases are connected with transformation of rapid MHD wave into the electromagnetic one, perpendicular to the uniform magnetic field, and Alfven into electromagnetic, along the magnetic field, at propagation in the direction of decreasing conductivity parameter. We have shown, that transformation of MHD into EM wave may happen only in highly magnetized plasma with $a > a_{thr}$. The MHDE equations describe both hydrodynamic and electromagnetic phenomena in the situation when the velocity of the matter remains non-relativistic. For description

PoS(FRAPWS2024)003

of the behavior in relativistic jets it is necessary to apply relativistic (Special Relativity) equations of the fluid motion. However, MHDE equations can be applied to phenomena in the X-ray pulsars and other flows near neutron stars, as well as in some AGN and magnetically driven jet models where the velocity of the fluid and the speed of sound have non-relativistic values. With account of the displacement current, the dispersion equation is valid for arbitrary small density and arbitrary large magnetic field. This transformation in the case of Alfven waves was used for construction of the universal numerical scheme in the frame of MHDE, by McGregor & Robinson [**?**] (under the name FMHD).

The matter in consideration could be plasma, metal, possibly liquid, for a highly conducting media. Very resistive materials could be cold neutral gas, transparent dielectric, like glass or plastic, very cold and relatively dense gas clouds [14–16]. These clouds consist of the neutral gas, and contain large scale magnetic fields. Collision of a rapid plasma wind from pulsars or blue supergiant stars with such cold magnetized cloud should lead to formation a very long electromagnetic wave, propagating through the cloud. To describe this phenomena the full nonlinear MHDE set of equations should be used. Similar process could be imitated in a laboratory, as a collision of the rapid particle beam with a magnetized gas reservoir filled with a cold neutral low density gas. Collision of a similar beam may happen with a glass wall, in presence of a magnetic field, producing electromagnetic wave, which may appear in the opposite side, crossing the glass layer.

The MHDE system of equations can be applied to mildly relativistic magnetized jets from magnetically-driven core-collapse supernovae and, possibly, for further evolution of a supernova remnant.

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PoS(FRAPWS2024)003

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DISCUSSION

ALEXANDR DOLGOV: How important is the effect of trace anomaly?

GENNADY BISNOVATYI-KOGAN: We are working iside the classical physics of Euler-Maxwell. We did not use the consept of energy-momentum tensor, as well as its trace anomaly, because we did not need them.