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Nonlocality in quantum cloning states

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This study investigates the nonlocality characteristics of output states produced by a universal quantum cloning machine, with particular emphasis on the Bell-CHSH inequality violations exhibited by the two-qubit reduced states of cloned outputs. Through systematic analysis, we identify the parameter regimes where these reduced states can effectively serve as quantum channels for teleportation protocols. Our results demonstrate that within specific operational ranges of the cloning machine parameters, the two-qubit reduced states are suitable for quantum teleportation but do not exhibit violations of Bell-CHSH inequalities. Furthermore, we quantitatively characterize the monogamy relations governing Bell-CHSH violations in quantum cloning states across different parameter configurations, revealing fundamental constraints on nonlocality distribution in cloned quantum systems.

Keywords Nonlocality, Quantum teleportation, Bell-CHSH inequality

Quantum cloning research is a core issue in quantum information science, involving both the theory and practice of quantum state replication. In classical information, information can be copied without loss of the original data. However, in quantum information systems, due to the no-cloning theorem, exact replication of quantum states is impossible. Since its proposal in 1982 by Wootters and Zurek[1], this theory has become a cornerstone of quantum information science. Nevertheless, despite the inability to achieve perfect cloning, researchers have explored models of approximate cloning states and quantum cloning machines to study how to approximate the replication of quantum states. The study of quantum cloning[2–6] not only helps us understand the limitations of quantum information transmission and replication, but also provides new ideas and methods for technological innovations in quantum communication[7,8], quantum computing[9,10], and quantum encryption[11,12].

The investigation of quantum cloning originated with the formulation of the no-cloning theorem[1]. This theorem asserts that for any unknown quantum state, there does not exist a universal quantum operation that can precisely replicate it onto another quantum system, i.e., a perfect cloning operation is non-existent. The work of Wootters and Zurek elucidated this point, highlighting the fundamental distinction between the non-clonability of quantum information and the replicability of classical information.

Nevertheless, scholars[2,13–16,18] have proposed models of quantum cloning machines to investigate how to approximate the replication of quantum states under certain conditions. The most prevalent models of quantum cloning machines include the $1 \rightarrow 2$ cloning machines[2,13] and the $1 \rightarrow N$ cloning machines[14–16], which aim to maximize cloning efficiency and define the measure of cloning quality-cloning fidelity. The core idea of the quantum cloning machines is that, through entanglement with auxiliary systems and appropriate quantum operations, an approximate replication of quantum states can be achieved.

The quantum no-cloning theorem establishes the foundation for the absolute security of quantum communication, while simultaneously presenting challenges for information processing. In practical applications, it is not always necessary to obtain an exact copy of a quantum state. Consequently, Bužek and Hillery[13,17] proposed the concept of a universal quantum cloning machine in 1996 to achieve approximate cloning. The quantum cloned states generated by this cloning machine are independent of the initial input state. For an arbitrary quantum state $|\phi\rangle = \alpha|0\rangle + \beta|1\rangle$, the optimal $1 \rightarrow 2$ universal quantum cloning machine can be expressed as follows:

$$|0\rangle_1|00\rangle_{23} \rightarrow \sqrt{\frac{2}{3}}|0\rangle_2|00\rangle_{13} + \sqrt{\frac{1}{3}}|1\rangle_2|\varphi^+\rangle_{13}, \tag{1}$$

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$$|1\rangle_1|00\rangle_{23} \rightarrow \sqrt{\frac{2}{3}}|1\rangle_2|11\rangle_{13} + \sqrt{\frac{1}{3}}|0\rangle_2|\varphi^+\rangle_{13}, \tag{2}$$

where $|\varphi^+\rangle_{13} = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$, qubit 1 represents the initial state to be cloned, qubit 2 serves as the auxiliary state of the system, and qubit 3 denotes the target blank state. In this case, the cloned state is given by:

$$\rho_1^{out} = \rho_3^{out} = \frac{5}{6}|\phi\rangle\langle\phi| + \frac{1}{6}|\phi_\perp\rangle\langle\phi_\perp|, \tag{3}$$

where $|\phi_\perp\rangle = \beta^*|0\rangle - \alpha^*|1\rangle$. α^* and β^* denote the complex conjugates of the complex numbers α and β, respectively. The optimal fidelity achieved is $\frac{5}{6} \approx 0.833$.

How can we further improve the fidelity of the cloned state? Current research focuses on two directions. On the one hand, reducing the requirement for the arbitrariness of the quantum state. Bruß et al.[19], and Fan et al.[20] and Kim and Chitambar[21]have proposed phase-covariant cloning, which involves cloning under the condition of some known information about the quantum state.

For the quantum state $|\phi_x\rangle = \frac{1}{\sqrt{2}}[|0\rangle + e^{ix}|1\rangle]$, where the phase angle $x \in [0,\ 2\pi)$, the optimal $1 \rightarrow 2$ phase-covariant cloning machine can be expressed as follows:

$$|0\rangle_1|00\rangle_{23} \rightarrow \sqrt{\frac{1}{2}}(|0\rangle_2|00\rangle_{13} + |1\rangle_2|\varphi^+\rangle_{13}), \tag{4}$$

$$|1\rangle_1|00\rangle_{23} \rightarrow \sqrt{\frac{1}{2}}(|1\rangle_2|11\rangle_{13} + |0\rangle_2|\varphi^+\rangle_{13}). \tag{5}$$

The optimal fidelity achieved is $\frac{1}{2} + \frac{1}{\sqrt{8}} \approx 0.854$, which surpasses that of the universal cloning machine.

On the other hand, by relaxing the conditions for the deterministic implementation of the quantum cloning to a probabilistic implementation, Duan and Guo[22] proposed probabilistic cloning of quantum states in 1998. Probabilistic cloning[23–26] requires that the input quantum states to be cloned must be linearly independent, and a selective measurement process must be introduced in order to probabilistically obtain cloned states with a fidelity of 1. For related works on quantum cloning, see references[27–29].

Nonlocality, a key feature of quantum mechanics, enables entangled states to violate Bell inequalities, challenging classical realism. It drives quantum technologies like teleportation, quantum key distribution, and computing algorithms. Quantum entanglement and Bell inequality are crucial topics in quantum information science. This paper focuses on the output cloned states of quantum cloning machines, investigating whether two-qubit states violate the Bell-CHSH inequality in the absence of noise and whether they can serve as quantum channels for quantum teleportation. Using Horodecki's criterion[30] and teleportation fidelity measures, we show that some Bell-local cloned states still enable nonclassical teleportation. This reveals a fundamental distinction between Bell nonlocality and teleportation utility, offering new insights into the role of cloned states in quantum information processing beyond fidelity considerations.

In Sect. "Quantum cloning machine and Bell-CHSH inequality", the quantum cloning machine and the Bell-CHSH inequality, as described in references[30,32–35], will be introduced. Section "Quantum cloned states, Bell-CHSH inequality, and teleportation fidelity" explores the relationship between the output cloned states, the Bell-CHSH inequality, and the fidelity of quantum teleportation. Section "Monogamy of Bell inequality violation" derives the Bell inequality violation values for quantum states based on the monogamy property of Bell inequalities. Finally, a summary of the paper is provided.

Quantum cloning machine and Bell-CHSH inequality

For any quantum state $|\phi\rangle$ that needs to be cloned, the following two quantum states are defined[17]:

$$|\Omega_{00}\rangle_{23} = \frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle), \tag{6}$$

$$|\Omega_{01}\rangle_{23} = \frac{1}{\sqrt{2}}|0\rangle(|0\rangle + |1\rangle). \tag{7}$$

The quantum circuit of the cloning machine comprises four controlled-NOT (CNOT) gates, as detailed in Fig. 1:

In the circuit, the quantum state of qubit 1 is $|\phi\rangle$, and the quantum states of qubits 2 and 3 are either$|\Omega_{00}\rangle$ or $|\Omega_{01}\rangle$. Consequently, the quantum circuit implements the following unitary transformation:

$$|\phi\rangle_1|\Omega_{00}\rangle_{23} \rightarrow |\phi\rangle_1|\Omega_{00}\rangle_{23}, \tag{8}$$

$$|\phi\rangle_1|\Omega_{01}\rangle_{23} \rightarrow |\phi\rangle_2|\Omega_{00}\rangle_{13}. \tag{9}$$

Let

$$|\Omega\rangle_{23} = k_0|\Omega_{00}\rangle_{23} + k_1|\Omega_{01}\rangle_{23}, \tag{10}$$

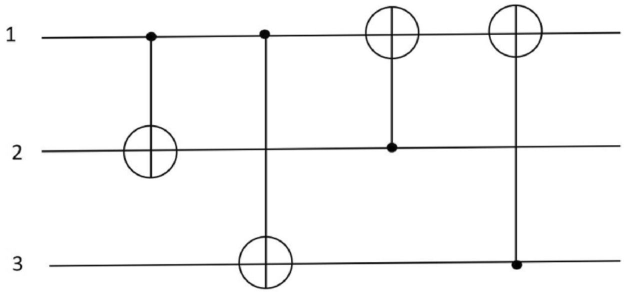

Fig. 1. Schematic diagram of the quantum circuit for the universal quantum cloning machine[17].

where $k_0,\ k_1 \in R$, and satisfy $k_0^2 + k_1^2 + k_0 k_1 = 1$. The quantum states at the outputs of qubits 1 and 2 are given by:

$$\rho_1^{out} = (k_0^2 + k_0 k_1)|\phi\rangle\langle\phi| + \frac{k_1^2}{2} I, \tag{11}$$

$$\rho_2^{out} = (k_1^2 + k_0 k_1)|\phi\rangle\langle\phi| + \frac{k_0^2}{2} I. \tag{12}$$

From Eqs. (11 and 12), it can be observed that when $k_0 = k_1 = \frac{1}{\sqrt{3}}$, we have $\rho_1^{out} = \rho_2^{out}$, achieving the optimal fidelity of universal cloning $\frac{5}{6}$.

Next, we introduce the Bell-CHSH inequality and its relationship with teleportation fidelity (for details, see references[30,32–35]). Consider a quantum state ρ in the Hilbert space $\mathscr{H} = \mathbb{C}^2 \otimes \mathbb{C}^2$, which can be expressed in the Hilbert-Schmidt basis as follows:

$$\rho = \frac{1}{4}(I \otimes I + r \cdot \sigma \otimes I + I \otimes s \cdot \sigma + \sum_{i,j=1}^{3} h_{ij} \sigma_i \otimes \sigma_j). \tag{13}$$

Here, I denotes the identity operator, $\{\sigma_i\}_{i=1}^3$ are the standard Pauli operators, and $r,\ s \in R^3$, with $r \cdot \sigma = \sum_{i=1}^3 r_i \sigma_i$. Additionally, $H = (h_{ij})$ where $h_{ij} = tr(\rho \sigma_i \otimes \sigma_j)$.

The Bell operator associated with the CHSH inequality is defined as:

$$\mathscr{B}_{CHSH} = \hat{a} \cdot \sigma \otimes (\hat{b} + \hat{b'}) \cdot \sigma + \hat{a'} \cdot \sigma \otimes (\hat{b} - \hat{b'}) \cdot \sigma, \tag{14}$$

where, $\hat{a},\ \hat{a'},\ \hat{b},\ \hat{b'}$ are unit vectors in R^3.

The Bell-CHSH inequality is expressed as

$$|\langle \mathscr{B}_{CHSH} \rangle_\rho| \leqslant 2, \tag{15}$$

where $\langle \mathscr{B}_{CHSH} \rangle_\rho \equiv tr(\rho \mathscr{B}_{CHSH})$.

Define the real-valued function $N(\rho)$ as:

$$N(\rho) = \max_{i>j}(\nu_i + \nu_j), \tag{16}$$

where ν_i and ν_j are the eigenvalues of the matrix $H^\dagger H$.

As demonstrated in references[30,31], the maximal violation of the Bell-CHSH inequality for a given quantum state ρ is given by

$$B_{\max}(\rho) = \max |\langle B_{\mathrm{CHSH}} \rangle_\rho| = 2\sqrt{N(\rho)}, \tag{17}$$

where $N(\rho)$ expressed in Eq. (16).

Based on the preliminary knowledge outlined above, it can be concluded that the density matrix ρ(as defined in (13)) violates the Bell-CHSH inequality (given in (15)) if and only if $N(\rho) > 1$. Furthermore, any quantum state that violates the Bell-CHSH inequality can be effectively utilized as a quantum channel to achieve teleportation. (Here, "effectively" means that the fidelity of teleportation surpasses the classical fidelity threshold of $\frac{2}{3}$).

Define the function $M(\rho)$ as:

$$M(\rho) := tr\sqrt{H^\dagger H}, \tag{18}$$

then for any quantum state, standard teleportation can be effectively achieved if and only if $M(\rho) > 1$. In this case, the fidelity of quantum teleportation is given[32] by:

$$F = \frac{1}{2}\left[1 + \frac{1}{3}M(\rho)\right]. \tag{19}$$

Quantum cloned states, Bell-CHSH inequality, and teleportation fidelity

When$|\Omega\rangle_{23}$(as defined in (10)) is chosen as the input quantum state of the cloning machine, the initial state $|\phi\rangle_1$ is processed through the quantum cloning circuit. Subsequently, by taking the partial trace over qubits 3, 2, and 1 respectively, the reduced density matrices $\rho_{12},\ \rho_{13}$ and ρ_{23} are obtained, where

$$\rho_{12} = \frac{1}{2}\begin{pmatrix} (k_0+k_1)^2|\alpha|^2 & k_1(k_0+k_1)\alpha^*\beta & k_0(k_0+k_1)\alpha\beta^* & 0 \\ k_1(k_0+k_1)\alpha\beta^* & k_0^2|\alpha|^2+k_1^2|\beta|^2 & k_0k_1 & k_0(k_0+k_1)\alpha\beta^* \\ k_0(k_0+k_1)\alpha^*\beta & k_0k_1 & k_1^2|\alpha|^2+k_0^2|\beta|^2 & k_1(k_0+k_1)\alpha\beta^* \\ 0 & k_0(k_0+k_1)\alpha^*\beta & k_1(k_0+k_1)\alpha^*\beta & (k_0+k_1)^2|\beta|^2 \end{pmatrix},$$

$$\rho_{13} = \frac{1}{2}\begin{pmatrix} (k_0+k_1)^2|\alpha|^2+k_1^2|\beta|^2 & k_0k_1\alpha^*\beta & k_0(k_0+k_1)\alpha\beta^* & k_1(k_0+k_1) \\ k_0k_1\alpha\beta^* & k_0^2|\alpha|^2 & 0 & k_0(k_0+k_1)\alpha\beta^* \\ k_0(k_0+k_1)\alpha^*\beta & 0 & k_0^2|\beta|^2 & k_0k_1\alpha^*\beta \\ k_1(k_0+k_1) & k_0(k_0+k_1)\alpha^*\beta & k_0k_1\alpha\beta^* & k_1^2|\alpha|^2+(k_0+k_1)^2|\beta|^2 \end{pmatrix},$$

$$\rho_{23} = \frac{1}{2}\begin{pmatrix} (k_0+k_1)^2|\alpha|^2+k_0^2|\beta|^2 & k_0k_1\alpha^*\beta & k_1(k_0+k_1)\alpha\beta^* & k_0(k_0+k_1) \\ k_0k_1\alpha\beta^* & k_1^2|\alpha|^2 & 0 & k_1(k_0+k_1)\alpha\beta^* \\ k_1(k_0+k_1)\alpha^*\beta & 0 & k_1^2|\beta|^2 & k_0k_1\alpha^*\beta \\ k_0(k_0+k_1) & k_1(k_0+k_1)\alpha^*\beta & k_0k_1\alpha\beta^* & k_0^2|\alpha|^2+(k_0+k_1)^2|\beta|^2 \end{pmatrix}.$$

For real coefficients $k_0,\ k_1$, the normalization condition is satisfied as follows:

$$k_0^2 + k_1^2 + k_0k_1 = 1. \tag{20}$$

Based on the knowledge of elliptic equations, we can define:

$$k_0 = \sqrt{\frac{1}{3}}\cos x - \sin x, \tag{21}$$

$$k_1 = \sqrt{\frac{1}{3}}\cos x + \sin x, \tag{22}$$

where $x \in [0,\ 2\pi)$.

In the following, we discuss the relationship between the quantum states $\rho_{12},\ \rho_{13},\ \rho_{23}$ and the Bell-CHSH inequality, as well as their fidelity as quantum channels for teleportation.

For the quantum state ρ_{12}, its correlation matrix T_{12} is given by:

$$T_{12} = \begin{pmatrix} k_0k_1 & 0 & 0 \\ 0 & k_0k_1 & 0 \\ 0 & 0 & k_0k_1 \end{pmatrix}.$$

The value of $N(\rho_{12})$ corresponding to the Bell-CHSH inequality is given by

$$N(\rho_{12}) = 2k_0^2k_1^2. \tag{23}$$

Substituting Eqs. (21 and 22) into Eq. (23) yields:

$$N(\rho_{12}) = \frac{2}{9}(1 - 4\sin^2 x)^2. \tag{24}$$

Therefore, when $\sin^2 x > \frac{\sqrt{\frac{9}{2}}+1}{4} \approx 0.7803$, that is, $|\sin x| > 0.8833$, we have $N(\rho_{12}) > 1$. In this case, *x* lies within the intervals $(62^\circ,\ 118^\circ) \cup (242^\circ,\ 298^\circ)$. It can be observed that the choice of parameters k_0 and k_1 for the cloning machine directly influences the Bell nonlocality properties of ρ_{12}.

When the quantum state ρ_{12} is utilized as a quantum channel for quantum teleportation, the corresponding $M(\rho_{12})$ is given by:

$$M(\rho_{12}) = 3 \mid k_0k_1 \mid = \mid 1 - 4\sin^2 x \mid. \tag{25}$$

From the above Eq. (25), it follows that when $\sin^2 x > \frac{1}{2}$, that is, $|\sin x| > 0.7071$, we have $M(\rho_{12}) > 1$. In this case, *x* lies within the intervals $(45^\circ,\ 135^\circ) \cup (225^\circ,\ 315^\circ)$.

When $\sin x = \pm 1$, which corresponds to $x = 90^\circ$ or 270°, according to Eq. (19), the fidelity of quantum teleportation F reaches its maximum value of 1. In this case, the quantum state of the universal cloning machine is given by: $|\Psi\rangle_{23} = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)|1\rangle$.

From the above analysis, it follows that when the parameter x satisfies $0.7071 <| \sin x |< 0.8833$, the quantum state ρ_{12} obtained at the output ports 1 and 2 of the cloning machine can be used as a quantum channel for teleportation, yet it does not violate the Bell-CHSH inequality. For different values of x within the range $[0,\ 2\pi)$, the nonlocal properties exhibited by ρ_{12} are illustrated as shown in Fig. 2.

For the quantum state ρ_{13}, its correlation matrix T_{13} is given by:

$$T_{13} = \begin{pmatrix} k_1(k_0+k_1) & 0 & 0 \\ 0 & -k_1(k_0+k_1) & 0 \\ 0 & 0 & k_1(k_0+k_1) \end{pmatrix},$$

The value of $N(\rho_{13})$ corresponding to the Bell-CHSH inequality is given by:

$$N(\rho_{13}) = 2k_1^2(k_0+k_1)^2. \tag{26}$$

Substituting Eqs. (21 and 22) into Eq. (26) yields

$$N(\rho_{13}) = \frac{8}{9}\left[\frac{1}{2} + \sin\left(2x + \frac{\pi}{6}\right)\right]^2. \tag{27}$$

Therefore, when $\sin(2x + \frac{\pi}{6}) > \sqrt{\frac{9}{8}} - \frac{1}{2} \approx 0.5607$, which corresponds to $x \in (2^\circ,\ 58^\circ) \cup (182^\circ,\ 238^\circ)$, we have $N(\rho_{13}) > 1$.

When the quantum state ρ_{13} is used as a quantum channel for teleportation, the corresponding $M(\rho_{13})$ is given by:

$$M(\rho_{13}) = 3 \mid k_1(k_0+k_1) \mid = \mid 1 + 2\sin\left(2x + \frac{\pi}{6}\right) \mid. \tag{28}$$

When $\sin(2x + \frac{\pi}{6}) > 0$, which corresponds to $x \in (0,\ 75^\circ) \cup (165^\circ,\ 255^\circ) \cup (345^\circ,\ 360^\circ)$, we have $M(\rho_{13}) > 1$. When $\sin(2x + \frac{\pi}{6}) = 1$, which corresponds to $x = 30^\circ$, according to (19), the fidelity of quantum teleportation F reaches its maximum value of 1. In this case, the quantum state of the universal cloning machine is given by: $|\Psi\rangle_{23} = \frac{1}{\sqrt{2}}|0\rangle(|0\rangle + |1\rangle)$.

It can be observed that when the parameter x satisfies $0 < \sin(2x + \frac{\pi}{6}) < 0.5607$, the quantum state ρ_{13} obtained at the output ports 1 and 3 of the cloning machine can be used as a quantum channel for teleportation without violating the Bell-CHSH inequality. For different values of x within the range $[0,\ 2\pi)$, the nonlocal properties exhibited by ρ_{13} are illustrated as shown in Fig. 3.

For the quantum state ρ_{23}, its correlation matrix T_{23} is:

$$T_{23} = \begin{pmatrix} k_0(k_0+k_1) & 0 & 0 \\ 0 & -k_0(k_0+k_1) & 0 \\ 0 & 0 & k_0(k_0+k_1) \end{pmatrix}$$

The value of $N(\rho_{23})$ corresponding to the Bell-CHSH inequality is given by:

$$N(\rho_{23}) = 2k_0^2(k_0+k_1)^2. \tag{29}$$

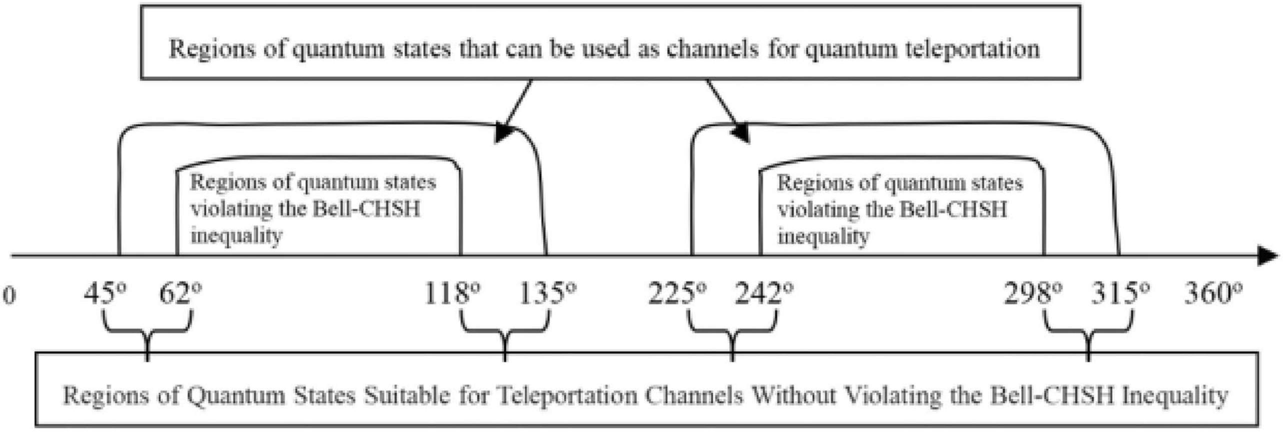

Fig. 2. Regions of quantum state ρ_{12} for teleportation and Bell-CHSH inequality violation.

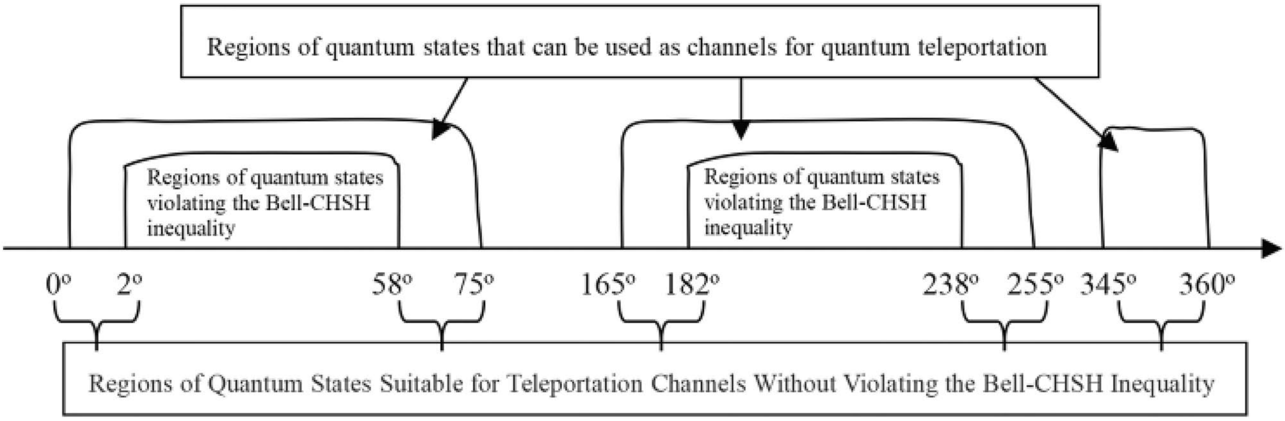

Fig. 3. Regions of quantum state ρ_{13} for teleportation and Bell-CHSH inequality violation.

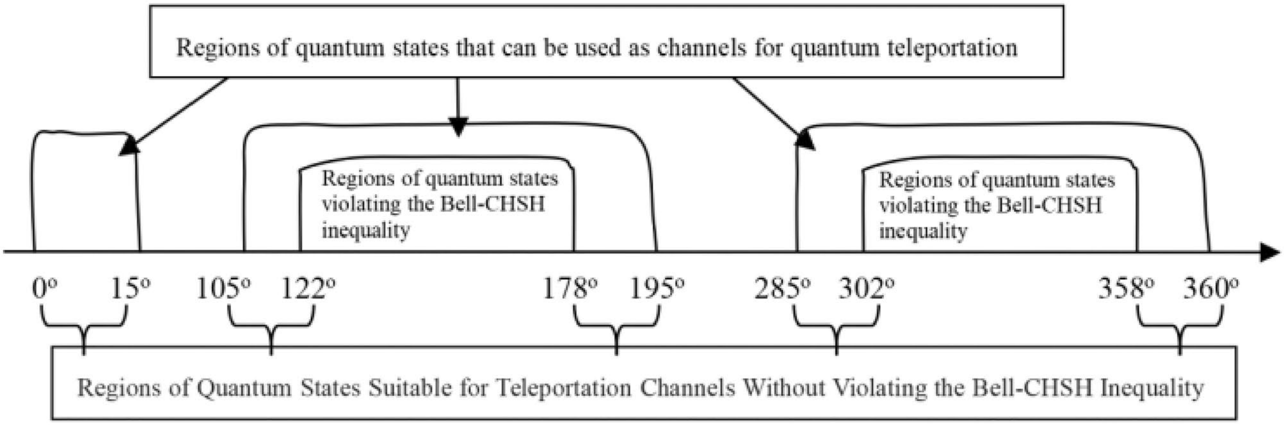

Fig. 4. Regions of quantum state ρ_{23} for teleportation and Bell-CHSH inequality violation.

Substituting Eqs. (21 and 22) into Eq. (29) yields

$$N(\rho_{23}) = \frac{8}{9}\left[\frac{1}{2} + \cos\left(2x + \frac{\pi}{3}\right)\right]^2. \tag{30}$$

Therefore, when $\cos(2x + \frac{\pi}{3}) > \sqrt{\frac{9}{8}} - \frac{1}{2} \approx 0.5607$, which corresponds to $x \in (122^\circ,\ 178^\circ) \cup (302^\circ,\ 358^\circ)$, we have $N(\rho_{23}) > 1$.

When the quantum state ρ_{23} is used as a quantum channel for teleportation, the corresponding $M(\rho_{23})$ is given by:

$$M(\rho_{23}) = 3 \mid k_0(k_0 + k_1) \mid = \mid 1 + 2\cos\left(2x + \frac{\pi}{3}\right) \mid. \tag{31}$$

From the above analyse, it can be concluded that when $\cos(2x + \frac{\pi}{3}) > 0$, which corresponds to $x \in (0,\ 15^\circ) \cup (105^\circ,\ 195^\circ) \cup (285^\circ,\ 360^\circ)$, we have $M(\rho_{23}) > 1$. When $\cos(2x + \frac{\pi}{3}) = 1$, which corresponds to $x = 330^\circ$, according to (19), the fidelity of quantum teleportation F reaches its maximum value of 1. In this case, the quantum state of the universal cloning machine is given by: $|\Psi\rangle_{23} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$.

For the quantum state ρ_{23}, when the parameter x satisfies $0 < \cos(2x + \frac{\pi}{3}) < 0.5607$, the quantum state ρ_{23} obtained at the output ports 2 and 3 of the cloning machine can be used as a quantum channel for teleportation without violating the Bell-CHSH inequality. For different values of x within the range $[0,\ 2\pi)$, the nonlocal properties exhibited by ρ_{23} are illustrated as shown in Fig. 4.

Monogamy of Bell inequality violation

As established in Sect. "Quantum cloned states, Bell-CHSH inequality, and teleportation fidelity", certain two-qubit mixed entangled states exist that do not exhibit violations of the Bell inequality. Prior research[37,40] has shown that the local behavior of such quantum correlations stems from the monogamous nature of bipartite

Bell correlations. Specifically, the monogamy of Bell-inequality violations stipulates that in a tripartite quantum system, if any two of the three bipartite subsystems exhibit a violation of the Bell inequality, then the remaining bipartite subsystem necessarily cannot. This monogamous constraint imposes fundamental limitations on the distribution of quantum nonlocality and entanglement across multipartite systems.

For the two-qubit state ρ_{AB}, define

$$\mathfrak{B}(\rho_{AB}) = \max\{0,\ N(\rho_{AB}) - 1\}, \tag{32}$$

then $\mathfrak{B}(\rho_{AB})$ quantitatively measures the degree to which the two-qubit state violates the Bell inequality, thus characterizing the degree of non-local correlations in the quantum state.

In this section, we analyze the monogamy of Bell-inequality violations within the reduced two-qubit subsystems derived from the cloned output states. Due to the monogamous restriction, at most one of the three reduced density matrices can violate the Bell inequality. Therefore, any observed bipartite violation in the output state must originate solely from one of the subsystems-either AB, AC, or BC. A direct consequence of this finding is that only one among the quantities $\mathfrak{B}(\rho_{AB})$, $\mathfrak{B}(\rho_{AC})$, or $\mathfrak{B}(\rho_{AB})$ can assume a non-zero value. Accordingly, the variable $\mathfrak{B}(\rho_{ABC})$ encapsulates the magnitude of this unique non-zero quantity.

For the three-qubit state $|\psi\rangle_{ABC}$, we define

$$\mathfrak{B}(|\psi\rangle_{ABC}) = \max\{\mathfrak{B}(\rho_{AB}), \mathfrak{B}(\rho_{BC}), \mathfrak{B}(\rho_{AC})\}, \tag{33}$$

where $\mathfrak{B}(\rho_{AB}), \mathfrak{B}(\rho_{BC})$, and $\mathfrak{B}(\rho_{AC})$ represent the measures of Bell inequality violation for the two-qubit reduced density matrices of $|\psi\rangle_{ABC}$.

According to the monogamy of Bell inequality violation[36], it is known that in a tripartite shared quantum state ρ_{ABC}, if the reduced state ρ_{AB} shared by two parties violates the Bell inequality, then ρ_{BC} and ρ_{AC} must not violate the Bell inequality.

For the quantum state ρ_{123} output by the cloning machine, the following expression for $\mathfrak{B}(\rho_{123})$ can be obtained:

$$\mathfrak{B}(\rho_{123}) = \begin{cases} N(\rho_{12}) - 1,\ x \in (62^\circ,\ 118^\circ) \cup (242^\circ,\ 298^\circ), \\ N(\rho_{13}) - 1,\ x \in (\ 2^\circ,\ \ 58^\circ) \cup (182^\circ,\ 238^\circ), \\ N(\rho_{23}) - 1,\ x \in (122^\circ,\ 178^\circ) \cup (302^\circ,\ 358^\circ). \end{cases}$$

Conclusion

In this paper, we investigate the output states of a universal quantum cloning machine and calculate the parameter ranges of the cloning machine for which the two-qubit states can be used as quantum channels for teleportation without violating the Bell-CHSH inequality. The results demonstrate that a significant subset of cloned states retain teleportation utility without violating local hidden variable models-a phenomenon previously observed in specific entangled states like GHZ and W states[37]. This reinforces the broader principle that nonclassical teleportation capability does not strictly require Bell nonlocality, aligning with alternative resource identification frameworks such as entropic uncertainty relations[38] and nonlocal advantages of quantum coherence[39].

The parametric tunability of cloning machines introduces a unique advantage: by systematically adjusting cloning operations, one can engineer output states with graded nonlocality tailored for specific communication protocols. Future investigations could explore synergies between cloning dynamics, Bell inequality thresholds, and emerging resource measures (e.g., coherence asymmetry, steering robustness) across multi-qubit systems. Such studies would advance both fundamental understanding of quantum resource interconversion and the development of noise-resilient teleportation architectures for scalable quantum networks.

Data availability

All data generated or analysed during this study are included in this published article.

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Author contributions

Jun Liu wrote and edited the manuscript. Zhiwen Mo supervised the study and provided guidance.

Declarations

Competing interests

The authors declare no competing interests.

Additional information

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