SPACETIME INFORMATION AS A GUIDING PRINCIPLE FOR COLOR RECONNECTION IN Herwig 7*

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A new model of color reconnection in the Monte Carlo generator Herwig 7 is presented. It is based on the minimization of a boost-invariant distance of the parton system, where all partons have momentum as well as spacetime position assigned. We test the influence of both types of variables, namely the rapidity span and transverse distance, on the actual need to reconnect the system to better describe soft physics measurements. We find reasonable agreement with the data and conclude that spacetime topology of the event can be useful for hadron collision modeling.

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1. Introduction

The investigation of physics of hadron collisions at the LHC experiments has reached unprecedented levels of precision especially with respect to perturbative QCD calculations. However, hadron collision modeling consists of number of other mechanisms, which proceed at low-energy regime, generally known as hadronization and underlying event. These phenomena are less understood and one method to study them is by using general-purpose Monte Carlo event generators [1–5], which become indispensable for current measurements.

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One of the problems in simulating hadronization is how to correlate multiple parton interactions (MPI) within a single hadron collision. The description of MPI in Monte Carlo programs has a relatively long history, see *e.g.* [1, 2, 4, 6–10]. Assuming we leave the parton correlations inside a hadron behind, we still need to unpuzzle the interactions of MPI among each other. One possible approach is to test an existence of would-be extra gluon exchanges exhibiting as a color changes in the outgoing partons. This type of algorithms are called color reconnection [11–15]. Such a mechanism is also motivated by a need of corrections for errors in the leading-color approximation of the parton shower. A summary of the history of color reconnection and the effects of such a mechanism on precise measurements is given in [16]. Herwig 7 first focused on reconnecting excited $q\bar{q}$ pairs, called clusters, by minimizing the sum of the invariant masses [12]. Later, work [14] expanded upon this model to introduce the possibility of forming so-called baryonic clusters qqq and $\bar{q}\bar{q}\bar{q}$ from three ordinary/mesonic clusters. Other methods have investigated color reconnection at the perturbative stages of event simulation or have taken inspiration from perturbative techniques [17–19].

In these proceedings, we summarize main points of [20], where we discuss the need of the introduction of spacetime coordinates for all partons inside the event simulation in Herwig 7. Such information allows us to deal with spacetime separation between the particles for the first time, since all the other stages of generation of pp event involve only the energy-momentum framework. In particular, spacetime picture provides a guidance what parts of the event are allowed to undergo color reconnection within a given slice of phase space, if one thinks that color reconnection needs to be a causal effect. We believe that this approach might be crucial especially for heavy-ion collisions, where the measured final state strongly depends on the position of interaction inside the overlap region, *e.g.* a phenomenon known as jet quenching [21–23]. As a result, pp-oriented event generators have also started to include more spacetime information, using these coordinates for various aspects of the simulation, such as collective hadronization effects [24, 25] and a spacetime evolution of the parton shower [26]. PYTHIA recently introduced a framework for generating spacetime coordinates [27] for quantitative studies of Lund string fragmentation [28]. The effects of introducing spacetime coordinates have been recently studied in dipole evolution in $\gamma^* A$ collisions [29].

2. Event simulation in Herwig 7

Since the color reconnection is mainly motivated by existence of MPI and deals with clusters, formed at the beginning of the hadronization step of the event generation, we now start with their short description. Herwig 7

distinguishes two classes of MPI: hard and soft, with the transverse momentum cut $p_\perp^{\min}$ separating them. This parameter is then left free for tuning to data.

The hard interactions are calculated perturbatively and initial- and final-state partons undergo showering. The color topology is motivated by the leading-color approximation used in the shower. The soft scatterings are modeled phenomenologicaly according to multi-peripheral MPI model [9, 10] and the created partons do not get showered.

Both types of interactions are bound together through the optical theorem [30] in the form of

$$\sigma_{\mathrm{tot}}(s) = 2\pi \int_0^\infty \mathrm{d}b^2 \left(1 - \mathrm{e}^{-(\chi_{\mathrm{hard}}(b,s)+\chi_{\mathrm{soft}}(b,s))}\right) , \tag{1}$$

where $\sigma_{\mathrm{tot}}(s)$ is the total cross section of the given collision at a c.m.s. energy squared, s. The integration goes over the impact parameter of the collision, b. The eikonal factor is split to hard and soft component, both given by the same prescription

$$\chi(b,s) = \frac{1}{2} A(b;\mu)\sigma^{\mathrm{inc}}\left(s; p_\perp^{\min}\right) = \frac{1}{2}\langle n(b,s)\rangle \tag{2}$$

but with the different values of its parameter μ, which has a meaning of inverse hadron radius. The total cross section σ_{tot} and the hard inverse radius μ_{hard} are tunable parameters and fully determine the soft inverse radius μ_{soft} through (1). $A(b;\mu)$ is a function describing an overlap of the colliding hadrons at given b and $\sigma^{\mathrm{inc}}(s; p_\perp^{\min})$ is the inclusive cross section to produce a pair of partons above $p_\perp^{\min}$. The same as inverse radius, also values of $\sigma^{\mathrm{inc}}(s; p_\perp^{\min})$ for hard and soft component differ. This formalism was developed in [31] and Herwig's implementation is built on the Jimmy framework [7].

With the exception of momentum conservation requirement, Herwig 7 generates all MPI independently and thus assumes a Poissonian distribution for their multiplicities (for both hard and soft MPI) at a given b[1]

$$\mathcal{P}_{i,k}(b) = \frac{(2\chi_{\mathrm{h}}(b))^i}{i!}\frac{(2\chi_{\mathrm{s}}(b))^k}{k!}\mathrm{e}^{-2(\chi_{\mathrm{h}}(b)+\chi_{\mathrm{s}}(b))} , \tag{3}$$

where indices i and k denote the actual number of hard and soft interactions. To understand this, we explicitly write in (2) that we consider the eikonal factor to be one half of the mean number of interactions $\langle n\rangle$ of the given

[1] We have dropped the functional dependence on s.

type. The last step is to integrate (3) over the b space to produce the exact probability to produce the corresponding number of hard and soft interactions in an event. Figure 1 shows the joint Poissonian distributions for several cases and confirms our expectations: the more scatters are produced, the more likely it is that the collision is central (small b). More details of the technicalities involved in the implementation of MPI algorithm can be found in [1].

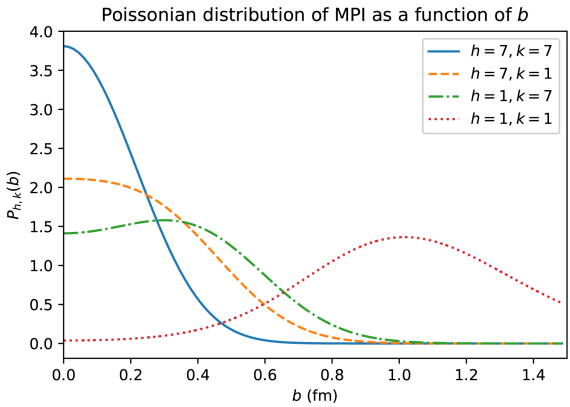

Fig. 1. Joint Poissonian distribution $\mathcal{P}_{h,k}(b)$, as a function of impact parameter b, for a number of h hard scatters and k soft scatters. We have used the following fixed values for the distributions: $\sigma^{\text{inc}}_{\text{hard}} = 83$ mb, $\sigma^{\text{inc}}_{\text{soft}} = 127$ mb, $\mu^2_{\text{hard}} = 0.71$ GeV^2, and $\mu^2_{\text{soft}} = 0.52$ GeV^2. All distributions are normalized to unit area.

Partons from the hard interactions are showered down to the parton shower cutoff scale, and the resulting color topology has triplets connected to anti-triplets via gluon connections. At the lower scales, Herwig 7 uses the cluster hadronization model [32] based on the pre-confinement property of angular-ordered showers [33]. This model requires as an input a set of quarks and anti-quarks, and thus all hadronizing gluons are non-perturbatively split into $q\bar{q}$ pairs. The algorithm then combines all partons into the $q\bar{q}$ pairs, which are the closest in the color space, and typically also nearest in momentum space due to pre-confinement. These $q\bar{q}$ pairs are, therefore, colorless objects called clusters, or more specifically mesonic clusters.

Due to the chosen leading-color topology for the additional scatters, MPI are color-connected to the beam remnant and other subprocesses. It is shown in [12], that this approximation performs significantly worse for the soft MPI and thus an extra treatment is required. The clusters undergo the color reconnection. It aims to minimize a chosen measure, typically momentum-based, of the given set of clusters. Herwig 7 has currently three available models [12, 14]: Plain, Statistical/Metropolis, and Baryonic.

3. Spacetime coordinates generation

There are two parts of how our new model generates coordinates to partons: individual MPI are moved to their own position and partons originating from them are smeared around these points. For the MPI position generation, we first need to specify the actual impact parameter of a collision. It is randomly sampled according to distribution drawn in Fig. 1. Once b is determined for a given event, we set the incoming beam positions to be at $(\pm b/2, 0, 0, 0)$, *i.e.* aligned along the x-axis, for simplicity. The overlap function then determines the sampling of MPI positions through the random radius b' with respect to the beam positions. In this work, an overlap function is chosen to be a Bessel function of the third kind

$$A(b;\mu) = \frac{\mu^2}{96\pi}(\mu b)^3 K_3(\mu b)\,. \tag{4}$$

Since the overlap region is given by the convolution of the two protons' transverse profiles $G(b';\mu)$, we get the necessary sampling function as an integrand of it[2]

$$A(b) = \int \mathrm{d}^2 b' G\left(b'\right) G\left(b - b'\right)\,. \tag{5}$$

The overlap function governs the density of MPI scattering centres in the transverse plane for a given offset between the protons. We note that hard and soft scatters use different μ^2 values. As a result, hard scatters are slightly more concentrated in the centre of the transverse plane, while soft scatters have a longer tail.

With regards to the spacetime coordinates generated by the parton shower algorithm, we recently showed that distances traveled in transverse space are mostly negligible [20]. They get significant only at small energy scales. Therefore, we only give positions to the partons that remain at the end of the shower and we assign them the same virtuality ν^2 to put the threshold on its minimal value. A parton of mass m thus receives the mean lifetime in its rest frame

$$\tau_0 = \frac{\hbar m}{\nu^2}\,. \tag{6}$$

The distance traveled by a parton is simply result of the appropriate Lorentz transformation, where time of the decay is sampled according to the exponential decay law

$$P_{\mathrm{decay}}(t < t^*) = 1 - \exp\left(-\frac{t^*}{\tau}\right)\,, \tag{7}$$

where t^* is the rest-frame decay time and t its equivalent in a lab-frame.

[2] We have suppressed the dependence on μ for clarity.

Once all the partons have their new coordinates with respect to their MPI scattering centre, we then shift these coordinates using the points produced from the MPI coordinate generator. One example of the final positions is shown schematically in Fig. 2.

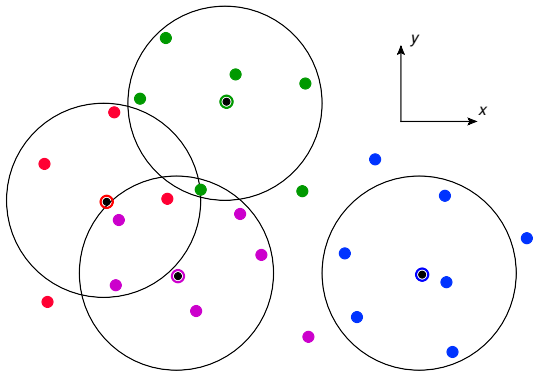

Fig. 2. (Color online) A schematic diagram for how our model introduces transverse spacetime coordinates for the multiple parton interactions (black points), and for the end of the parton shower. Different colored points are partons from different, respectively ring-colored MPI centres. The thin black circles represent a characteristic scale for parton propagation about the MPI centre.

4. Spacetime color reconnection

The first model of color reconnection in Herwig was the so-called Plain color reconnection. For simplicity, we start with its description, where we include the new information from spacetime coordinates and then we move to the model, which was actually used to describe the experimental data: the baryonic spacetime model.

The classical Plain color reconnection is based on the check if the sum of clusters' invariant masses after the reconnection is smaller than the sum before the reconnection. If this is the case, the reconnection is accepted with a constant probability which is a tunable parameter of the model. In other words, only parton momenta are used in such a measure and it may lead to a connection of causally-disconnected partons. In the Plain model extended by incorporating the spacetime information, we still deal with mesonic clusters but instead of invariant mass we use measure defined as

$$R_{ij}^2 = \frac{\Delta r_{ij}^2}{d_0^2} + \Delta y_{ij}^2 \,, \tag{8}$$

which can be calculated for two cluster constituents i, j. This measure contains a combination of spacetime transverse distance between the con-

stituents $\Delta r_{ij}^2 = (\vec{x}_{\perp,i} - \vec{x}_{\perp,j})^2$ and rapidity difference Δy_{ij}. d_0 is the characteristic length scale for color reconnection in our spacetime model, which is a tunable parameter. This parameter can also be viewed as the strength of the transverse component of the spacetime measure relative to the rapidity component. The Plain color reconnection based on the measure from (8) recursively checks the sum of the pairing of cluster constituents and searches for the minimal variant. If the sum of the cluster separations is smaller after a possible reconnection

$$R_{q\bar{q}'} + R_{q'\bar{q}} < R_{q\bar{q}} + R_{q'\bar{q}'} \,, \tag{9}$$

then we accept the reconnection with a flat probability. A similar model was studied earlier in [34].

Let us now move to baryonic spacetime color reconnection. It uses the algorithm from [14], where the partners for mesonic and baryonic reconnection are found by using the projection onto a given cluster's quark axis. In the case of baryonic cluster pair, we cannot directly compare the sum of measures (8), since we would be starting with 3 clusters — each with 2 partons — and ending with 2 clusters with 3 partons. For the latter situation, the transverse distance measure is actually not defined. The algorithm deals with two baryonic clusters as with a set of three quarks and three antiquarks. A pair of quark–antiquark is found with the lowest possible measure (8). A mean spacetime position and rapidity is then calculated for the remaining diquark (anti-diquark) system. We allow baryonic reconnection if the following criterion is true:

$$R_{q,qq} + R_{\bar{q},\bar{q}\bar{q}} < R_{q,\bar{q}} + R_{qq,\bar{q}\bar{q}} \,. \tag{10}$$

The reconnection is accepted with probability w_b, which is a new tunable parameter. If the reconnection is rejected, all three candidate clusters remain ordinary mesonic clusters.

We note that the above-described model already relies on the preselection of clusters for reconnection based on parton rapidities. It is meant as a first discriminating factor when searching for potential partners. The transverse separation between constituents in measure (8) provides extra information and thus improves the original baryonic color reconnection model, especially in larger systems like heavy-ion collisions.

At the end of the model description, we would like to note that the building of baryonic spacetime color reconnection model in Herwig 7 required additional changes to the original code. These changes are of more general nature than the specifics of our model and we refer the reader to a separate publication [35].

5. Results

Before obtaining results, there are eight parameters of the model which have to be set to match the experimental data [36–40] as best as possible. This so-called tuning procedure was performed within the Autotunes [41] framework that internally makes use of the Rivet and Professor frameworks [42, 43]. Figure 3 shows χ^2 values for two pairs of the parameters, namely ($\mu^2_{\rm hard}$) and ($p_\perp^{\rm min}$). The white area in the parameter space means that the model fails to fit the data without violating the total cross section.

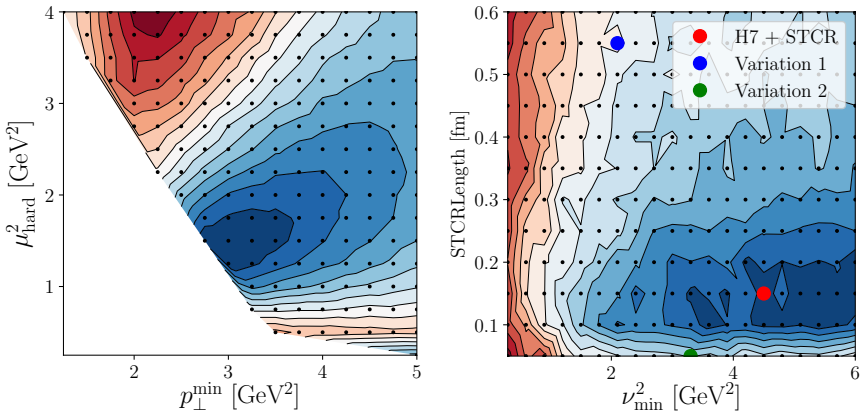

Fig. 3. (Color online) χ^2 contour planes for parameter sets ($\mu^2_{\rm hard}$, $p_\perp^{\rm min}$) and (ν^2, d_0). Darker areas correspond to smaller χ^2 values. In the right plot, we show three points corresponding to the chosen variations of (ν^2, d_0).

TABLE I

The newly tuned parameters for Minimum Bias simulation and our baryonic spacetime color reconnection model. The top row is the retuned parameters of the old Herwig 7 Minimum Bias model. The bottom row is the three new parameters of the spacetime components of our model, and a determined parameter of the old model.

$\sigma_{\rm tot}$ [mb]	$R_{\rm Diff}$	$p_\perp^{\rm min}$ [GeV]	$\mu^2_{\rm hard}$ [GeV^2]
96.0	0.2	3.0	1.5
ν^2 [GeV^2]	d_0 [fm]	w_b	$\mu^2_{\rm soft}$ [GeV^2]
4.5	0.15	0.98	0.254

Besides the best-fit values (labeled "H7 + STCR"), written in Table I, we also choose two other points from the (ν^2, d_0)-plane in order to show the variability of our spacetime model. These points are "Variation 1" ($\nu^2 =$

2.1 GeV^2, $d_0 = 0.55$ fm) and "Variation 2" ($\nu^2 = 3.3$ GeV^2, $d_0 = 0.05$ fm). Most of the parameters have already been explained. We add an extra parameter R_{Diff}, the amplitude of the non-diffractive cross section. We reiterate that the μ^2_{soft} parameter is not tuned itself but calculated by the model with respect to other parameters

Let us make two extra remarks on tuning. First, the baryonic color reconnection probability w_b got the value of 0.98, which seems too large. This is actually caused by the preselection of candidate clusters for the color reconnection, as described in detail in [14]. Second, we avoid to retune the probability for strangeness production during the hadronization and keep the value from [14], despite the recent developments described in [44]. We leave a full retune of all the hadronization parameters to future work.

As an example of results obtained from the newly tuned Herwig 7 with the spacetime color reconnection model, we show the rapidity and transverse momentum distributions as measured in [38], see Fig. 4. Here, we put several distributions in each plot. They differ only in cuts on the track momentum and/or on minimum number of tracks in the event. These observables are crucial for the description of Minimum Bias and soft physics, and we find that the model is perfectly capable at describing the distributions. Moreover, we add two gray lines (solid and dashed) to each distributions corresponding to the chosen variation points as described above ("Variation 1" and "Variation 2"). They form a very nice envelope of the experimental data and demonstrate further adaptability of the model. More results of our work can be found in [20].

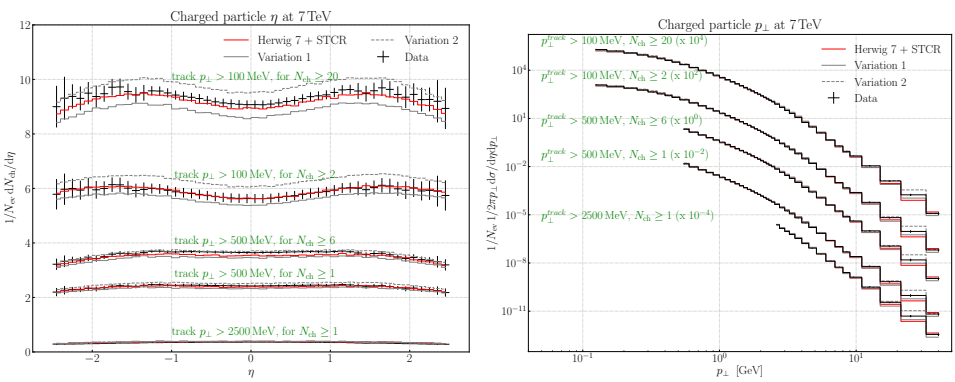

Fig. 4. Charged particle spectra against rapidity and transverse momentum for various leading track $p_\perp$ and number of charged particle N_{ch} slices. An overall good agreement with data is found. The variations are purely in the spacetime length and minimum virtuality parameters of our model.

6. Conclusions

We probed the possibility of using spacetime positions of partons in the hadronization step of proton collision simulation in Herwig 7 for the first time. In order to do so, we first implemented two new mechanisms how to generate the spacetime coordinates: the positions of MPI scattering centers, and the propagation distances at the end of the parton shower. Then we used the transverse plane distances among partons in combination with their rapidities to define a measure that we minimize when performing baryonic color reconnection. We call this model as baryonic spacetime color reconnection.

We observe that this model provides meaningful results for many observables in Minimum Bias events in *pp* collisions at the LHC. These optimistic results open a completely new branch of studies. One can further develop either the spacetime position generation algorithms or the color reconnection step. For instance, one can allow only certain MPI subsystems to reconnect with each other, see [45], or one can use the coordinates to decrease the computation complexity of soft-gluon-evolution model [18] by focusing on the small neighbourhood in spacetime. Another possibility might be to study the final state of the event in more detail using spacetime coordinates, as started in [27]. One interesting idea is the interplay between Bose–Einstein correlations, and hadron position and extent [46]. Moreover, we would like to point out that this work might also be taken as a step towards larger system description, as heavy-ion collision.

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