



SDC SOLENOIDAL DETECTOR NOTES

**DETERMINATION OF THE RADIATION LENGTH OF HV
CAPACITORS FOR THE SDC STRAW TRACKER**

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Determination of the Radiation Length of HV Capacitors for the SDC Straw Tracker

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In the SDC straw tracker, the blocking capacitors which isolate the front end electronics from the anode wires are potentially a major source of material in terms of radiation lengths. This note describes a measurement of photon absorption by sample capacitors to determine the effective radiation length of their material. This work was performed as a student exercise in an undergraduate laboratory at the University of Colorado.^[1]

In our prototypes we are using a surface mount, 270 pf, 3 kV capacitor from Johanson Dielectrics. Its dimensions are $3.91 \times 3.68 \times 2.69$ mm, and the density (by our measurement) is 5.16 ± 0.02 g cm⁻³. The material is not known for sure, but available materials of high dielectric constant include barium titanate (BaTiO₃) and neodymium titanate (Nd₂(TiO₃)₃). Barium at $Z = 56$ and neodymium at $Z = 60$ are strong scatterers. For compounds with P_i atoms of element i per molecule the radiation length X_0 is given by

$$\frac{1}{\rho X_0} = \sum \frac{W_i}{(\rho X_0)_i},$$

$$W_i = P_i A_i / A_{\text{mol}},$$

where ρ is the mass density, A_i the atomic weight of element i , and A_{mol} the molecular weight, given by $\sum P_i A_i$. Table 1 shows the results of this calculation for the titanates of magnesium, calcium, barium, and neodymium. For both barium and neodymium the result is about 11.4 g cm⁻². The numbers change only very slightly if we assume up to 1:1 admixture of copper, for the electrodes.

The measurement by gamma ray absorption is described in the appended report.^[1] Fig. 1 here shows curves of the photon absorption coefficient μ/ρ vs photon energy for several materials (from Ref. 2 of Ref. 1). In the energy range around 80 – 400 keV the coefficients are sensitive to the atomic number Z . By interpolating to the measurements one can extract an effective Z for the unknown samples. Fig. 2 shows the absorption coefficient vs Z for $E_\gamma = 100$ and 200 keV on the same plot as $1/\rho X_0$. The radiation length curve was generated by evaluating the formula^[2]

$$X_0 = \frac{716.4 \text{ g cm}^{-2} A}{Z(Z+1) \ln(287/\sqrt{Z})}.$$

From this curve one can read off $1/\rho X_0$ at the effective Z value determined by the μ/ρ data, at least for pure substances (see below).

Fig. 1 also shows the data from Ref. 1 for the unknown and for reference absorbers of lead, copper, iron, and aluminum. The agreement between the reference absorbers and the curves is quite good for copper and lead. It is rather poor for aluminum. The unknown is nicely bracketed by the copper and lead samples, which gives confidence that the technique works for this range of Z values. The interpolation yields the effective atomic number for photon absorption $Z_\mu \simeq 47 \pm 3$.

For compounds μ/ρ is computed by the same weighted sum as $1/\rho X_0$ given above. However, the effective Z value will be strictly consistent only if the Z dependences of the two processes are the same. For our measurement, the most precise data for μ/ρ were obtained for E_γ near 100 keV. The curve fits approximately $\mu/\rho \propto Z^{2.3}$, whereas $1/\rho X_0$ is more nearly $\propto Z^1$ (see Fig. 2). For either of the hypothetical heavy dielectrics (barium and neodymium titanate), the effect of this difference is that $Z_X \simeq 0.83 Z_\mu = 39 \pm 3$. Fig. 3 shows $1/\rho X_0$ vs Z on a linear scale, from which we read off $1/\rho X_0 = 0.094 \pm 0.005$, or $\rho X_0 \simeq 10.6 \pm 0.8 \text{ g cm}^{-2}$ (with allowance for some systematic error), consistent with the expected value of 11.4 g cm^{-2} .

The measurement confirms that these capacitors contain rather high- Z mate-

rial.

The formulations used for capacitor dielectrics comprise various mixtures of compounds to control voltage and temperature dependence, dielectric constant, dielectric strength, etc., and these are generally proprietary. For discussions with the vendors we propose a figure of merit that reflects the radiation lengths contributed by the capacitors. The quantity we wish to minimize is the ratio x/X_0 , where x is the thickness traversed by a particle (say, at normal incidence). We have one capacitor per detector of area A_{det} , so we can write this in terms of a “radiation cross section” defined as follows:

$$\begin{aligned}\frac{x}{X_0} &= \frac{\sigma_0}{A_{det}}, \\ \sigma_0 &\equiv \left\langle \frac{1}{\rho X_0} \right\rangle M_{Cap} \\ &= [0.012 + 2.42 \times 10^{-3} \langle Z \rangle_w - 8.0 \times 10^{-6} \langle Z^2 \rangle_w] M_{Cap} \text{ cm}^2 \text{ g}^{-1}.\end{aligned}$$

Here M_{Cap} is the mass of the capacitor in grams. With knowledge of the dielectric’s formulation, one can compute the average (by weight) atomic number $\langle Z \rangle_w$ and the average squared atomic number $\langle Z^2 \rangle_w$. The expression in brackets is the result of a fit to the data in Fig. 3.

The capacitors we tested have a mass of 0.197 g, from which we find $\sigma_0 = 0.0185 \text{ cm}^2$. For the 4 mm diameter straws the hexagonal area $A_{det} = 2\sqrt{3}r_{straw}^2 = 0.138 \text{ cm}^2$, so the capacitors contribute 13% of a radiation length at normal incidence out the end of a superlayer. This is what is given in the SDC TDR.^[3] It remains to be seen whether a capacitor with more optimal characteristics can be found.

REFERENCES

1. R. Honke, **Determination of an Approximate Value for the Atomic Number Z of an Unknown Capacitor Material by Means of Photon Attenuation**, student report for Physics 4430 (Senior Laboratory), Univ. of Colorado (1991); appended. The hand-drawn figures in this report were not legibly reproducible; their content appears in this note (Figs. 1 and 2), along with the related discussion replacing sections 6 and 7 of Honke's report.
2. K. Hikasa et al., (Particle Data Group), *Phys. Rev. D* **45** (1992) S1.
3. Solenoidal Detector Collaboration, **Technical Design Report**, SDC Report SDC-92-201 (1992).

Table 1. Radiation Length of Capacitor Dielectrics

Element	Z	A	X0
			gm/cm ²
O	8	16.00	34.24
Mg	12	24.30	25.26
Ca	20	40.08	16.43
Ti	22	47.88	16.17
Cu	29	63.55	12.86
Ba	56	137.33	8.45
Nd	60	144.24	7.82
MgTiO ₃		120.18	22.57
CaTiO ₃		135.96	19.99
BaTiO ₃		233.21	11.31
Nd ₂ (TiO ₃) ₃		576.12	11.52

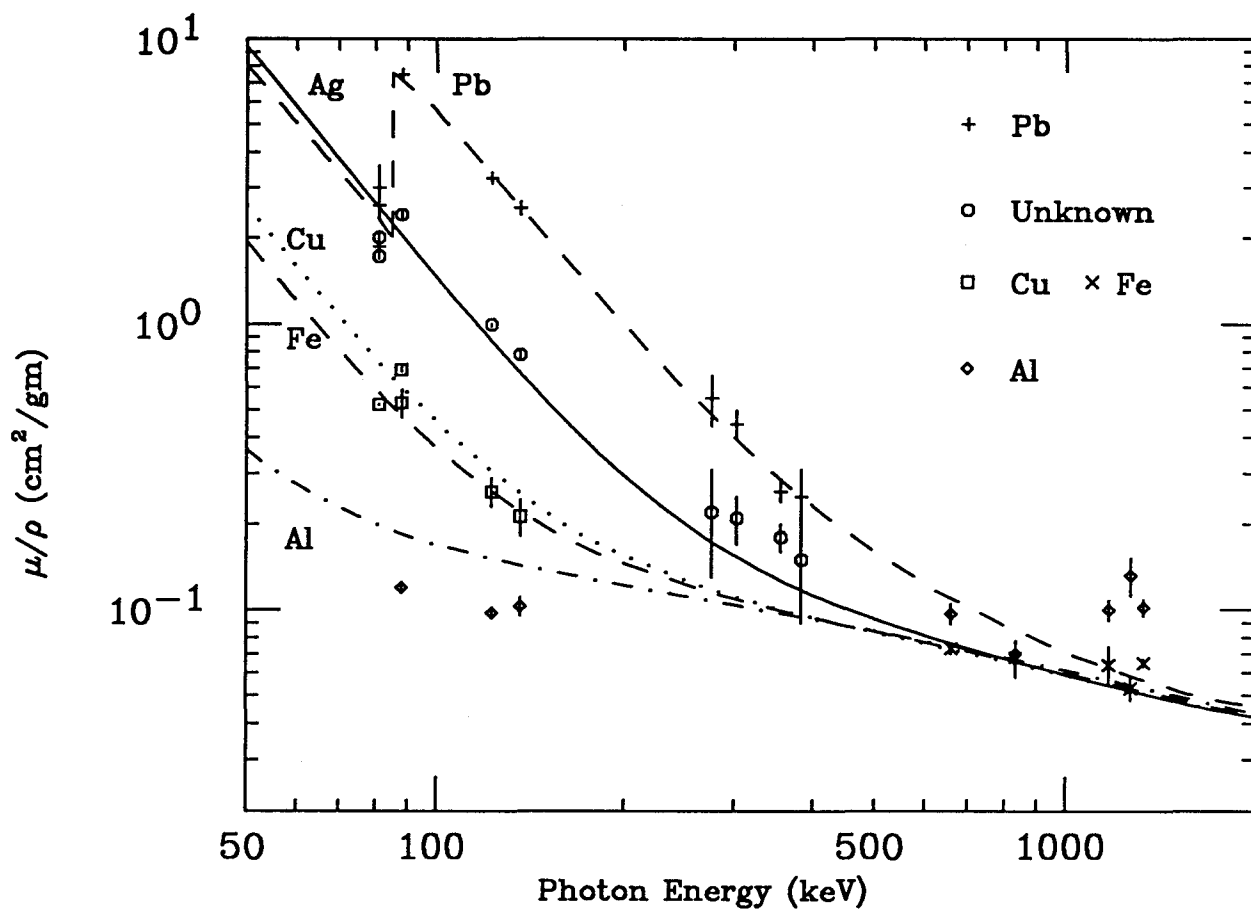


Fig. 1

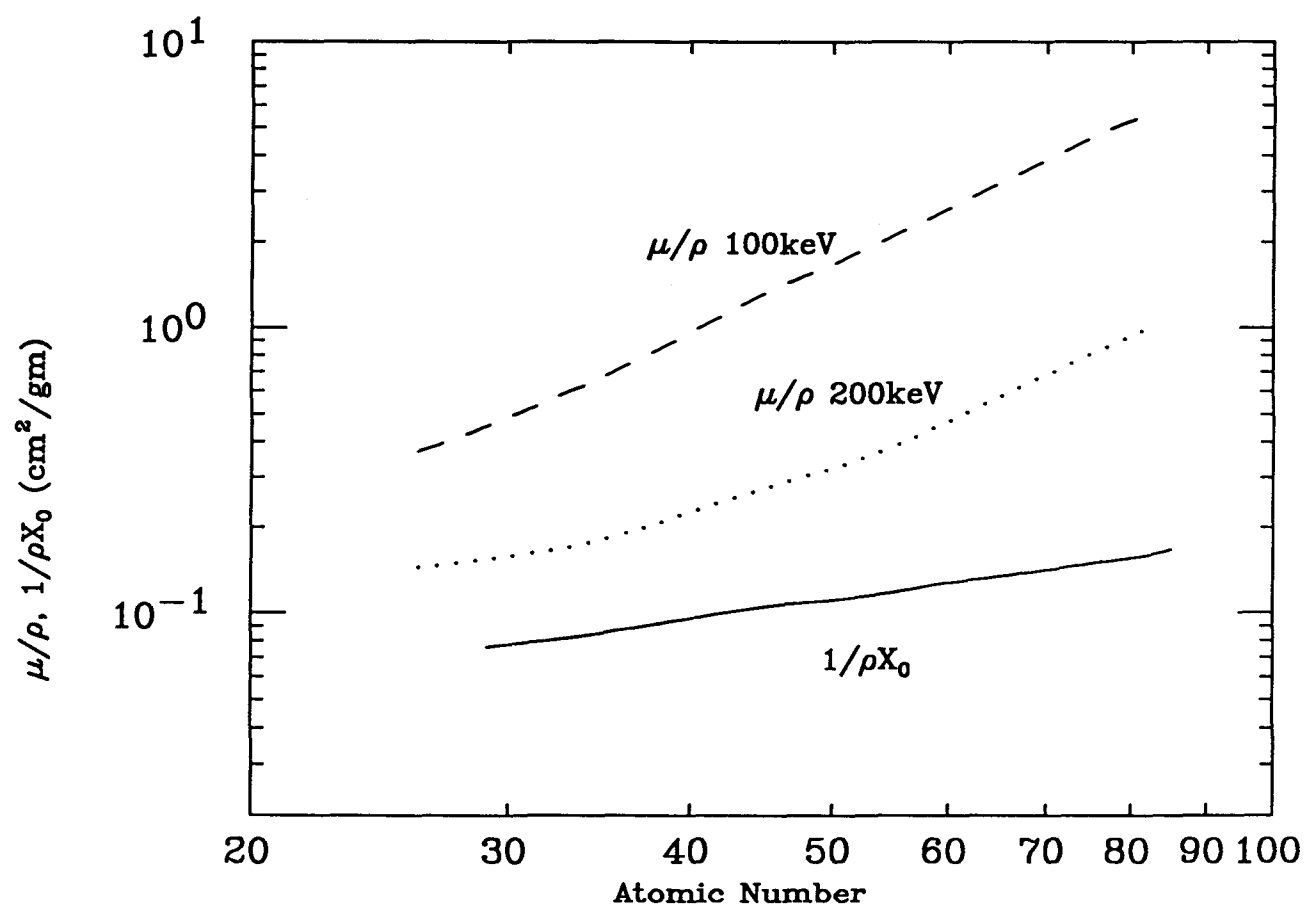


Fig. 2

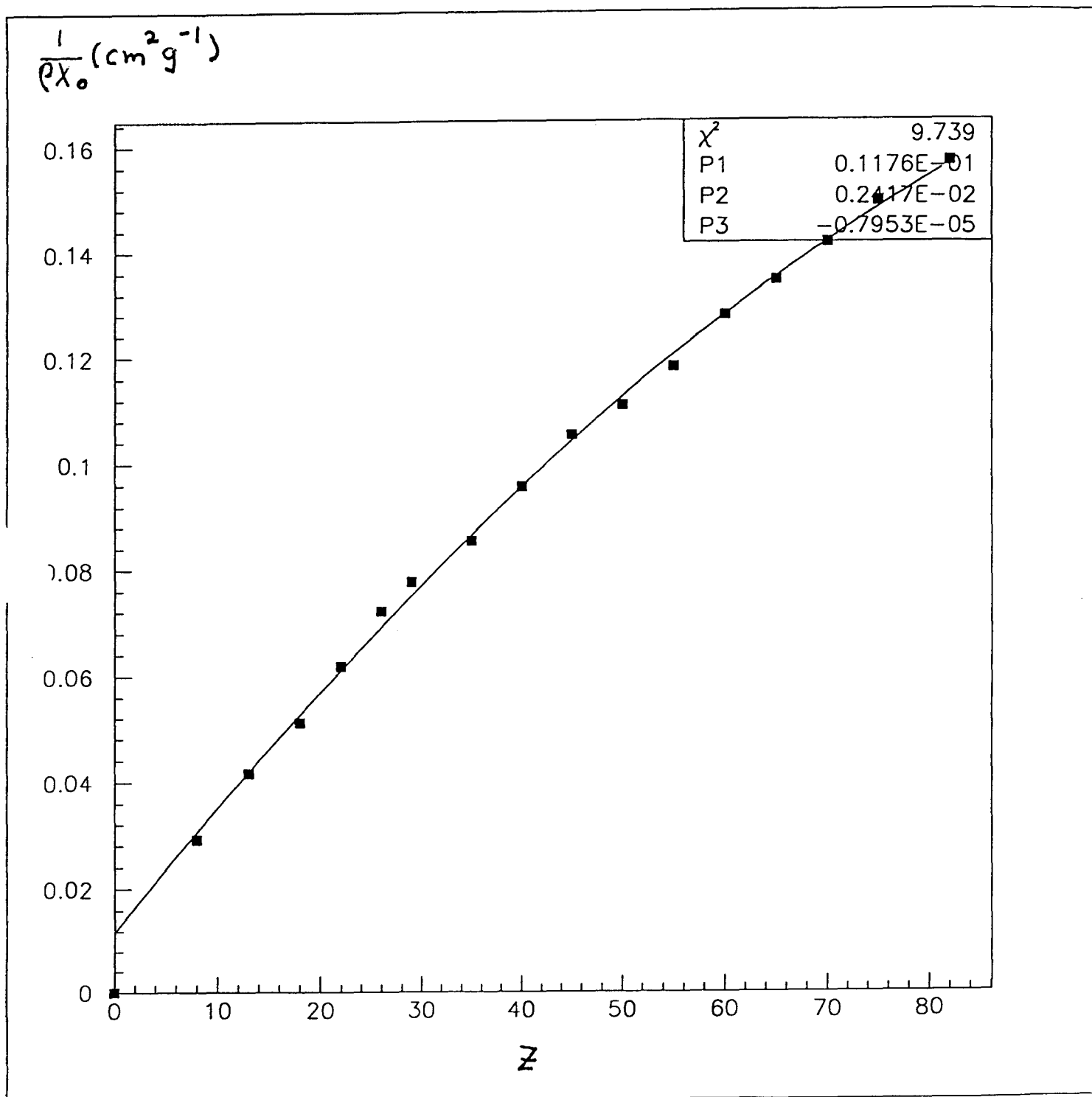


Fig. 3

Determination of an Approximate Value for the Atomic Number Z of an Unknown Capacitor Material by Means of Photon Attenuation

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Abstract

The mass absorption coefficient of an unknown sample was measured for various photon energies to determine its mean atomic number by comparing them with literature values. The result was $Z = 51 \pm 2$, but only guesses could be made what are the constituents of the unknown material.

1 Introduction

The goal of this experiment was to determine approximate values (or at least a mean value) of Z for the constituents of an unknown sample made of high voltage capacitors being considered for use with the read out electronics for a wire drift chamber in an experiment to be built at the Superconducting Super Collider (SSC). In that experiment excessive material (measured in terms of the number of radiation length) is deleterious, because electrons that lose some of their energy by bremsstrahlung will not be accurately measured in the magnetic spectrometer.¹ The energy loss is given by [5]

$$-\left(\frac{dE}{dx}\right)_{average} = \sum_{constituents} \frac{Z_i^2 n_i}{137} E(x) r_o^2 \left(4 \ln \frac{183}{Z_i^{\frac{1}{3}}} + \frac{2}{9}\right) \quad (1)$$

hence the radiation length L_o by (neglecting $\frac{2}{9}$)

$$\frac{1}{L_o} = \sum_{constituents} \frac{Z_i^2 n_i r_o^2}{137} 4 \ln \left(\frac{183}{Z_i^{\frac{1}{3}}}\right) \quad (2)$$

where r_o is the classical electron orbit radius of the H-atom and $Z_{(i)}$ the atomic number (of the i^{th} constituent of a compound). Thus it is obvious important to know the atomic numbers Z_i for compounds and mixtures. We pursued our goal by studying the absorption of gamma rays from radioactive sources.

¹In principle it is possible to take the radiation length directly by measuring the transmission of a high energy electron beam with a magn. spectrometer, but such facilities were not available.

2 The Mass Attenuation Coefficient $\frac{\mu}{\rho}$

A narrow beam of mono energetic photons is attenuated to an intensity I from an incident intensity I_o in passing through a layer of material with mass per unit area x according to the exponential law

$$\frac{I}{I_o} = \exp(-\frac{\mu}{\rho}x) \quad (3)$$

in which $\frac{\mu}{\rho}$ is the mass absorbtion coefficient. Thus $\frac{\mu}{\rho}$ can be obtained by measuring $\frac{I}{I_o}$ and x (= thickness of the absorber times the density ρ ; units $[\frac{g}{cm^2}]$). $\frac{\mu}{\rho}$ is heavily related to the total cross section σ_{tot} :

$$\frac{\mu}{\rho} = \sigma_{tot}(\frac{N_A}{A_r}) \quad (4)$$

$$\sigma_{tot} = \sigma_{incoh} + \sigma_{coh} + \tau + \kappa + \sigma_{ph.n} \quad (5)$$

in which $\sigma_{(in)coh}$ is the coherent (Reyleigh) (incoherent (Compton)) scattering cross section; τ is the photo effect cross section, κ is the positron-electron pair production cross section an $\sigma_{ph.n}$ is the photonuclear cross section. N_A is the Avogadro number and A_r the relative atomic mass of the absorber element. From this, measured values can be compared with the theory (e.g. [1]). However in this experiment our measure values were compared with those measured and published by J.H. Hubbell (see copy in appendix)([2]).

The values of $\frac{\mu}{\rho}$ for mixtures an compounds can be obtained from those of pure elements using the mixture rule

$$\frac{\mu}{\rho} = \sum_i w_i(\frac{\mu}{\rho})_i \quad (6)$$

in which w_i is the proportion by weight of the i^{th} constituent with mass attenuation coefficient $(\frac{\mu}{\rho})_i$.²

3 Method

Particularly for photon energies $< 1MeV$ $\frac{\mu}{\rho}$ shows a strong dependence of E_γ , mainly due to the photoelectric contribution to σ_{tot} , which is also quite different as a function of Z .

Thus we measured $\frac{\mu}{\rho}$ over a range of about 80 to 400keV and compared the values with those of known substances in Hubbell's table. We also measured $\frac{\mu}{\rho}$'s of iron, lead, copper and aluminum over a energy range of 80 to 1300keV to check, if those values are consistent with literature values and to make sure that our results were reasonable. Fig.1 shows a sketch of the apparatus used. The detector was a EG&G Ortec germanium gamma ray detector (model Gem 2112-12175-5) with a built in preamplifier. The signals were amplified by a Ortec TC203BLR linear amplifier, digitalized by a ADC interfaced to a Zenith computer. Thus the spectra could be stored on disc and evaluated with the help of the software also provided by EG&G. In particular it was possible to calculate the "net areas" below the gamma ray peaks, where net area means the area without background. It was calculated by the software as follows:

A region of interest around the peak could be marked and then the program considered the mean of the three utter data points on the left and the right hand side of the peak, respectively, as

² Whenever values for w_i (or relative atomic masses) were needed they were taken from [3].

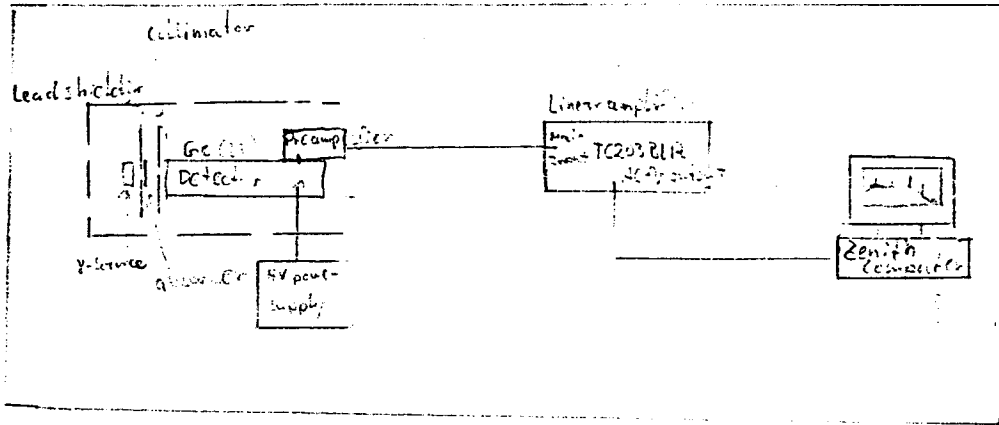


Figure 1: Sketch of the Apparatus

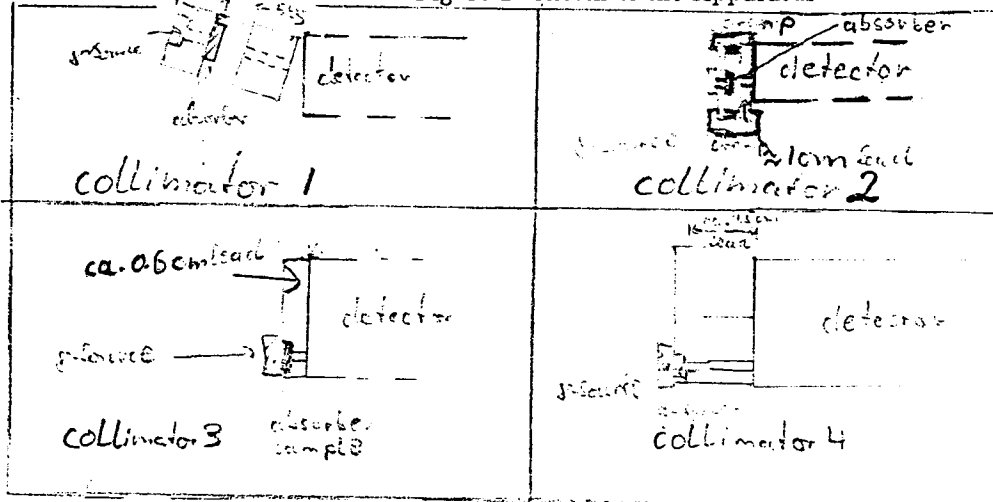


Figure 2: Collimators 1 to 4

the level of background counts, drew a straight line between those points and calculated the area between this line and the peak contour. It also calculated the error, but no proof had been done on this.

This area is proportional to the intensities I . Thus it was possible to measure $\frac{I}{I_0}$ and we have

$$\frac{\mu}{\rho} = \frac{\ln \frac{\text{net area without absorber}}{\text{net area with absorber}}}{\text{thickness of absorber } d \cdot \rho_{\text{absorber}}} \quad (7)$$

In order to try to insure that the detector sees the same solid angle from the source with and without absorber we used four different collimators (fig.2) depending on the particular photon energies, the size of the absorbers and simply the fact that we optimized the apparatus for energies of the order 100keV advancing in our work. Thinner collimators gave better counting rates but they couldn't be used for higher energies because they would have let too much photons pass through the lead rather through the absorbers. Collimator 1 used for energies $> 500\text{keV}$ consisted of two five cm thick lead blocks each with a hole 1.25cm in diameter. It was placed at a slight angle to the detector face. This resulted in a greater counting rate (probably due to the geometry of

source	energies[keV]
^{133}Ba	88.04 276.379 302.85 383.85
^{109}Cd	80.04
^{57}Co	122.66 136.48
^{137}Cs	661.038
^{54}Mn	834.75
^{60}Co	1173.208 1322.491
^{22}Na	1270.82

Table 1: Gamma Sources

the detector surface that was not known). Collimators 2 (2 layers of lead about 1cm thick; holes about 6mm) and 4 (about 1.5cm thick) were used for energies smaller than 400keV and the thinnest collimator (collimator 3) only for energies smaller than 140 keV.³ Collimator 3 and 4 consisted of one layer of lead 0.6 and about 1.5cm thick, respectively, a hole of about 2 mm in diameter and a milled recess (about 0.125"X0.12"X0.11") tailored for the capacitor, but also lead and copper pieces were cut to such dimensions.

Collimators 2 to 4 were placed tightly, surface to surface, to the detector.

4 Available Photon Energies, Settings

We could use six different γ -sources. Those, together with the photon energies belonging to them ([3]) are listed in tab.1.

To identify the particular peaks we first had to calibrate the spectrometer. This could be done by the software. We just had to find two appropriate peaks of known energy. But first, in order to get a good resolution, we did the settings so that the 1322.491 peak of ^{57}Co was at the right edge of the spectrum. The settings were then (and remained for the rest of the experiment):

High voltage power supply: 1996V

TC203BLR: Coarse gain 50; fine gain 8.03; time constant $2\mu\text{s}$

After we got familiar with some of the γ -source pulse height spectra, we were able to calibrate the system with the 88.036keV peak of ^{109}Cd and the 1270.82keV peak of ^{22}Na . With this calibration we were able to identify all of the energies listed in tab.1.

5 Data

For high energies we took the run with several sources on place at once: ^{137}Cs ^{54}Mn ^{22}Na ^{60}Co . All other data was taken with only one source respectively.

Absorbers:

The different absorbers used in this experiment are listed in tab.2. The errors of the thickness are estimations; it was taken into account how smooth and even the surface of each sample was. With those absorbers we obtained the data listed in tab.3. The deviations for $\frac{\mu}{\rho}$ are calculated by

³E.g. for a shielding of 3mm lead one can calculate $\frac{I}{I_0} = 0.1\%$ at about 150 keV

See SDC Note

No.	description	thickness d [cm]	$\rho[\frac{g}{cm^3}][4]$
1	Fe	1.09 ± 0.05	7.87
2	Al	0.288 ± 0.005	2.7
3	Al	1.00 ± 0.05	2.7
4	Cu	0.17 ± 0.02	8.96
5	Cu(coll.3 and 4)	0.158 ± 0.0014	8.96
6	Pb(coll.3)	0.0203 ± 0.0005	11.35
7	Pb	0.0203 ± 0.0005	11.35
8	Pb(coll.3 and 4)	0.110 ± 0.005	11.35
9	capacitor	0.269 ± 0.002	5.16 ± 0.02

Table 2: Absorber Samples

$$\sigma_{\frac{\mu}{\rho}} = \sqrt{\left(\frac{\Delta A_o}{A_o \rho d}\right)^2 + \left(\frac{\Delta A}{A \rho d}\right)^2 + \left(\left(\frac{\bar{\mu}}{\rho}\right) \frac{\Delta d}{d}\right)^2}$$

6 Evaluation

The measure values for known absorbers and some appropriate literature values (black curves) are plotted in graph 1. Measure values for $\frac{\mu}{\rho}$ of lead and iron match the literature curves very well and those of copper are quite acceptable. No explanation was found, why the values for aluminum differ so much from the literature values.

The data for the unknown capacitor material is plotted in graph 2. In order to get a mean value for the atomic number Z_{cap} a streight line (mean; red) and two (blue) error lines were drawn through the 2nd to the 4th (in energy) data point (graph 3). The first measure value at 80.993keV lies significant below this line. But note also that there is a absorbtion edge between 88.04 and 80.993 keV, confirmed by the measurments for lead (graph 1). The lines are parallel to the literature curves assuming the same power law is valid for all curves in this range.

From Hubbell's table three plots of $\frac{\mu}{\rho}$ vs the atomic number Z were drawn for 80, 100 and 150keV (corresponding to the pink vertical lines in graph 3), respectively (graphs 4 to 6). Then the values for $\frac{\mu}{\rho}$ at the intersections of those vertical with the read line and the error lines were transferred to the according diagrams (graph 4 to 6) and one could read the corresponding value for Z_{cap} at the Z -axis. The result was:

$E[keV]$	Z_{cap}
80.993	50.9 ± 2.0
100	51.4 ± 2.5
150	51.3 ± 2.2

Therefore

$$Z_{cap} = \frac{\frac{51.4}{2.5} + \frac{50.9}{2.0} + \frac{51.3}{2.2}}{2.5^{-1} + 2.2^{-1} + 2.0^{-1}} = 51.18$$

life time[s]	ph. energy[keV]	absorber	collimator	net area no abs. A_0	net a. with abs. A	$\frac{\mu}{\rho} [\frac{cm^2}{g}]$
5460	661.083	1(<i>Fe</i>)	1	20042 ± 258	10739 ± 176	0.073 ± 0.003
"	834.75	1	1	2938 ± 168	1638 ± 101	0.068 ± 0.01
"	1173.282	1	1	20147 ± 216	11621 ± 163	0.064 ± 0.01
"	1270.82	1	1	3758 ± 99	2377 ± 74	0.053 ± 0.005
"	1332.491	1	1	19085 ± 151	1092 ± 123	0.065 ± 0.002
4100	88.04	2(<i>Al</i>)	2	9548 ± 109	7526 ± 100	0.120 ± 0.001
4000	122.66	2	2	84485 ± 298	69496 ± 276	0.098 ± 0.003
4000	136.48	2	2	1016 ± 111	8285 ± 100	0.103 ± 0.008
5460	661.083	3	1	20241 ± 258	15337 ± 204	0.097 ± 0.008
"	834.75	3	1	2938 ± 168	2432 ± 146	0.0700 ± 0.005
"	1173.208	3	1	20147 ± 216	15377 ± 195	0.100 ± 0.008
"	1270.82	3	1	3258 ± 99	2627 ± 95	0.132 ± 0.02
"	1332.491	3	1	19085 ± 151	14510 ± 141	0.102 ± 0.007
4200	80.993	5(<i>Cu</i>)	3	10572 ± 155	9350 ± 116	0.522 ± 0.001?
4100	88.04	4	2	9548 ± 109	4298 ± 79	0.53 ± 0.06
4100	88.04	5	3	22789 ± 164	8540 ± 108	0.693 ± 0.011
4000	122.66	4	2	84485 ± 298	56876 ± 249	0.26 ± 0.03
4000	136.48	4	2	10161 ± 111	7360 ± 97	0.213 ± 0.03
4100	80.993	7(<i>Pb</i>)	3	19572 ± 155	12726 ± 129	1.86 ± 0.07?
4200	80.993	8	3	19572 ± 155	465 ± 35	2.99 ± 0.6
6000	80.993	8	4	3564 ± 71	134 ± 22	2.59 ± 0.17
4100	88.04	8	3	22789 ± 164	4078 ± 70	7.46 ± 0.2
3000	122.60	6	2	84485 ± 298	40042 ± 29	3.24 ± 0.08
3000	136.48	6	2	10161 ± 111	5652 ± 87	2.54 ± 0.14
6000	276.43	8	4	465 ± 39	234 ± 23	0.55 ± 0.11
"	303.04	8	4	1343 ± 49	769 ± 38	0.446 ± 0.05
"	356.26	8	4	6124 ± 90	4403 ± 82	0.26 ± 0.02
"	384.09	8	4	825 ± 42	602 ± 36	0.25 ± 0.06
4200	80.993	9(<i>Cap.</i>)	3	19572 ± 155	1789 ± 64	1.72 ± 0.03?
6000	80.993	9	4	3564 ± 71	210 ± 28	2.0 ± 0.1
4100	88.04	9	3	22789 ± 164	805 ± 52	2.41 ± 0.05
3000	122.6	9	3	116660 ± 347	29229 ± 177	0.994 ± 0.01
"	136.48	9	3	14266 ± 127	4831 ± 79	0.78 ± 0.02
6000	276.43	9	4	465 ± 39	343 ± 33	0.22 ± 0.09
"	303.09	9	4	1343 ± 49	1009 ± 50	0.21 ± 0.04
"	356.26	9	4	6124 ± 90	4790 ± 91	0.18 ± 0.02
"	384.09	9	4	825 ± 42	669 ± 39	0.15 ± 0.06

and

$$Z_{cap} = \bar{Z}_{cap} \left(1 + \frac{1}{2.5^{-1} + 2.0^{-1} + 2.2^{-1}} \sqrt{\frac{2.5}{51.3^2} + \frac{2.5}{51.4^2} + \frac{2.0}{50.9^2}} \right)$$

$$Z_{cap} = 51 \pm 2$$

Discussion (and some thoughts):

Unfortunately a mean atomic number is not enough for the determination of the electron radiation length L_o because of the particle density n_i or better the index i in eq. 2. Thus it is important to know the Z_i of each of the elements the capacitor is made of.

The measure value at 80.993keV indicates that there is with high probability a certain amount of lead in it. I did this measurement even twice, because I didn't believe that at first. And note that the values for $\frac{\mu}{\rho}$ at 276, 303, 356 and 384keV are gained from the same measurement with ^{133}Ba and they fit my assumed (red) curve in graph 3 at least to some degree.

Some guesses about the capacitor material were already made before our measurements: Probably a compound or a mixture with Ba (perhaps BaTiO_3) and Cu in it. The density of BaTiO_3 (hex.) is $5.8 \frac{\text{g}}{\text{cm}^3}$ [3] and the values for dielectric constants of titanates (note also of PbTiO_3) are 15 - 12000 (at 10^6 cycles) [3]. Hence it is a good capacitor material. The $\frac{\mu}{\rho}$ - curve is also plotted in graph 2 (it was gained from Hubbell's table using eq.20). It doesn't fit the data very well. Also the density of the capacitor material ($5.16 \frac{\text{g}}{\text{cm}^3}$) differs somewhat from that of BaTiO_3 . Thus it can't be pure BaTiO_3 .

It was also assumed that there was some copper in it. A curve for Ba_2CuO_3 (calculated as for BaTiO_3) is given in graph 2. It fits the data (except the value for 80.993keV) quite well.

In terms of mechanical properties the capacitor seems to be ceramics: it broke like it and the site of fracture was smooth but somewhat convex or concave, respectively (thus it is at least no crystal).

7 Summary

We could derive a "mean atomic number" for the unknown capacitor material of

$$Z_{cap} = 51 \pm 2$$

but only guess which elements and which amount of them are in it. Because it is crucial to know the Z_i and the n_i for the calculation of the radiation length and the results for those in this experiment are only speculations, it is of no use to give a value for it here. Hence further work has to be done. Perhaps other available methods could give more information.

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