A STUDY OF DARK MATTER AND DARK ENERGY

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ABSTRACT

Recent cosmological observations, including the observations of distant Type-Ia supernovae, the Large Scale Structure, Wilkinson Microwave Anisotropy Probe in the Cosmic Microwave Background Radiation, the mass-energy density estimates from galaxy clusters, weak lensing etc. suggest that ordinary matter and energy constitutes only about 4% of the universe and the remaining 96% of the universe is yet unknown. Out of these 96%, about 23% is some form of hitherto unknown matter called dark matter and about 73% is dubbed as dark energy. The present observed accelerated expansion of the universe has been attributed to this exotic component, dark energy, with negative pressure which can induce repulsive gravity causing the accelerated expansion of the universe. The question of the actual nature and composition of the dark matter and dark energy of the universe today is one of the most exciting and challenging problems.

This Ph. D. thesis on a study of dark matter and dark energy consists of seven chapters.

In **Chapter 1**, a brief introduction to dark matter and dark energy is given.

In **Chapter 2**, we have considered Raychaudhuri equation. Though A. K. Raychaudhuri formulated his equation without the knowledge of accelerated expansion of the universe, we have shown that dark energy can be incorporated into Raychaudhuri equation through enlarged law of gravitation. In this chapter we have considered the dynamical cosmological term, quintessence, k-essence, tachyon and phantom scalar field as the candidates of dark energy.

In **Chapter 3**, we have obtained analytical solutions of the evolution of mass of black holes and worm holes immersed in a generalised Chaplygin gas (GCG) model and calculated the evolution of the mass of black hole and worm hole embedded in a universe filled with GCG. The GCG model represents the unification of dark matter and dark energy of the universe. For the equation of state (EOS) $\omega = \frac{p}{\rho} = -1$ of the dark component in GCG model it is found that the mass of the black hole increases and the mass of the worm hole decreases as the universe expands and both the masses become constant when the dark energy component of the GCG model becomes dominant in the universe.

In **Chapter 4**, we have studied the anisotropic expansion and acceleration of the universe driven by tachyonic matter. Though the present day universe appears to be isotropic, there is no evidence that the early universe was also of the same type. There is a cosmological view that the universe might have been anisotropic and also inhomogeneous in the very early era and that in the course of its evolution these characteristics have been wiped out under the action of some process or mechanism, and finally an isotropic and homogeneous universe had resulted. Hence the expansion and acceleration of the universe driven by tachyonic matter are studied by taking the anisotropic models of the universe Bianchi type I, Kantowski Sachs and Bianchi type III metrics in four dimensions and also Bianchi type I metric in higher dimensions. A set of solutions is obtained and their physical implications are discussed.

In **Chapter 5**, we have studied a homogeneous and anisotropic universe filled with matter and holographic dark energy components. Assuming deceleration parameter to be a constant, an exact solution to Einstein's field equations in axially symmetric Bianchi type-I line element is obtained. A correspondence between the holographic dark energy models with the quintessence dark energy models is also established.

Quintessence potential and the dynamics of the quintessence scalar field are reconstructed, which describe accelerated expansion of the universe.

In **Chapter 6,** a homogeneous and anisotropic universe filled with new agegraphic dark energy (NADE) and cold dark matter components is studied. Here also, assuming deceleration parameter to be a constant, we obtained some exact solutions to Einstein's field equations in axially symmetric Bianchi type-I line element. We established a correspondence between the interacting NADE models with the tachyon and bulk viscous dark energy models. Tachyon potential, the dynamics of the tachyonic scalar field and bulk viscous coefficient are reconstructed, which describe accelerated expansion of the universe.

In the last Chapter, viz. **Chapter 7,** we have considered the dark energy model with the equation of state $p_{DE} = -\rho_{DE} - A\rho_{DE}^{\alpha}$ which leads to four finite life time future singularities of the universe for different values of the parameters A and α. Since from the matter dominated era to the dark energy dominated era the ratio of the dark energy density to the matter energy density increases as the universe expands for these future singularities, the universe passes through a significant time when the dark energy density and the matter energy density are nearly comparable. Considering $\frac{1}{r_0} < r = \frac{\rho_{DE}}{\rho_M} < r_0$, where r_0 is any fixed ratio, we have calculated the fraction of total life time of the universe when the universe passes through the coincidental stage for these singularities. It has been found that the fractional time varies as α varies within the range for which these finite life time future singularities occur and the fraction is smaller for smaller values of r_0. Importance of the fractional time and observational limits onto the values of the parameter A and α has also been discussed.

List of symbols

s	Action
a	Scale factor
a_0	Present value of the scale factor
Λ	Cosmological constant
q	Deceleration parameter
φ	Scalar
G	Gravitational constant
p	Matter pressure
p_φ	Pressure of the scalar field
R	Ricci scalar
$R_{\mu\nu}$	Ricci tensor
$R^{\alpha}_{\mu\nu}$	Riemann tensor
T	Temperature
$T_{\mu\nu}$	The energy momentum tensor
$g_{\mu\nu}$	The metric tensor
τ	Relaxation time for transient bulk viscous effects

ξ	Bulk viscosity coefficient
Π	Bulk viscous stress
θ	Expansion
σ^2	Shear
v^μ	unit velocity vector
$\dot{v}^\mu$	Acceleration (departure of v^μ from geodesicity). It arises from the pressure gradient in the case of a perfect fluid.
ω^2	Vorticity
G	Gravitational constant
H	Hubble parameter
φ	The scalar
$\dot{\varphi}^2$	The scalar field kinetic term
$V(\varphi)$	The Scalar field potential
ρ_m	Matter energy density
ρ_b	Energy density of the baryon
ρ_r	Energy density of the radiation
ρ_φ	Density of the scalar field
ρ_Λ	Vacuum energy density
ρ_{ch}	Energy density of the Chaplygin gas
Ω_m	Matter energy density parameter

Ω_{m0}	Present value of the matter energy density parameter
Ω_{Λ}	Vacuum energy density parameter
$\Omega_{\Lambda 0}$	Present value of the vacuum energy density parameter
Ω_{φ}	Scalar field energy density parameter
ω	Equation of state
ω_m	Matter equation state
ω_{φ}	Scalar field equation state

Contents

Chapter - 1

Introduction

The speculations about the universe are as old as man himself. That is why, Cosmology, the science of the universe, attracts and fascinates us all. Cosmologists have long been dedicated a large amount of time and effort to reveal the true nature of the universe. But history of Cosmology shows that the study of the universe evolves only in a piecemeal way. Until 1915, the universe was thought to be static. Even Albert Einstein was so sure about this that to describe a static universe he modified the field equations in his General Theory of Relativity by introducing a constant Λ, called the cosmological constant. The picture of expanding universe dates back to only 1929 when the American astronomer Edwin Hubble detected that the distant galaxies were receding faster than the closer ones with a speed v proportional to its distance d given by $v = Hd$, known as the Hubble's law, where the constant H is called the Hubble parameter or Hubble constant. The Hubble parameter H changes with time and its value at the present epoch is denoted by H_0.

Now, whether the expansion of the universe will continue forever or not, it is necessary to find out what the universe is made of and what fraction of the density is made of the visible matter. Since it is easy to calculate how much matter is needed for its total gravitational influence to bring the observational expansion a halt, cosmologists have therefore put a great effort to determine the matter density of the universe. The density of visible matters like bright stars and galaxies can be inferred by counting the number of galaxies in our region of space and also by measuring the way in which the galaxies move. Just as the speed of the earth in its orbit around the sun is related to the mass of the sun, so is true for the relative speeds of different galaxies in a cluster of galaxies. It has been observed in clusters of galaxies that the motion of galaxies within a cluster suggests that they are bound by a gravitational force due to about 5-10 times as much matter as can be accounted for from luminous matter in said galaxies. Thus, when the total mass of cluster of galaxies is compared to its total luminosity, most of the mass is found to be missing. This missing mass neither emits light nor scatters light. It also does not absorb light and is not even made out of atoms. A Swiss Astrophysicist Fritz Zwicky [56] of the California Institute of Technology was the first to propose the existence of such missing mass.

In 1933, when Zwicky estimated the total amount of mass in a cluster of galaxies, known as the Coma cluster, based on the motion of the galaxies near the edge of the clusters and compared it to one based on the number of galaxies and total brightness of the cluster, he found that 9/10 of the matter in the Coma cluster was not luminous and therefore could not be seen. Also the gravity of the visible galaxies in the cluster should be far too small for such fast orbits, so something extra was required. Zwicky inferred that there must be some other form of matter existent in the cluster which provides enough of the mass and gravity to hold the cluster together, Zwicky called this "dark matter".

In the 1970s, Rubin and Ford [184] played a major role in establishing the existence of dark matter in spiral galaxies. Much like Zwicky's findings with galaxy clusters, Rubin found that the stars near the bulge behaved as expected, but stars on the edge of the spiral galaxies were moving much more quickly than one would have predicted. Without dark matter, spiral galaxies could fly apart, but the dark matter stabilizes these galaxies.

There is also evidence of dark matter in elliptic galaxies, in particular, coming from strong gravitational lensing measurements. Einstein's theory of General Relativity predicts that light always travels in a straight path and bend only to follow the curvature of space-time. The curvature of space-time depends on the mass of the gravitating object. This effect can be used to gravitationally ascertain the existence of mass even when it emits no light. Lensing measurements confirm the existence of enormous quantities of dark matter both in galaxies and in clusters of galaxies [105].

Another piece of gravitational evidence for dark matter is the hot gas in clusters. The existence of this hot gas in the cluster can only be explained by a large dark matter component that provides the potential well to hold on to the gas [181].

In most regions of the universe, dark matter and visible matter are found together due to their mutual gravitational attraction. But in the Bullet Cluster (a cluster formed out of a collision of two smaller clusters), a collision between two galaxy clusters appears to have caused a separation of dark matter and ordinary matter [90]. For this reason the Bullet Cluster is considered as the most direct observational evidence for dark matter.

Thus dark matter is some form of hitherto unknown matter that is inferred to exist from gravitational effects on visible matter but is undetectable by emitted or scattered electromagnetic radiation. On the basis of the observations [9, 28, 52, 75, 115, 171] dark matter is recognised today to be nearly 23% of all the mass in the universe while the ordinary matter accounts for only about 4%. From these figures, dark matter constitutes about 85% of the matter in the universe. Though observations suggest that a large amount of the matter in the universe is dark, the composition of dark matter is not yet clear. However, the cosmologists speculate that dark matter may be baryonic or non-baryonic [57].

Baryonic dark matter are astrophysical objects that are not to be exotic, such as planets, brown dwarfs, white dwarfs or black holes. These are bodies that have been either never managed to become stars or are the remnants of a star such as white dwarfs or black holes. However, large masses, like galaxy-sized black holes can be ruled out on the basis of gravitational lensing data. Studies of big bang nucleosynthesis have convinced us that only a small fraction of the total dark matter can be baryonic dark matter [28]. Possibilities involving normal baryonic matter include astronomical bodies like brown dwarfs or perhaps small, dense clouds of heavy elements that are composed of ordinary matter which emit little or no electromagnetic radiation. Such objects are known as massive compact halo objects or "MACHOS" [32, 91, 143]. Though a small proportion of dark matter may be baryonic, search for MACHOS has been started utilizing the gravitational lens effect predicted by General Theory of Relativity, as baryonic matter seems adequate to at least explain the rotation curves of galaxies.

Non-baryonic dark matter is supposed to be made of one or more elementary particles other than the usual electrons, protons and neutrons [57]. However, the vast majority of the dark matter in the universe is believed to be non-baryonic and thus not formed out of atoms. It is also believed not to interact with ordinary matter via electromagnetic forces. Non-baryonic dark matter is classified in terms of the mass of the particles that is assumed to make it up and the velocity dispersion of those particles. Based on these there are three prominent hypotheses on non-baryonic dark matter, called Hot dark matter (HDM), Warm dark matter (WDM) and cold dark matter (CDM) [6]. Hot dark matter has zero or near zero mass particles allowing it to move with a velocity approximately over $0.95c$ which is very close to the velocity of light c. Thus HDM are particles that travel at ultra-relativistic velocities. This high velocity causes a high energy state that is not conductive to building structure. A known example of HDM already exists are the neutrinos. Neutrinos have a very small mass, at least 10^5 times less massive than an electron. Neutrinos do not interact via electromagnetic or the strong nuclear force. Other than gravity, neutrinos only interact with normal matter via the weak nuclear force and so are incredibly difficult to detect. This is what makes them appealing as dark matter [159].

Warm dark matter are particles travelling at relativistic speeds, basically with a velocity greater than $0.1c$ but less than $0.95c$. No particle has been discovered so far which can be categorised as WDM. A postulated candidate for the WDM is the sterile neutrino which is a heavier but slower form of neutrinos that does not even interact through the weak nuclear force unlike regular neutrinos. Like HDM, WDM also cannot explain how individual galaxies are formed from the Big Bang because it moves too quickly to be bound to galaxies or cluster of galaxies [135].

Cold dark matter is massive enough to move at sub-relativistic or non-relativistic velocities. Generally its velocity is less than $0.1c$. Therefore cold dark matter can explain how galaxies form and stay the way they are. It is thought that the dark matter in our universe is cold and it was likely created during the Big Bang. Cold dark matter is the most favoured of the three types of dark matter [192].

Some non-baryonic candidates for dark matter found in literature are given in Table 1.

There are many experiments currently running or planned aiming to detect dark matter. These can be divided into two classes: direct detection experiment and indirect detection experiment. Direct detection experiment search for the scattering of dark matter particles of atomic nuclei within a detector and operate in deep underground laboratories to reduce the background from cosmic rays [18, 144]. The majority of present experiments use one of the two detector technologies: Cryogenic detectors and Noble liquid detectors. Indirect detection experiment search for the products of WIMP (weakly interacting massive particles) annihilations [5, 47]. If WIMPs are the particles whose particle and antiparticle are the same, then two WIMPs colliding would annihilate to produce gamma rays and particle-antiparticle pairs. However, such detection is not conclusive evidence for dark matter as the backgrounds from other sources are not yet fully understood.

At the time while the prospect of a universe filled with dark matter has itself challenges our understanding of the physical world; an even more startling cosmological discovery has come to light in 1998. Until the late 1990's cosmologists took it for granted that the expansion of the universe was slowing down under the influence of gravitation. A dramatic breakthrough happened in 1998 when two independent teams of astronomers, one led by S. Perlmutter [171] and the other by A. G. Riess [9], were searching for distant supernovae hoping to measure the rate at which the expansion of the universe was slowing down. They traced the expansion of the universe over the past five billion years and were in a shock to find that the cosmic expansion is not slowing down but speeding up. This discovery has created a confusing situation among the cosmologists because although the standard cosmological models have been confirmed by data from Wilkinson Microwave Anisotropy Probe (WMAP) and by other telescope surveys of the large-scale structure of the universe, it was not known why the cosmic expansion is accelerating. To unveil the truth, intensive search is going on both in the theoretical and observational level. Many researchers suggested modifications and changes to Einstein's General Theory of Relativity. Some others expected a conventional explanation for the accelerating expansion of the universe based on Astrophysics, e.g. the effect of dust on difference between young and the old supernovae. But to the cosmologists around the world, a kind of repulsive force which acts as anti- gravity is responsible for gearing up the universe some five billion years ago [154]. This hitherto unknown physical entity is dubbed as "dark energy" which has negative pressure and makes up about three quarters of the total present cosmic energy density.

Many cosmologists like to select the cosmological constant Λ, introduced by Einstien in his field equations, as a suitable candidate for dark energy because of its weired repulsive gravity. The cosmological constant provides a pretty good explanation to the expansion of the universe being accelerated.

But selection of the cosmological constant as dark energy faces some serious problems. Due to its non-evolving nature, the cosmological constant Λ is plagued with the fine-tuning problem which demands that the value of Λ must be 123 orders of magnitude and 55 orders of magnitude larger respectively in Planck Scale ($\sim 10^{19}$Ge) and Electroweak scale ($\sim 10^{2}$Ge) than its present observed value. Apart from fine-tuning problem, equation of state for Λ is $\omega = \frac{p}{\rho} = -1$ and various observations indicate that the present value of ω is closer to -1, not necessarily equal to -1. Again the dynamical nature of dark energy introduces a new cosmological problem. Various recent cosmological observations like the cosmic microwave temperature fluctuations, the luminosity-redshift relation from supernova light-curves etc. have converged on a cosmological model which is expanding and whose energy density is dominated by dark energy ($\sim$73%) but contains a comparable amount of matter ($\sim$27%) and some radiation [41]. The energy of these components drives the expansion of the universe via the Friedmann equation and in turn responds to expansion via their equations of state [80]. If Einstein's cosmological constant Λ is considered to be the dark energy, then radiation dilutes as a^{-4} and matter dilutes as a^{-3} while the dark energy density remains constant, where a is the scale factor of the universe. But different observations suggest that matter and dark energy dilutes at different rates during cosmic expansion and the matter energy density ρ_M and the dark energy density ρ_Λ happen to be of the same order today [150]. Thus, we are faced with a problem, known as the "Cosmic Coincidence Problem"- why the present observed density of dark energy and matter are nearly comparable! For these reasons at present, Λ with a dynamical character is preferred over a constant Λ, especially a time dependent Λ which has decreased slowly from its large initial value to reach its present small value [80].

Some phenomenological Λ decay laws available in the literature are given in Table 2.

As an alternative to the cosmological constant as dark energy, a number of dynamically evolving scalar field models of dark energy are also considered to explain the present accelerated expansion of the universe. Quintessence is such a possible suspects of dark energy [30]. Quintessence is different in comparison to the cosmological constant in the sense that it can vary in space and time. Average energy density and pressure of the quintessence decay slowly with time. This feature might help to explain the tuning and sudden onset of cosmic acceleration. Though actual evidence is not available in support of the existence of quintessence, but it is still a very possible candidate of dark energy and should not be ruled out. In comparison to cosmological constant, quintessence predicts a slightly slower acceleration of the expansion of the universe [51]. Some researchers think that the best evidence for quintessence would come from violations of Einstein's equivalence principle and variation of the fundamental constants in space or time. However quintessence cosmologies have been exhaustively tested by using CMB and SN Ia data.

K-essence is another candidate of dark energy. Originally, kinetic-energy driven inflation, called K-inflation, was proposed to explain early universe inflation at high energies [33]. This scenerio was later applied to dark energy and was called K-essence [79]. K-essence is characterised by a scalar field with a non-canonical kinetic energy. The difference between quintessence and K-essence is that K-essence cosmologies unlike quintessence ones are derived from Lagrangian with non-canonical kinetic energy terms. For both these models the value of the equation of state parameter ω lies between -1 and $-\frac{1}{3}$.

The tachyon field is proposed as another source of the dark energy. For Tachyon scalar field, ω lies between -1 and 0. The tachyon is an unstable field which has become important in string theory through its role in the Dirac-Born-Infeld (DBI) action which is used to describe the D-brane action. There have been a number of works done concerning the cosmology of Tachyon's [26, 85, 178].

All these scalar field models mentioned above correspond to an equation of state parameter $\omega \geq -1$. But the recent observational data suggest that the parameter ω has a very narrow range around $\omega = -1$ and is quite consistent with being below this value. On this observational basis Caldwell et al., [152] first proposed the phantom dark energy where the value of ω is less than -1. The simplest explanation for the phantom dark energy is provided by a scalar field with a negative kinetic energy. The energy density of the phantom field increases with increasing scale factor and the phantom energy density becomes infinite at a finite time [153].

The phantom field with a negative kinetic term has a problem with quantum instabilities. To solve this problem another kind of dark energy model was proposed called the dilaton field φ. The coefficient of the kinematic term of the dilaton can be negative in the Einstein frame, which means that the dilaton behaves as a phantom-type scalar field. The presence of higher-order derivative terms for the dilaton field φ the stability of the system is satisfied even when the coefficient of $\dot{\varphi}^2$ is negative [55, 116].

Besides these scalar field models there are some other kinds of models which are proposed to describe the universe with dark energy, such as: Chaplygin gas [17], a linear equation of state [53], the holographic dark energy models [121, 190], the original agegraphic dark energy [147] and new agegraphic dark energy models [68].

Recently, a great variety of interacting dark energy models [21, 22, 31, 63, 103, 107, 146, 149, 174, 176, 194, 199] have been constructed to solve the coincidence problem. The unified inflation/acceleration universe also occurs in some complicated EOS of the universe [185].

Another equivalent approach is to assume that dark energy and dark matter sectors interact in such a way that the dark matter particles acquire a varying mass, dependent on the scalar field which reproduces dark energy [65]. This consideration allows for a better theoretical justification, since a scalar-field-dependent varying-mass can arise from string or scalar-tensor theories [175]. The dark energy with varying-mass dark matter particles models have been studied detail in Refs. [40, 59, 97, 98, 99, 100, 124, 182, 195].

The generalized Chaplygin gas (GCG) model [109] is an interesting candidate for the unification of the dark matter and dark energy. In the GCG approach an exotic background fluid is considered, described by the equation of state $p_{ch} = -\frac{A}{\rho_{ch}^{\alpha}}$, where A and α are positive constants with $0 < \alpha \leq 1$. The case $\alpha = 1$ corresponds to the Chaplygin gas. The GCG model has been successfully confronted with many phenomenological tests such as Supernovae data, CMB data etc. An attractive feature of these models is that at early times, the energy density behaves as a matter and as a cosmological constant at a later stage.

The cosmic viscosity, an effective quantity caused mainly by the non-perfect cosmic contents interactions may also play a role as dark energy candidate causing the observed acceleration of the universe [84, 108, 193].

In some of these models of dark energy there have been another theoretical possibility that expanding universe can come to a violent end at a finite future time. Phantom dark-energy models, with the equation of state $\omega < -1$, are characterised by a finite lifetime future singularity of the universe [153]. Barrow [74] showed that a different type of future singularity can appear at a finite time even when the strong energy condition $\rho + 3p \geq 0$, $\rho + p \geq 0$, is satisfied. This sudden future singularity corresponds to the one in which the pressure p, density ρ and the scale factor a are finite. S. Nojiri et al. first proposed that the dark energy model with the equation of state $p_{DE} = -\rho_{DE} - A\rho_{DE}^{\alpha}$, is characterised by four types of future singularity for a finite life time of the universe for different values of the parameter A and α [170]. It is to be noted that the singularity issue has the fundamental importance in the modern cosmology [139, 164, 170].

At present various attempts have also been made to modify Einstein's general theory of relativity, and therefore avoid the need for exotic matter to drive the accelerated expansion. The contribution of the matter content of the universe is represented by the energy momentum tensor on the right hand side of Einstein equations, whereas the left hand side is represented by pure geometry. There are then two ways to give rise to an accelerated expansion: (i) either by supplementing the energy momentum tensor by an exotic form of matter such as a cosmological constant or scalar field; or (ii) by modifying the geometry itself [50]. The geometrical modifications can arise from quantum effects such as higher curvature corrections to the Einstein Hilbert action. Such curvature corrections are used to avoid the above singularities in the presence of a dark fluid [167]. Here dark fluid combines dark matter and dark energy in a single energy field that produces different effects at different scales.

The alternative approach of modified gravity (or dark gravity) provides a new angle on the problem, but also faces serious difficulties, including in all known cases like severe fine-tuning and the problem of explaining why the vacuum energy does not gravitate. The lack of an adequate theoretical framework for the late-time acceleration of the universe represents a deep crisis for the theory—but also an exciting challenge for theorists. It seems likely that an entirely new paradigm is required to resolve this crisis [145]. A variety of different aspects of f(R) gravity and associated cosmological dynamics is discussed in Ref. [36, 39, 59, 93, 155, 160]. An interesting possibility of obtaining late time acceleration from modified Gauss-Bonnet gravity is discussed in Ref. [166].

The other exciting possibility of obtaining accelerated expansion of the universe is provided by theories with large extra dimensions known as braneworlds. Being inspired by string theory, our four dimensional space-time (brane) is assumed to be embedded in a higher dimensional bulk space-time. In these scenarios all matter fields are confined on the brane whereas gravity being a true universal interaction can propagate into anti de sitter bulk [50]. In the braneworld model of Dvali-Gabadadze-Prrati there is a cross-over scale around which gravity manifests these higher-dimensional properties [62]. This scenario is a simple one parameter model which can account for the current acceleration of the universe provided the cross-over scale is fine tuned to match observations.

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Chapter – 2

Dark energy and Raychaudhuri equation

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2. Dark energy and Raychaudhuri equation

2.1 Introduction

Recent Astrophysical data obtained from high red shift surveys of Supernovae (SnIa) [9, 171] indicate that the present universe is passing through a phase of accelerated expansion. This expansion has been attributed to an exotic component, dubbed "dark energy", with negative pressure which can induce repulsive gravity causing the accelerated expansion. The evidence for dark energy has been indirectly verified by the recent measurements of Wilkinson Microwave Anisotropy Probe (WMAP) in the Cosmic Microwave Background Radiation (CMB) and the Large -Scale Structure (LSS) observations [113]. These observations made it clear that the current matter-energy density of the universe is closed to its critical value of which 73% is attributed to dark energy, 23% to cold dark matter and only 4% is ordinary matter of baryonic nature. Naturally a cosmological model is required to explain acceleration in the present universe. Many cosmologists like to select the cosmological constant Λ, introduced by Einstien in his field equations, as a suitable candidate for dark energy because of its weired repulsive gravity. But due its non-evolving nature, it is plagued with the fine-tuning problem which demands that the value of Λ must be 123 orders of magnitude and 55 orders of magnitude larger respectively in Planck Scale ($\sim 10^{19}$Ge) and Electroweak scale ($\sim 10^{2}$Ge) than its present observed value. Apart from fine-tuning problem, equation of state for Λ is $\omega = \frac{p}{\rho} = -1$ and various observations indicate that the present value of ω is closer to -1, not necessarily equal to -1. For these two reasons at present, Λ with a dynamical character is preferred over a constant Λ, especially a time dependent Λ which has decreased slowly from its large initial value to reach its present small value [80]. A number of dynamically evolving scalar field models of dark energy such as Quintessence, K-essence, Tachyon, Phantom etc. are also considered to explain the present accelerated expansion of the universe.

Quintessence is a homogeneous minimally coupled scalar field φ which is a plausible alternative to the cosmological constant Λ [173]. Quintessence model provides a solution to the fine-tuning problem and by means of tracker solutions it provides a solution to the coincidence problem also. But with particular potentials the scalar field φ lead to late time inflation. Since Quintessence relics on the potential energy of the scalar fields to lead to the late time acceleration of the universe, therefore it is possible to have a situation where the accelerated expansion arises out of modifications to the kinetic energy of the scalar fields. Originally, kinetic-energy driven inflation, called K-inflation, was proposed to explain early universe inflation at high energies [33]. This scenario was later applied to dark energy and was called K-essence [34]. K-essence is characterised by a scalar field with a non-canonical kinetic energy. The difference between Quintessence and K-essence is that K-essence cosmologies unlike Quintessence ones are derived from Lagrangian with non-canonical kinetic energy terms. Due to the simplicity and economy, these two models are popular models of dark- energy. Quintessence cosmologies have been exhaustively tested using CMB and SN Ia data. Recently, on investigation of the degree of resemblance between these two theoretical set ups it was found that every Quintessence model can be viewed as a K-essence model by a kinetic linear function [79]. For both these models the value of the equation of state parameter ω lies between -1 and $-1/3$.

Meanwhile in the search of exotic matter, being source for dark energy, Tachyon also has been considered as a possible candidate for dark energy. For Tachyon scalar field, ω lies between -1 and 0. There have been a number of works done concerning the cosmology of Tachyon's [66, 85]. All these scalar field models mentioned above correspond to an equation of state parameter $\omega \geq -1$. But on the basis of recent observational data, R. R. Caldwell [152] noted that the parameter ω has a very narrow range around $\omega = -1$ and is quite consistent with being below this value. The region where the value of ω is less than -1 is typically referred to as being due to some form of phantom (ghost) dark energy. The simplest explanation for the phantom dark energy is provided by a scalar field with a negative kinetic energy [153]. Such a field may also be motivated from S-brane construction in string theory [37]. The energy density of the phantom field increases with increasing scale factor and the phantom energy density becomes infinite at a finite time known as Big Rip condition [161]. However, this problem may be avoided in some models which meet the current observations fairly well. Besides these scalar field models there are some more alternatives such as a Chaplygin gas, Dilatonic dark energy etc. which may be considered as suitable candidates for dark energy.

Recently, Krori et al. have shown that with appropriate inputs, the Raychaudhuri equation can provide physically rational clues to certain cosmological phenomena [89]. When A. K. Raychaudhuri, in 1955, formulated his equation, the accelerated expansion of the universe was not known. It would therefore be reasonable to incorporate dark energy into his equation. Krori et al. have shown how dark energy can be incorporated into Raychaudhuri equation. Here in this chapter we have shown that all the above mentioned candidates of dark energy can be incorporated into the Raychaudhuri equation through enlarged law of gravitation causing the accelerated expansion of the universe.

2.2 Raychaudhuri Equation

Raychaudhuri Equation is given by

$$\dot{v}^{\mu}_{;\mu} + 2\omega^2 - 2\sigma^2 - \frac{1}{3}\theta^2 - \theta_{,\alpha}v^{\alpha} - 4\pi G(\rho + 3p) = 0 \quad (1)$$

where v^{μ} = unit velocity vector

$\dot{v}^{\mu}$ = acceleration (departure of v^{μ} from geodesicity). It arises from the pressure gradient in the case of a perfect fluid.

θ = expansion

ω^2 = vorticity

σ^2 = shear

ρ = matter energy density and

p = fluid pressure

The last term in Eq. (1) represents the gravity effect and is abetted by shear.

With $\theta_{,\alpha}v^{\alpha} = \frac{d\theta}{ds}$, where s parametrises a geodesic, Eq. (1) can be written in the form

$$\frac{d\theta}{ds} + \frac{1}{3}\theta^2 = \dot{v}^{\mu}_{;\mu} + 2\omega^2 - 2\sigma^2 - 4\pi G(\rho + 3p) \quad (2)$$

For a general metric, defining a scalar function, R, such that $\theta = 3\frac{\dot{R}}{R}$, Raychaudhuri et al [16] have written Eq.(2) in the alternative form

$$\frac{\ddot{R}}{R} = \frac{1}{3}\dot{v}^{\mu}_{;\mu} + \frac{2}{3}\omega^2 - \frac{2}{3}\sigma^2 - \frac{4\pi G}{3}(\rho + 3p) \quad (3)$$

2.3.1 Dynamical cosmological constant term Λ

Einstein's law of gravitation is given by [172]

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \Lambda g_{\mu\nu} = -8\pi G T_{\mu\nu} \quad (4)$$

If we take Eq. (4) in the form

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi G\left(T_{\mu\nu} - \frac{\Lambda}{8\pi G}g_{\mu\nu}\right) \quad (5)$$

then we can interpret Λ as a part of the matter content of the universe and hence a function of the time t. The incorporation of a time-dependent Λ in Einstein's field equations amounts to assuming that in the presence of the usual energy-momentum tensor of the universe, there is an additional term $-\frac{\Lambda}{8\pi G} g_{\mu\nu}$ which may be regarded as the energy-momentum tensor of the vacuum.

Using Eq. (5) the Raychaudhuri equation (3) is generalised into the form

$$\frac{\ddot{R}}{R} = \frac{1}{3}\dot{v}^{\mu}_{;\mu} + \frac{2}{3}\omega^2 - \frac{2}{3}\sigma^2 - \frac{4\pi G}{3}(\rho + 3p) + \frac{1}{3}\Lambda \quad (6)$$

From Eq. (6) it is clear that $\Lambda = \Lambda(t)$ opposes the gravity effect due to $4\pi G(\rho + 3p)$. If $\frac{1}{3}\Lambda(t)$ is large enough to make the R.H.S. of Eq. (6) positive the acceleration of the universe should occur.

2.3.2 Quintessence

The action for quintessence scalar field is [173]

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)\right] \quad (7)$$

where $V(\varphi)$ is the quintessence potential.

The energy-momentum tensor for quintessence scalar φ has the form

$$T_{\mu\nu} = \partial_\mu\varphi\partial_\nu\varphi + g_{\mu\nu}\left[-\frac{1}{2}g^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi - V(\varphi)\right] \quad (8)$$

which yield

$$\rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + V(\varphi)$$

$$p_\varphi = \frac{1}{2}\dot{\varphi}^2 - V(\varphi)$$

with the equation of state

$$\omega_\varphi = \frac{\dot{\varphi}^2 - 2V(\varphi)}{\dot{\varphi}^2 + 2V(\varphi)}$$

Incorporating quintessence scalar field into the Raychaudhuri equation, from Eq. (3), we obtain

$$\frac{\ddot{R}}{R}=\frac{1}{3}\dot{v}^{\mu}_{;\mu}+\frac{2}{3}\omega^2-\frac{2}{3}\sigma^2-\frac{4\pi G}{3}\left[\rho+3p+\left(1+3\omega_{\varphi}\right)\rho_{\varphi}\right] \tag{9}$$

As $-1<\omega_{\varphi}<-\frac{1}{3}$, from Eq. (9) it is clear that the quintessence scalar field term $-\frac{4\pi G}{3}(1+3\omega_{\varphi})\rho_{\varphi}$ opposes gravity due to the term $-\frac{4\pi G}{3}(\rho+3p)$ and hence in a quintessence scalar field dominated universe, the accelerated expansion is inevitable.

2.3.3 K-essence

The action for K-essence characterised by a scalar field with a non-canonical kinetic energy is given by [33]

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2}\frac{R}{k^2}+p(\varphi_k,X)\right] \tag{10}$$

where $k^2=8\pi G$ and $X=-\frac{1}{2}\nabla^{\mu}\varphi_k\nabla_{\mu}\varphi_k$

The pressure of the K-essence scalar-field is given by $p(\varphi_k,X)$ itself and $p(\varphi_k,X)=f(\varphi_k)(-X+X^2)$

The energy-density ρ_k is given by

$\rho_k=f(\varphi_k)(-X+3X^2)$

and the equation of state is

$\omega_k=\frac{1-X}{1-3X}$

Incorporating K-essence scalar field into the Raychaudhuri equation, from Eq. (3) we obtain

$$\frac{\ddot{R}}{R}=\frac{1}{3}\dot{v}^{\mu}_{;\mu}+\frac{2}{3}\omega^2-\frac{2}{3}\sigma^2-\frac{4\pi G}{3}[\rho+3p+(1+3\omega_k)\rho_k] \tag{11}$$

As in 2.3.2, like quintessence, accelerated expansion of the universe is also inevitable in a K-essence scalar field dominated universe.

2.3.4 Tachyon Scalar

An action for tachyon scalar φ_t is given by Born-Infeld like action [66]

$$s = -\int d^4x\sqrt{-g}\, V(\varphi_t)\sqrt{1 - g^{\mu\nu}\partial_\mu\varphi_t\partial_\nu\varphi_t} \qquad (12)$$

where $V(\varphi_t)$ is the tachyon potential.

Energy-momentum tensor component for tachyon scalar φ_t are

$$T^t_{\mu\nu} = V(\varphi_t)\left[\frac{\partial_\mu\varphi_t\partial_\nu\varphi_t}{\sqrt{1-g^{\alpha\beta}\partial_\alpha\varphi_t\partial_\beta\varphi_t}} + g_{\mu\nu}\sqrt{1 - g^{\alpha\beta}\partial_\alpha\varphi_t\partial_\beta\varphi_t}\right] \qquad (13)$$

which yield

$$\rho_t = \frac{V(\varphi_t)}{\sqrt{1-(\varphi_t)^2}}$$

$$p_t = -V(\varphi_t)\sqrt{1 - \dot{\varphi_t}^2}$$

with the equation of state

$$\omega_t = \frac{p_t}{\rho_t} = \dot{\varphi_t}^2 - 1$$

Now, incorporating tachyon scalar field into the Raychaudhuri equation, from Eq. (3) we obtain

$$\frac{\dot{R}}{R} = \frac{1}{3}\dot{v}^{\mu}_{;\mu} + \frac{2}{3}\omega^2 - \frac{2}{3}\sigma^2 + \frac{8\pi G}{3}\frac{V(\varphi_t)}{\sqrt{1-\dot{\varphi_t}^2}}\left[1 - \frac{3}{2}\,\dot{\varphi_t}^2\right] \qquad (14)$$

From Eq.(14) it is obvious that the tachyon scalar field causes accelerated expansion of the universe provided $\dot{\varphi_t}^2 < \frac{2}{3}$. But $\dot{\varphi_t}^2 = 1 + \omega_t$ and $-1 < \omega_t < 0$, therefore it is within the range.

2.3.5 Phantom Scalar

The action for phantom scalar field is [152]

$$s = \int d^4x\sqrt{-g}\,[\tfrac{-1}{2} g^{\mu\nu}\partial_\mu\varphi_{ph}\partial_\nu\varphi_{ph} - V(\varphi_{ph})] \qquad (15)$$

where $V(\varphi_{ph})$ is the phantom potential.

The energy-momentum tensor for phantom scalar φ_{ph} has the form

$$T_{\mu\nu}^{ph} = -\partial_\mu \varphi_{ph} \partial_\nu \varphi_{ph} + g_{\mu\nu}[\frac{1}{2} g^{\alpha\beta} \partial_\alpha \varphi_{ph} \partial_\beta \varphi_{ph} - V(\varphi_{ph})] \quad (16)$$

which yield

$$\rho_{ph} = -\frac{1}{2} \dot{\varphi}_{ph}{}^2 + V(\varphi_{ph})$$

$$p_{ph} = -\frac{1}{2} \dot{\varphi}_{ph}{}^2 - V(\varphi_{ph})$$

with the equation of state

$$\omega_{ph} = \frac{\dot{\varphi}_{ph}{}^2 - 2V(\varphi_{ph})}{\dot{\varphi}_{ph}{}^2 + 2V(\varphi_{ph})}$$

Incorporating phantom scalar field into the Raychaudhuri equation, from Eq. (3), we obtain

$$\frac{\ddot{R}}{R} = \frac{1}{3} \dot{v}^{\mu}_{;\mu} + \frac{2}{3}\omega^2 - \frac{2}{3}\sigma^2 - \frac{4\pi G}{3}[\rho + 3p + (1 + 3\omega_{ph})\rho_{ph}] \quad (17)$$

Obviously the phantom scalar field term in (17) opposes gravity as $\omega_{ph} < -1$ and in a phantom-dominated universe the acceleration of the universe should occur.

2.4 Discussion

From the above five examples, it is evident that with appropriate inputs, the Raychaudhuri equation indeed reveals interesting cosmological phenomena. Though Raychaudhuri formulated his equation without the knowledge of accelerated expansion of the universe, it is possible to incorporate dark energy into his equation through enlarged law of gravitation.

Chapter - 3

Analytical solutions of the evolution of mass of black holes and worm holes immersed in a Generalized Chaplygin Gas model

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3. Analytical solutions of the evolution of mass of black holes and worm holes immersed in a Generalized Chaplygin Gas model

3.1 Introduction

Recent observations from distant Type Ia supernovae as well as the combination of the anisotropies of the Cosmic Microwave Background Radiation and the mass-energy density estimates from galaxy clusters, weak lensing and large-scalar structure suggest that the universe is going through a phase of accelerated expansion [9, 171]. The cause for the mysterious acceleration is however some-kind of anti-gravitational force with negative pressure. Present observation indicates that about 96% energy density of the universe is dark out of which 73% energy density is for dark energy (DE) and 23% energy density for dark matter (DM). The first evidence for DM stemmed from observations of cluster of galaxies by Zwicky in 1933. From 1970 scientists began to realize that only large amount of dark matter could explain many of their observations. Scientists also realize that the existence of some unseen mass would also support theories regarding the structure of the Universe [118].

Many cosmologists like to select the cosmological constant Λ, introduced by Einstien in his field equations, as a suitable candidate for dark energy because of its weired repulsive gravity [80]. A number of dynamically evolving scalar field models of dark energy such as Quintessence, K-essence, Tachyon, Phantom etc. are also considered to explain the present accelerated expansion of the universe [34, 13, 85, 173]. Some other models have also been considered such as Quartessence, which proposes a unified fluid with the characteristics of both dark matter and dark energy [60].

The generalized Chaplygin gas (GCG) model is an interesting candidate for the unification of the dark matter and dark energy [110]. In the GCG approach an exotic background fluid is considered, described by the following equation of state

$$p_{ch} = -\frac{A}{\rho_{ch}^{\alpha}} \tag{1}$$

where A and α are positive constants with $0 < \alpha \leq 1$. The case $\alpha = 1$ corresponds to the Chaplygin gas. The GCG model has been successfully confronted with many phenomenological tests such as Supernovae data, CMB data etc [111]. Regarding the latest supernova data, M. C. Bento et al. [110] have shown that the GCG model is degenerated with a dark energy model involving a phantom like equation of state. They have also shown that GCG can be considered as a unique mixture of interacting dark matter and dark energy and because of the interaction there is a flow of energy from dark matter to dark energy.

Recently Lima et al [71] have studied the evolution of the mass of a black hole embedded in a universe filled with DE and cold dark matter (CDM) in a closed form within a test fluid model in a Schwarzschild metric taking into account the cosmological evolution of both fluids. In this chapter we have studied the evolution of the mass of black hole and worm hole for the universe filled with generalized Chaplygin gas, baryon and radiation. In section 2 we solve of the field equations. Section 3 deals with the Accretion process in GCG model. The evolution of the mass of black hole and worm hole is represented in section 4. We conclude the chapter in section 5.

3.2 The field equations and solutions

We start with the Einstein's equations coupled to the baryon, radiation and GCG fluids. They read

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi G\left(T^{b}_{\mu\nu} + T^{r}_{\mu\nu} + T^{ch}_{\mu\nu}\right) \quad (2)$$

$$T^{\mu\nu}_{b;\mu} = 0, T^{\mu\nu}_{r;\mu} = 0, \ T^{\mu\nu}_{ch;\mu} = 0$$

The superscripts (subscripts) $b, r,\ ch$ stand for baryon, radiation and the generalised Chaplygin gas. It is assumed that GCG represents non-relativistic cold dark matter and dark energy i.e. dark component of the universe. We assume a perfect fluid structure for the cosmic medium as a whole and also for each of the components so that

$$T^{\mu\nu}_{\epsilon} = (\rho_{\epsilon} + p_{\epsilon})u^{\mu}_{\epsilon}u^{\nu}_{\epsilon} - p_{\epsilon}g^{\mu\nu}\,, \epsilon = b, r, ch \quad (3)$$

The Friedmann Robertson Walker (FRW) metric for a homogeneous and isotropic flat universe is given by

$$ds^2 = -dt^2 + a^2(t)[dr^2 + r^2d\theta^2 + r^2sin^2\theta d\phi^2] \quad (4)$$

where $a(t)$ is the scale factor and t represents the cosmic time.

The Friedmann equation for the universe filled with baryons, radiation and generalised Chaplygin gas is

$$3H^2 = 8\pi G(\rho_b + \rho_r + \rho_{ch}) \quad (5)$$

And the energy-conservation law can be written as

$$\dot{\rho_b} + 3H\rho_b = 0 \quad (6)$$

$$\rho_b = \rho_{b0}a^{-3} \quad (7)$$

$$\dot{\rho_r} + 4H\rho_r = 0 \quad (8)$$

$$\rho_r = \rho_{r0}a^{-4} \quad (9)$$

$$\dot{\rho_{ch}} + 3H(\rho_{ch} + p_{ch}) = 0 \quad (10)$$

where ρ_{b0} and ρ_{r0} are the values of the energy density of baryons and radiation at present.

Within the framework of FRW cosmology, insertion of equation (1) into the relativistic energy conservation equation, leads to an energy-density evolving as [111]

$$\rho_{ch} = \left[A + \frac{B}{a^{3(1+\alpha)}}\right]^{\frac{1}{1+\alpha}} \quad (11)$$

where B is a constant of integration which should be positive for a well-defined ρ_{ch} at all times.

From (1) and (11) we then get the pressure and density of the Chaplygin gas as

$$p_{ch} = -\frac{Aa^{3\alpha}}{\left[B+Aa^{3(1+\alpha)}\right]^{\frac{\alpha}{1+\alpha}}} \tag{12}$$

$$\rho_{ch} = \frac{\left[B+Aa^{3(1+\alpha)}\right]^{\frac{1}{1+\alpha}}}{a^3} \tag{13}$$

The equation of state parameter of the GCG can be written as

$$\omega = \frac{p_{ch}}{\rho_{ch}} = \frac{p_{DE}}{\rho_{DM}+\rho_{DE}} = \frac{\omega_{DE}\rho_{DE}}{\rho_{DM}+\rho_{DE}} \tag{14}$$

Using (12)-(14), we obtain

$$\rho_{DE} = -\frac{\rho_{DM}}{1+\omega_{DE}\left[1+\frac{B}{A}a^{-3(1+\alpha)}\right]} \tag{15}$$

Now, for the entire history of the universe, we should have $\omega_{DE} \leq -1$. Again for $\omega_{DE} < -1$, the GCG behaves as phantom like DE. The DE accretion process for black hole and worm hole for $\omega_{DE} < -1$ has already been studied by P.Martin-Moruno [137]. The mass of the black hole in this case decreases and the mass of the worm hole increases as the Hubble parameter increases as the universe expands in this case. So, in our work, we take the case $\omega_{DE} = -1$. In this case the energy density ρ_{ch} can be split in a unique way [110] as

$$\rho_{ch} = \rho_{DM} + \rho_{\Lambda} \tag{16}$$

where $$\rho_{DM} = \frac{Ba^{-3}}{\left[B+Aa^{3(1+\alpha)}\right]^{\frac{\alpha}{1+\alpha}}} \tag{17}$$

and $$\rho_{\Lambda} = -p_{\Lambda} = \frac{Aa^{3\alpha}}{\left[B+Aa^{3(1+\alpha)}\right]^{\frac{\alpha}{1+\alpha}}} \tag{18}$$

from which one obtains the scaling behaviour of the energy densities as

$$\frac{\rho_{DM}}{\rho_{\Lambda}} = \frac{B}{A}a^{-3(1+\alpha)} \tag{19}$$

Now by setting $a_0 = 1$ at present, from equations (16)-(18), it follows that

$$\rho_{ch0} = \rho_{DM0} + \rho_{\Lambda0} = (A+B)^{\frac{1}{1+\alpha}} \tag{20}$$

where $\rho_{ch0}, \rho_{DM0}, \rho_{\Lambda 0}$ are the present values of $\rho_{ch}, \rho_{DM}, \rho_{\Lambda}$ respectively. Parameters A, B can then be written as a function of ρ_{ch0} as follows:

$$A = \rho_{\Lambda 0}\rho_{ch0}^{\alpha} \; ; \; B = \rho_{\mathrm{DM}0}\rho_{ch0}^{\alpha} \tag{21}$$

Using (7), (9) and (11), from (5) the Friedmann equation is

$$3H^2 = 8\pi G\left[\left\{A + Ba^{-3(1+\alpha)}\right\}^{\frac{1}{1+\alpha}} + \rho_{b0}a^{-3} + \rho_{r0}a^{-4}\right] \tag{22}$$

Using the definition of the energy densities parameter $\Omega_{DM} = \frac{\rho_m}{\rho_c}$ and $\Omega_{\Lambda} = \frac{\rho_{\Lambda}}{\rho_c}$, where ρ_c is the critical energy density given by $\rho_c = \frac{3H^2}{8\pi G}$ we can write [110]

$$A \approx \Omega_{\Lambda 0}\rho_{c0}^{1+\alpha} \; ; \; B \approx \Omega_{\mathrm{DM}0}\rho_{c0}^{1+\alpha} \tag{23}$$

Using (23) in equation (22) we get

$$3H^2 = H_0^2\left[\left(\Omega_{\Lambda 0} + \Omega_{DM0}a^{-3(1+\alpha)}\right)^{\frac{1}{1+\alpha}} + \Omega_{b0}a^{-3} + \Omega_{r0}a^{-4}\right] \tag{24}$$

The energy density parameters Ω_{DM}, Ω_{Λ}, and Ω_b, Ω_r can be written as

$$\Omega_{DM} = \frac{\Omega_{DM0}a^{-3(1+\alpha)}}{\left[\Omega_{\Lambda 0}+\Omega_{DM0}a^{-3(1+\alpha)}\right]^{\frac{\alpha}{1+\alpha}}X} \tag{25}$$

$$\Omega_{\Lambda} = \frac{\Omega_{\Lambda 0}}{\left[\Omega_{\Lambda 0}+\Omega_{DM0}a^{-3(1+\alpha)}\right]^{\frac{\alpha}{1+\alpha}}X} \tag{26}$$

$$\Omega_{\mathrm{b}} = \frac{\Omega_{\mathrm{b}0}a^{-3}}{X} \tag{27}$$

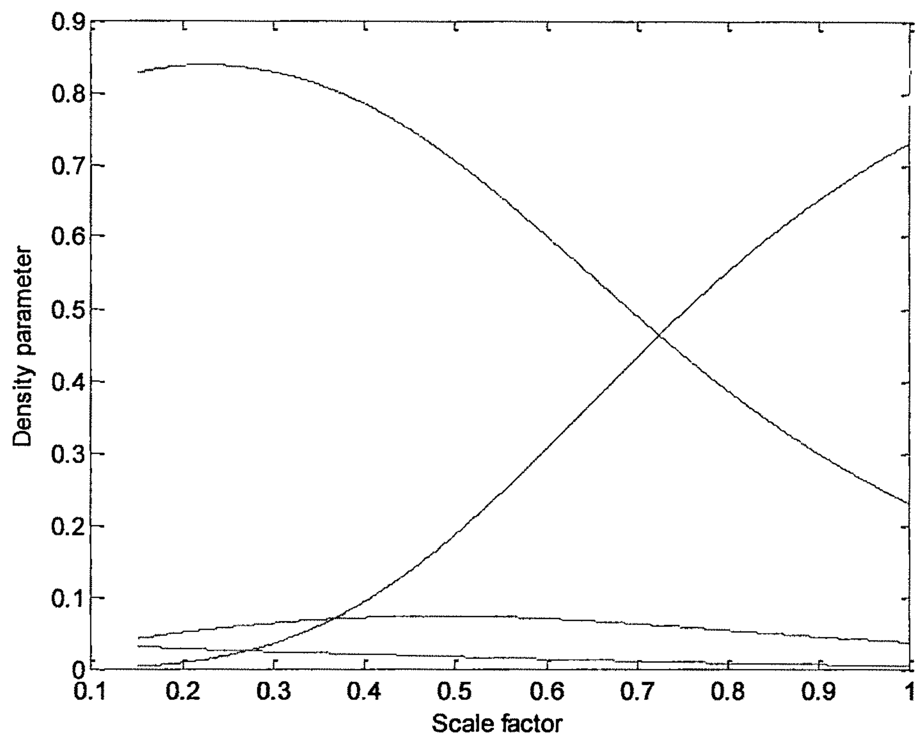
$$\Omega_{\mathrm{r}} = \frac{\Omega_{\mathrm{r}0}a^{-4}}{X} \tag{28}$$

$$X = \left(\Omega_{\Lambda 0} + \Omega_{DM0}a^{-3(1+\alpha)}\right)^{\frac{1}{1+\alpha}} + \Omega_{b0}a^{-3} + \Omega_{r0}a^{-4} \tag{29}$$

Using (10), (16) and (18) the energy conservation equation for the GCG can be written as

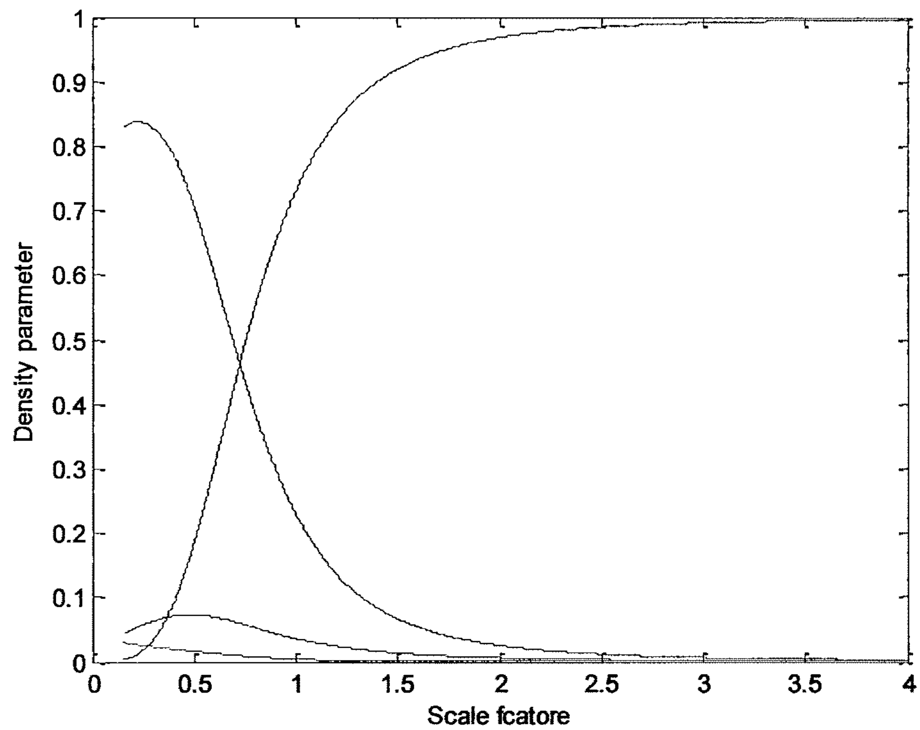
$$\dot{\rho}_{DM} + 3H\rho_{DM} = -\dot{\rho}_{\Lambda} \tag{30}$$

The variations of the different energy density parameters are shown in the following graphs:



In the graph the solid line, dashed line, dash dotted line and dotted line represent the values of the density parameter of dark energy, dark matter, baryon and radiation respectively. We take the present value of the density parameters as $\Omega_{b0} = .036$, $\Omega_{r0} = .004$, $\Omega_{DM0} = .23$, $\Omega_{\Lambda 0} = .73$, *set it for the graph and* $\alpha = 0.2$*)*

From the graph it is seen that the energy is transformed from the dark matter sector to the dark energy sector as the universe expands where the energy transforms from vacuum to matter sector is almost zero in ΛCDM model. Therefore the GCG model is quite different in this case from the ΛCDM model [130]. We assume that no matter of black hole and worm hole is transformed from the dark matter to the dark energy of the universe as black hole and worm hole are not dynamically important for the universe.



Again it is seen from the graph that as the universe expands the dark energy component will dominate the energy density of the universe and the energy density of the dark component becomes constant.

In the next section we use this result to compute the evolution of the mass of black hole and worm hole in a universe containing baryon, radiation and GCG.

3.3 Accretion process in GCG model

To study the accretion process of black hole, it is considered that the whole space is homogeneous and isotropic filled with baryons, radiations and GCG. The black hole mass rate for an asymptotic observer can be expressed as [111, 138]

$$\dot{m} = 4\pi D m^2(\rho + p) \tag{31}$$

where D is a constant of order unity and m is the mass of the black hole. ρ and p are the total energy density and the pressure of the universe.

The wormhole mass rate for an asymptotic observer can be expressed as [140, 141]

$$\dot{M} = -4\pi Q M^2(\rho + p) \tag{32}$$

where Q is a constant of order unity and M is the mass of the worm hole.

Equation (31) may be rewritten through a change of variable as [54]

$$\frac{dm}{d\rho}\dot{\rho} = 4\pi D m^2(\rho + p) \tag{33}$$

After substituting the term $(\rho + p)$ using the conservation equation

$$\dot{\rho} + 3H(\rho + p) = 0$$

in (33) we get

$$\frac{dm}{d\rho}\dot{\rho} = -\frac{4\pi D m^2 \dot{\rho}}{3H} \tag{34}$$

with a first integral

$$-\frac{1}{m} = -\left(\frac{8\pi}{3G}\right)^{\frac{1}{2}} D\rho^{\frac{1}{2}} + C \tag{35}$$

To find the integration constant we set the initial value for the primordial black hole mass as m_i at the instant with the fluid density ρ_i so that

$$C = \left(\frac{8\pi}{3G}\right)^{\frac{1}{2}} D\rho_i^{\frac{1}{2}} - \frac{1}{m_i} \tag{36}$$

Inserting the value of the constant in equation (35) we find the black hole mass as function of the background density

$$m(\rho) = \frac{m_i}{1 + m_i\sqrt{\frac{8\pi}{3G}}D\left(\rho^{\frac{1}{2}} - \rho_i^{\frac{1}{2}}\right)} \tag{37}$$

where $m_i = \dfrac{m_0}{1 - m_0\sqrt{\frac{8\pi}{3G}}D\left(\rho_0^{\frac{1}{2}} - \rho_i^{\frac{1}{2}}\right)}$ (38)

In a similar way one can get the expression of mass for wormhole

$$M(\rho) = \frac{M_i}{1 + M_i\sqrt{\frac{8\pi}{3G}}Q\left(\rho_i^{\frac{1}{2}} - \rho^{\frac{1}{2}}\right)} \tag{39}$$

3.4 Evaluation of the mass of black hole and worm hole

(i) For the early universe the Chaplygin gas behaves like a pressure less dark matter due to large densities. Therefore we have from equation (22) for matter dominated universe

$$\rho_{ch} = \rho_{DM} = B^{\frac{1}{1+\alpha}}a^{-3} \tag{40}$$

The black hole and worm hole accreting matter in this scenario will evolve as

$$m_{DM}(t) = \frac{m_i}{1+m_i\sqrt{\frac{8\pi}{3G}}D\left[\left(B^{\frac{1}{1+\alpha}}a^{-3}+\rho_{b0}a^{-3}+\rho_{r0}a^{-4}\right)^{\frac{1}{2}}-(\rho_i)^{\frac{1}{2}}\right]} \tag{41}$$

$$M_{DM}(t) = \frac{M_i}{1+M_i\sqrt{\frac{8\pi}{3G}}Q\left[(\rho_i)^{\frac{1}{2}}-\left(B^{\frac{1}{1+\alpha}}a^{-3}+\rho_{b0}a^{-3}+\rho_{r0}a^{-4}\right)^{\frac{1}{2}}\right]} \tag{42}$$

As the universe expands the mass of the black hole increases and the mass of the worm hole decreases as seen from the equations (41) and (42).

(ii) At present, in the universe filled with both dark matter and dark energy, we have the mass of the black hole and the worm hole as

$$m(t) = \frac{m_i}{1+m_i\sqrt{\frac{8\pi}{3G}}D\left\{\left[\{A+Ba^{-3(1+\alpha)}\}^{\frac{1}{1+\alpha}}+\rho_{b0}a^{-3}+\rho_{r0}a^{-4}\right]^{\frac{1}{2}}-(\rho_i)^{\frac{1}{2}}\right\}} \tag{43}$$

$$M(t) = \frac{M_i}{1+M_i\sqrt{\frac{8\pi}{3G}}Q\left\{(\rho_i)^{\frac{1}{2}}-\left[\{A+Ba^{-3(1+\alpha)}\}^{\frac{1}{1+\alpha}}+\rho_{b0}a^{-3}+\rho_{r0}a^{-4}\right]^{\frac{1}{2}}\right\}} \tag{44}$$

From the equation (22) it is seen that the Hubble parameter decreases as the universe expands. Therefore the total energy density of the universe decreases as the universe expands. Again since the dark matter is diluted as the large amount of energy transfer from dark matter to dark energy for the evolution of the scale factor and we assume that no matter of black hole and worm hole is transformed from the dark matter to the dark energy of the universe, the mass of the black hole increases and the mass of the worm hole decreases as seen from the equations (43) and (44).

(iii) In future the generalised Chaplygin gas behaves like a cosmological constant. From the graph it is clear that the dark energy component of the GCG will dominate the total energy density of the universe in future. Therefore the universe will be filled with dark energy only and ρ_Λ will become constant as seen from the conservation equation (30). Therefore the black hole and worm hole with their maximum and minimum mass will immerse in such a fluid.

3.5 Conclusion

In this work we have seen that in four dimensional, homogeneous and isotropic universe, the mass of black hole increases and the mass of worm hole decreases as the universe expands in GCG model with equation of state $\omega = -1$ with the assumption that no matter of black hole and worm hole is transformed to dark energy. When the energy density of the universe is dominated by the dark energy component of the universe the mass of the black holes and the worm holes become constants. The primordial black holes with maximum mass and worm holes with minimum mass will immerse in such a GCG fluid.

Chapter - 4

Anisotropic expansion and acceleration of the universe driven by tachyonic matter

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4. Anisotropic expansion and acceleration of the universe driven by tachyonic matter

4.1 Introduction

Though the present day universe appears to be isotropic, there is no evidence that the early universe was also of the same type. There is a cosmological view that the universe might have been anisotropic and also inhomogeneous in the very early era and that in the course of its evolution these characteristics have been wiped out under the action of some process or mechanism, and finally an isotropic and homogeneous universe had resulted. It would therefore be of some interest to study different types of anisotropic cosmological models [179-191]. It is well known that the relativistic cosmological models for spatially homogeneous and anisotropic space-time belong either to the Bianchi types or Kantowski - Sachs cosmological models. There is also fairly good evidence from the BOOMERANG observations of CMB (Cosmic Microwave Background) that the universe underwent a period of acceleration, so called Primordial Inflation at early times. Recent astrophysical data obtained from high red shift surveys of Supernovae (SnIa) [9, 171] indicate that the present universe is also passing through a phase of accelerated expansion. Friedmann equation can be consistent with such an accelerated expansion only if the universe is populated by some medium with negative pressure [24, 25, 26, 37, 80, 109, 113, 153, 173]. One of the possible sources which could provide such a negative pressure is a scalar field φ with the Lagrangian of the type L = -V $(\varphi)\sqrt{1 - g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi}$. Recently it has been suggested that the tachyonic condensate in a class of string theories can be described by an effective scalar field with a Langrangian of the above form [66]. The evolution of this condensate can have worth exploring cosmological significance. Here the basic idea is that the usual open string vacuum is unstable but there exists a stable vacuum with zero energy density which is stable. This state is associated with the condensation of electric flux tubes of closed strings which can be described successfully by using an effective Born-Infeld action.

Attempts have been made to construct viable cosmological model using rolling tachyon field as a suitable candidate for inflation, dark matter and dark energy. For this the tachyon potential field is considered as a viable model of dark energy [85, 177].

In this chapter, we study the role of tachyonic matter in Bianchi type I, Kantowski Sachs, Bianchi type III anisotropic cosmological models of the universe. The chapter is organized as follows. In section 2, the Einstein field equations for tachyon in case of four dimensions are written down. In section 3, the solutions of the field equations for the three types of cosmologies are derived. In section 4, the field equations and solutions for higher dimensional Bianchi type I cosmologies are obtained. We conclude this chapter with a brief physical interpretation of the solutions in section 5.

4.2 Field equations:

The action for the tachyon scalar φ is given by Born-Infeld like action

$$s = \int \sqrt{-g}\, dx^4 \left(\frac{R}{16\pi G} - V(\varphi)\sqrt{1 - g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi}\right) \tag{1}$$

where $V(\varphi)$ is the tachyon potential.

Taking variation of s w.r.t. $g_{\mu\nu}$ we get the field equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G T_{\mu\nu} \tag{2}$$

The energy-momentum tensor $T_{\mu\nu}$ for tachyonic matter is

$$T_{\mu\nu} = V(\varphi)\left[\frac{\partial_\mu\varphi\partial_\nu\varphi}{\sqrt{1+g^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi}} - g_{\mu\nu}\sqrt{1+g^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi}\right] \quad (3)$$

The general (3+1) metric for axially symmetric Bianchi-I, Bianchi-III and Kantowski-Sachs space-time can be described by [61].

$$ds^2 = dt^2 - a_1{}^2(t)dx_1{}^2 - a_2{}^2(t)d\Omega_k{}^2 \quad (4)$$

where $d\Omega_k{}^2 = dx_2{}^2 + dx_3{}^2$ for $k = 0$ (Bianchi-I Model),

$d\Omega_k{}^2 = d\theta^2 + sin^2\theta d\phi^2$ for $k = 1$ (Kantowski-Sachs Model),

$d\Omega_k{}^2 = d\theta^2 + sin^2\theta d\phi^2$ for $k = -1$ (Bianchi-III Model),

For the line element (4) using (2) we get,

$$2\frac{\dot{a_1}}{a_1}\frac{\dot{a_2}}{a_2} + \left(\frac{\dot{a_2}}{a_2}\right)^2 + \frac{k}{a_2{}^2} = 8\pi G\left(\frac{V(\varphi)}{\sqrt{1-\dot{\varphi}^2}}\right) = 8\pi G\rho_\varphi \quad (5)$$

$$\frac{\ddot{a_1}}{a_1} + \frac{\ddot{a_2}}{a_2} + \frac{\dot{a_1}}{a_1}\frac{\dot{a_2}}{a_2} = 8\pi GV(\varphi)\sqrt{1-\dot{\varphi}^2} = -8\pi Gp_\varphi \quad (6)$$

$$2\frac{\ddot{a_2}}{a_2} + \left(\frac{\dot{a_2}}{a_2}\right)^2 + \frac{k}{a_2{}^2} = 8\pi GV(\varphi)\sqrt{1-\dot{\varphi}^2} = -8\pi Gp_\varphi \quad (7)$$

Here $\rho_\varphi = \left(\frac{V(\varphi)}{\sqrt{1-\dot{\varphi}^2}}\right)$ and $p_\varphi = -V(\varphi)\sqrt{1-\dot{\varphi}^2}$ are the density and pressure of the tachyonic matter.

The equation of state for tachyonic matter is

$$p_\varphi = \omega_\varphi\rho_\varphi \text{ where } \omega_\varphi = \dot{\varphi}^2 - 1 \quad (8)$$

The scalar field equation of motion is

$$\frac{\ddot{\varphi}}{1-\dot{\varphi}^2} + \left(\frac{\dot{a_1}}{a_1} + 2\frac{\dot{a_2}}{a_2}\right)\dot{\varphi} + \frac{1}{V}\frac{dV}{d\varphi} = 0 \quad (9)$$

Here we have two independent equations having four unknowns a_1, a_2, ρ_φ, p_φ.Two more equations are required to find an exact solution.

4.3 Solutions:

As majority of cosmological models belong to either power law form or exponential form, we presently considering a power law relation between scale factors and time co-ordinates as,

$$a_1 = a_{01}t^n \quad \text{and} \quad a_2 = a_{02}t^m \tag{10}$$

Now since the right hand sides of (6) and (7) are identical, we have

$$\frac{\ddot{a_1}}{a_1} + \frac{\ddot{a_2}}{a_2} + \frac{\dot{a_1}}{a_1}\frac{\dot{a_2}}{a_2} = 2\frac{\ddot{a_2}}{a_2} + \left(\frac{\dot{a_2}}{a_2}\right)^2 + \frac{k}{{a_2}^2} \tag{11}$$

which gives $n = m$ or $n + 2m = 1$ for $k = 0$ and $m = 1$, $n^2 - 1 = \frac{k}{a_{02}^2}$ for $k \neq o$.

4.3.A Considering $k = 0$ and $n = m$ from (10) we have

$$a_1 = a_{01}t^n \quad \text{and} \quad a_2 = a_{02}t^n \tag{12}$$

Using these conditions we get from (5), (6) and (7)

$$\varphi(t) = \left(\frac{2}{3n}\right)^{\frac{1}{2}} t + \varphi_0 \tag{13}$$

$$V(\varphi) = \frac{3n^2}{8\pi G}\left(1 - \frac{2}{3n}\right)^{\frac{1}{2}} t^{-2} \tag{14}$$

where $n > \frac{2}{3}$

Combining (13) and (14) we get,

$$V(\varphi) = \frac{n}{4\pi G}\left(1 - \frac{2}{3n}\right)^{\frac{1}{2}} (\varphi - \varphi_0)^{-2} \tag{15}$$

For such a potential, it is possible to have arbitrarily rapid expansion with large n.

Using equation (13), (14) and (15) we have

$$\rho_\varphi = \frac{n}{4\pi G}(\varphi - \varphi_0)^{-2} = \frac{3n}{8\pi G} t^{-2} \tag{16}$$

From equation (8) we get,

$$p_\varphi = (\dot{\varphi}^2 - 1)\frac{n}{4\pi G}(\varphi - \varphi_0)^{-2} = \frac{2-3n}{8\pi G} t^{-2} \tag{17}$$

The deceleration parameter is,

$$q = -1 - \frac{\dot{H}}{H^2} = -1 + \frac{1}{n} \tag{18}$$

Now for positive ρ_φ we must have $n > 0$ and also for accelerating universe we must have $n > 1$. So for $n > 1$, we have the positive ρ_φ and negative p_φ of the accelerating universe.

The CMBR and Supernovae observation suggest $0.85 \leq H_0 t_0 \leq 1.13$ that gives $0.85 \leq n \leq 1.13$. So the result agrees with the present observational data of the universe.

Also when $0 < n < 1$ then ρ_φ and q is positive which gives the past deceleration of the universe.

This also agrees with the FRW result.

4.3.B We now consider $k = 0$ and $n + 2m = 1$. For this relation we have from (10)

$$a_1 = a_{01} t^n \quad \text{and} \quad a_2 = a_{02} t^{\frac{1-n}{2}} \tag{19}$$

Using these conditions from (5), (6), and (7) we get

$$\varphi(t) = \varphi_0 \tag{20}$$

$$V(\varphi) = \frac{(1-n)(3n+1)}{32\pi G} t^{-2} \tag{21}$$

Using (20) and (21) we have

$$\rho_\varphi = \frac{(1-n)(3n+1)}{32\pi G} t^{-2} \tag{22}$$

$$p_\varphi = (\dot{\varphi}^2 - 1)\frac{(1-n)(3n+1)}{32\pi G} t^{-2} = -\frac{(1-n)(3n+1)}{4} t^{-2} \tag{23}$$

The deceleration parameter is,

$$q = -1 - \frac{\dot{H}}{H^2} = 2 \tag{24}$$

Also we have $H_0 t_0 = \frac{1}{3}$.

For positive ρ_φ we must have $n < 1$. This gives the negative value of the pressure of the tachyon scalar field and the positive value of the deceleration parameter. The potential function is positive and decreases w.r.t. time and φ is constant. So in this case it is not possible to construct a present accelerating model of the Universe.

4.3.C Lastly for $k \neq o, m = 1, n^2 - 1 = \frac{k}{a_{02}^2}$ we have the scale factors in the form

$$a_1 = a_{01} t^n \quad \text{and} \quad a_2 = a_{02} t \tag{25}$$

Using these conditions from (5), (6) and (7) we get

$$\varphi(t) = \sqrt{\frac{2}{n+2}}t + \varphi_0 \tag{26}$$

$$V(\varphi) = \frac{n\sqrt{n(n+2)}}{8\pi G}t^{-2} \tag{27}$$

where $n(n+2) > 0$

Combining (26) and (27) we get,

$$V(\varphi) = \frac{n^{\frac{3}{2}}}{4\pi G\sqrt{n+2}}(\varphi - \varphi_0)^{-2} \tag{28}$$

Using (26) and (27) we get,

$$\rho_\varphi = \frac{n(n+2)}{8\pi G}t^{-2} \tag{29}$$

From equation (8) we get,

$$p_\varphi = -\frac{n^2}{8\pi G}t^{-2} \tag{30}$$

The deceleration parameter is,

$$q = -1 - \frac{H}{H^2} = -1 + \frac{3}{n+2} \tag{31}$$

Now for positive ρ_φ we must have $n > 0$ and also for accelerating universe we must have $n > 1$. So for $n > 1$, we have the positive ρ_φ and negative p_φ of the accelerating universe.

The CMBR and Supernovae observation suggest $0.85 \leq H_0 t_0 \leq 1.13$ that gives $0.55 \leq n \leq 1.39$. So the result agrees with the present observational data of the universe.

4.4 Bianchi type I metric in higher dimensions:

The Bianchi type-I metric in case of $(n+1)$- dimensions is

$$ds^2 = dt^2 - \sum_{i=1}^{n} a_i^2(t)\left(dx^i\right)^2 \tag{32}$$

where a_i's are functions of t only. The Einstein equations for the metric (32) are

$$\frac{\dot{a}_1}{a_1}\sum_{i=2}^{n}\frac{\dot{a}_i}{a_i}+\frac{\dot{a}_2}{a_2}\sum_{i=3}^{n}\frac{\dot{a}_i}{a_i}+\cdots+\frac{\dot{a}_{n-1}}{a_{n-1}}\frac{\dot{a}_n}{a_n}=8\pi G\rho_\varphi \tag{33}$$

$$\sum_{i=1}^{n}\frac{\ddot{a}_i}{a_i}-\frac{\ddot{a}_1}{a_1}+\frac{\dot{a}_1}{a_1}\sum_{i=2}^{n}\frac{\dot{a}_i}{a_i}+\frac{\dot{a}_2}{a_2}\sum_{i=3}^{n}\frac{\dot{a}_i}{a_i}+\cdots+\frac{\dot{a}_{n-1}}{a_{n-1}}\frac{\dot{a}_n}{a_n}-\frac{\dot{a}_1}{a_1}\sum_{i=2}^{n}\frac{\dot{a}_i}{a_i}=8\pi Gp_\varphi$$

$$\sum_{i=1}^{n}\frac{\ddot{a}_i}{a_i}-\frac{\ddot{a}_2}{a_2}+\frac{\dot{a}_1}{a_1}\sum_{i=2}^{n}\frac{\dot{a}_i}{a_i}+\frac{\dot{a}_2}{a_2}\sum_{i=3}^{n}\frac{\dot{a}_i}{a_i}+\cdots+\frac{\dot{a}_{n-1}}{a_{n-1}}\frac{\dot{a}_n}{a_n}-\frac{\dot{a}_2}{a_2}\sum_{i=2}^{n}\frac{\dot{a}_i}{a_i}-\frac{\dot{a}_2}{a_2}\frac{\dot{a}_1}{a_1}=8\pi Gp_\varphi$$

$$\sum_{i=1}^{n}\frac{\ddot{a}_i}{a_i}-\frac{\ddot{a}_3}{a_3}+\frac{\dot{a}_1}{a_1}\sum_{i=2}^{n}\frac{\dot{a}_i}{a_i}+\frac{\dot{a}_2}{a_2}\sum_{i=3}^{n}\frac{\dot{a}_i}{a_i}+\cdots+\frac{\dot{a}_{n-1}}{a_{n-1}}\frac{\dot{a}_n}{a_n}-\frac{\dot{a}_3}{a_3}\sum_{i=4}^{n}\frac{\dot{a}_i}{a_i}-\frac{\dot{a}_3}{a_3}\sum_{i=1}^{2}\frac{\dot{a}_i}{a_i}=8\pi Gp_\varphi$$

$$\sum_{i=1}^{n}\frac{\ddot{a}_i}{a_i}-\frac{\ddot{a}_4}{a_4}+\frac{\dot{a}_1}{a_1}\sum_{i=2}^{n}\frac{\dot{a}_i}{a_i}+\frac{\dot{a}_2}{a_2}\sum_{i=3}^{n}\frac{\dot{a}_i}{a_i}+\cdots+\frac{\dot{a}_{n-1}}{a_{n-1}}\frac{\dot{a}_n}{a_n}-\frac{\dot{a}_4}{a_4}\sum_{i=5}^{n}\frac{\dot{a}_i}{a_i}-\frac{\dot{a}_4}{a_4}\sum_{i=1}^{3}\frac{\dot{a}_i}{a_i}=8\pi Gp_\varphi$$

....

....

$$\sum_{i=1}^{n}\frac{\ddot{a}_i}{a_i}-\frac{\ddot{a}_r}{a_r}+\frac{\dot{a}_1}{a_1}\sum_{i=2}^{n}\frac{\dot{a}_i}{a_i}+\frac{\dot{a}_2}{a_2}\sum_{i=3}^{n}\frac{\dot{a}_i}{a_i}+\cdots+\frac{\dot{a}_{n-1}}{a_{n-1}}\frac{\dot{a}_n}{a_n}-\frac{\dot{a}_r}{a_r}\sum_{i=r+1}^{n}\frac{\dot{a}_i}{a_i}-\frac{\dot{a}_r}{a_r}\sum_{i=1}^{r-1}\frac{\dot{a}_i}{a_i}=8\pi Gp_\varphi$$

....

....

$$\sum_{i=1}^{n}\frac{\ddot{a}_i}{a_i}-\frac{\ddot{a}_n}{a_n}+\frac{\dot{a}_1}{a_1}\sum_{i=2}^{n}\frac{\dot{a}_i}{a_i}+\frac{\dot{a}_2}{a_2}\sum_{i=3}^{n}\frac{\dot{a}_i}{a_i}+\cdots+\frac{\dot{a}_{n-1}}{a_{n-1}}\frac{\dot{a}_n}{a_n}-\frac{\dot{a}_n}{a_n}\sum_{i=1}^{n-1}\frac{\dot{a}_i}{a_i}=8\pi Gp_\varphi$$

and the Bianchi identity is

$$\dot{\rho_\varphi}+\rho_\varphi\left(\frac{\dot{a}_1}{a_1}+\frac{\dot{a}_2}{a_2}+\frac{\dot{a}_3}{a_3}\right)+(\rho_\varphi-p_\varphi)\left(\frac{\dot{a}_4}{a_4}+\frac{\dot{a}_5}{a_5}+\cdots+\frac{\dot{a}_n}{a_n}\right)=0 \tag{34}$$

We consider the case of $(n+1)$-dimensions with

$a_1=a_2=a_3=a$ and $a_4=a_5=\cdots=a_n=\acute{a}$ where $a=a_{01}t^k$ and $\acute{a}=a_{02}t^{\acute{k}}$. We get

$$\rho_\varphi=\left(\frac{V(\varphi)}{\sqrt{1-\dot{\varphi}^2}}\right)=\frac{1}{t^2}\left[3k^2+3(n-3)k\acute{k}+\frac{(n-3)(n-4)}{2}\acute{k}^2\right] \tag{35}$$

$$p_\varphi=-V(\varphi)\sqrt{1-\dot{\varphi}^2}=\frac{1}{t^2}\left[6k^2-3k+3(n-4)k\acute{k}-(n-4)\acute{k}+\frac{(n-3)(n-4)}{2}\acute{k}^2\right] \tag{36}$$

The relation between k and $\acute{k}$ is given by the equality of the above equations as

$$3k^2-k+(n-6)k\acute{k}+\acute{k}-(n-3)\acute{k}^2=0 \tag{37}$$

The isotropic three-space will expand as t increases if $k>0$ and the extra dimensions contract as t increases if $\acute{k}<0$.

For $n=4$, from (35) and (36) we get

$$\rho_\varphi = \left(\frac{V(\varphi)}{\sqrt{1-\dot{\varphi}^2}}\right) = \frac{3}{t^2}k(k+\acute{k}) \tag{38}$$

$$p_\varphi = -V(\varphi)\sqrt{1-\dot{\varphi}^2} = \frac{3}{t^2}k(2k-1) \tag{39}$$

And from (37) we get

$$3k^2 - k - 2k\acute{k} + \acute{k} - \acute{k}^2 = 0 \tag{40}$$

which gives $k = \acute{k}$ or $3k + \acute{k} = 1$

For $k = \acute{k}$ we get

$$\rho_\varphi = \left(\frac{V(\varphi)}{\sqrt{1-\dot{\varphi}^2}}\right) = \frac{6}{t^2}k^2$$

$$p_\varphi = -V(\varphi)\sqrt{1-\dot{\varphi}^2} = \frac{3}{t^2}k(2k-1)$$

For $k > 1$ both the dark energy density and pressure are positive. So in this case it is not possible to construct present accelerating model of the universe.

And for $3k + \acute{k} = 1$ we get

$$\rho_\varphi = \left(\frac{V(\varphi)}{\sqrt{1-\dot{\varphi}^2}}\right) = \frac{3}{t^2}k(-2k+1)$$

$$p_\varphi = -V(\varphi)\sqrt{1-\dot{\varphi}^2} = -\frac{3}{t^2}k(-2k+1)$$

which gives

$\varphi(t) = \varphi_0(\text{constant})$

and $V(\varphi) = \frac{3}{t^2}k(-2k+1)$

In this case also for positive ρ_φ , we have $k < \frac{1}{2}$

which gives the past deceleration of the universe.

4.5 Discussion

Since the exponential form of the scale factors give the uniform values of the parameters which do not satisfy the present observational data of the universe, so considering the scale factors in the power law form of time we obtain the solutions of the field equations.

Firstly, we have found the solution in axially symmetric Bianchi type I cosmology for the case $n = m$ in four dimensions. Positive density of the tachyonic matter indicates that $n > 0$. It is found that as the universe expands the tachyon rolls down to the minimum of its potential. When $n > 1$ we get accelerated expanding cosmological model of the universe. In this case the tachyon can be considered as an alternative candidate for dark energy with negative pressure. Secondly for the case $n + 2m = 1$ in axially symmetric Bianchi type I cosmology, we get the deceleration parameter positive which shows the past deceleration of the universe. So this model in this case failed to explain the present acceleration of the universe. For both Kantowski–Sachs and Bianchi type III metrics, we get the present accelerating model of the universe for $n > 1$. In Bianchi type I with higher dimensions we get the past deceleration of the universe and do not get the present accelerating model of the universe.

Chapter – 5

Holographic quintessence model of dark energy in Bianchi type-I anisotropic universe

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5.1 Holographic quintessence model of dark energy in Bianchi type-I anisotropic universe

5.1 Introduction

Observations of distant type-Ia supernovae and cosmic microwave background suggest that our universe has entered a phase of accelerated expansion in the recent past [9, 171]. This is attributed to the contribution of an unknown component, dubbed dark energy, which has negative pressure and makes up about three quarters of the total cosmic density. The simplest candidate for dark energy is the cosmological constant (Λ) with equation of state parameter $\omega = -1$ since it fits the observational data well, but it needs to be extremely fine-tuned to satisfy the present value of dark energy [51]. To solve cosmological constant problem at present epoch, Λ with a dynamical character is preferred over a constant Λ, especially a time dependent Λ which has decreased slowly from its large initial value to reach its present small value [80]. To further investigate the properties of dark energy, many dynamical dark energy models have been proposed, such as quintessence [173], phantom [153], tachyon [26, 85, 178], k-essence [34], dilatonic ghost condensate [116], quartessence [60] and so forth. The cosmic viscosity is also an effective quantity as caused mainly by the non-perfect cosmic contents interactions and may play a role as dark energy candidate causing the observed acceleration of the universe [84, 108, 193].

The holographic principle is considered as another alternative to the solution of the dark energy problem. This principle was first put forward by G. 't Hooft [64] in the context of black hole physics. According to the holographic principle, the entropy of a system scales not with its volume, but with its surface area. In the cosmological context, Fischler and Susskind [190] have proposed a new version of the holographic principle, viz. at any time during cosmological evolution, the gravitational entropy within a closed surface should not be always larger than the particle entropy that passes through the past light-cone of that surface. In the context of the dark energy problem, though the holographic principle proposes a relation between the holographic dark energy density ρ_Λ and the Hubble parameter H as $\rho_\Lambda = H^2$, it does not contribute to the present accelerated expansion of the universe. In [102], Granda and Oliveros proposed a holographic density of the form $\rho_\Lambda \approx \alpha H^2 + \beta \dot{H}$ where H is the Hubble parameter and α, β are constants which must satisfy the restrictions imposed by the current observational data. They showed that this new model of dark energy represents the accelerated expansion of the universe and is consistent with the current observational data. In [101], they study the correspondence between the quintessence, tachyon, k-essence and dilaton dark energy models with this holographic dark energy model in the flat FRW universe.

But there is a cosmological view that the universe might have been anisotropic and also inhomogeneous in the very early era and that in the course of its evolution these characteristics might have been wiped out under the action of some processes or mechanism, resulting in an isotropic and homogeneous universe. Spatially homogeneous and anisotropic cosmological models play a significant role in the description of large scale behaviour of the universe and such models have been widely studied by many authors in search of a relativistic picture of the early universe. Anisotropic Bianchi type-I, Bianchi type-III, Bianchi type-V dark energy models with the usual perfect fluid have also been extensively studied in the literature [19, 20, 23, 133, 162]. So it will be interesting to study the evolution of holographic dark energy in an anisotropic model like Bianchi type- I.

In this Chapter we consider the holographic dark energy model in the axially symmetric Bianchi type-I model to investigate the correspondence with quintessence models of the universe. We obtain the Equation of state parameter for the holographic dark energy model in axially symmetric Bianchi type-I in section 2. In section 3, we solve the field equations by considering the deceleration parameter to be a constant. The correspondence between the holographic dark energy with quintessence is shown in section 4. We conclude this chapter in section 5.

5.2 Holographic dark energy model in Bianchi type-I universe

The metric for axially symmetric Bianchi type I is

$$ds^2 = dt^2 - {a_1}^2(t){dx_1}^2 - {a_2}^2(t){dx_2}^2 - {a_2}^2(t){dx_3}^2 \tag{1}$$

Einstein's field equation is given by

$$R_{ij} - \frac{1}{2}g_{ij}R = -(T_{ij} + \bar{T}_{ij}) \tag{2}$$

where R_{ij} is the Ricci tensor.

The energy momentum tensor for matter and the holographic dark energy are defined as

$$T_{ij} = \rho_m u_i u_j \text{ and } \bar{T}_{ij} = (\rho_\Lambda + p_\Lambda)u_i u_j + g_{ij}p_\Lambda \tag{3}$$

where ρ_m , ρ_Λ are the energy densities of matter and the holographic dark energy and p_Λ is the pressure of the holographic dark energy.

The field equations for the axially symmetric Bianchi type-I metric are

$$2\frac{\dot{a}_1}{a_1}\frac{a_2}{a_2} + \left(\frac{\dot{a}_2}{a_2}\right)^2 = \rho_m + \rho_\Lambda \tag{4}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2} = -p_\Lambda \tag{5}$$

$$2\frac{\dot{a}_2}{a_2} + \left(\frac{a_2}{a_2}\right)^2 = -p_\Lambda \tag{6}$$

The average scale factor a and the average Hubble's parameter are defined as

$$a = (a_1{a_2}^2)^{\frac{1}{3}} \tag{7}$$

$$H = \frac{1}{3}\left(\frac{\dot{a}_1}{a_1} + 2\frac{\dot{a}_2}{a_2}\right) \tag{8}$$

The scalar expansion θ, deceleration parameter q, shear scalar σ^2 and the average anisotropy parameter A_m are defined by

$$\theta = \frac{\dot{a}_1}{a_1} + 2\frac{\dot{a}_2}{a_2} \tag{9}$$

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 - \frac{H}{H^2} \tag{10}$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - \frac{1}{3}\theta^2\right) \tag{11}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 \tag{12}$$

where $\Delta H_i = H_i - H \ (i = 1,2,3)$

The holographic dark energy density is given by

$$\rho_\Lambda = 3\left(\alpha H^2 + \beta \dot{H}\right) \tag{13}$$

i.e. $\rho_\Lambda = 3\left(\alpha H^2 + \beta \dot{H}\right)$ with $M_p^{-2} = 8\pi G = 1$ [101].

Combining (4)-(6) the continuity equation can be obtained as

$$\dot{\rho}_m + \left(\frac{\dot{a}_1}{a_1} + 2\frac{\dot{a}_2}{a_2}\right)\rho_m + \dot{\rho}_\Lambda + \left(\frac{\dot{a}_1}{a_1} + 2\frac{a_2}{a_2}\right)(\rho_\Lambda + p_\Lambda) = 0 \tag{14}$$

The continuity equation of the matter is

$$\dot{\rho}_m + \left(\frac{\dot{a}_1}{a_1} + 2\frac{\dot{a}_2}{a_2}\right)\rho_m = 0 \tag{15}$$

The continuity equation of the holographic dark energy is

$$\dot{\rho}_\Lambda + \left(\frac{\dot{a}_1}{a_1} + 2\frac{a_2}{a_2}\right)(\rho_\Lambda + p_\Lambda \quad) = 0 \tag{16}$$

The barotropic equation of state is

$$p_\Lambda = \omega_\Lambda \rho_\Lambda \tag{17}$$

Using equation (13) and (17) in equation (16) we get

$$\omega_\Lambda = -1 - \frac{2\alpha H H + \beta \ddot{H}}{3H(\alpha H^2 + \beta \dot{H})} \tag{18}$$

5.3 Cosmological Solutions

Since the right hand sides of (5) and (6) are identical, we have

$$\frac{\ddot{a}_1}{a_1}+\frac{\ddot{a}_2}{a_2}+\frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2}=2\frac{\ddot{a}_2}{a_2}+\left(\frac{\dot{a}_2}{a_2}\right)^2 \tag{19}$$

Solving equation (19) and using (7) we get,

$$\frac{a_1}{a_2}=D_2exp(D_1\int a^{-3}dt) \tag{20}$$

The metric functions, therefore can be read as

$$a_1(t)=D_2^2a\ exp(2D_3\int a^{-3}dt) \tag{21}$$

$$a_2(t)=D_2^{-1}a\ exp(-D_3\int a^{-3}dt) \tag{22}$$

where D_1, D_2 and D_3 are constants and $3D_3=D_1$

To solve these field equations we assume the average Hubble parameter H to be related to the average scale factor a, by the relation following Berman [128]

$$H=ma^{-\frac{1}{m}} \tag{23}$$

where $m>0$ is a constant.

Solving the equation (23) we get

$$a(t)=(t+c_1)^m \tag{24}$$

Using (24) in (21) and (22) we get the metric functions as

$$a_1(t)=D_2^2(t+c_1)^m exp\left[\frac{2D_3}{1-3m}(t+c_1)^{1-3m}\right] \tag{25}$$

$$a_2(t)=D_2^{-1}(t+c_1)^m exp\left[\frac{-D_3}{1-3m}(t+c_1)^{1-3m}\right] \tag{26}$$

This gives a constant value of the deceleration parameter

$$q=-1+\frac{1}{m} \tag{27}$$

Since recent observational data indicates that the universe is accelerating and the value of deceleration parameter lies somewhere in the range $-1<q<0$, so we have $m>1$ for the accelerating universe.

The Hubble parameter, the scalar expansion, shear scalar and the average anisotropy parameter are therefore

$$H = m(t + c_1)^{-1} \tag{28}$$

$$\theta = 3m(t + c_1)^{-1} \tag{29}$$

$$\sigma^2 = 3D_3^2(t + c_1)^{-6m} \tag{30}$$

$$A_m = 2D_3^2 m^{-2}(t + c_1)^{2-6m} \tag{31}$$

The matter density parameter (Ω_m) and holographic dark energy density parameter (Ω_Λ) are given by

$$\Omega_m = \frac{\rho_m}{3H^2} \text{ and } \Omega_\Lambda = \frac{\rho_\Lambda}{3H^2} \tag{32}$$

Using (24)-(26), (28) and (32) in (4) we get

$$\Omega_m + \Omega_\Lambda = 1 - D_3^2 m^{-2}(t + c_1)^{2-6m} \tag{33}$$

Equation (33) shows that the sum of the energy density parameters approaches 1 at late times. So at late times the universe becomes flat. Therefore for sufficiently large time, this model predicts that the anisotropy of the universe will damp out and universe will become isotropic. This result also shows that in the early universe i.e. during the radiation and matter dominated era the universe was anisotropic and the universe approaches to isotropy as dark energy starts to dominate the energy density of the universe. This result is totally different from Pradhan et al.'s result [23] where they found in their model that the universe does not approach isotropy through its whole evolution.

Using (28) in equation (18) we get

$$\omega_\Lambda = -1 + \frac{2}{3m} \tag{34}$$

This shows that for $m > 1$, we have $-1 < \omega_\Lambda < -\frac{1}{3}$. In this case the holographic dark energy EOS behaves like quintessence.

5. 4 Correspondence between the holographic and quintessence scalar field model of dark energy

To establish the correspondence between the holographic dark energy with quintessence dark energy models, we compare the EOS and the dark energy density for the corresponding models of dark energy.

The action for the quintessence scalar φ is given by

$$s = \int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)\right] \tag{35}$$

The energy density and pressure for the quintessence scalar field are represented by

$$\rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + V(\varphi) \tag{36}$$

$$p_\varphi = \frac{1}{2}\dot{\varphi}^2 - V(\varphi) \tag{37}$$

where $V(\varphi)$ is the quintessence potential.

The equation of state for the scalar field is given by

$$\omega_\varphi = \frac{\varphi^2 - 2V(\varphi)}{\dot{\varphi}^2 + 2V(\varphi)} \tag{38}$$

For the accelerated expansion of the universe, the equation of state parameter for quintessence must be less than $-\frac{1}{3}$.

From (34) and (38) we have

$$-1 + \frac{2}{3m} = \frac{\dot{\varphi}^2 - 2V(\varphi)}{\varphi^2 + 2V(\varphi)} \tag{39}$$

Also comparing equations (13) and (36) one can write

$$\rho_\Lambda = 3\left(\alpha H^2 + \beta\dot{H}\right) = \frac{1}{2}\dot{\varphi}^2 + V(\varphi) \tag{40}$$

From equations (38) and (39), the kinetic energy term and the quintessence potential in power law form can be obtained as follows

$$\varphi - \varphi_0 = \sqrt{2(m\alpha - \beta)}\; In\left(\frac{t + c_1}{t_0 + c_1}\right) \tag{41}$$

$$V(\varphi) = (3m - 1)(m\alpha - \beta)(t + c_1)^{-2} \tag{42}$$

Using (41) in (42) we get the potential in the exponential form as

$$V(\varphi) = (3m-1)(m\alpha-\beta)(t_0+c_1)^{-2}exp\left(-\frac{1}{\sqrt{2(m\alpha-\beta)}}(\varphi-\varphi_0)\right) \tag{43}$$

For $m > 1$, this type of exponential potential can produce an accelerated expansion of the universe [51]. Thus one can establish a correspondence between the holographic dark energy and quintessence scalar field, and describe holographic dark energy by making use of quintessence.

The dynamics and the potentials of tachyon and k-essence scalar field models of the anisotropic Bianchi type-I universe can be obtained in the same way.

5.5 Conclusion

Though the present day universe appears to be isotropic, there is no evidence that the early universe was also of the same type. There is a cosmological view that the universe might be inhomogeneous and anisotropic in the very early era and that in the course of its evolution these characteristic have been wiped out under the action of some process or mechanism, and finally an isotropic and homogeneous universe had resulted. Observational data also suggest that dark energy is responsible for gearing up the universe some five billion years ago. But at that time the universe need not to be isotropic. So in this chapter we assume the universe to be anisotropic and consider a homogeneous axially symmetric Bianchi type-I universe filled with matter and holographic dark energy. Assuming the deceleration parameter to be a constant, we have obtained an exact solution of Einstein's field equations. Under certain conditions the solution describes the accelerated expansion of the universe. The EOS parameter of the holographic dark energy also behaves like quintessence EOS. Using these results we have established a correspondence between the holographic dark energy model with the quintessence scalar field dark energy models in the Bianchi type-I universe. Quintessence potential and the dynamics of the quintessence scalar field are reconstructed for this anisotropic accelerating model of the universe. Our result shows that the universe was anisotropic in the early stage and at the late time dynamics anisotropy of the universe damps out and the present day universe becomes isotropic as suggested by different observational data.

Chapter – 6

Interacting new agegraphic tachyon and bulk viscous models of dark energy in Bianchi type-I anisotropic universe

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6. Interacting new agegraphic tachyon and bulk viscous models of dark energy in Bianchi type-I anisotropic universe

6.1 Introduction

Observations of distant type-Ia supernovae and cosmic microwave background anisotropy indicate in favour of the accelerating expansion of the universe [9, 171]. This is attributed to the contribution of an unknown component, dubbed dark energy, which has negative pressure and makes up about three quarters of the total cosmic density. Many cosmologists believe that the simplest candidate for the dark energy is the cosmological constant (Λ) since it fits the observational data well, but it needs to be extremely fine-tuned to satisfy the present value of dark energy [51]. To solve cosmological constant problem at present epoch, Λ with a dynamical character is preferred over a constant Λ, especially a time dependent Λ which has decreased slowly from its large initial value to reach its present small value [80]. To further investigate the properties of dark energy, many dynamical dark energy models have been proposed, such as quintessence [173], phantom [153], tachyon [26, 85, 178], k-essence [34], dilatonic ghost condensate [116], holographic dark energy model [121], quartessence [60]and so forth. The cosmic viscosity is also an effective quantity as caused mainly by the non-perfect cosmic contents interactions and may play a role as dark energy candidate causing the observed acceleration of the universe [84, 108, 193].

In [147], R. G. Cai proposed the agegraphic dark energy model to explain the present accelerated expansion of the universe. This new model of dark energy had suffered some problems to justify the matter dominated era. To solve this problem, H. Wei and

R. G. Cai proposed new agegraphic dark energy (NADE) model [68]. This new agegraphic dark energy model is actually a single-parameter model and the coincidence problem can naturally be solved in this model. The new agegraphic dark energy model can also fit the cosmological observations of type Ia supernovae, cosmic microwave background and large scale structure very well. The equation of state of the interacting new agegraphic dark energy models behaves both like quintessence and phantom scalar field models of dark energy. Recently, a great deal of effort has gone into reconstruction for the holographic and agegraphic scalar field models of dark energy [82, 87, 88, 94, 95, 101, 197].

Although different observational data suggest that our present day universe appears to be isotropic, there is no evidence that the early universe was also of the same type. There is a cosmological view that the universe might have been anisotropic and also inhomogeneous in the very early era and that in the course of its evolution these characteristics have been wiped out under the action of some process or mechanism, and finally an isotropic and homogeneous universe had resulted. Spatially homogeneous and anisotropic cosmological models play a significant role in the description of large scale behaviour of universe. Barman in [128], proposed a special law of variation of the average Hubble's parameter which yields a constant value of deceleration parameter. Anisotropic Bianchi type-I, Bianchi type-III, Bianchi type-V dark energy models with the usual perfect fluid and with constant deceleration parameter have been extensively studied in the literature [19, 20, 23, 133, 162].

In this Chapter we consider the interacting NADE model in the axially symmetric Bianchi type-I to investigate the correspondence with tachyon and bulk viscosity models of the universe. We obtain the Equation of state parameter for the interacting NADE model in axially symmetric Bianchi type-I in section 2. In section 3, we solve the field equations by considering the deceleration parameter to be a constant. The correspondence between the interacting NADE with tachyon and bulk viscous dark energy is shown in section 4. We conclude this chapter in the last section.

6.2 Interacting NADE model in Bianchi type-I universe

The metric for axially symmetric Bianchi type I is

$$ds^2 = dt^2 - a_1{}^2(t)dx_1{}^2 - a_2{}^2(t)dx_2{}^2 - a_2{}^2(t)dx_3{}^2 \quad (1)$$

Einstein field equation is given by

$$R_{ij} - \frac{1}{2}g_{ij}R = -\left(T_{ij} + \bar{T}_{ij}\right) \quad (2)$$

where R_{ij} is the Ricci tensor and we take $c = 8\pi G = 1$ for relativistic unit.

The energy momentum tensor of pressure less cold dark matter (CDM) and NADE are defined as

$$T_{ij} = \rho_{dm}u_i u_j \text{ and } \bar{T}_{ij} = (\rho_{\text{de}} + p_{\text{de}})u_i u_j + g_{ij}p_{\text{de}} \quad (3)$$

where ρ_{dm} , ρ_{de} are the energy densities of CDM and NADE and p_{de} is the pressure of NADE.

The field equations for the axially symmetric Bianchi type-I metric are

$$2\frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2} + \left(\frac{\dot{a}_2}{a_2}\right)^2 = \rho_{dm} + \rho_{\text{de}} \quad (4)$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2} = -p_{\text{de}} \quad (5)$$

$$2\frac{\ddot{a}_2}{a_2} + \left(\frac{\dot{a}_2}{a_2}\right)^2 = -p_{\text{de}} \quad (6)$$

The average scale factor a and the average Hubble parameter H are defined as

$$a = (a_1 a_2{}^2)^{\frac{1}{3}} \quad (7)$$

$$H = \frac{1}{3}\left(\frac{\dot{a}_1}{a_1} + 2\frac{\dot{a}_2}{a_2}\right) \quad (8)$$

The scalar expansion θ, shear scalar σ^2 and the average anisotropy parameter A_m are defined by

$$\theta = \frac{\dot{a}_1}{a_1} + 2\frac{\dot{a}_2}{a_2} \quad (9)$$

$$q = -\frac{a\ddot{a}}{a^2} = -1 - \frac{\dot{H}}{H^2} \quad (10)$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - \frac{1}{3}\theta^2\right) \quad (11)$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 \quad (12)$$

where $\Delta H_i = H_i - H \ (i = 1,2,3)$

The energy density of the NADE is given by [68]

$$\rho_{\text{de}} = \frac{3n^2}{\eta^2} \quad (13)$$

The conformal time η for Bianchi type-I universe is defined by

$$\eta = \int \frac{dt}{a} \quad (14)$$

The CDM density parameter (Ω_{dm}) and NADE density parameter (Ω_{de}) are given by

$$\Omega_{\text{dm}} = \frac{\rho_{dm}}{3H^2} \quad \text{and} \quad \Omega_{\text{de}} = \frac{\rho_{\text{de}}}{3H^2} \quad (15)$$

Using the definition $\rho_{de} = 3H^2\Omega_{\text{de}}$, we get

$$\eta = \frac{n}{H\sqrt{\Omega_{\text{de}}}} \quad (16)$$

Combining (4)-(6) one can easily obtain the continuity equation as

$$\dot{\rho}_{dm} + \left(\frac{\dot{a}_1}{a_1} + 2\frac{\dot{a}_2}{a_2}\right)\rho_{dm} + \dot{\rho}_{\text{de}} + \left(\frac{\dot{a}_1}{a_1} + 2\frac{\dot{a}_2}{a_2}\right)(\rho_{\text{de}} + p_{\text{de}}) = 0 \quad (17)$$

For a universe evolving through the cosmic substratum would consist of two interacting components: NADE and pressure less cold dark matter. So we write the continuity equation as [68]

$$\dot{\rho}_{\text{de}} + \left(\frac{a_1}{a_1} + 2\frac{\dot{a}_2}{a_2}\right)(\rho_{\text{de}} + p_{\text{de}}) = -Q \quad (18)$$

and $$\dot{\rho}_{dm} + \left(\frac{\dot{a}_1}{a_1} + 2\frac{\dot{a}_2}{a_2}\right)\rho_{dm} = Q \quad (19)$$

where Q represent the interaction term between dark matter and dark energy.

The barotropic equation of state is

$$p_{\text{de}} = \omega_{\text{de}}\rho_{\text{de}} \quad (20)$$

Using equation (20) in equation (18) we get

$$\dot{\rho}_{\mathrm{de}} + (1+\omega_{\mathrm{de}})\left(\frac{\dot{a}_1}{a_1} + 2\frac{\dot{a}_2}{a_2}\right)\rho_{\mathrm{de}} = -Q \tag{21}$$

Taking time derivatives of the equation (13) and using $\dot{\eta} = \frac{1}{a}$ (which can be obtained from equation (14)), we get

$$\dot{\rho}_{\mathrm{de}} = \frac{-2H\sqrt{\Omega_{\mathrm{de}}}}{na}\,\rho_{de} \tag{22}$$

Substituting equation (22) in (21), we get the equation of state parameter of the interacting NADE model as

$$\omega_{\mathrm{de}} = -1 + \frac{2\sqrt{\Omega_{\mathrm{de}}}}{3na} - \frac{Q}{3H\rho_{de}} \tag{23}$$

Since from the continuity equation, the interaction term should be proportional to a quantity with units of inverse of time, so we choose $Q = 3\zeta H\rho_{\mathrm{de}}$ as an interaction term where $3\zeta H$ is the decay rate of the NADE component into CDM with a coupling constant ζ. A detailed study of different Q-classes are done in [45, 58, 96, 97, 130, 200]. Therefore equation (23) becomes

$$\omega_{\mathrm{de}} = -1 + \frac{2\sqrt{\Omega_{\mathrm{de}}}}{3na} - \zeta \tag{24}$$

Taking $\Omega_{\mathrm{de}} = 0.73$, $n = 2.7$ [69] and the average scale factor of the present universe $a(t_0) = 1$, we get from equation (24)

$$\omega_{\mathrm{de}} = -0.79 - 1.37\zeta \tag{25}$$

It is observed from equation (25) that for $\zeta > 0.21$ and $\zeta < 0.21$ the EOS behaves like phantom and quintessence EOS respectively:

6.3 Cosmological Solutions

Since the right hand sides of (5) and (6) are identical, we have

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2} = 2\frac{\ddot{a}_2}{a_2} + \left(\frac{\dot{a}_2}{a_2}\right)^2 \tag{26}$$

Solving equation (26) and using (7) we get,

$$\frac{a_1}{a_2} = D_2 exp(D_1 \int a^{-3} dt) \qquad (27)$$

The metric functions, therefore can be read as

$$a_1(t) = D_2^2 a\ exp(2D_3 \int a^{-3} dt) \qquad (28)$$

$$a_2(t) = D_2^{-1} a\ exp(-D_3 \int a^{-3} dt) \qquad (29)$$

where D_1, D_2 and D_3 are constants and $3D_3 = D_1$

6.3.1 To solve these field equations first we assume that the average Hubble parameter H is related to the average scale factor a, by the relation

$$H = Da^{-l} \qquad (30)$$

where $D > 0$ and $l \geq 0$ are constants. This relation is first proposed by Berman in [128].

Case (i): solution of the field equations for $l \neq 0$:

Solving the equation (30) we get

$$a(t) = (lDt + c_1)^{\frac{1}{l}} \qquad (31)$$

Using (31) in (28) and (29) we get the metric functions as

$$a_1(t) = D_2^2 (lDt + c_1)^{\frac{1}{l}} exp\left[\frac{2D_3}{D(l-3)} (lDt + c_1)^{\frac{l-3}{l}}\right] \qquad (32)$$

$$a_2(t) = D_2^{-1} (lDt + c_1)^{\frac{1}{l}} exp\left[\frac{-D_3}{D(l-3)} (lDt + c_1)^{\frac{l-3}{l}}\right] \qquad (33)$$

This gives a constant value of the deceleration parameter

$$q = l - 1 \qquad (34)$$

Since recent observational data indicates that the universe is accelerating and the value of deceleration parameter lies somewhere in the range $-1 < q < 0$, so we have $0 < l < 1$ for the accelerating universe. The Hubble parameter, the scalar expansion, shear scalar and the average anisotropy parameter are

$$H = D(lDt + c_1)^{-1} \quad (35)$$

$$\theta = 3D(lDt + c_1)^{-1} \quad (36)$$

$$\sigma^2 = 3D_3^2(lDt + c_1)^{\frac{-6}{l}} \quad (37)$$

$$A_m = \frac{2D_3^2}{D^2}(lDt + c_1)^{\frac{2l-6}{l}} \quad (38)$$

Using (30)-(33), (35) and (15) in (4) we get

$$\Omega_{\rm m} + \Omega_{\rm de} = 1 - \frac{D_3^2}{D^2}(lDt + c_1)^{\frac{2l-6}{l}} \quad (39)$$

Equation (39) shows that the sum of the energy density parameters approaches 1 at late times. So at late times the universe becomes flat. Therefore for sufficiently large time, this model predicts that the anisotropy of the universe will damp out and universe will become isotropic. This result also shows that in the early universe i.e. during the radiation and matter dominated era the universe was anisotropic and the universe approaches to isotropy as dark energy starts to dominate the energy density of the universe. This result is totally different from Pradhan et al.'s result [23] where they found in their model that the universe does not approach isotropy through its whole evolution.

Using equations (19), (31) and $Q = 3\zeta H \rho_{\rm de}$ we get the interacting CDM energy density of the universe as

$$\rho_{dm} = 9\zeta\Omega_{\rm de}\left(\frac{Dl}{3-2l}\right)H^2 + 3{H_0}^2\left(\Omega_{\rm dm,0} - \frac{3Dl}{3-2l}\zeta\Omega_{\rm de,0}\right) \quad (40)$$

Case (ii): Solution of the field equation for $l = 0$.

Solving the equation (30) we get

$$a(t) = c_2 e^{Dt} \quad (41)$$

Using (41) in (27) and (28) we get the metric functions as

$$a_1(t) = D_2^2 c_2 exp\left(Dt - \frac{2D_3}{3Dc_2^3}e^{-3Dt}\right) \quad (42)$$

$$a_2(t) = D_2^{-1} c_2 exp\left(Dt + \frac{D_3}{3Dc_2^3}e^{-3Dt}\right) \quad (43)$$

The deceleration parameter, Hubble parameter, the scalar expansion, shear scalar and the average anisotropy parameter are

$$q = -1 \tag{44}$$

$$H = D \tag{45}$$

$$\theta = 3D \tag{46}$$

$$\sigma^2 = \frac{3D_3^2}{c_2^6} e^{-6Dt} \tag{47}$$

$$A_m = \frac{2D_3^2}{D^2 c_2^6} e^{-6Dt} \tag{48}$$

Using (41)-(43), (45) and (15) in (4) we get

$$\Omega_{\rm m} + \Omega_{\rm de} = 1 - \frac{D_3^2}{D^2 c_2^6} e^{-6Dt} \tag{49}$$

The Equation (49) indicates that at late times the universe becomes flat. The constant value of the Hubble's parameter and the constant negative value of the deceleration parameter indicate the fastest rate of expansion of the universe. This model can be used to describe the dynamics of the late time evolution of the universe dominated by dark energy.

6.3.2 Here we take one more extra condition that the deceleration parameter is a constant and $a_2(t)$ to be an arbitrary function of $a_1(t)$ to get the solution of the field equations [157]. Let

$$a_2 = \chi\{a_1(t)\} \tag{50}$$

Using equation (50) in the equation (25) we get

$$\left(\frac{\chi'}{\chi} - \frac{1}{a_1}\right)\ddot{a}_1 + \left\{\frac{\chi''}{\chi} + \left(\frac{\chi'}{\chi}\right)^2 - \frac{\chi'}{\chi a_1}\right\}\dot{a}_1{}^2 = 0 \tag{51}$$

The prime over χ denotes ordinary differentiation w. r. t. $a_1(t)$.

This equation results in the following possibilities

Case (i):

$$\dot{a}_1 = 0 \tag{52}$$

Solving equation (52) we get

$$a_1 = c_{11} \tag{53}$$

where c_{11} is a constant

From equation (53) and equation (25) we get

$$a_2 = c_{12}^{\frac{1}{2}}(t + c_{13})^{\frac{1}{2}} \tag{54}$$

This gives $q = 2 > 0$. This represents the deceleration of the universe. So in this case it is not possible to obtain the accelerating model of the universe.

Case (ii):

$$\ddot{a}_1 = 0 \tag{55}$$

$$\text{and } \frac{\chi^{//}}{\chi} + \left(\frac{\chi^{/}}{\chi}\right)^2 - \frac{\chi^{/}}{\chi a_1} = 0 \tag{56}$$

Solving equation (56) we get

$$a_1 = c_{21}t + c_{22} \tag{57}$$

where c_{21} and c_{22} constants.

Using equations (57) and (25) we get

$$a_2 = \{c_{23}(c_{21}t + c_{22})^2 + c_{24}\}^{\frac{1}{2}} \tag{58}$$

where c_{23} and c_{24} constants.

For constant deceleration parameter we have $c_{24} = 0$. This gives $q = 0$. So in this case also it is not possible to obtain an accelerating model of the universe.

Case (iii):

$$\frac{\chi^{/}}{\chi} - \frac{1}{a_1} = 0 \tag{59}$$

$$\text{and } \frac{\chi^{//}}{\chi} + \left(\frac{\chi^{/}}{\chi}\right)^2 - \frac{\chi^{/}}{\chi a_1} = 0 \tag{60}$$

Solving equation (58) and (59) we get

$$a_2 = \chi = c_{31} a_1 \tag{61}$$

where c_{31} is a constant

Solving $q = constant = -\frac{a\ddot{a}}{\dot{a}^2}$ we get $(t) = (q+1)^{\frac{1}{q+1}}(c_1 t + c_2)^{\frac{1}{q+1}}$. Therefore keeping equation (61) in mind we presently considering a power law relation between scale factors and time co-ordinates as,

$$a(t) = (a_{02}a_{03}^2)^{\frac{1}{3}}(t + a_{01})^m \tag{62}$$

$$a_1 = a_{02}(t + a_{01})^m \tag{63}$$

and $a_2 = a_{03}(t + a_{01})^m$ (64)

where a_{01}, a_{02} and $a_{03} = c_{31}a_{02}$ are constants.

Therefore the average Hubble parameter H is

$$H = \frac{m}{t + a_{01}} \tag{65}$$

The deceleration parameter is,

$$q = -1 + \frac{1}{m} \tag{66}$$

Here $m > 1$ gives the present acceleration of the universe.

Using (62)-(65) and (15) in equation (4) we get

$$\Omega_{\rm m} + \Omega_{\rm de} = 1 \tag{67}$$

This represents a flat homogeneous and isotropic universe i.e. FRW universe. So the constant deceleration parameter with the extra condition $a_2 = \chi\{a_1(t)\}$ gives only one isotropic solution for the accelerating universe. Using equations (19), (62) and $Q = 3\zeta H \rho_{\rm de}$ we get the interacting CDM energy density of the universe as

$$\rho_{dm} = 9\zeta\Omega_{\rm de}\left(\frac{m}{3m-2}\right)H^2 + 3{H_0}^2\left(\Omega_{\rm dm,0} - \frac{3m}{3m-2}\zeta\Omega_{\rm de,0}\right)a^{-3} \tag{68}$$

6.4 The correspondence between the interacting NADE with tachyon and bulk viscous dark energy models:

To establish the correspondence between the interacting NADE with tachyon and bulk viscous dark energy models, we compare the equation of state (EOS) and the dark energy density for the corresponding models of dark energy. The average scale factor of the universe for this case is in the following from: $a(t) = (lDt + c_1)^{\frac{1}{l}}$ where $0 < l < 1$ or $a(t) = c_2 e^{Dt}$ or $a(t) = (a_{02}a_{03}^2)^{\frac{1}{3}}(t + a_{01})^m$ where $m > 1$.

6.4.1 Correspondence between the interacting NADE and tachyon scalar field model of dark energy

The action for the tachyon scalar φ is given by Born-Infeld like action

$$s = \int \sqrt{-g}\, dx^4 \left(-V(\varphi)\sqrt{1 - g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi}\right) \tag{69}$$

The energy density and pressure for the tachyon field is represented by [85]

$$\rho_\varphi = \frac{V(\varphi)}{\sqrt{1-\dot{\varphi}^2}} \tag{70}$$

$$p_\varphi = -V(\varphi)\sqrt{1-\dot{\varphi}^2} \tag{71}$$

where $V(\varphi)$ is the tachyon potential.

The equation of state for tachyonic matter is

$$\omega_\varphi = \frac{p_\varphi}{\rho_\varphi} = \dot{\varphi}^2 - 1 \tag{72}$$

From Eq. (72) we see that $-1 < \omega_\varphi < 0$. For the correspondence between the tachyon scalar field and the NADE we must have $-1 < \omega_\varphi = \omega_{\text{de}} < 0$. Using (24) and (72) we have

$$\omega_{\text{de}} = -1 + \frac{2\sqrt{\Omega_{\text{de}}}}{3na} - \zeta = \dot{\varphi}^2 - 1 \tag{73}$$

Also comparing equations (13) and (70) one can write

$$\rho_{\text{de}} = \frac{3n^2}{\eta^2} = \frac{V(\varphi)}{\sqrt{1-\varphi^2}} \tag{74}$$

From equations (73) and (74), we can obtain the kinetic energy term and the tachyon potential energy as follows

$$\dot{\varphi}^2 = \frac{2\sqrt{\Omega_{\text{de}}}}{3na} - \zeta \tag{75}$$

$$V(\varphi) = 3H^2\Omega_{\text{de}}\sqrt{1 - \frac{2\sqrt{\Omega_{\text{de}}}}{3na} + \zeta} \tag{76}$$

From (74), we obtain the evolutionary form of the tachyon scalar field as

$$\varphi(t) - \varphi(t_0) = \int_{t_0}^{t} \left(\frac{2\sqrt{\Omega_{\text{de}}}}{3na} - \zeta\right)^{\frac{1}{2}} dt \tag{77}$$

The dynamics and the potentials of other anisotropic scalar field models such as quintessence, phantom, k-essence and dilaton field can be obtained in a similar way.

6.4.2 Correspondence between the interacting NADE and Bulk viscous model of dark energy

For a universe to be described by the stress energy tensor of an imperfect fluid, the energy momentum tensor is

$$T_{eff}^{ij} = (\mu + P)u^i u^j + Pg^{ij} \tag{78}$$

where μ is the energy density and P is the effective pressure of the universe given by

$$P = p + \Pi \tag{79}$$

Here p is the equilibrium pressure and $p \geq 0$ for gaseous matter. Π represents the pressure of the non-equilibrium part of the universe. For perfect fluid we have $\Pi = 0$, i.e., $P \geq 0$ and for a conventional viscous fluid $\Pi \leq 0$ is valid during expansion. For a universe filled with cold dark matter and dark energy, one can decompose the total energy density into two parts as [193]

$$\mu = \rho_{dm} + \rho_v \tag{80}$$

where ρ_v is the energy density of the non-equilibrium part of the universe.

In the first-order Eckart theory we get [35]

$$\Pi = -3\xi H \tag{81}$$

where ξ is the coefficient of bulk viscosity.

The equation of state of the non-equilibrium part is

$$\omega_v = \frac{\Pi}{\rho_{de}} = \frac{-3\xi H}{\rho_v} \tag{82}$$

This EOS behaves quintessence like EOS and for large bulk viscous pressure it behaves like phantom.

Using equations (82) and (24) we have

$$\omega_{\rm de} = -1 + \frac{2\sqrt{\Omega_{\rm de}}}{3na} - \zeta = \frac{-3\xi H}{\rho_v} \tag{83}$$

Also comparing equations (13) and (80) we can write

$$\rho_{\rm de} = \frac{3n^2}{\eta^2} = \rho_v \tag{84}$$

From equations (83) and (84) we get

$$\xi = \left(1 - \frac{2\sqrt{\Omega_{\rm de}}}{3na} + \zeta\right) H\Omega_{\rm de} \tag{85}$$

6.5 Conclusion

Here we have considered a homogeneous axially symmetric Bianchi type-I universe filled with interacting new agegraphic dark energy and cold dark matter. Assuming the deceleration parameter to be a constant, we get two exact solutions of Einstein's field equations. Both the solutions represent the present acceleration of the universe and predict a flat homogeneous isotropic universe at late time evolution. It is shown that applying one more condition $a_2 = \chi\{a_1(t)\}$ along with the constant deceleration parameter only one accelerating solution can be obtained which represent a flat FRW universe. The EOS parameter of NADE, ω_{de}, indicate that in the presence of interaction between NADE and CDM, it can cross the phantom divide. Using these results we have established a correspondence between the interacting NADE model with the tachyon and bulk viscous dark energy models in the Bianchi type-I universe. Tachyon potential, the dynamics of the tachyonic scalar field and bulk viscous coefficient are reconstructed for these anisotropic accelerating models of the universe. Results indicate that these scalar field and bulk viscous models of dark energy are effective theories of an underlying theory of dark energy.

Chapter – 7

Dark energy, cosmic coincidence and future singularity of the universe

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7. Dark energy, cosmic coincidence and future singularity of the universe

7.1 Introduction

Recent cosmological observations, including the observations of the Supernovae of the type Ia [9, 171], the Large Scale Structure, Wilkinson Microwave Anisotropy Probe (WMAP) in the Cosmic Microwave Background Radiation (CMB) indicate that the present universe is passing through a state of accelerating expansion. The cause of this acceleration is supposed to be some kind of anti-gravitational force. A large majority of cosmological models explain the acceleration of the universe in terms of a component with the negative pressure, the so called dark energy. The present universe appears to consist of approximately 27% non-relativistic matter including both baryons and dark matter and 73% dark energy. Many cosmologists like to select the cosmological constant Λ, introduced by Einstien in his field equations, as a suitable candidate for dark energy because of its weired repulsive gravity. A number of dynamically evolving scalar field models of dark energy such as Quintessence, K-essence, Tachyon etc. are also considered to explain the present accelerated expansion of the universe. Some other models have also been considered such as Quartessence, which proposes a unified fluid with the characteristics of both dark matter and dark energy [60].

The dynamical nature of dark energy introduces a new cosmological problem, known as "Cosmic Coincidence Problem"- though matter energy density ρ_M and dark energy density ρ_{DE} diluted at different rates during cosmic expansion, the present observed density of dark energy and matter energy density are nearly comparable. In early universe $\rho_M \gg \rho_{DE}$ while in the far future, $\rho_M \ll \rho_{DE}$ as in all the models for the dark energy, the matter energy density decreases more rapidly than the dark energy density. To solve the coincidence problem at present epoch, Λ with a dynamical character is preferred over a constant Λ, especially a time dependent Λ which has decreased slowly from its large initial value to reach its present small value [80]. Various forms of unification and interacting dark energy models also have been constructed in order to solve the coincidence problem [63, 149, 174, 176]. Phantom dark energy cosmologies, first proposed by Caldwell, significantly ameliorate the coincidence problem of the universe [152].

Again there have been many recent investigations into the theoretical possibility that expanding universe can come to a violent end at a finite future time. This singularity issue has the fundamental importance in the modern cosmology [139, 164, 170]. Phantom dark-energy models, with $\omega < -1$, are characterised by a finite lifetime future singularity of the universe [153]. The existence of this future singularity is often considered as a negative feature of phantom dark-energy models, so a great deal of effort has gone into constructing models with $\omega < -1$ to avoid this future singularity [49, 165, 169]. S. Nojiri et al. in [170] first proposed that the dark energy model with the above equation of state (EOS), is characterised by four types of future singularities for a finite life time of the universe for different values of the parameter A and α. But the presence of these finite time future singularities may cause various problems in the current black holes and stellar astrophysics. At present it is expected that near to singularity the quantum gravity effects may play the significant role and such effects may contribute to the occurrence or remove the singularity. In [164] S. Nojiri et. al., have shown that if in the vicinity of singularities the quantum effects come into play then quantum effects may reduce or even completely remove the singular behaviour at classical singularities. Recently, in [167] S. Nojiri et. al., showed that modified gravity of special form or inhomogeneous dark energy fluid may offer the universal scenario to cure the future singularity of these types.

Since such singularities produce a finite life time of the universe, so it is possible to calculate the fraction of time that universe spends in a "coincidental state" where the matter energy density and dark energy density are nearly comparable. For a positive feature of phantom dark-energy model with future singularity, R. J. Scherrer examines the time evolution of phantom cosmologies with $\omega < -1$, and calculates the fraction of the total lifetime of the universe for which the matter and dark-energy densities are roughly similar [150].

In this chapter we study a positive feature of the above EOS with four types of finite life future singularities for a flat homogeneous and isotropic universe i.e. flat FRW universe. We calculate the fraction of total life time of the universe for the coincidental stage in section 2. In section 3, we represent our results graphically for type I singularity. Results for type II, III, IV singularities respectively are represented graphically in section 4. We conclude the chapter in section 5.

7.2 Fraction of total life time of the universe for the coincidental stage:

The FRW metric for a flat, homogeneous and isotropic universe is

$$ds^2 = -dt^2 + a^2(t)[\ dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2] \tag{1}$$

where $a(t)$ is the scale factor and t represents the cosmic time.

We assume that the universe contains two components, non-relativistic matter (including both baryons and dark matter) and dark energy.

The Friedmann field equation is

$$3\left(\frac{\dot{a}}{a}\right)^2 = 8\pi G(\rho_M + \rho_{DE}) \tag{2}$$

where ρ_M is the non-relativistic matter density and ρ_{DE} is the dark energy density.

The energy conservation is

$$\dot{\rho_M} + 3H\rho_M = 0 \quad (3)$$

$$\dot{\rho_{DE}} + 3H(\rho_{DE} + p_{DE}) = 0 \quad (4)$$

Solving equation (3) we get

$$\rho_M = \rho_{M0}\left(\frac{a}{a_0}\right)^{-3} \quad (5)$$

where ρ_{M0} is the present value of the non-relativistic matter density and a_0 is the present value of the scale factor.

The equation of state of dark energy $p = -\rho - f(\rho)$, where $f(\rho)$ is a function in terms of density ρ is characterised by various types of future singularities on the choice of the function $f(\rho)$. In this present work we consider $f(\rho) = A\rho^{\alpha}$ where A and α are constants [170].

Therefore the pressure of the dark energy is

$$p_{DE} = -\rho_{DE} - A\rho_{DE}^{\alpha} \quad (6)$$

Solving equation (4) using (6) we obtain the dark energy density

$$\rho_{DE} = \rho_{DE0}\left(1 + 3\tilde{A}(1-\alpha)In\frac{a}{a_0}\right)^{\frac{1}{(1-\alpha)}} \quad (7)$$

where $\tilde{A} = A\rho_{DE,0}^{\alpha-1}$.

Here A and α are real parameters. Here we consider $\alpha \neq 1$ because for $\alpha = 1$ the dark energy EOS reduces to the dark energy with the constant EOS of either the quintessence $A < 0$ or the phantom $A > 0$ type and for $A = 0$ it represent the cosmological constant.

Differentiating both sides of the equation (7) we get the speed of change of the dark energy as

$$\frac{d\rho_{DE}}{da} = 3\tilde{A}\rho_{DE,0}\left(1 + 3\tilde{A}(1-\alpha)In\frac{a}{a_0}\right)^{\frac{1}{(1-\alpha)}-1}\frac{1}{a} \quad (8)$$

The equation (8) shows that for $\tilde{A} > 0$ the dark energy density grows, for $\tilde{A} = 0$ the dark energy density is constant, while for $\tilde{A} < 0$ it decreases with the expansion of the universe [67].

For the analysis of the dynamics of the universe we assume as in [67] that the dark energy density increases, remains constant or decreases more slowly than the non-relativistic matter energy density, and for $\alpha = \frac{1}{2}$ and $\alpha \neq \frac{1}{2}$ the expressions for the time depended scale factor are respectively

$$In\frac{1+\frac{3}{2}\tilde{A}In\frac{a_1}{a_0}}{1+\frac{3}{2}\tilde{A}In\frac{a_2}{a_0}} = \frac{3}{2}\tilde{A}\Omega_{DE,0}^{\frac{1}{2}}H_0(t_1 - t_2) \quad (9)$$

and

$$\left(1 + 3\tilde{A}(1-\alpha)In\frac{a_1}{a_0}\right)^{\frac{1-2\alpha}{2(1-\alpha)}} - \left(1 + 3\tilde{A}(1-\alpha)In\frac{a_2}{a_0}\right)^{\frac{1-2\alpha}{2(1-\alpha)}} = \frac{3}{2}\tilde{A}(1-2\alpha)\Omega_{DE,0}^{\frac{1}{2}}H_0(t_1 - t_2) \quad (10)$$

Since at $\alpha = \frac{1}{2}$ and $\alpha = 1$ the equations (7), (9) and (10) have singularities, so these values of α represent the bordering points of the intervals on the α line when we are going to determine the evolution of the universe.

The ratio of the dark energy density and the matter energy density is

$$r \equiv \frac{\rho_{DE}}{\rho_M} = \frac{\rho_{DE0}\left(1+3\tilde{A}(1-\alpha)In\frac{a}{a_0}\right)^{\frac{1}{(1-\alpha)}}}{\rho_{M0}\left(\frac{a}{a_0}\right)^{-3}} \quad (11)$$

At present the ratio $r \approx 2.7$. But $\frac{1}{3} < \frac{\rho_{DE}}{\rho_M} < 3$ can also be considered for coincidental stage of the universe. So the ratio r can be considered as a free parameter in the calculation. We take $\frac{1}{r_0} < r < r_0$, where r_0 is any fixed ratio for our calculation of the fractional time.

Using the definition of the energy densities parameter $\Omega_{M,0} = \frac{\rho_{M,0}}{\rho_{c,0}}$ and $\Omega_{DE,0} = \frac{\rho_{DE,0}}{\rho_{c,0}}$, where $\rho_{c,0}$ is the critical energy density of the present universe given by $\rho_{c,0} = \frac{3H_0^2}{8\pi G}$ we can write equation (11) as

$$r = \frac{\Omega_{DE,0}\left(1+3\tilde{A}(1-\alpha)In\frac{a}{a_0}\right)^{\frac{1}{(1-\alpha)}}}{\Omega_{M,0}\left(\frac{a}{a_0}\right)^{-3}} \qquad (12)$$

Using equation (5) and equation (7) in equation (2) we get

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G\left(\rho_{M,0}\left(\frac{a}{a_0}\right)^{-3} + \rho_{DE,0}\left(1+3\tilde{A}(1-\alpha)In\frac{a}{a_0}\right)^{\frac{1}{(1-\alpha)}}\right) \qquad (13)$$

Integrating equation (13) we get the total time of the universe from the big bang to the future singularity as

$$t_U = \int_0^{a_{max}} a^{-1}\left[\frac{8}{3}\pi G\left(\rho_{M,0}\left(\frac{a}{a_0}\right)^{-3} + \rho_{DE,0}\left(1+3\tilde{A}(1-\alpha)In\frac{a}{a_0}\right)^{\frac{1}{(1-\alpha)}}\right)\right]^{-1/2} da \qquad (14)$$

The total time that the universe spends in expanding from a given initial scale factor, a_1, to a given final scale factor a_2 is

$$t_{12} = \int_{a_1}^{a_2} a^{-1}\left[\frac{8}{3}\pi G\left(\rho_{M,0}\left(\frac{a}{a_0}\right)^{-3} + \rho_{DE,0}\left(1+3\tilde{A}(1-\alpha)In\frac{a}{a_0}\right)^{\frac{1}{(1-\alpha)}}\right)\right]^{-1/2} da \qquad (15)$$

Therefore the fraction f of the total lifetime of the universe that it spends in expanding from a_1 to a_2 is

$$f = \frac{\int_{a_1}^{a_2} a^{-1}\left[\frac{8}{3}\pi G\left(\rho_{M,0}\left(\frac{a}{a_0}\right)^{-3} + \rho_{DE,0}\left(1+3\tilde{A}(1-\alpha)In\frac{a}{a_0}\right)^{\frac{1}{(1-\alpha)}}\right)\right]^{-1/2} da}{\int_0^{a_{max}} a^{-1}\left[\frac{8}{3}\pi G\left(\rho_{M,0}\left(\frac{a}{a_0}\right)^{-3} + \rho_{DE,0}\left(1+3\tilde{A}(1-\alpha)In\frac{a}{a_0}\right)^{\frac{1}{(1-\alpha)}}\right)\right]^{-1/2} da} \qquad (16)$$

To calculate the value of f for different singularities we have set $a_0 = 1$ for the present universe [109].

7.3 Type I singularity

For $\widetilde{A} > 0$ and $\frac{1}{2} < \alpha < 1$, let the universe be dominated by dark energy from a time t_* onwards and a_* be the corresponding scale factor of the universe. Then the equation (10) is in the form

$$\left(1 + 3\widetilde{A}(1-\alpha)\ln\frac{a}{a_0}\right) = \left[\left(1 + 3\widetilde{A}(1-\alpha)\ln\frac{a_*}{a_0}\right)^{\frac{1-2\alpha}{2(1-\alpha)}} - \frac{3}{2}\widetilde{A}(2\alpha-1)\Omega_{DE,0}^{\frac{1}{2}}H_0(t-t_*)\right]^{\frac{2(\alpha-1)}{2\alpha-1}} \quad (17)$$

at the instant of time

$$t_s = t_* + \frac{2}{3\widetilde{A}(2\alpha-1)\Omega_{DE,0}^{\frac{1}{2}}H_0}\left(1 + 3\widetilde{A}(1-\alpha)\ln\frac{a_*}{a_0}\right)^{\frac{1-2\alpha}{2(1-\alpha)}} \quad (18)$$

At this time the scale factor diverges. The dark energy density and the pressure diverge at the same instant. Therefore in this model, type I singularity ("Big Rip") where for $t \to t_s$, $a \to \infty$, $\rho \to \infty$, and $|p| \to \infty$ occurs when $\tilde{A} > 0$ and $\frac{1}{2} < \alpha < 1$.

From equations (5) and (7) we can find that for type I singularity where $\tilde{A} > 0$ and $\frac{1}{2} < \alpha < 1$ dark energy density increases whereas the matter energy density decreases as the universe expands and the ratio of the dark energy density to the matter energy density increases rapidly with the expansion of the universe. Considering these ranges of the parameters A and α and taking the ratio r as $r_1 = \frac{1}{r_0}$ and $r_2 = r_0$, we calculate the values of the scale factors a_1 and a_2 from equation (12). Using these values of a_1 and a_2 in (16) we find out the fraction of time f that the universe spends in coincidental state for type I singularity. In Fig.1 we show f as a function of r_0 for some representative values of α.

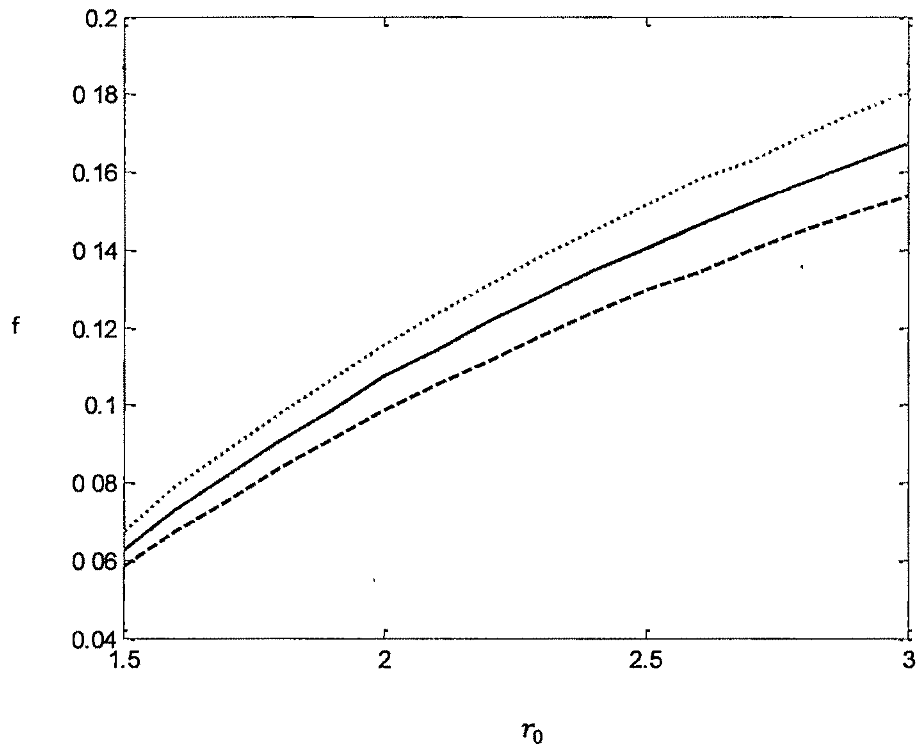

Fig.1 *In the graph the dotted, solid and dashed curve line represent the fraction of time f that the universe spends in a coincidental state for type I singularity, defined as* $\frac{1}{r_0} < \frac{\rho_{DE}}{\rho_M} < r_0$ *for* $\alpha = .9,\ \alpha = .85,\ \alpha = .8$.

From the graph it is seen that the fraction of time increases as the value of α increses. For $\alpha = .8,\ .85,\ .9$ fraction of time is 15.4%, 16.77%, 18.05% of the total life time of the universe where $\frac{1}{3} < r < 3$. The fraction of time increases when the parameter α closes to 1.

7.4 Type II, Type III, Type IV singularities

These singularities are classified in the following way [170]:

Type II ("Sudden"): $t \to t_s$, $a \to a_s$, $\rho \to \rho_s$, and $|p| \to \infty$.

Type III: $t \to t_s$, $a \to a_s$, $\rho \to \infty$, and $|p| \to \infty$.

Type IV (Mild): $t \to t_s$, $a \to a_s$, $\rho \to 0$, and $|p| \to 0$ and higher derivatives of H diverge.

where t_s, a_s and ρ_s are constants with $a_s \neq 0$.

The scale factor of the universe in the above three types of future singularities of the universe is finite. The dark energy density diverges for type III singularity whereas for type II and type IV singularities the dark energy density is finite.

The equation (7) shows that the finite value of the scale factor for which the dark energy density diverge or zero is

$$a_{max} = a_0 e^{\frac{1}{3\tilde{A}(\alpha-1)}} \quad (19)$$

From equation (19) it is found that the type II, III and IV singularities of the universe occur when

$$\alpha = 1 + \frac{1}{\tilde{A}} \quad (20)$$

The different ranges of the parameters $\tilde{A}$ and α for which these three singularities occur are: for type II, $\tilde{A} < 0$ and $\alpha < 0$; for Type III, $\tilde{A} > 0$ and $\alpha > 1$; for type IV, $\tilde{A} < 0$ and $0 < \alpha < \frac{1}{2}$ where $\frac{1}{1-2\alpha}$ is not an integer [67]. The equation (20) satisfies these different ranges of the parameters $\tilde{A}$ and α.

From equations (5) and (7) one can find that for type II singularity and type IV singularity where $\tilde{A} < 0$, $\alpha < 0$ and $\tilde{A} < 0$, $0 < \alpha < \frac{1}{2}$ respectively, both the matter and the dark energy densities decrease as the universe expands whereas the matter energy density decreases but the dark energy density increases with the expansion of the universe for type III singularity where $\tilde{A} > 0$, $\alpha > 1$. The ratio of the dark energy density to the relativistic matter energy density increases for all these type II, type III, type IV singularities. Taking the ratio r as $r_1 = \frac{1}{r_0}$, $r_2 = r_0$ and considering these ranges of the parameters A and α, we calculate the values of the scale factors a_1 and a_2 from equation (12). Using these values of a_1 and a_2 in (16) we find out the fraction of time f that the universe spends in coincidental state for type II, III and IV singularity. In Fig.2, Fig.3 and Fig4 we show f as a function of r_0 for some representative values of α.

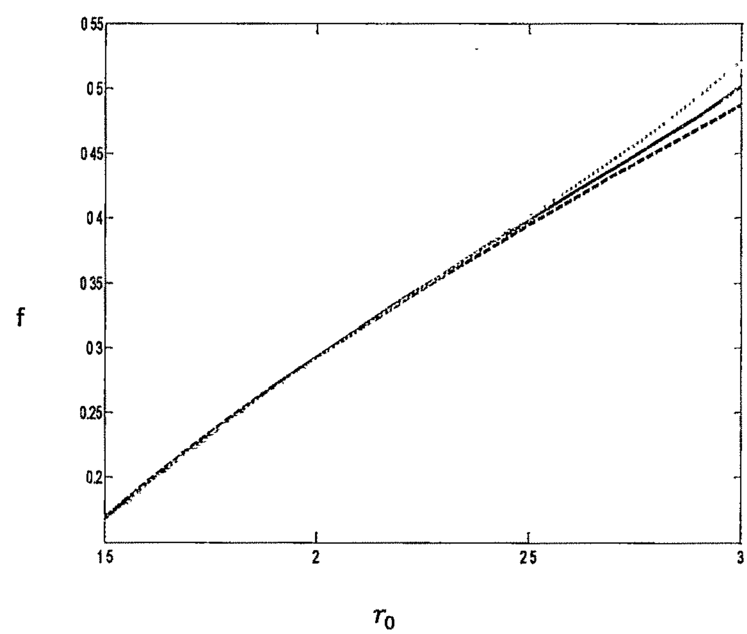

Fig.2: *In the graph the dotted, solid and dashed curve line represent the fraction of time f that the universe spends in a coincidental state for type II singularity, defined as* $\frac{1}{r_0} < \frac{\rho_{DE}}{\rho_M} < r_0$ *for* $\alpha = -1$, $\alpha = -1.5$, $\alpha = -2$.

In type II singularity the fraction of time increases as the value of α increases. For $\alpha = -2, -1.5, -1$ fraction of time is 48.73%, 50.11%, 52.06% of the total life time of the universe where $\frac{1}{3} < r < 3$.

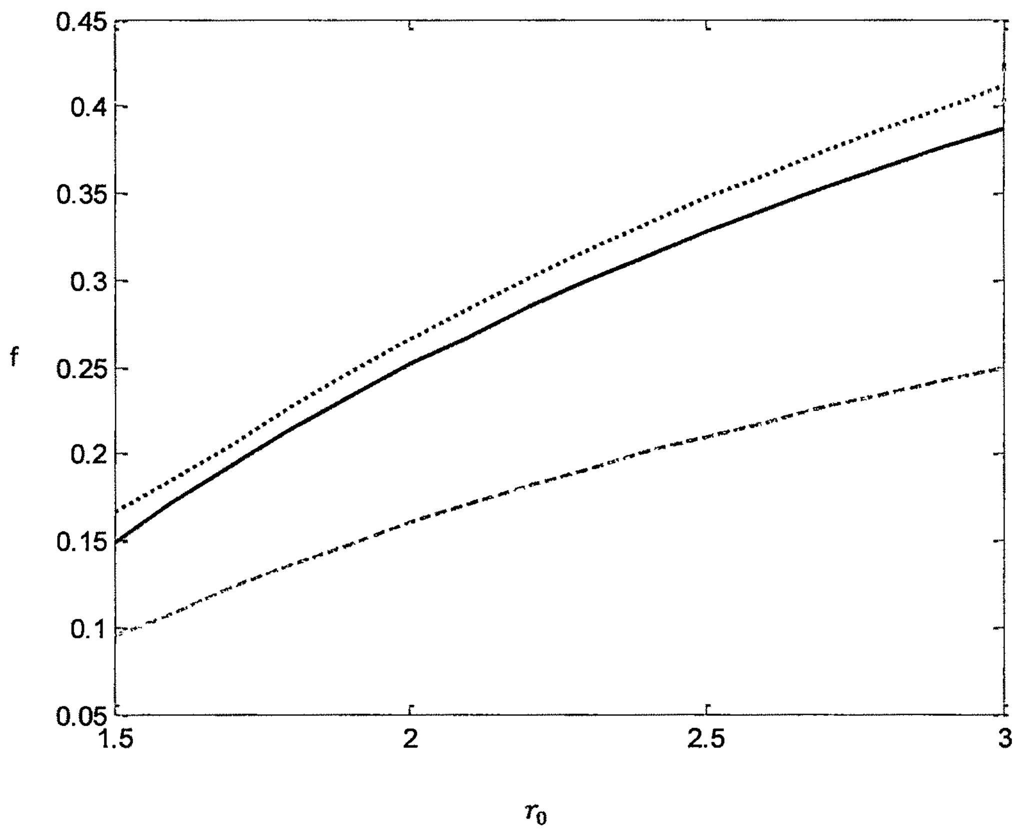

Fig.3: *In the graph the dotted, solid and dashed curve line represent the fraction of time f that the universe spends in a coincidental state for type III singularity, defined as* $\frac{1}{r_0} < \frac{\rho_{DE}}{\rho_M} < r_0$ *for* $\alpha = 4.5$, $\alpha = 3$, $\alpha = 1.5$.

The fraction of time f for type III singularity also increases as the value of α increases within the range for this singularity. The fraction is 25%, 38.75%, 41.22% for $\alpha = 1.5, 3, 4.5$ respectively where $\frac{1}{3} < r < 3$.

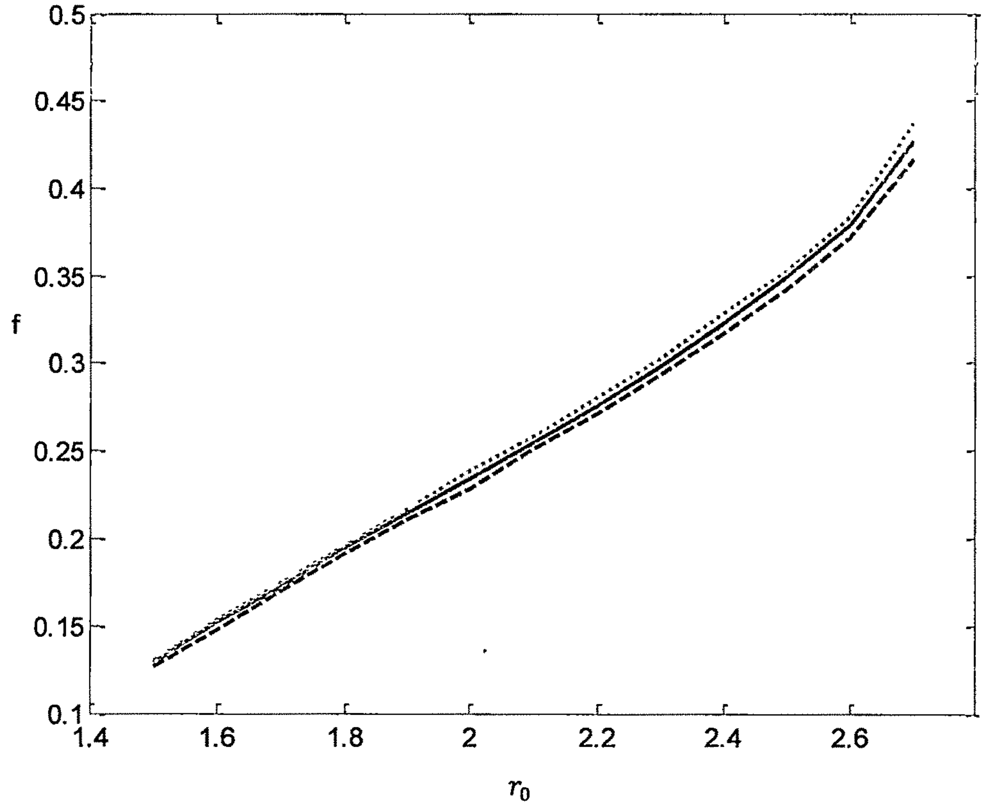

Fig.4: *In the graph the dotted, solid and dashed curve line represent the fraction of time f that the universe spends in a coincidental state for type IV singularity, defined as* $\frac{1}{r_0} < \frac{\rho_{DE}}{\rho_M} < r_0$ *for* $\alpha = \frac{1}{13}$, $\alpha = \frac{1}{11}$, $\alpha = \frac{1}{9}$.

In case of Type IV singularity the fraction f decreases as the value of α increases. For $\alpha = \frac{1}{13}, \frac{1}{11}, \frac{1}{9}$ the fraction f is 43.75%, 42.02%, 41.97% respectively where $\frac{1}{2.7} < r < 2.7$.

7.5 Conclusion

In this chapter, we considered the dark energy model with the EOS (6), which leads to four finite life time future singularities of the universe for different values of the parameters A and α in the late time evolution of the universe. Different ranges of the parameters A and α for which these four singularities occur, show that the ratio of the dark energy density to the matter energy density increases as the universe expands from the matter dominated era to the dark energy dominated era. This alleviates the cosmic coincidence problem of the universe. Our results indicate that for this dark energy model the universe spends a significant fraction of its total lifetime in a state for which ρ_M and ρ_{DE} differ by less than an order of magnitude for all these singularities. These results clearly depend on the fact that the universe with these four singularities obtained from this dark energy EOS have a finite life time. Here the fractional life time f of the universe does not represent the universe being in a state which can support life but it represent the important time that the universe spends of its total life time when the matter and the dark energy densities are nearly comparable. By giving the restrictions on the parameters A and α obtained by comparison with the observational data one may select the destiny of the universe. The probability of living in a "coincidental state" for this dark energy model then could be determined.

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Table 3

Values of Ω_{m0} and $\Omega_{\Lambda 0}$ from various observational sources

Sources	Year	Ω_{m0}	$\Omega_{\Lambda 0}$
A. G. Riess et al., [9] (SNela + MLCS)	1998	$0.24^{+0.56}_{-0.24}$	$0.72^{+0.72}_{-0.48}$
A. Balbi et al., [2] (MAXIMA-1 + COBE)	2001	0.25-0.50	0.45-0.75
L. V. E. Koopman et al., [106] (lens + SD)	2003	0.30	0.70
B. J. Barris et al., [29] (If A deep Survey)	2004	0.33	0.67
D. J. Einstein et al. [41] (SDSS + LRG + BAO)	2005	0.27 ± 0.03	$0.73^{+0.03}_{-0.03}$
P. Astier et al. [134] (SNLS)	2006	0.31 ± 0.21	0.80 ± 0.31
A. G. Riess et al.; [10] (HST+SNLS+ ESSENCE)	2007	0.268	0.732
M. Kowalski et al., [119] (UNION + BAO + OHD)	2008	0.268	0.732
R. Kessler et al., [151] (MLCS + BAO +OHD)	2009	0.296	0.704
R. Amanullah et al., [142] (UNION2)	2010	0.272	0.728

Table 1

Some non-baryonic candidates for dark matter:

Particles approximate mass predicted by Astrophysical effect [83]

G(R)	---------	Non-Newtonian gravitation	Apparent DM on large scales
Cosmological constant Λ	----------	General relativity	$\Omega = 1$ without dark matter
Axion,majoron,Goldstone boson	$10^{-5}eV$	QCD;PQ-symmetry breaking	Cold DM
Normal neutrinos	10-100eV	GUTs	hot DM
Light Higgsino, photino,gravitino, axino,sneutrino	10-100eV	SUSY/SUGRA	hot DM
Para-photon	20-400eV	Modified QED	Hot, warm DM
Right-handed neutrinos	500eV	Super weak interaction	warm DM
Gravitino etc	500eV	SUSY/SUGRA	warm DM
Photino,gravitino,axino,mirror particles, Simpson neutrino	keV	SUSY/SUGRA	Warm/cold DM
Photino,sneutrino,Higgsino, gluino,heavy neutrino	MeV	SUSY/SUGRA	Cold DM
Shadow matter	MeV	SUSY/SUGRA	Hot/cold DM(baryon-like)
Preon	20-200TeV	Composite models	Cold DM
Monopole	$10^{16}GeV$	GUTs	Cold DM
Pyrgon,maximon,perry pole,Newtorite,Schwarzschild	$10^{19}Gev$	Higher dimensional theories	Cold DM
Super symmetric strings	$10^{19}Gev$	SUSY/SUGRA	Cold DM
Quark-nuggets ,nuclearites	$10^{15}g$	QCD,GUTs	Cold DM
Primordial black holes	$10^{15-30}g$	General relativity	Cold DM
Cosmic strings, domain walls	$10^{8-10}M_0$	GUTs	Support for galaxy formation, but without large contribution to Ω

Table 2

Some Phenomenological Λ decay laws

Decay law	References
1. $\Lambda \propto t^{-2}$	[4, 77, 114, 129, 183, 198]
2. $\Lambda \propto T^4$	[183]
3. $\Lambda \propto e^{-\beta a}$	[158]
4. $\frac{d\Lambda}{dt} \propto \Lambda^{\beta}$	[188]
5. $\Lambda \propto a^{-2}$	[77, 122, 123, 189]
6. $\Lambda \propto a^{-4(1+\epsilon)}$	[92, 107, 117]
7. $\Lambda \propto a^{-m}$	[46, 112, 180, 186, 187]
8. $\frac{d\Lambda}{dt} \propto aH^n\Lambda$	[120]
9. $\frac{d\Lambda}{dt} \propto H^3$	[120]
10. $\Lambda \propto C + \beta a^{-m}$	[78]
11. $\Lambda \propto t^{l-2} + \beta t^{2(l-1)}$	[42]
12. $\Lambda \propto \beta a^{-2} + H^2$	[13, 73]
13. $\Lambda \propto t^{-2} + \beta t^{\frac{-2}{l}}$	[4]
14. $\Lambda \propto C + e^{-\beta t}$	[4, 136]
15. $\Lambda \propto \beta a^{-m} + H^2$	[81]
16. $\Lambda \propto H^2$	[14, 38, 70]
17. $\Lambda \propto (1+\beta H)\left(H^2 + \frac{k}{a^2}\right)$	[72]
18. $\Lambda \propto t^{-1}(\beta + t)^{-1}$	[43]
19. $\frac{d\Lambda}{dt} \propto \beta\Lambda - \Lambda^2$	[86]
20. $\Lambda \propto a^{-2} + \beta a^{-4}$	[131]
21. $\Lambda \propto H^2 + \beta aH\left(\frac{H}{da}\right)$	[27]
22. $\Lambda \propto \rho$	[148]
23. $\Lambda \propto \frac{a}{a}$	[11, 12, 15]