REPORT ON W BOSON MODEL OF WEAK INTERACTIONS WITH MAXIMAL CP VIOLATION

R.E. Marshak, City College of New York

§1 Introduction

The Universal (V-A) current-current theory of weak interactions was formulated in 1957 - in the face of several contradictory experiments - and in the ensuing fifteen years, the predictions of this current-current model (with the addition of the Cabibbo angle) have been confirmed in an enormous number of leptonic, semi-leptonic and hadronic weak processes[1]. The first derivation of this current-current theory by Sudarshan and the author[2] was based on the principle of chirality invariance for spin 1/2 Dirac fields and this was soon followed by Feynman and Gell-Mann's derivation[3] on the basis of the non-derivative interaction of two-component Klein-Gordon spinor fields for spin 1/2 particles. Both derivations of the correct theory were carried out within the current-current framework and can only artificially be applied to the semi-weak W boson-current interaction which can be used to generate the (V-A) current-current theory in the limit of $m_W \to \infty$ (m_W is the W boson mass).

Unfortunately, the remarkably successful (V-A) current-current theory has one defect - it predicts CP conservation in all weak processes. While the CP violation effects associated with the decay of the K_L^o meson are small (of the order of 10^{-3}) and it is possible to introduce a phenomenological parameter into the (V-A) current-current theory to explain these effects, such an approach tells us very little about the origin of CP - violation in weak interactions.

There is a second major question which must be faced in connection with further refinements of the universal (V-A) current-current theory and that has to do with the very existence of the W boson. Experiment has already demonstrated[4] that if the W boson exists at all, its mass $m_W \geq 2$ Gev. This large mass explains why the experiments carried out until now (with $q^2 << m_W^2$ where q^2 is the four-momentum transfer squared) can not decide whether the current-current interaction is the basic interaction (whose field-theoretic content must still be delineated through a deeper study of higher order weak interactions[5] and a refined analysis of lowest order weak interactions for large q^2) or whether the current-current interaction is itself a second order effect resulting from the more fundamental semi-weak Yukawa-type interaction involving the W boson (or bosons). If we adopt the latter viewpoint and postulate the existence of a massive W boson (or bosons), we open up the very real possibility of developing a unified theory of CP-conserving and CP-violating weak processes on the basis of a single semi-weak W boson-current interaction. The strong cubic W boson model of weak interactions as developed by Okubo and the author[6], is the best example of such a theory and the one whose consequences have been most fully explored. The interest of this W boson model is further enhanced by the fact that it is the only extant theory which offers any hope of explaining the recent $K_L \to 2\mu$ puzzle[7] in a "natural" fashion.

In what follows we shall describe the essential features of the strong cubic W boson model (§ 2) and then show how this model can explain both the low rate for $K_L \to 2\mu$ decay and the high rate for $K_S \to 2\mu$ decay (§ 3). Finally, we shall summarize other experimental tests of the strong cubic W boson model (§ 4).

§ 2. Strong Cubic W Boson Model

The strong cubic W boson model of weak interactions arose out of the observation that the CP - violating parameter ϵ (which appears in the definition of $|K_L\rangle = |K_2\rangle + \epsilon\,|K_1\rangle$, with $|K_1\rangle$ and $|K_2\rangle$ the CP = +1 and CP = -1 combinations of $|K^o\rangle$ and $|\bar{K}^o\rangle$ respectively) is of the order[8] of the semi-weak coupling constant g. This line of argument leads to writing down a "pure" CP = -1 semi-weak W boson-current interaction which is capable of duplicating the results of the usual CP - conserving (V-A) current-current theory in order g^2 (in the limit $m_W \to \infty$) and the CP-violating effects (in K_L decay) in order g^3. This can be accomplished by postulating the existence of a triplet of W bosons[9] (with total charge 0) interacting strongly among themselves via a cubic interaction (hence the expression "strong cubic W boson model") and writing down a CP = -1 semi-weak interaction between this triplet of W bosons and suitable lepton and hadron currents.

More explicitly, we assume that the triplet of W bosons (W^o, W^-, W^+) are described by the following Lagrangian:

$$\mathcal{L}_o = -\tfrac{1}{2}\left[\partial_\nu \overline{W}_\mu^{(a)}(x) - \partial_\mu \overline{W}_\nu^{(a)}(x)\right]\left[\partial_\nu W_\mu^{(a)}(x) - \partial_\mu W_\nu^{(a)}(x)\right]$$

$$- m_o^2\, \overline{W}_\mu^{(a)}(x)\, W_\mu^{(a)}(x) - i f_o \epsilon_{abc}\left[W_\mu^{(a)}(x)\, W_\nu^{(b)}(x)\, W_\lambda^{(c)}(x) - \overline{W}_\mu^{(a)}(x)\, \overline{W}_\nu^{(b)}(x)\, \overline{W}_\lambda^{(c)}\right] \quad (1)$$

where a=1, 2, 3 (corresponding to charges 0, -1, +1), $W_\mu^{(a)}$ represents the vector W boson field, f_o is the strong coupling constant for the three W's and ϵ_{abc} is the usual antisymmetric tensor. Eq. (1) can be given a more convincing origin if we note that a quasi-Yang-Mills approach to the W Lagrangian suggests a new definition for the fields $F_{\mu\nu}^{(a)}(x)$ (note the bars over the W's in the second term):

$$F_{\mu\nu}^{(a)}(x) = \left[\partial_\mu W_\nu^{(a)}(x) - \partial_\nu W_\mu^{(a)}(x)\right] + i f_o \epsilon_{abc}\, \overline{W}_\mu^{(b)}(x)\, \overline{W}_\nu^{(c)}(x) \quad (2)$$

It is easy to show that if the $W_\mu^{(a)}$ fields transformaccording to the triplet representation of SU_3, the same will be true of the quasi-Yang-Mills fields $F_{\mu\nu}^{(a)}$. The W Lagrangian (1) can then be rewritten in terms of the $F_{\mu\nu}^{(a)}$ fields as[10]:

$$\mathcal{L}_o = -\tfrac{1}{2}\, \overline{F}_{\mu\nu}^{(a)}(x)\, F_{\mu\nu}^{(a)}(x) - m_o^2\, \overline{W}_\mu^{(a)}(x)\, W_\mu^{(a)}(x) \quad (3)$$

where $\mathcal{L}_o$ is manifestly invariant under SU_3.

Another important property of Eq. (1) [and Eq. (3)] is its invariance under the transformation:

$$W_\mu^{(a)}(x) \to \lambda_a W_\mu^{(a)}(x) \;, \quad \overline{W}_\mu^{(a)}(x) \to \lambda_a^* \overline{W}_\mu^{(a)}(x) \quad (4)$$

where λ_a is a complex constant satisfying the cubic equation:

$$\lambda_a^3 = 1 \quad (4a)$$

The parameters λ_a , λ_b , λ_c are the cube roots of unity and effectively assign different "cubic parities" to the three W fields. The concept of "cubic parity" is basic to the strong cubic W boson model.

If we further define the charge conjugation operation for the W field by:

$$C : \quad W_\mu^{(a)}(x) \rightarrow -\overline{W}_\mu^{(a)}(x) \tag{4b}$$

the simplest CP= -1 total semi-weak interaction which can be written down is (θ is the Cabibbo angle):

$$\begin{aligned} H_{S.W.} = ig \Big\{ \Big[& W_\mu^{(o)} \left(\alpha J_{\mu 3}^2 + \beta J_{\mu 3}^3 \right) \\ & + W_\mu^{(-)} \left(\gamma J_{\mu 2}^1 + \delta \ell_\mu \right) \\ & + W_\mu^{(+)} \left(\gamma' J_{\mu 1}^3 + \delta' \bar{\ell}_\mu \right) \Big] - h.c. \Big\} \end{aligned} \tag{5}$$

where α, β, γ, δ, γ' and δ' are real coefficients, $\ell_\mu = i\bar{e}\gamma_\mu(1+\gamma_5)\nu_e + i\bar{\mu}\gamma_\mu(1+\gamma_5)\nu_\mu$ is the total (V-A) charged lepton current and $J_{\mu j}^i$ (i, j = 1, 2, 3) is the octet hadron current (in tensor notation). Note the coefficient i and the subtraction of the hermitian conjugate in Eq. (5); these features account for the CP= -1 property of the semi-weak interaction since C and P are defined by virtue of the strong cubic self-interaction of the W's. The interaction (5) also possesses the property that it is the most general semi-weak interaction which forbids $\Delta Y=2$ (Y is the hypercharge) transitions[11] to order g^2 and g^3.

It is now possible to show that the first-order effects[12] in g are forbidden by the invariance of Eq. (5) under the "cubic parity" transformation (4); this forbiddenness extends to any process which is first-order in g and of arbitrary order in e (electric charge) and therefore excludes the occurrence of an electric dipole moment of the neutron in this order[13]. The first non-vanishing effects in the strong cubic W boson model occur in order g^2 since a term like $\langle W_\mu(x) \overline{W}_\nu(x) \rangle_o$ is consistent with cubic parity conservation. In this way, one can derive an effective CP= +1 current-current interaction in order g^2 (in the limit $m_W \rightarrow \infty$) which can explain the whole range of CP-conserving leptonic, semi-leptonic and hadronic weak processes. Indeed, one can fix the coefficients in Eq. (5) by the requirement that they reproduce in order g^2 the results of the universal (V-A) current-current weak interaction:

$$H_W = \frac{G}{\sqrt{2}} \bar{\mathcal{J}}_\mu(x) \mathcal{J}_\mu(x) \quad (6)$$

where

$$\mathcal{J}_\mu(x) = \cos\theta\, J^1_{\mu 2}(x) + \sin\theta\, J^1_{\mu 3}(x) + \ell_\mu(x) \quad (6a)$$

is the Cabibbo current and $G/\sqrt{2} = g^2/m_W^2$. Eq. (6) yields a contribution of order g^2 to purely leptonic processes and $g^2 \cdot \cos^2\theta$ and $g^2 \sin^2\theta$ contributions to the $\Delta Y=0$ and $\Delta Y=1$ semi-leptonic weak processes respectively. These features are readily recaptured by the iteration of Eq. (5) through the unique choice:

$$\delta = \delta' = \tfrac{1}{2} \;, \quad \gamma = \cos\theta \;, \quad \gamma' = \sin\theta \quad (7)$$

The choice of (α and β) in Eq. (5) can not so easily be determined by comparing the iteration of Eq. (5) with the $g^2 \cos\theta \sin\theta$ contribution of Eq. (6) to the $\Delta Y=1$ weak hadron processes since we are asked to equate:

$$\alpha\beta \langle f | J^2_{\mu 3} J^3_{\mu 3} | i \rangle = \cos\theta \sin\theta \langle f | J^2_{\mu 1} J^1_{\mu 3} | i \rangle \quad (8)$$

Eq. (8) does not yield a simple determination of the coefficients α and β since its L.H.S. (originating from the W boson model) involves neutral hadron currents exclusively[14] and its R.H.S. (originating from the current-current theory) involves only charged hadron currents; consequently, at the present stage of the strong cubic W boson theory, the choice of α and β is dictated by experiment (we shall find below that $\alpha \sim \beta \sim 1$). The structure of the L.H.S. of Eq. (8) has the very desirable consequence that the $\Delta Y=1$ weak hadron processes automatically obey the $\Delta I=\frac{1}{2}$ rule (I is the isospin) in the W boson theory - in contrast to the artificial suppression of the $\Delta I=3/2$ contribution (through octet enhancement or some other mechanism) required in the current-current theory.

The CP = -1 semiweak W boson interaction which is consistent with the

CP = +1 weak current-current interaction thus becomes:

$$H_{S.W.} = \frac{ig}{\sqrt{2}} \left\{ \left[W_\mu^{(o)} \left(\alpha J_{\mu 3}^{2} + \beta J_{\mu 3}^{3} \right) \right.\right.$$
$$+ W_\mu^{(-)} \left(\sqrt{2} \cos\theta J_{\mu 2}^{1} + \ell_\mu / \sqrt{2} \right) \tag{9}$$
$$\left.\left. + W_\mu^{(+)} \left(\sqrt{2} \sin\theta J_{\mu 1}^{3} + \bar{\ell}_\mu / \sqrt{2} \right) - h.c. \right\}$$

Eq. (9) will be used to compute the matrix elements following from the W boson model. However, it is illuminating to recast Eq. (9) into the form:

$$H_{S.W.} = ig' \left\{ \left[W_\mu^{(o)} \left(\alpha' J_{\mu 3}^{2} + \beta' J_{\mu 3}^{3} \right) \right.\right.$$
$$+ \frac{\left(W_\mu^{(-)} - W_\mu^{(+)} \right)}{\sqrt{2}} \left(\cos\theta J_{\mu 2}^{1} + \sin\theta J_{\mu 3}^{1} + \ell_\mu \right) \tag{10}$$
$$\left.\left. + \frac{\left(W_\mu^{(-)} + \overline{W}_\mu^{(+)} \right)}{\sqrt{2}} \left(\cos\theta J_{\mu 2}^{1} - \sin\theta J_{\mu 3}^{1} \right) \right] - h.c. \right\}$$

where $g' = g/\sqrt{2}$, $\alpha' = \sqrt{2}\,\alpha$, $\beta' = \sqrt{2}\,\beta$. The second term now contains the interaction of the usual Cabibbo current (consisting of charged hadron and lepton currents) with the normalized combination $\frac{\left(W_\mu^{(-)} - \overline{W}_\mu^{(+)} \right)}{\sqrt{2}}$

In the third term, the orthogonal combination of the $W_\mu^{(-)}$ and $\overline{W}_\mu^{(+)}$ fields interacts with a purely charged hadron current while in the first term the neutral vector field $W_\mu^{(o)}$ interacts with a purely neutral hadron current. It is worth remarking that the combinations of $W_\mu^{(-)}$ and $\overline{W}_\mu^{(+)}$ which enter in Eq. (10) are precisely the ones that interact with the electromagnetic field when one adds this field to the quasi-Yang-Mills Lagrangian (3). This places the semi-weak interaction of the W boson model on an attractive theoretical foundation.

The basic difference between the current-current interaction model (6) and the strong cubic W boson model first arises in order g^3 (in the W boson model) where one encounters a term of the type $\langle W_\mu(x)\, W_\nu(x)\, W_\lambda(x) \rangle_o$ which conserves "cubic parity" and is large because of the strong cubic self-coupling of the W boson triplet. Thus, the strong cubic W boson model allows certain weak processes to occur in order g^3 which can only occur in the current-current theory in order $g^4 \sim \left(Gm_W^2\right)^2$. Moreover, the weak processes occurring in order g^3 receive CP = -1 contributions in this order and can exhibit CP-violation effects under suitable circumstances. When the CP = -1 g^3 amplitude interferes with the CP = +1 g^2 amplitude, the CP-violating effect will be of the order $g \sim 10^{-3}$ whereas if it interferes with a CP = +1 ⟵ $g^2 e^n$ amplitude (a combined weak-electromagnetic amplitude), the CP-violating effect can be much larger (gross CP violation[15]). The CP-violating effect in $K_L \to 2\pi$ decay is an example of the former type of interference effect (and was the reason for proposing the strong cubic W boson model in the first place) while $K_L \to 2\mu$ decay would be an example of the latter type of interference effect. We conclude this section with a sketch of the calculation for $K_L \to 2\pi$ decay - to indicate the nature of the approximations invoked - and in the next section apply the strong cubic W boson model to the $K_L \to 2\mu$ problem.

The diagram contributing to $K_L \to 2\pi^0$ decay is given in Fig. 1 (a similar diagram can be drawn for $K_L \to \pi^+\pi^-$) and the matrix element following from Eq. (9) is:

$$M = \frac{g^3 \cos\theta \sin\theta\,\beta}{\sqrt{8p_o k_{10} k_{20}}} \int d^4q\, \Delta^{W(+)}_{\mu\alpha'}(p-q)\, \Delta^{W(0)}_{\nu\beta'}(k_2)\, \Delta^{W(-)}_{\lambda\gamma'}(p-q-k_2) \tag{11}$$

$$\times\, \Gamma_{\alpha'\beta'\gamma'}(p,q,k_2)\, \Delta^{\pi}(q)\, \langle K_2(p)|V^3_{\mu 1}|\pi^-(q)\rangle \langle\pi^-(q)|V^1_{\lambda 2}|\pi^0(k_1)\rangle \langle\pi^0(k_2)|A^3_{\nu 3}|0\rangle$$

where $\Delta^W_{\mu\nu}$ is the W boson propagator, $\Gamma_{\alpha'\beta'\gamma'}$ is the triple W vertex and $V^i_{\mu j}$ and $A^i_{\mu j}$ are the vector and axial vector hadron currents with suitable tensor indices respectively. From symmetry considerations:

$$\Gamma_{\alpha'\beta'\gamma'} = f(q^2, q\cdot p, q\cdot k_2)\left[\delta_{\alpha'\beta'}(p-q+k_2)_{\gamma'} - \delta_{\alpha'\gamma'}(2p-2q-k_2)_{\beta'} + \delta_{\beta'\gamma'}(p-q)_{\alpha'}\right] \tag{12}$$

We assume that $f(q^2, q\cdot p, q\cdot k_2) \simeq f_o$ and, retaining the most divergent contributions, we get

$$M \simeq -\frac{3\Lambda^4}{256\pi^2}\,\frac{g^3}{m_W^6}\,\cos\theta \sin\theta\,\beta\, f_o f_\pi m_K^2 \tag{13}$$

where f_π is the pion decay amplitude. Hence:

$$| M (K_L \rightarrow 2\pi) / M (K_S \rightarrow 2\pi) | \simeq 10^{-3} f_o \left(\frac{\Lambda}{m_W} \right)^4 g\beta \tag{14}$$

Another relation between f_o and Λ is derived by calculating the self-mass of the W boson with cubic interaction; using the same approximations for the triple W vertex, one gets:

$$\delta m_W \simeq \frac{3}{4\sqrt{2}\pi} \frac{f_o \Lambda^2}{m_W} \simeq m_W \tag{15}$$

or

$$f_o \Lambda^2 / m_W^2 \simeq 2\pi \tag{16}$$

These approximate calculations show that the correct order of magnitude of the CP-violating amplitude for $K_L \rightarrow 2\pi$ decay can be obtained from the strong cubic W boson model for a reasonable choice of parameters: $g \sim 3 \times 10^{-2}$ (corresponding to $m_W \simeq 10$ Gev), $\Lambda \simeq 2 m_W$ and $\beta \simeq 1$.

§ 3. Application of the Strong Cubic W Boson Model to $K_L \rightarrow 2\mu$ Puzzle

The strong cubic W boson model was not invented to explain the recent $K_L \rightarrow 2\mu$ puzzle[7]. It was put forward as the simplest W boson model capable of providing a unified description of both CP-conserving and CP-violating weak processes. However, it turns out that the same feature of the model which predicts the existence of the CP-violating $K_L \rightarrow 2\pi$ decay in order g^3 also predicts the existence of effective neutral lepton currents in the same order and this, when combined with the symmetry properties of the model, enables us to understand the low rate for $K_L \rightarrow 2\mu$ decay and a much higher rate for $K_S \rightarrow 2\mu$ decay. This interesting prediction of the strong cubic W boson model is now examined in some detail.

Let us write in the usual fashion:

$$|K_L\rangle = |K_2\rangle + \epsilon |K_1\rangle \tag{17a}$$

$$|K_S\rangle = |K_1\rangle + \epsilon |K_2\rangle \tag{17b}$$

where $\epsilon \simeq \bar{\epsilon}_o e^{i\pi/4}$ ($\bar{\epsilon}_o = 2 \times 10^{-3}$) and terms of higher order in ϵ have been dropped. It is easy to show that in the "local" approximation[16] to the g^3 matrix element, the CP = -1 $|K_2\rangle$ state does not decay into 2μ whereas the CP = +1 $|K_1\rangle$ state does. Indeed, the effective CP = -1 g^3 interaction in the local limit takes the CP = +1 $|K_1\rangle$ state into the CP = -1 (^1S_o) state of the 2μ system with the amplitude:

$$M(K_1 \to 2\mu) = i\bar{u}\, b\, \gamma_5\, v \tag{18}$$

where u and v are the Dirac spinors for the muons and b is computed from the diagram in Fig. 2. Using the same approximations made in computing $K_L \to 2\pi$ decay (see § 2), one obtains:

$$b = 2\sqrt{2}\, m_\mu\, G\, Z\, \sin\theta\, a\, f_K \tag{19}$$

where f_+ is the K^+ decay amplitude and

$$Z = -\frac{3}{16\pi^2} f_o \left(\frac{\Lambda}{m_W}\right)^2 g\cos\theta \simeq -\frac{3}{8\pi} g\cos\theta \tag{19a}$$

[using relation (16)]. The rate becomes:

$$\Gamma(K_1 \to 2\mu) = (G \sin\theta\, a\, |z|\, f_K\, m_\mu)^2 (m_K{}^2 - 4m_\mu{}^2)^{1/2} \tag{20}$$

$$= 12\, |z|^2\, \Gamma_L \qquad (a \sim 1) \tag{21}$$

where we have expressed the decay rate in Eq. (21) in terms of $\Gamma_L = \Gamma(K_L \to \text{all})$. It is important to note that the amplitude (18) for $K_1 \to 2\mu$ is pure imaginary since the diagram in Fig. 2 gives a real contribution and the i comes from the fact that the CP = -1 part of the interaction is responsible for this contribution. It is this feature which enables (18) to partially cancel the CP = +1 absorptive contribution to the amplitude for $K_2 \to 2\mu$ given by the weak-electromagnetic diagram in Fig. 3.

Let us be more explicit: we may write the total amplitudes for $K_L \to 2\mu$ and $K_S \to 2\mu$ as follows:

$$M\left(K_{L,S} \to 2\mu_\pm\right) \equiv \langle 2\mu_\pm|T|K_{L,S}\rangle = \langle 2\mu_\pm|T|K_{2,1}\rangle + \epsilon\,\langle 2\mu_\pm|T|K_{1,2}\rangle \tag{22}$$

where the subscripts ± on μ denote the CP = ± 1 final states of the $\mu\bar{\mu}$ system respectively. Separating the real and imaginary parts of the amplitudes, we write:

$$a_\pm = \mathcal{I}m \langle 2\mu_\pm |T_\mp| K_2\rangle$$
$$b_\pm = \mathcal{I}m \langle 2\mu_\pm |T_\pm| K_1\rangle$$
$$c_\pm = \mathrm{Re} \langle 2\mu_\pm |T_\pm| K_1\rangle \qquad (23)$$
$$d_\pm = \mathrm{Re} \langle 2\mu_\pm |T_\mp| K_2\rangle$$

where the subscripts $\pm$ on T denote the CP= ± 1 character of the effective interaction respectively. Using the definitions (23), we get:

$$\mathcal{I}m \langle 2\mu_\pm |T| K_L\rangle = a_\pm + \epsilon_0 b_\pm$$
$$\mathrm{Re} \langle 2\mu_\pm |T| K_L\rangle = d_\pm - \epsilon_1 b_\pm \qquad (24)$$

where ϵ_0 = Re (ϵ), ϵ_1 = Im (ϵ). The corresponding relations for the $K_S \to 2\mu$ amplitudes are:

$$\mathcal{I}m \langle 2\mu_\pm |T| K_S\rangle = b_\pm + \epsilon_0 a_\pm$$
$$\mathrm{Re} \langle 2\mu_\pm |T| K_S\rangle = c_\pm - \epsilon_1 a_\pm \qquad (25)$$

We repeat that $2\mu_+$ are in a $^3P_0$ state and $2\mu_-$ in a $^1S_0$ state.

Four of the quantities in Eq. (23), b_-, c_-, a_+, d_+, involve the CP=-1 part of the interaction and must be estimated on the basis of the g^3 diagram in Fig. 2: b_- is precisely the amplitude b defined by Eq. (18); $c_- \simeq 0$ since m_W is large; $a_+ \simeq 0$ in the local limit; $d_+ \simeq 0$ for the same reason as c_-. The other four quantities, a_-, d_-, b_+, c_+ involve the CP= +1 part of the interaction and must be estimated on the basis of the $g^2 e^4$ diagram in Fig. 3: a_- is the quantity computed by Sehgal[17]; d_- is the dispersive part of the $g^2 e^4$ contribution and is not expected to exceed the absorptive part a_- (although a good calculation has not yet been carried out as yet); b_+ is estimated to be the same order as a_- and the same can be said of c_+. Inserting our results into Eqs. (24) and (25), we get (all amplitudes are given in units of $\sqrt{\Gamma_L}$):

$$\begin{cases} \mathcal{I}m \ \langle 2\mu_+ | T | K_L \rangle = 7 \times 10^{-5} \, \epsilon_0 \\ Re \ \langle 2\mu_+ | T | K_L \rangle = - D_L \, \epsilon_1 \end{cases} \tag{26}$$

$$\begin{cases} \mathcal{I}m \ \langle 2\mu_- | T | K_L \rangle = 7 \times 10^{-5} - 3.5 |z| \, \epsilon_0 \\ Re \ \langle 2\mu_- | T | K_L \rangle = D_L + 3.5 |z| \, \epsilon_1 \end{cases} \tag{27}$$

$$\begin{cases} \mathcal{I}m \ \langle 2\mu_+ | T | K_S \rangle = 1 \times 10^{-4} \\ Re \ \langle 2\mu_+ | T | K_S \rangle = D_S \end{cases} \tag{28}$$

$$\begin{cases} \mathcal{I}m \ \langle 2\mu_- | T | K_S \rangle = - 3.5 |z| + 1 \times 10^{-4} \, \epsilon_0 \\ Re \ \langle 2\mu_- | T | K_S \rangle = - D_S \, \epsilon_0 \end{cases} \tag{29}$$

where D_L and D_S are the real parts of the weak-electromagnetic amplitudes for K_L and K_S decay into 2μ respectively[18] (via the two-photon mechanism of Fig. 3).

From Eqs. (26) - (29), we obtain the estimated partial transition rates for the 2μ decays of K_L and K_S into the CP = +1 ($^3P_0$) and CP = -1 ($^1S_0$) states:

$$\Gamma (K_L \to 2\mu_+) \simeq 5 \times 10^{-9} \cdot 2 \times 10^{-6} \cdot \Gamma_L \tag{30}$$

$$\Gamma (K_L \to 2\mu_-) \simeq \left(7 \times 10^{-5} - 3.5 \cdot 1.4 \times 10^{-3} |z|\right)^2 \Gamma_L \tag{31}$$

$$\Gamma (K_S \to 2\mu_+) \simeq 10^{-8} \cdot \Gamma_L \tag{32}$$

$$\Gamma (K_S \to 2\mu_-) \simeq 12 \, |z|^2 \, \Gamma_L \tag{33}$$

and hence for the total rates:

$$\Gamma (K_L \to 2\mu) \simeq \left(7 \times 10^{-5} - 3.5 \cdot 1.4 \times 10^{-3} \, |z|\right)^2 \Gamma_L \tag{34}$$

$$\Gamma (K_S \to 2\mu) \simeq 12 \, |z|^2 \, \Gamma_L \tag{35}$$

It is clear from Eqs. (34) and (35) that the $K_L \to 2\mu$ puzzle can readily be resolved by a suitable choice of the parameter $|z|$ (and therefore of m_W). Thus, we can match (34) to the upper limit 1.8×10^{-9} for the branching ratio

B.R. $(K_L \rightarrow 2\mu)$ by choosing $|z| \simeq 5.5 \times 10^{-3}$ (and a fortiori $m_W \approx 15$ Gev). Inserting this value into (35) yields $\Gamma(K_S \rightarrow 2\mu)/\Gamma_S \simeq 0.6 \times 10^{-6}$ which is fairly close to Steinberger's upper limit[19] 0.8×10^{-6}. It should be emphasized that the predicted B.R. $(K_S \rightarrow 2\mu)$ is very sensitive to the observed B.R. $(K_L \rightarrow 2\mu)$; thus, if B.R. $(K_L \rightarrow 2\mu)$ turned out to be 3×10^{-9} (a small change in such a difficult experiment), we would get $|z| \simeq 3 \times 10^{-3}$ ($m_W \simeq 10$ Gev) and the predicted $\Gamma(K_S \rightarrow 2\mu)/\Gamma_S$ would be 0.2×10^{-6}. It should also be pointed out that if only the $g^2 e^4$ diagram contributes to $K_S \rightarrow 2\mu$, the predicted $\Gamma(K_S \rightarrow 2\mu)/\Gamma_S$ would be $\sim 2 \times 10^{-10}$, an extremely small value; any evidence for a substantially larger branching ratio would require a modification of the (V-A) current-current theory.

§4. Further Predictions of the Strong Cubic W Boson Model

We have seen that one consequence of the strong cubic W boson model is the automatic prediction of an effective $(\mu\bar{\mu})$ current interaction with the K_1 meson in order g^3 which interferes destructively with the weak-electromagnetic current interaction in order $g^2 e^4$ for the K_L meson but not for the K_S meson. This is a negative sort of triumph and the true test of this model will lie with positive confirmation of a variety of predictions for weak processes involving the effective interaction of neutral lepton currents with hadrons in order g^3 and the competition, where appropriate, with weak-electromagnetic transitions to the same final states in order $g^2 e^2$ or $g^2 e^4$. Many of the relevant calculations in this regard have been given in previous papers[6] and only the more interesting results will be mentioned here.

A more perspicuous way to deduce the consequences of the strong cubic W boson model for neutral lepton pair effects in semi-leptonic processes is to derive the local limit (together with first-order corrections[20]) for the effective neutral lepton current-hadron current interaction. One derives the following effective interaction:

$$- \frac{iG\sin\theta}{\sqrt{2}} |z| \left\{ J^3_{\lambda 2} - J^2_{\lambda 3} \right\} \cdot \left\{ \left[i\bar{\nu}_e \gamma_\lambda (1+\gamma_5)\nu_e + i\bar{e}\gamma_\lambda(1+\gamma_5)e + i\bar{\nu}_\mu \gamma_\lambda (1+\gamma_5)\nu_\mu + i\bar{\mu}\gamma_\lambda(1+\gamma_5)\mu \right] - 2\rho' \left[m_e i\partial_\lambda (\bar{e}\gamma_5 e) + m_\mu i\partial_\lambda (\bar{\mu}\gamma_5 \mu) \right] \right\} \quad (36)$$

The first term in brackets in (36) is the strict local interaction whereas the second term is of order $\frac{q}{m_W}$ (q is the four-momentum transfer); terms of order q^2/m_W^2 (and m_ℓ^2/m_W^2) have been neglected. The current $\left[J^3_{\lambda 2}(x) - J^2_{\lambda 3}(x) \right] \equiv 2i\, J^7_\lambda(x)$ (in the octet notation) is a $\Delta Y=1$, $CP=-1$ neutral hadronic current which can be written in the form:

$$J^7_\lambda(x) = A^7_\lambda + \delta V^7_\lambda \quad (37)$$

The parameters $|z|$, ρ (we set $\rho' m_K^2 = \rho - 1$) and δ can, in principle, be determined from three experiments and the results then used to make further predictions. However, in view of the unreliability of the experimental data (and the fact that only upper limits are actually known for the relevant transition rates), we shall make the reasonable assumption that $\delta \sim 1$ and see whether the estimated values of $|z|$ and ρ are consistent with the present data.

We can determine $|z|$ from B.R. ($K_S \to 2\mu$) if we choose that value, $|z| \sim 4 \times 10^{-3}$, which most plausibly reconciles B.R. ($K_L \to 2\mu$) and B.R. ($K_S \to 2\mu$) at the present time. A value of ρ can then be found from the predicted ratio for B.R. ($K_S \to 2\mu$) to B.R. ($K^+ \to \pi^+ \nu\bar{\nu}$), namely:

$$\frac{\text{B.R.}(K_S \to 2\mu)}{\text{B.R.}(K^+ \to \pi^+ \nu\bar{\nu})} = 0.23\,\rho^2 \quad (38)$$

The experimental upper limit[21] for B.R. ($K^+ \to \pi^+ \nu\bar{\nu}$) is 1.2×10^{-6} and if we insert this value into Eq. (38), we obtain $\rho^2 \sim 1.5$. The branching ratio $\Gamma(K^+ \to \pi^+ \nu\bar{\nu}) / \Gamma(K^+ \to \pi^0 e^+ \nu_e)$ itself gives an independent determination of $|z|$ through the relation:

$$\frac{\Gamma(K^+ \to \pi^+ \nu\bar{\nu})}{\Gamma(K^+ \to \pi^0 e^+ \nu_e)} = 2\,|z|^2 \quad (39)$$

whence $|z| \lesssim 3.5 \times 10^{-3}$ which is of the right magnitude.

We now list some predictions for several other interesting K meson decays involving the emission of a neutral lepton pair. Consider the decay $K_L \to \pi^o \ell \bar{\ell}$ which proceeds in order g^3 without violating any symmetries of the basic interaction. This implies that the CP = +1 $g^2 e^4$ weak-electromagnetic contribution to this decay may be neglected with respect to the CP = -1 g^3 contribution. The rate for $K_L \to \pi^o \mu \bar{\mu}$ decay involves ρ^2 whereas we can neglect this term for $K_L \to \pi^o e \bar{e}$ decay; we predict (using $|z| \sim 3 \times 10^{-3}$):

$$B.R.\ (K_L \to \pi^o e \bar{e}) = 0.41\ |z|^2 \simeq 3 \times 10^{-6} \tag{40}$$

$$\frac{\Gamma(K_L \to \pi^o \mu \bar{\mu})}{\Gamma(K_L \to \pi^o e \bar{e})} = \frac{1+\rho^2}{2} \left\{ 0.51 + \frac{\rho^2 - 1}{\rho^2 + 1} (0.17) + 1.5 \lambda_+ - \frac{2\rho^2}{\rho^2+1} \left(0.16 - 0.13\,\xi - 0.03\,\xi^2 \right) \right\} \tag{41}$$

which, for $\lambda_+ = 0.03$, $\xi = -0.6$ and $\rho^2 \approx 1.5$ yields the value[22] 0.40 so that the predicted $B.R.(K_L \to \pi^o \mu \bar{\mu}) \simeq 1.2 \times 10^{-6}$. It would be interesting to have measurements of these rare decay modes of K_L.

The decay process $K^+ \to \pi^+ \ell \bar{\ell}$ is more complicated because now the CP = +1 weak-electromagnetic amplitude involves one photon (and is therefore of order $g^2 e^2$) and is comparable with the CP = -1 g^3 amplitude. Estimates of the weak-electromagnetic contributions to the branching ratios have been made with the results:

$$B.R.\ (K^+ \to \pi^+ \mu \bar{\mu}) = 3.5 \times 10^{-7} \tag{42}$$

$(g^2 e^2)$:

$$B.R.\ (K^+ \to \pi^+ e \bar{e}) = 8.5 \times 10^{-7} \tag{43}$$

The g^3 contributions to $K^+ \to \pi^+ \ell \bar{\ell}$ are identical with the corresponding contributions to $K^+ \to \pi^+ \nu \bar{\nu}$ so that we have [cf. Eq. (39)]

$$B.R.\ (K^+ \to \pi^+ \mu \bar{\mu}) \simeq 5 \times 10^{-7} \tag{44}$$

(g^3):

$$B.R.\ (K^+ \to \pi^+ e \bar{e}) \simeq 12 \times 10^{-7} \tag{45}$$

If we recall that Eqs. (42) and (43) were obtained from absorptive amplitudes and that Eqs. (44) and (45) were derived from real amplitudes multiplied by i (to represent the CP= -1 nature of the W boson interaction), it follows that destructive interference may occur between the corresponding amplitudes and thereby reduce the actual branching rations. This may account for the measured lower upper bound for B.R. $(K^+ \to \pi^+ e\bar{e}) \lesssim 0.4 \times 10^{-6}$ compared to B.R. $(K^+ \to \pi^+ \nu\bar{\nu}) \lesssim 1.2 \times 10^{-6}$. If the above estimates are at all reasonable, the strong cubic W boson model would also predict gross CP violation effects for these rare decay modes of K^+. The magnitude of these effects and their modes of detection have already been discussed in considerable detail.(15) In the same paper will be found a discussion of gross CP violation effects in the related rare decay mode: $\Sigma^+ \to pe\bar{e}$ as well as in the W boson production reaction itself, namely, $\nu_\mu + N \to \mu^- + W + N$. In the latter case, the detection of an appreciable transverse polarization of the muon from the decaying W boson would provide evidence for gross CP violation.

We conclude our status report on the strong cubic W boson model by remarking that a search for neutral lepton pairs from decaying bosons or baryons at the g^3 level is of great interest independent of our model. CP violation is, so to speak, a large effect at this (g^3) level and it is difficult to believe that this departure from the "classical" (V, A) current-current interaction theory will not reflect itself in the occurrence of other unexpected phonomena at the same (g^3) level.

Acknowledgements:

I wish to thank Prof. K. Nishijima for emphasizing the possible importance of the approximate equality of the CP-violating parameter and the semi-weak coupling constant. Many valuable discussions with Prof. S. Okubo concerning the incorporation of this idea into the W boson model led to the triplet W boson model which is highlighted in this report. This report includes the results of more recent calculations (with special emphasis on the $K_L \to 2\mu$ puzzle) in which I was assisted by Prof. N.P. Chang and Dr. T. Ma, of The City College of New York. Conversations with Prof. B. Sakita are also gratefully acknowledged.

References

1. For a full account see R.E. Marshak, Riazuddin and C.P. Ryan, "Theory of Weak Interactions in Particle Physics," John Wiley (1969).

2. E.C.G. Sudarshan and R.E. Marshak, Proc. Padua-Venice Conf. on Mesons and Recently Discovered Particles (1957) and Phys. Rev. 109, 1860 (1958); reprinted in P.K. Kabir, Development of Weak Interaction Theory, pp. 118-128, Gordon and Breach (1963).

3. R.P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958).

4. Cf. M.M. Block et al, Phys. Letters 12, 281 (1966) and G. Bernardini et al, Phys. Letters 13, 86 (1966).

5. Cf. B.L. Ioffe and E.P. Shabalin, Soviet J. Nucl. Phys. 6, 603 (1968) and R.N. Mohapatra, J.S. Rao and R.E. Marshak, Phys. Rev. 171, 1502 (1968).

6. S. Okubo, Nuovo Cim. 54A, 491 (1968); Ann. Phys. (N.Y.) 49, 219 (1968); R. Marshak, R. Mohapatra, S. Okubo and J. Rao, Nucl. Phys. B11, 253 (1969); R.E. Marshak, Y.W. Yang and J.S. Rao, Phys. Rev. D3, 1640 (1971).

7. A.R. Clark et al, Phys. Rev. Letters 26, 1667 (1971).

8. K. Nishijima and L.J. Swank Phys. Rev. 146, 1161 (1966) wrote down a CP = -1 semi-weak hadronic interaction in the form $f\dot{\partial}_\lambda N_\lambda$ ($\dot{N}_\lambda$ is a CP = +1 neutral hadron current) where $f \sim g$. This term is in addition to the CP=+1 (V-A) current-current interaction.

9. One need not work with a triplet of W bosons - one can develop an equivalent strong cubic W boson model with an octet of W bosons as Okubo (ref. 6) first showed. I prefer the triplet formulation since it is simpler and requires the minimum number of new quantum numbers.

10. Eq. (3) contains an additional quartic interaction among the W's which prevents an infinite lower bound to the energy implied by a cubic interaction alone. Since the quartic interaction contributes to the W boson self-energy, it will

change the estimate of the coupling constant f_o; however, the qualitative conclusions will remain unchanged.

11. A $\Delta Y=2$ transition in order g^3 is forbidden by the small value of the (K_L-K_S) mass difference (which is of order g^4).

12. The typical term which appears is $\langle W_\mu(x)\rangle_o$ (where the expectation value is understood with respect to the vacuum state of the W bosons defined by the W. Lagrangian) and this must vanish by "cubic parity" conservation.

13. Cf. P.D. Miller et al., Phys. Rev. Letters 19, 381 (1967); C.G. Shull and R. Nathans, Phys. Rev. Letters 19, 391 (1967).

14. This follows from the fact that $W^{(-)}$ and $\overline{W^{(+)}}$ are not antiparticles of each other - since they possess different "cubic parities" - and so there is no $J_1^2 J^1_3$ contribution to the hadronic weak processes.

15. See R.E. Marshak, Y.W. Yang and J.S. Rao, Phys. Rev. D3, 1640 (1971).

16. If SU_3 symmetry holds for the W bosons or even if $m(W^{(-)}) = m(W^{(+)})$ with $m(W^{(o)})$ being different, the g^3 amplitude for $K_2 \to 2\mu$ will vanish identically; neither condition is necessary in the "local" approximation to the matrix element.

17. Cf. L.M. Sehgal, Phys. Rev. 183, 1511 (1969).

18. The dispersive parts of the weak-electromagnetic contribution are difficult to compute although they are estimated to be less than the absorptive parts -Cf. Ref. 17 and G.V. Dass and L. Wolfenstein, Phys. Letters 38B, 435 (1972).

19. J. Steinberger (private communication).

20. S. Okubo and M. Bace (to be published in Nuclear Phys.) have calculated the local limit; N.P. Chang and T. Ma (private communication) have also computed the first-order correction.

21. J.H. Klems, R.H. Hildebrand and R. Stiening, Phys. Rev. D4 66 (1971).

22. This is to be compared with the value of 0.32 obtained by Okubo and Bace (ref. 20). Independent computations of other neutral lepton processes are also carried out in this paper.

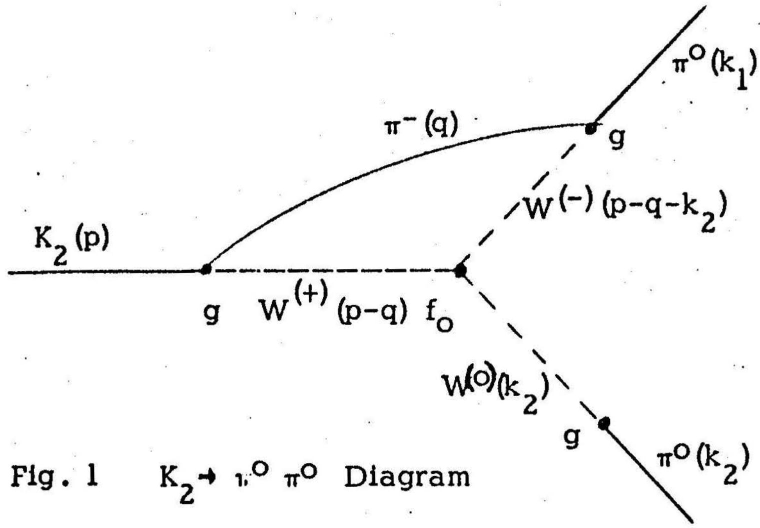

Fig. 1 $K_2 \rightarrow \pi^o \pi^o$ Diagram

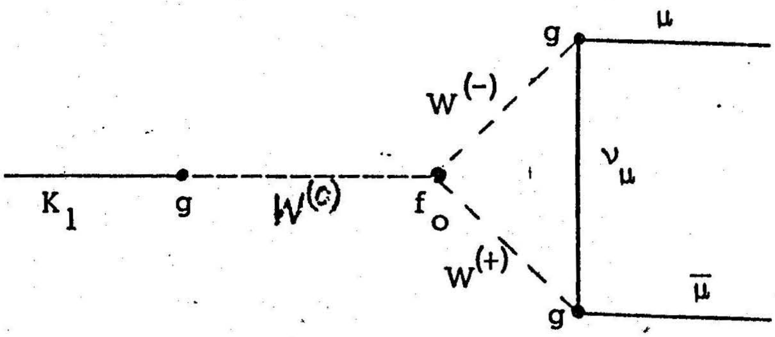

Fig. 2. $K_1 \rightarrow 2\mu$ Diagram (g^3)

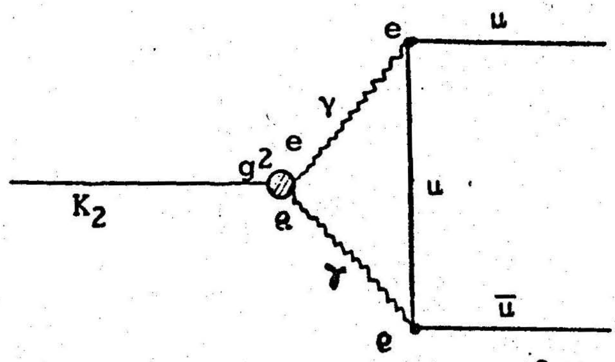

Fig. 3 $K_2 \rightarrow 2\mu$ Diagram $(g^2 e^4)$