ISBN: 978-3-95450-211-0 doi:10.18429/JACoW-SRF2019-THP104

NUMERICAL ESTIMATION OF BEAM BREAK-UP INSTABILITY IN TESLA CAVITIES*

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Abstract

In this article the numerically estimated BBU instability behaviors of a 9 cell superconducting TESLA cavity are presented for first two pass-band trapped dipole modes (18 in all). The given BBU threshold current values are calculated by the method of beam energy gain averaging on phases of dipole mode fields. BBU instability behaviors in cases of applying the cavities in Linacs as well in Energy Recovery Linacs (ERLs) are considered. The BBU influence on beam emittance degradation is demonstrated. Examples for suppression of beam BBU oscillations by a solenoid focusing and applying of an external RF generator with a feedback are visualized.

INTRODUCTION

The method of BBU instability threshold current calculations by averaging of beam energy gain or loss on all RF phases of dipole mode fields is described in [1]. The energy gain or loss is calculated with help of ASTRA cod [2] by tracking particles along the axis of the cavity enclosing dipole mode field. Due to RF transversal oscillations appeared the particles hits to the longitudinal electric dipole RF field near the axis and so gets some energy gain or loss. The beam current I_Q at which BBU instability begins with unit value of the dipole mode quality factor ($Q=1$) is named "Threshold current" [1]:

$$I_Q = I \cdot Q = -\frac{\omega \cdot U}{E_{BBU}} \qquad (1)$$

Here ω is angular frequency of the dipole mode, E_{BBU} is energy gain/loss in Volts averaged on RF phases, Q is loaded quality factor of the dipole mode, I is the beam current value at which BBU instability begins. The beam always be stable if it get an energy gain only ($E_{BBU}>0$, i.e. $I_Q<0$), and it can be instable if it get enough value of energy loss, i.e. $E_{BBU}<0$ and $I > I_Q/Q > 0$

DIPOLE MODES IN TESLA CAVITY

Two trapped dipole mode pass-bands (18 in all) are formed from H_{110} and E_{110} dipole modes (see Fig. 1).

The dipole mode fields are calculated by CLANS2 cod [3]. The pass-bend frequencies are presented in Fig. 2.

The number of initial beam energies E_b = 0.5, 1, 2, 4, 8, 16, 32 MeV is accepted. The threshold current values are normalized to the value of γ/β^2 ("normalized threshold current" [1]), i.e. $I_{QD} = I_Q \cdot \beta^2/\Upsilon$ that presented in Table 1.

There are two conclusions. First is that only half of all mods be stable absolutely ($I_{QD}<0$) and other mods becomes instable at the beam current value overriding the threshold current values. Second is that all normalized dipole threshold current values (if beam energy increases) converges to some constant values not depending on the beam characteristics. This indicates the fundamentality of dipole mode "threshold current" effect.

These also true for a single cell TESLA cavity. It`s normalized threshold currents depending on beam energy are presented in Fig. 3. There is interesting that the dipole threshold current of the single cell (200 kA) is more by 300 times than the smallest one of the 9 cell tesla cavity. The role of the cell quantity for BBU is presented below.

The light lines in Table 1 are the #6 mode that is most stable one and #7 that is the most instable one. Their threshold current of 700 A denotes that BBU instability will be appeared for the beam current of $I>700 \cdot 3.34/10^9=2.3$ mkA at $Q=10^9$ and beam energy of $E_b=1$ MeV ($\gamma/\beta^2=3.34$). To guarantee of BBU stability at the beam current of 100 mA the quality factor must be less than 25,000.

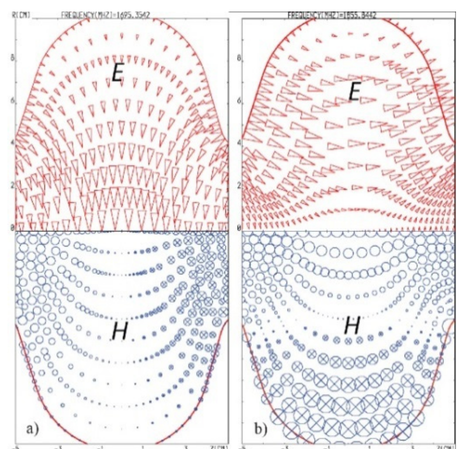

Figure 1: a) H_{110} and b) E_{110} trapped dipole modes in a single cell TESLA type cavity.

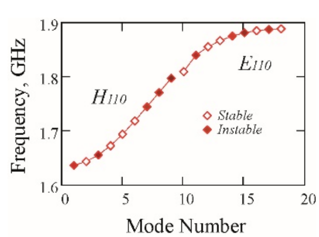

Figure 2: Trapped dipole mode pass-bend frequencies in TESLA cavities.

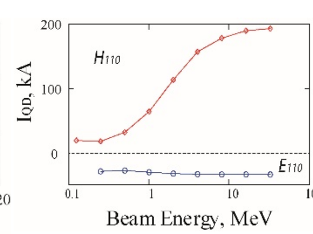

Figure 3: Normalized threshold currents of the single cell TESLA type cavity versus beam energy.

* Work supported by Russian Science Found. (Project N 14-50-00080)

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ISBN: 978-3-95450-211-0 doi:10.18429/JACoW-SRF2019-THP104

Table 1: Normalized BBU Threshold Currents (I_{QD}, κA) of Trapped Dipole Modes in TESLA Cavities

	E_b, *MeV*	**0.25**	**0.5**	**1**	**2**	**4**	**8**	**16**	**32**
N	γ/β^2	*2.71*	*2.66*	*3.34*	*5.13*	*8.94*	*16.72*	*32.34*	*63.6*
1	1636.80	18.7	53.1	-77.3	1527	197	166	160	158
2	1644.09	-9.68	-9.88	22.6	-259	-45.3	-38.0	-36.5	-36.2
3	1656.13	4.06	5.70	-7.66	-35.6	53.1	32.4	29.4	28.7
4	1673.17	1.08	-1.09	5.18	15.6	-34.3	-18.1	-16.1	-15.6
5	1694.27	-9.86	2.40	-0.77	-1.86	-4.23	-6.50	-7.6	-7.96
6	1718.62	34.6	1.04	1.69	-1.43	-0.95	-0.88	-0.87	-0.86
7	1744.93	85.1	-12.2	1.03	0.66	0.68	0.705	0.71	0.72
8	1771.26	-75.14	31.3	-11.9	84.4	5.29	3.84	3.55	3.47
9	1797.69	44.8	34.6	30.9	7.67	10.0	12.3	13.3	13.6
10	1809.75	-37.8	-20.5	-74.8	-8.54	-8.32	-8.73	-8.91	-8.96
11	1839.88	-500	68.5	-18.9	184	27.6	22.7	21.8	21.5
12	1855.31	654	-26.5	16.9	-15.6	-5.19	-4.14	-3.91	-3.85
13	1866.85	-98.0	75.6	-2.8	-1.29	-1.19	-1.19	-1.19	-1.19
14	1875.30	162	-8.76	-1.4	-3.46	-15.5	212	41.6	34.2
15	1881.27	-40.9	-1.67	6.5	2.53	2.71	2.85	2.9	2.91
16	1885.06	45.1	-12.6	4.57	469	-24.2	-20.3	-19.6	-19.5
17	1887.35	-6.19	4.39	-24.5	314	54.8	46.8	45.2	44.7
18	1888.43	-8.58	-58.1	81.7	860	-2.60	-241	-228	-225

BBU INSTABILITY FOR ERL

There are two beams with identical currents and different energies propagates through ERL cavity simultaneously. We considered the field of main mode are E_0=*10, 30, 50 MV/m*. The beam with initial energy of 6 MeV is accelerated and 50 MeV beam is decelerated. These beams have different BBU threshold currents of $I_{Q1}(6)$ and $I_{Q2}(50)$ (see Table 2) but the common BBU threshold current $I_Q(6+50)$ is defined in Ref. [1] as

$$1/I_Q = 1/I_{Q1} + 1/I_{Q2}, \tag{2}$$

Table 2: BBU Threshold Currents (in kA) in ERL TESLA Cavities. Dipole mode numbers are the same as in Table 1

	$I_{Q1}(6)$	$I_{Q2}(50)$		$I_Q(6+50)$	
N	E_0**: 10**	**10**	**10**	**30**	**50**
1	2696	14880	2275	2910	3327
2	-720	-3336	-592.5	-785.5	-881
3	529.7	2698	442.3	547	619.5
4	-369.3	-1415	-293.1	-363.4	-380.1
5	-115.6	-755.1	-100.3	-145	-173
6	-17.1	-79.3	-14.1	-19.5	-22.1
7	11.5	68.3	9.9	13	15.5
8	71.5	321.1	58.5	80.2	94.5
9	460.3	1136	327.4	856.5	759.5
10	-125.6	-876.2	-109.9	-130.2	-147.8
11	269.3	2227	240	290.8	373.2
12	-81.9	-352.5	-66.5	-92.3	-104.9
13	-4583	-14280	-3469	-595	-7425
14	-29.5	-144.4	-24.5	-34	-38.9
15	369	3502	333.4	356.6	652
16	48.2	263.8	40.7	56.1	66.5
17	-415.4	-1923	-341.8	-485.7	-575.5
18	935	5320	794.5	989	1132

The initial phase of main mode for 50 MeV beam and time delay for 6 MeV beam was chosen such that the first beam was maximally accelerated and second one maximally decelerated. The phase of dipole mode field was varied for averaging of the energy gain or loss that appeared in the cavity due to this mode.

From three columns with E_0=10 MV/m there is followed that *Eq. (2)* holds with high accuracy.

COMPARISON OF #6, #7 DIPOLE MODES

For visual comparison there is chosen the most stable *#6* dipole mode and most instable *#7* one. The beam with initial energy of 2 MeV propagates through two axially oriented TESLA cavities with identical dipole modes having the maximum axis rf field of *B*=0.25 mT (U_6=0.245 mJ , U_7=0.239 mJ). Between the cavities there is a double solenoid with inverse fields which does not change the polarization of transversal oscillations. Al fields for the solenoid and for the dipole modes are linear on radius, so these calculations are true for small oscillation amplitudes only.

As shown in Fig. 4 the beam energy is changed periodically with double frequency.

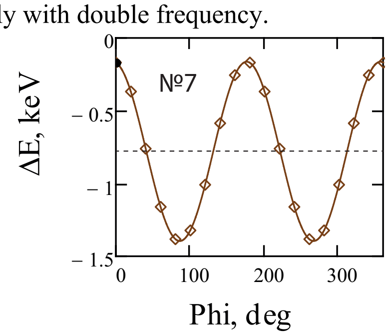

Figure 4: Beam energy depending on RF phase of #7 dipole mode at the first cavity exit.

ISBN: 978-3-95450-211-0 doi:10.18429/JACoW-SRF2019-THP104

The average energy change is dual one $\Delta E = E_{BBU} + E_{MOD}$, where E_{BBU} is the energy gain or loss due to BBU effect, E_{MOD} is the one due to excitation of the dipole mode by transversely modulated beam. Solenoid focusing strength is chosen such that ΔE energy loss to be minimal. The mode RF phase of second cavity is such that the ΔE loss to be maximal, i.e. it is the field that may be excited by the beam oneself (see "Beam excited" on Figures). This phase and ΔE depends on the solenoid strength, this are shown in Fig. 5 for #7 mode. Figure 5 shows that the excitation of dipole modes can be minimized by proper solenoid focusing. By the same way the BBU instability growth rate in Linacs can be reduced substantially as it is shown in [1]: the less dipole field or beam oscillations the less BBU growth rate.

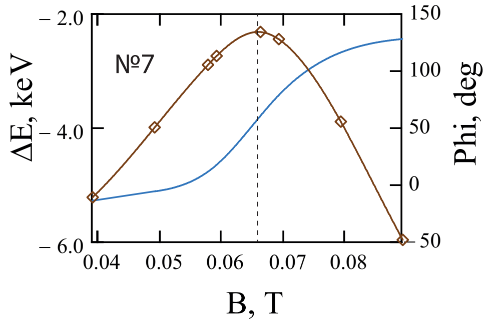

Figure 5: Beam average energy loss and excitation phase of #7 modes depending on the solenoid strength.

Let us remind that the excited dipole mode field energy is expressed in terms of the oscillation amplitude (A) as follows [1]:

$$\boldsymbol{U}(t)^{1/2} = \frac{A}{2}\frac{IQ\sqrt{(R_{II}/Q)/\omega}}{IQ/I_Q - 1} \times \left(1 - exp\left(IQ/I_Q - 1\right)\frac{\omega t}{2Q}\right), \quad (3)$$

where R_{II}/Q is the transverse impedance of the dipole mode in units of Ohm/m^2.

At the initial moment of a stepped excitation

$$\boldsymbol{U}(t \to 0)^{1/2} = \frac{A}{4} \cdot I\omega t\sqrt{(R_{II}/Q)/\omega}, \quad (4)$$

and in a stationary case with I lower than the threshold

$$\boldsymbol{U}(t \to \infty)^{1/2} = \frac{A}{2}\frac{IQ\sqrt{(R_{II}/Q)/\omega}}{IQ/I_Q - 1}. \quad (5)$$

The beam modulated amplitude envelops are shown in Figs. 6a, 6b. If the second cavity is absent then this is signed by "Drift" label. The excited stable #6 mode damps the oscillation (see Fig. 6a, "Beam excited" part) but the instable #7 one rocks the oscillation (see Fig. 6b). The "BBU suppression" part signs the excitation of second cavity by external RF generator having a special feedback such that the BBU oscillations are damped completely. It is interesting; the generator phase has the counter phase value relative to the "Beam excited" one with unstable #7 mode but is the one-phase with the phase of stable #6 mode.

The same suppression effect is obtained in the combination of #7+#6 excited modes without of the external generator where #6 frequency is tuned of equal to #7 one.

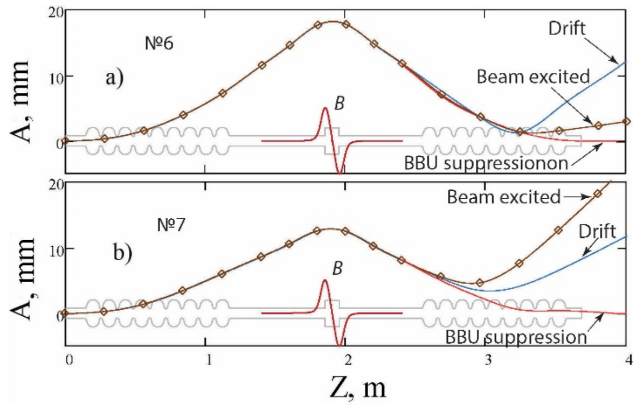

Figure 6: Beam amplitude modulation envelope of stable #6 mode and instable #7 one.

Beam energy changing is shown in Fig. 7 and beam emittance variations are in Fig. 8. Beam oscillations in transversal phase space (x, $x' = dx/dz$) have view of an ideal ellipsis as shown in Fig. 9. The emittance or area of this ellipsis is preserved on a drift space (without cavities). We see the emittance heavily deteriorates by instable modes (see Fig. 8) but recovers fully after the BBU suppression procedure.

Beam energy losses ($\Delta E = E_{BBU1} + E_{BBU2} + E_{MOD2}$) depending on RF #7 dipole field in second cavity is shown in Fig. 10. As can be seen the energy loss due to the excitation (E_{MOD2}) is linear dependent on the dipole field but the BBU loss (E_{BBU2}) has the quadratic dependency as it predicts in Ref. [1].

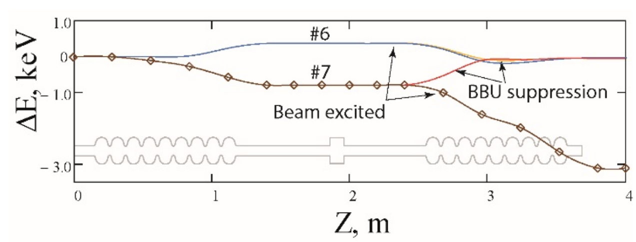

Figure 7: Beam energy loses in #6 and #7 dipole modes.

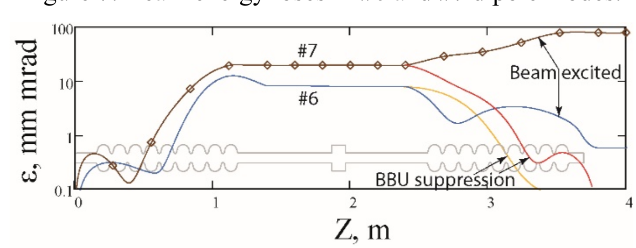

Figure 8: Beam traces space emittance variations.

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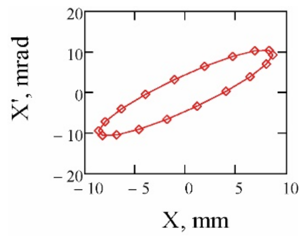

Figure 9: Beam oscillation in transversal phase space with #7 dipole modes. The same form is for #6 modes.

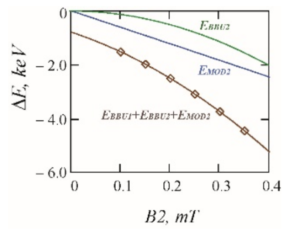

Figure 10: Beam energy losses depending on RF field amplitude of dipole #7 mode in second cavity.

ACCUMULATIVE BBU INSTABILITY

Let consider a Linac with identical cavities and solenoide focusing lenses between earch cavities (see Fig.11). The accumulative BBU instability can be analyzed in terms of the oscillation amplitudes by the Fig.11a, b with the switched off solenoides as following.

A transverse displacement of the particle beam or any dipole mode field fluctuation in the first cavity deflects the beam sideways. This beam excites the same dipole mode in the second cavity (see Eq.3) more strongly due to the increased oscillating amplitude. This dipole mode further deflects the beam more heavily, reinforcing the mode itself. As a result there is exponential growth of the beam oscillations that takes place even with stable modes.

To suppress this instability there have to switch on solenoids that must focus the oscillations exactly to the following cavity as by the method shown in Figs. 6. It`s focusing accuracy is independent on dipole mode types and can be used for all dipole modes simultaneously. The oscillations will be damped as it shown in Fig. 11b even if without of an external generator equipped by the feedback for the case with stable dipole modes.

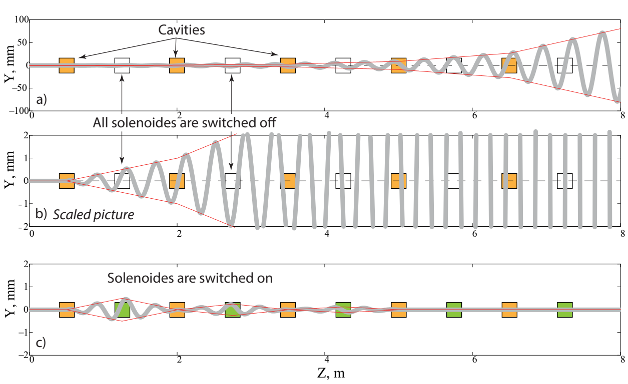

Figure 11: Demonstration of accumulative BBU instability suppression with the special beam focusing.

It seems that the described BBU instability suppression method with this special focusing system is more effective than those one having a total strong focusing system of all beam path especially for superconducting cavities sensitive to magnetic fields. Note that the solenoid system there can be replaced by quadrupole one.

BBU INSTABILITY VS CELL QUANTITY

Numerical calculations of BBU instability threshold currents also has made in [4] for the E110 type dipole 4140÷4280 MHz pass-band modes of normal conducting multi-cell accelerating cavities such as SLAC traveling wave structure 2856 MHz with three different cell quantities: N = 9, 27, and 89 cells. The calculated minimal values of threshold currents for the beam energy of 300 keV are presented in Fig. 12. This dependency is following to the formula $I_Q=718/N^2$, kA. It is interesting that there is enough to have the average beam current of about 8 mA to reach the BBU instability growth in the 89 cell normal conducting cavity with its quality factor of about 10^4. It is really to observe this experimentally.

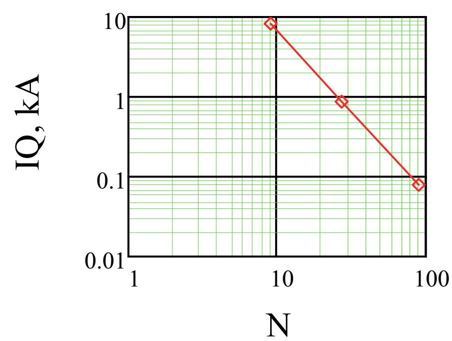

Figure 12: BBU instability threshold currents vs cell quantity of the multi-cell 2856 MHz cavity.

CONCLUSION

The assumptions for the existence of stable and instable dipole modes at the equal percentage and for its fundamentality are objectively confirmed by the numerical simulations. A special focusing may be the instrument for control under beam BBU oscillations and for BBU instability reducing in Linacs or even for suppression it especially with external RF generators equipped by appropriate feedback. BBU threshold currents in ERL cavities are always be less than those one in Linac cavities. And BBU threshold currents of multicell cavities are always be less than of the single cell cavity one as back proportionally to the cell quantity squared.

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