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Analysis of Laser Tracker and Total Station Surveys [*]

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Abstract

The SMX 4500 laser tracker has proven itself to be a beneficial addition to SLAC's set of alignment tools. Through actual field surveys and laboratory testing, a question regarding some unknown scale factor appearing after adjusting the combined tracker and total station results arose. Through a simple line survey and instrument analysis and then progressing through more and more rigorous network adjustments, the authors deduced that the apparent scale problem was in fact more likely attributed to wrong offset values introduced into the adjustment by mixing TC2002 total station data with the laser tracker measurements. A recalibration procedure of all of SLAC's total station prisms was designed and analyzed.

I. INTRODUCTION

Having an instrument that very accurately measures distances has added considerable functionality to SLAC's metrological tool set. Various generations of SMX's laser trackers have been used at SLAC providing an excellent source of accurate tacheometer data. Recent field use and laboratory testing at SLAC of SMX's 4500 laser tracker has shown some questionable data in terms of apparent problems with the trackers distance scaling.

The authors of this study were given the task of investigating this problem. They decided to repeat some previous testing of the tracker and then went on to tediously and systematically add more complexity to the analysis. The most essential and fundamentally important task of the authors was to keep track of all variables by only measuring one prism at a time and by keeping the data separate for proper understanding of the analysis results. This paper traces the authors' steps through this systematic analysis leading to the conclusion that by mixing observation types and by having outdated or non-representative a priori data, the apparent scale problem that was

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suggested was in fact an offset problem with the total station prisms that were mixed in with the tracker analysis.

II. THE LINE SURVEY

The first step in investigating the tracker's functionality is to use an existing test network. This very simple network consists of four targets in a line being about 8 meters distant from its neighbor (Figure 1). These targets are used to compare actual measured distances between specific stations with those same distances computed using a small triangular network. The expectation is to see similar results independent of the method used (of course considering the inherent accuracy limitations). Reproducing this test should confirm and verify previous results.

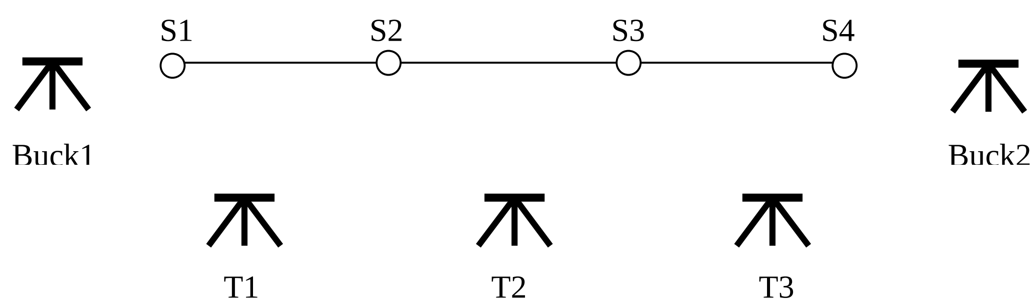


Figure 1. Layout of Stations and Targets
(Not To Scale)

The figure shows that the survey consists of four target points located on brackets mounted to one of the concrete walls located in the SLAC Sector 10 laboratory. Two bucked in stations are located on the outer ends of this line and three instrument stands are approximately in line and offset from the target points.

A decision to study one long distance S_1S_4 and one short distance S_2S_3 is made to simply compare the results of the tracker's measured and computed distances with those found using a Wild TC2002 total station. Only one target prism is used while using the tracker (PLX 678) and only one other prism is used while using the total station (labeled SC4). Each prism has a known offset that had been determined prior to this survey. The TC2002 prism (SC4) was calibrated on July 22, 1996. Associated with the total station used in this study (359906, called gun 4 at SLAC) its offset is 18.450 mm. This represents the combined instrument constant (C ~ 35mm) and the particular prism characteristics. The tracker prism (PLX 678) is designed to have no offset. This is accomplished by the manufacturer and has been reproduced at SLAC.

Metrological corrections are applied when either the tracker or the total station are used. With the tracker, SMX's Insight program applies the corrections for temperature, pressure and humidity using wavelength compensation. When using the total station, GEONET metrological corrections are applied based on the formulae of Owen, Sprung and Goff-Gratch where a refractive index of the air is computed.

LEGO (a SLAC adjustment package) is used to adjust the observations with the following a priori standard deviations: σ_d=50 µm σ_h=σ_v=0.5 mgon for the tracker and σ_d=250 µm σ_h=σ_v=0.3 mgon for the total station. The network is divided into three distinct surveys: one run for each bucked in instrument and one for the three remaining stations. The first two runs use the station fixed and are just made to confirm the measured distances. They check perfectly. For the third run, a free net approach is chosen. In this particular study, where only two resulting distances are analyzed, the choice of the datum is quite irrelevant as long as nothing is over-constrained. For the total station measurements, the computations have been executed twice. First without the assumption that the total stations have been leveled. (There are then 3 rotation unknowns per station, just like in the tracker case). Then with the level assumption, which makes the degree of freedom of the survey jump from 12 to 16, as each total station station as well as the datum orientation has only one rotation. Both approaches produce similar results. As expected the a posterior standard deviations on the vertical angles increased with the level option, but the overall statistics do not show any special discrepancy. The effect is -17 µm on the long distance and 23 µm on the short one.

Tables 1 and 2 provide a summary of the experiment. The first line records the differences between the measured distances from both bucked in stations. The second line present the result obtained once by averaging the two bucked in distances, then from the 3-station survey. Finally the third line records the difference between the two approaches.

Table 1. Long Distance Summary

S_1S_4 Study	Tracker	Difference	Total station
Δ between 2 bucked in dists.	31 µm		80 µm
Average bucked in distance	25.623999	-69 µm	25.623930
3 station survey	25.624002	-448 µm	25.623554
Δ between 2 approaches	3 µm		-376 µm

Table 2. Short Distance Summary

S_2S_3 Study	Tracker	Difference	Total station
Δ between 2 bucked in dists.	2 µm		69 µm
Average bucked in distance	8.507898	+77 µm	8.507975
3 station survey	8.507890	-111 µm	8.507779
Δ between 2 approaches	-8 µm		-196 µm

From the original version of these tests a suggestion of scale (or offset) discrepancies had been noted. Considering the geometry of the survey, those distances that are measured near perpendicular to the line of four targets (see Figure 2) contribute a much smaller portion to the error than those more parallel. These tests indicate that the prism offset for the TC2002 test is wrong. It is this error that appears as a scale error in a network adjustment simply because every distance is wrong by the same amount (see Section III). In the line test we can see how measuring within the bucked in line gives a "true" distance because the offset cancels out in the computation.

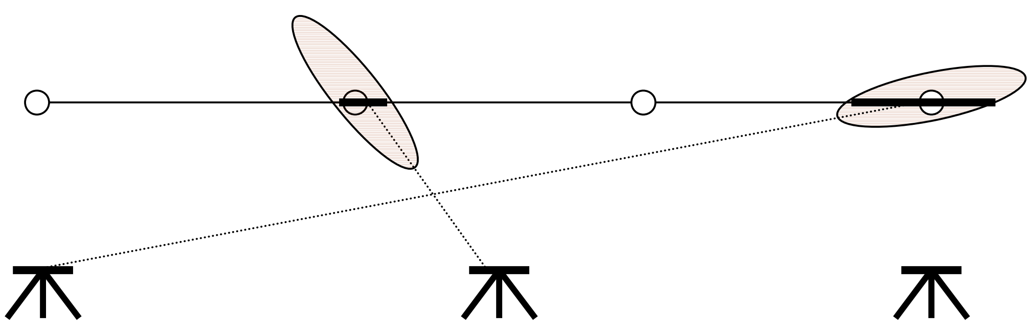


Figure 2. Effect of Geometry

The line survey alone presents a strong case as to why past discrepancies in network surveys exist. To investigate those factors creating a discordant network, the calibration of the instruments is next checked and a prism offset reestablished. Since only one prism is tested, the benefit is that the number of parameters can be well controlled. Of course this still leaves the question about the remaining prisms and a new R0 study has been requested by the authors for all the remaining total station prisms.

III. CALIBRATION STUDY

3.1 Tracker Calibration Model

The calibration study of the SMX 4500 laser tracker first involves some basic understanding of the internal optical path. The path includes pentaprisms and retroreflectors that each has their own degree of freedom. Various parameters are associated with certain optical and mechanical effects related to the tracker. Three additional parameters R0, *ast* and *act* are used to correct the raw observations d_0 and az_0 as follows:

$$d_{cor} = d_0 + \mathrm{R0}$$
$$az_{cor} = az_0 + ast\sin(az_0) + act\cos(az_0)$$

Table 3 summarizes the tracker compensation parameters and gives their approximate magnitudes (reference). The first four groups account for both optical and mechanical effects while the fifth contains two purely mechanical parameters related to the elevation axis.

Table 3. Tracker Compensation Parameters

Parameter Group	**Description**	**Parameter**	**Typical Value**
Upstream Optics	Beam deviations which occur upstream of AZPP.	AZTX	.00003 m
		AZTY	.00001 m
		AZRX	.00001 rad
		AZRY	.00004 rad
		QTX	.00015 m
		QTY	.00003 m
LTR Optics	Optics associated with LTR, between AZPP and ELPP.	ELTX	.000015 m
		ELTZ	.000015 m
		ELRX	.000005 rad
		ELRZ	.00002 rad
Final Optics	Beam deviations which occur as the beam exits the elevation pentaprism	TX	.000005 m
		TY	.0002 m
		RX	.0001 rad
		RY	.000007 rad
Elevation Axis	Configuration of elevation axis	AXOF	.000008 m
		AXNS	.00005 rad

The laser tracker requires a new calibration whenever it is moved to a new environment or when the conditions of an existing one change substantially. The procedure, using a suite of programs from SMX called CompIT, involves several steps:

- Pointing Test: it determines if a recalibration is necessary or not.
- Squareness Test: it produces the parameters AXOF, R0, ELRX, AXNS. There is also a simplified version called R0 Test which gives only R0 and AXOF
- Level compensation (not appropriate here because no level inside the tracker)
- ADM compensation (not appropriate here because the ADM was not used in the network survey)

The tracker was calibrated a total of three times in this study. Before the first calibration the tracker was set-up in the Sector 10 laboratory for a number of days to "soak". This was convenient and allowed all internal components of the tracker to reach metrological equilibrium. The first calibration was conducted just before starting the small line survey discussed in the previous section. The next calibration took place just before starting the large survey (see Section IV) and the last just after completing that same survey. The large survey lasted through two days.

The calibration values reported by the software are essentially identical. The decision was made to perform an additional calibration even though the post calibration results indicated that all tests had passed as shown in Table 4:

Table 4. Post Calibration Tracker

	d (inch)	Az°	El°	Tol (inch)	Error (inch)	Status
1	232	-90	90	0.0067	0.0047	PASS
2	229	-3	90	0.0066	0.0035	PASS
3	232	93	90	0.0067	0.0019	PASS
4	71	-2	139	0.0040	0.0038	PASS

The results of the additional calibration are reported in Table 5. As expected, the changes in the values were negligible.

Table 5. Tracker Optical Squareness

	Old Value	New Value	Δ	Tolerance
AXOF	-0.000016 m	-0.000014	2.2 μm	10.0
R0	0.141493 m	0.141497	3.6 μm	25.4
ELRX	0.000018 rad	0.000016	2.2 μrad	13.0
AXNS	-0.000150 rad	-0.000145	-2.2 μrad	13.0

3.2 R0 Determination Using the TC2002 Total Station

The determination of the R0 parameter provides instrument and prism offsets. The authors decided to follow this path and perform an offset determination for both the TC2002 total station and the SMX 4500 laser tracker using the associated prisms. This follows the strong implication for Section II that there is an offset problem. The set-up consists of four heavy-duty aluminum stands set in a line. These four positions are incorporated into the large network adjustment discussed in Section IV below.

This first offset determination uses the total station to calibrate its associated glass prism (SC4). These distances are compared to the tracker prism (#678) that was manufactured to have a zero offset. It is important to consider that the quality of the calibration is limited to the quality of the instrument. In Table 6 the results show a consistent difference between the two prisms of about 0.0162 meters. This value represents the reflector constant alone. The instrument constant of the total station cancels out due to the total station sitting at the end of the line instead of between points.

- R0 for TC2002

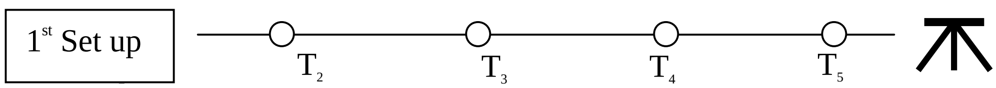


Figure 3. First Instrument Position

Table 6. Offset Determination Using Total Station (Position 1)

Station	Average Distance (m)	Vertical and Horizontal Angles	Δ Distance (m)
T_2	d = 17.1846 (#678) d = 17.2008 (SC4)	v = 100.00115 h = 0.00002	0.0162
T_3	d = 12.1893 (#678) d = 12.2056 (SC4)	v = 99.99299 h = 0.00971	0.0163
T_4	d = 7.1872 (#678) d = 7.2034 (SC4)	v = 99.98080 h = 0.01229	0.0162
T_5	d = 2.1921 (#678) d = 2.2082 (SC4)	v = 100.04463 h = 0.05969	0.0161

Computing the distance between two points gives a check of the quality of the measurements. One short T_3T_4 and one long distance T_2T_5 are chosen. The long and the short distance results both agree to a tenth of a millimeter.

Next the instrument is positioned at the middle position between points T_3 and T_4. This gives a distance that does not isolate the prism constant from the instrument constant; thus it is considered the combined constant. This value is actually found by comparing the sum of the two distances measured from between the targets to the known distance computed above. The difference is twice the value of the combined constant (Table 7).

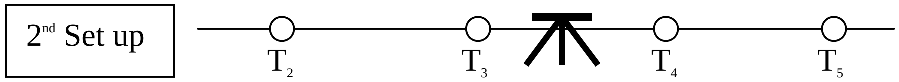


Figure 4. Second Instrument Position

Table 7. Offset Determination Using Total Station (Position 2)

Station	Average Distance (m)	Vertical and Horizontal Angles	Δ Distance (m)
T_2	d = 7.4543 (#678) d = 7.4705 (SC4)	v = 100.03946 h = 0.00012	0.0162
T_3	d = 2.4593 (#678) d = 2.4753 (SC4)	v = 100.07508 h = 0.03296	0.0160
T_4	d = 2.4733 (#678) d = 2.4895 (SC4)	v = 100.05607 h = 199.99454	0.0162
T_5	d = 7.4684 (#678) d = 7.4846 (SC4)	v = 100.05098 h = 199.99788	0.0162

The short distance computation using the SC4 prism produces a combined offset of 18.7 mm. The long distance 18.75 mm. The first number is used in this study instead of 18.45 mm which is the actual value stored in Geonet and used for the reduction of total station distances.

The next step is to extract the instrument constant of the total station from this data. From Table 7, the values calculated from the SMX #678 entries are T_3T_4 = 4.9326 and T_2T_5 = 14.9227. The SMX prism is a zero-offset prism. When it is measured using the total station, the isolated instrument constant is found. The short distance computation produces a value of 34.75 mm, the long one 34.9mm. These two results are very close to 35mm, the expected value of the instrument constant as noted in the manual.

As a final step the instrument is simply set-up on the other end of the line of targets as shown in Figure 5. This third and last step is simply a check of the first one.

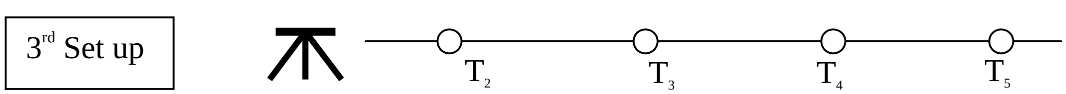


Figure 5. Third Instrument Position

Table 8. Offset Determination Using Total Station (Position 3)

Station	Average Distance (m)	Vertical and Horizontal Angles	Δ Distance (m)
T_2	d = 2.2623 d = 2.2784	v = 99.97303 h = 399.99518	0.0161
T_3	d = 7.2573 d = 7.2735	v = 99.97809 h = 399.98841	0.0162
T_4	d = 12.2595 d = 12.2757	v = 99.98342 h = 399.99992	0.0162
T_5	d = 17.2548 d = 17.2710	v = 100.00239 h = 399.99982	0.0162

This third set-up is only a confirmation test for the first one and they compare perfectly with the first set.

3.3 R0 Determination Using the Laser Tracker

The determination of the R0 parameter is limited to the accuracy of the TC2002 total station as illustrated in Section 3.2. In this section the offset determination is not only verified but also refined in quality by using the laser tracker to measure the distances.

The first task is to calibrate the ADM (Absolute Distance Measurement) of the laser tracker and see how it compares to the conventional tracking value. Then, with the ADM running, the beam is broken and the prisms are again measured. As the data shows, the only value that changes by any significant amount is the SC4 prism where a difference of 0.015949 meters is found. This is the prism constant and the value matches that from the previous determination of about 0.0161 meters.

SC4 Offset measurement with the Tracker (Use ADM):

When tracking	SMX	8.112721
ADM disabled	SC4	8.112705
AFTER breaking beam	SMX	8.112711
ADM enabled	SC4	8.128657

This test corroborates the previous findings in a simple manner. The combined offset for the total station and the SC4 prism is the sum of the prism constant of about 0.0161 meters and the instrument constant of about -0.0348 meters. An interesting side test was made while conducting this offset analysis. The glass cube (prism SC4) is angled in its nest relative to the instrument. Readings do not repeat perfectly; values vary from around 15 µm when placed normally to over 100 µm when skewed almost sideways.

The SMX prism is also tested in this manner giving the expected very small values of around 13 µm to 31 µm depending on the distance between the test points.

3.4 Establishing a Good Future Calibration Line

The previous calibration was based on an existing line of four instrument stands roughly placed in a line. Translation stages are used at the time of the measurement to refine the alignment. This whole process can be time consuming and the set-up is also non-permanent due to its central position in the laboratory. Regular prism calibration being essential to ensure good quality adjustments, it would be beneficial to build a permanent set-up. The end goal is to have a set of permanent targets all in line and a method to put an instrument anywhere along this line.

To simplify the problem, assume that there are only two targets (A and B) and study the influence of a misalignment of the instrument. The exact position of the instrument along the target lines is of no importance, as the measured distances are not used individually but in pairs to produce the distance AB. A displacement x from an ideal position on the line will have no effect when it is positioned along the line and the most effect when it is perpendicular to the line (Figure 6.)

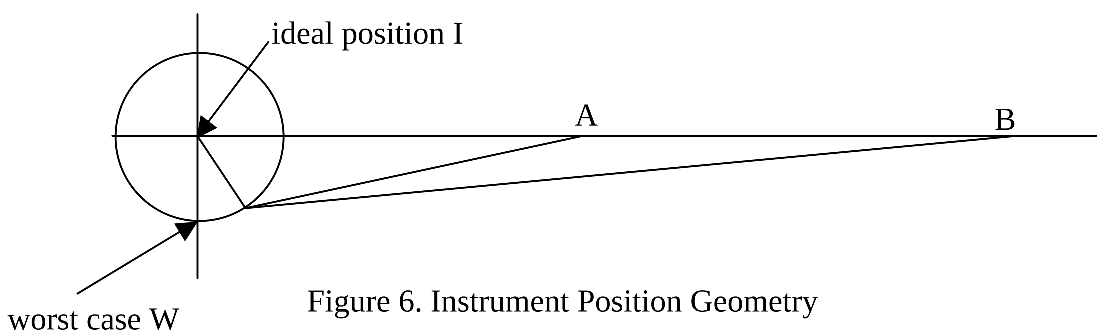
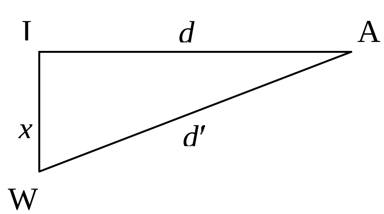


Figure 6. Instrument Position Geometry

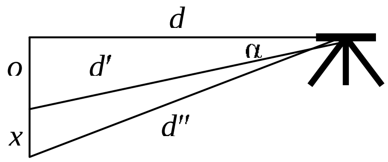
Note that d is the theoretical distance and d' is the true (measured) distance. The difference between the two is more important for short distances. At 2 meters for example, a difference $d' - d$ of 1µm is obtained for x equal 2mm.



$$d' - d \cong \frac{x^2}{2d}$$

Figure 7. Effect of Perpendicular Offset

Since one cannot be sure when the instrument is actually in line, the next illustration (Figure 8) shows how starting with an existing offset of σ and adding another offset *x* can change the measured distance of d' to d''. In this case to match the actual experiment, the instrument is fixed and the target moves.



$$d'' - d' \cong \frac{x^2}{2d} + \frac{o\,x}{d}$$

$$d'' - d' \cong \frac{x^2}{2d} + x\tan\alpha$$

Figure 8. Offset Geometry

This was tested using a translation stage situated on a monitored target point. The displacements were first made along the line instrument-target to "calibrate" the translation stage and then perpendicular to the line to prove the above developments. In a first experiment using a very short distance (0.9m) the measured difference was 81µm, and the predicted difference was 10µm leading to an original angular displacement of 0.97 degrees. A careful set-up on a short distance (1.9m) still shows discrepancies between the measured difference and the computed one (38µm versus 11µm, 101µm versus 44µm) leading to an assessment of the angle α of 0.05 degrees. These tests provide a good assessment of how careful one should be in aligning stations in a linear offset calibration set-up.

IV. NETWORK TEST

4.1 Network Geometry

The calibration study provides a level of confidence in the reliability of the SMX Laser Tracker leading to the next step, a network test. The goal is to have a rigorous and redundant set of observations from both the Laser Tracker and the TC2002 total station in a controlled environment. This allows for highly reliable comparisons of the two instruments through various adjustment scenarios and it also limits overlooked or unknown systematic or random errors found in non-laboratory conditions.

The network used in this study is based on a significant part of the existing test network found in SLAC's Sector 10 laboratory. Compared to the full set of points found in Sector 10, this network is truncated in length although six extra points are added, three high on the wall and three on the ceiling for better overall geometry (points M_{19},M_{20},M_{21} on the wall; points M_{22},M_{23},M_{24} on the ceiling). The magnetic nests used are the latest type with three solid supports holding the spherically mounted retroreflector (SMR). These are reported to be the nests that are most resistant to deformation due to the stress of the SMR being pulled towards the magnet.

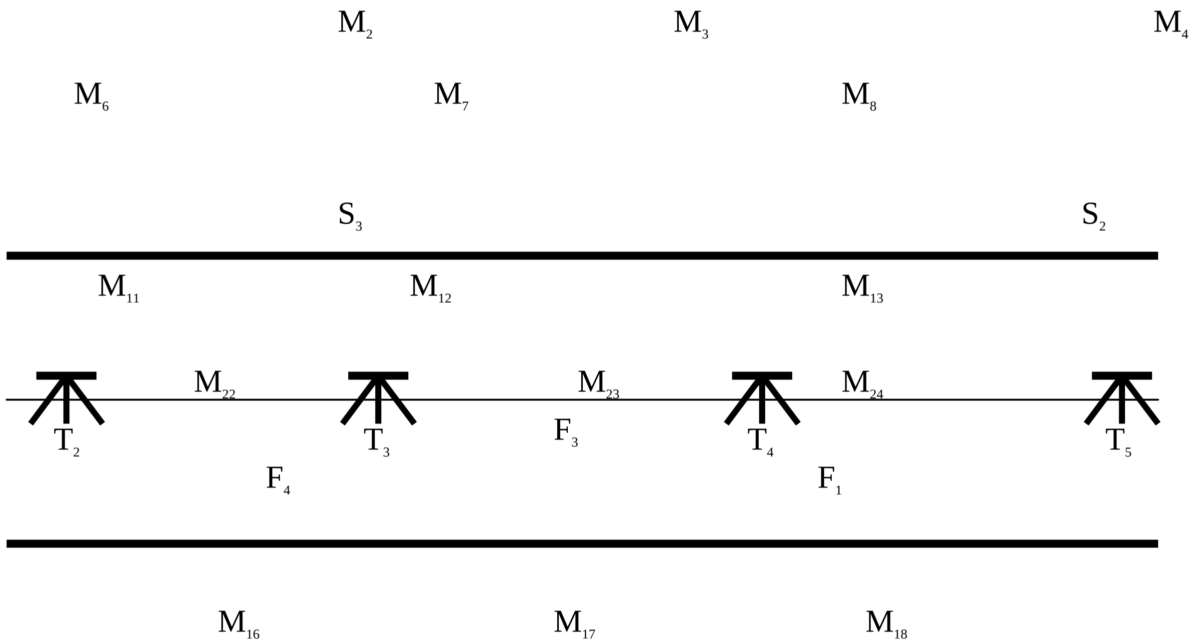


Figure 9. Full Network Point Distribution

The final network consists of twenty-three target nests and four tripod stands (T_2,T_3,T_4,T_5) that are also used for R0 calibrations which are used to determine instrument and target additive constants. Eight observation stations are used where the sequence of observation gathering proceeds in a “leap-frog” manner where the tracker is used at one station and is immediately replaced with the total station once the observations are complete. The tracker is moved to the next station immediately ready for observation gathering once the total station observations are complete. This ensures that the two sets of instrument observations are as close as possible in time to each other.

It should be noted that metrological conditions in Sector 10 are reasonably stable although some variance was found and included in all computations. Tracker and total station observation stations are very close to the same in position but of course have minor positioning and observation differences due to such factors such as instrument height differences, different angular range limitations, and thus the resulting occlusions of different target stations.

Two observation stations are located on the T_2,T_3,T_4,T_5 line at either end; Station 1 is next to T_2 and Station 5 is next to T_5. Stations 2, 3, and 4 are situated in the main isle in a zigzag manner and Stations 6, 7 and 8 are found in the space between the calibration tape bench and the west wall of the laboratory. The essential element of this survey is to have a network with similar geometrical and metrological conditions for both instruments. Keeping the same Laser Tracker, the same total station, and the same corresponding SMRs during the entire survey is not only desirable but is essential if one wishes to isolate any discrepancies between the two instruments. In all of the following results only one SMX Laser Tracker and one TC2002 total station are used with SMR #678 and prism SC4 respectively.

On March 27, 2000 the combined survey was conducted with the authors assisting Hans Imfeld who has considerable experience with using both the SMX Laser Tracker and the TC2002. His knowledge and care for his equipment are important additions to this series of tests. Due to the size of the combined network, two days were required for gathering observations from all eight stations. Stations 1, 2, 3, 4 and 5 were measured from on the first day using both the tracker and total station. Stations 6, 7 and 8 completed the combined survey on the following day. Metrological conditions were fairly stable although pressure was noted as dropping a bit due to weather changes outside. As noted above, these are compensated for, as are all pressure, temperature and humidity conditions.

4.2 A priori Statistical Determination

The combined instrument network test requires a priori standard deviations for each type of observation. Since the test is in a controlled environment, some very basic repeatability tests of the observations were run to pick realistic a priori values. A more rigorous series of tests would be necessary if the adjustment indicated such, but as can be seen in Section 4.3, our refined a priori standard deviations are in fact quite good. The quality of the observed angles from the laser tracker were found to repeat slightly *better* than those of the total station. This is interesting since the consensus had been that the tracker's angles were less repeatable possibly based on the abilities of an older SMX laser tracker model.

The initial a priori values are determined in the same manner as the ones used in regular surveys at SLAC. For distances, standard deviations are kept constant (independent of the distance) with determined values of 8 μm for the laser tracker and 80 μm for the total station. Angular standard deviations are identical for horizontal and vertical and are inversely proportional to the distance. The authors decided to also use only one formula for both angles but to test the use of a step function. This is to limit the impact of those angular observations that are very distant and to give back some weight to the very short shots that still warrant some contribution to the adjustment.

Table 9. Tracker – SMR Pointing Repeatability Test (Results)

Target	σ of ρ (m)	σ of θ (gon)	σ of φ (gon)
M7 (3m)	0.000001	0.000110	0.000127
M12 (2m)	0.000001	0.000239	0.000142
M19 (3m)	0.000001	0.000120	0.000125
S1 (15m)	0.000002	0.000073	0.000066
S1 (rot cup)	0.000005	0.000070	0.000137

From these results, a value of 8μm is chosen for the distance and the following curve is applied to find the standard deviations on the angles:

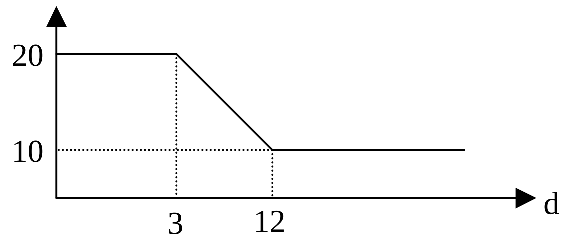


Figure 10. Tracker Angular Step Function

The step function for angular observations is chosen with a large distance range so that weighting would be consistent with previous tests conducted outside of this study. Further studies are necessary to refine the range. In this network, the purpose of using a step function is to see if using one is a better choice than just using a constant angular standard deviation or the SLAC choice of a simple linear function. The range of distances is based on:

- the size of the network,
- the knowledge that very short and steep observation shots are generally subject to numerous problems such as refraction and pointing errors,
- the concept that angular pointing to far targets stops improving after some set distance (i.e., pointing is as good as it gets).

Tracker repeatability is generally operator independent while the TC2002 requires an experienced operator for a fair comparison.

Table 10. TC2002 – SC4 Pointing Repeatability Test (Results)

Target	horizontal	σ	vertical	σ
M7 (3m)	291.12884	0.00027	102.59717	0.00042
M12 (2m)	297.58412	0.00034	148.72967	0.00036
M19 (3m)	133.02870	0.00025	67.62207	0.00021
S1 (15m)	381.19181	0.00017	101.34120	0.00026

From these results, a value of 80µm is chosen for the distance and the following curve is applied to find the standard deviations on the angles:

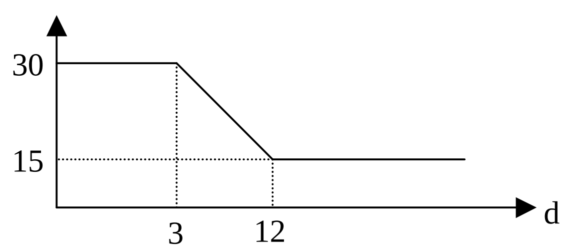


Figure 11. Total Station Angular Step Function

4.3 Adjustment Results

Once the appropriate a priori statistics have been established, the next step with this rigorous network is to confirm these choices. Since the statistics prove to be an accurate representation of the instrument abilities, analysis can proceed as to differentiating whether scale, offset or a combination of both might contribute to the network model. This is an extension of the very limited mini-network/line survey in Section II that indicated that almost the entire contribution comes from an out-of-date offset assigned to the SC4 prism.

4.3.1: LEGO Results for Three Adjustment Cases

The choice for an adjustment datum is to use a free network approach. This is the best adjustment choice, especially considering the goal of this study that is to extract any actual scale or offset problems. Through the use of variance analysis where the global tests pass each time, the results indicate that this is a well-determined network. Analyses of the normalized residuals indicate that the pre-analysis statistics are good choices and that the goodness-of-fit (normal curve choice) is correct. (See Figures 12 through 14 below.)

<u>Laser Tracker Only:</u>

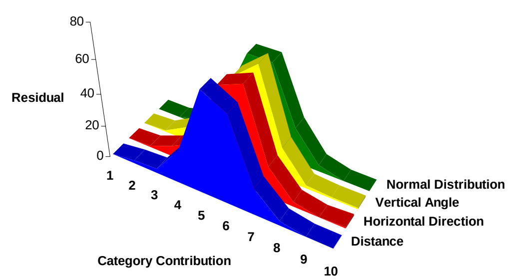


Figure 12. Tracker-Only Residual Distribution (File: newtrak10.out)

The test results and histograms show that all of the a priori standard deviations were reasonable. Since this network is for the tracker only, no offsets (or scale) are expected nor found in the analysis. In the next section the old prism offsets are first corrected.

Total Station Only:

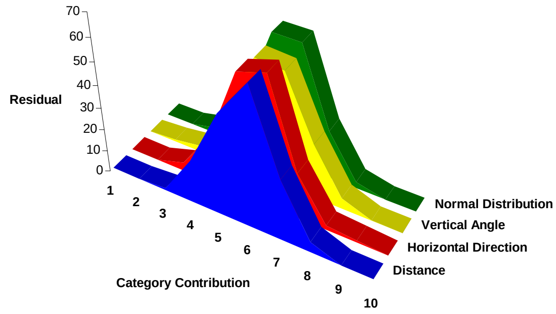


Figure 13. Total Station Residual Distribution (File: newtc10mod.out)

The test results and histograms for just the total station observations show that all of the a priori standard deviations were again reasonable. Since this network is for the total station prism SC4, the offset was first corrected. The adjustment results indicate that this is the correct course of action. In the next section both of the instruments are combined creating a larger network in terms of observations.

Combined Network Results:

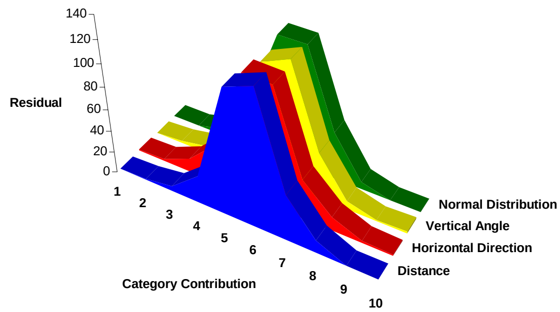


Figure 14. Combined Residual Distribution (File: newsector10.out)

The test results and histograms for just the combined observations show, as expected, that all of the a priori standard deviations were again reasonable. Essentially this is the result of combining two good surveys together where the number of instruments has increased to sixteen.

Statistical Summary:

The two following tables summarized the characteristics of the three runs described above. The three numerical columns refer to tracker only, total station only and combined:

Table 11. LEGO Summary 3D Problem

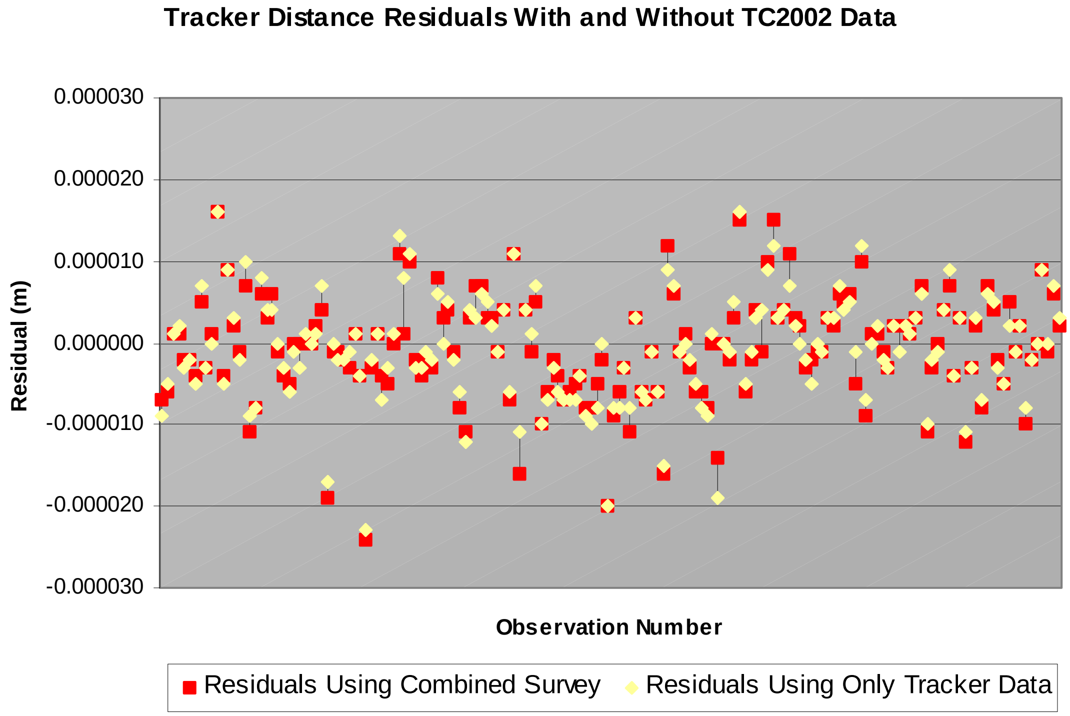
Number of instruments	8	8	16
Number of points	27	27	27
Number of distances	164	163	327
Number of horizontal directions	164	163	327
Number of vertical directions	164	163	327
Number of coordinates unknowns	105	105	129
Number of nuisance parameters	24	24	48
Number of datum parameters	6	6	6

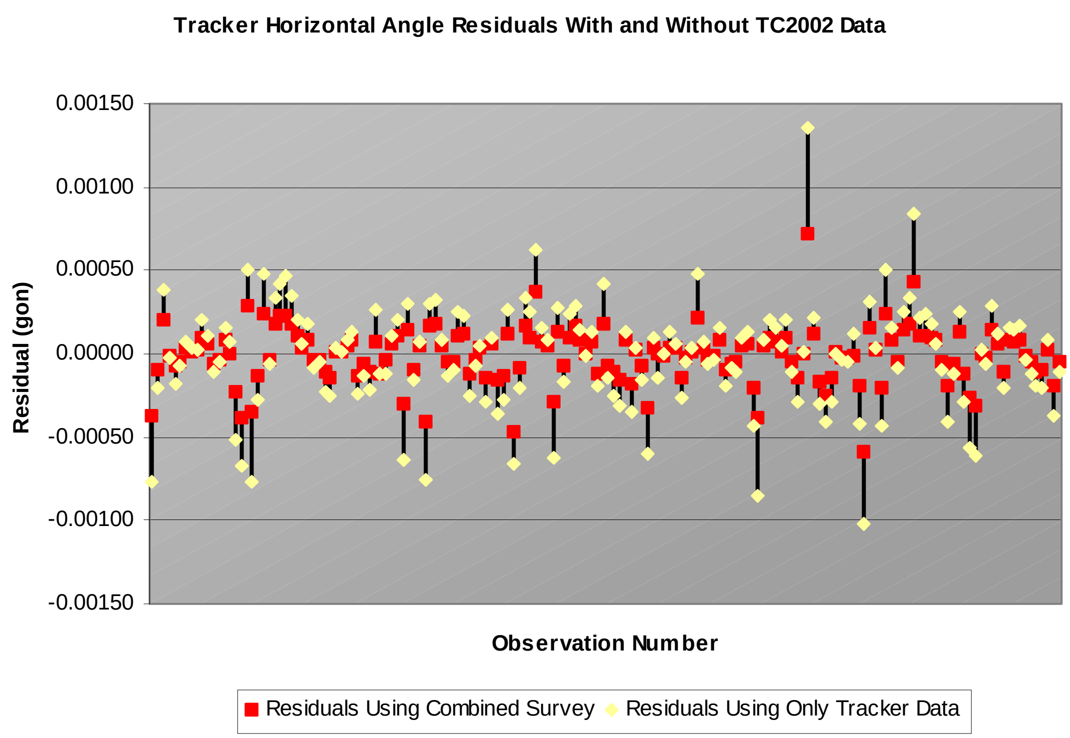
Table 12. LEGO Statistical Summary

Degree of freedom	369	366	810
Variance	0.974389	0.983446	1.067034
Sigma a posteriori	0.987112	0.991689	1.032973
Variance component for distances	0.916086	0.973137	0.934046
Sigma a posteriori for distances	0.957124	0.986477	0.966461
Variance component for horizontal	1.024935	0.861513	1.167052
Sigma a posteriori for horizontal	1.012391	0.928177	1.080302
Variance component for vertical	0.978018	1.107827	1.109537
Sigma a posteriori for vertical	0.988948	1.052534	1.053345

4.3.2: Adding Total Station Measurements to Tracker Only Network.

The tracker-only network is very rigorous on its own. Adding observation data from a less accurate instrument is questionable. The impact of adding the total station observations to the adjustment actually does improve the results but this is relatively obvious in almost any well-designed network. What isn't as necessarily obvious is by how much the additional data adds to the results. Since extra fieldwork is required the question of return—in terms of better results—versus the investment in time must not be ignored.





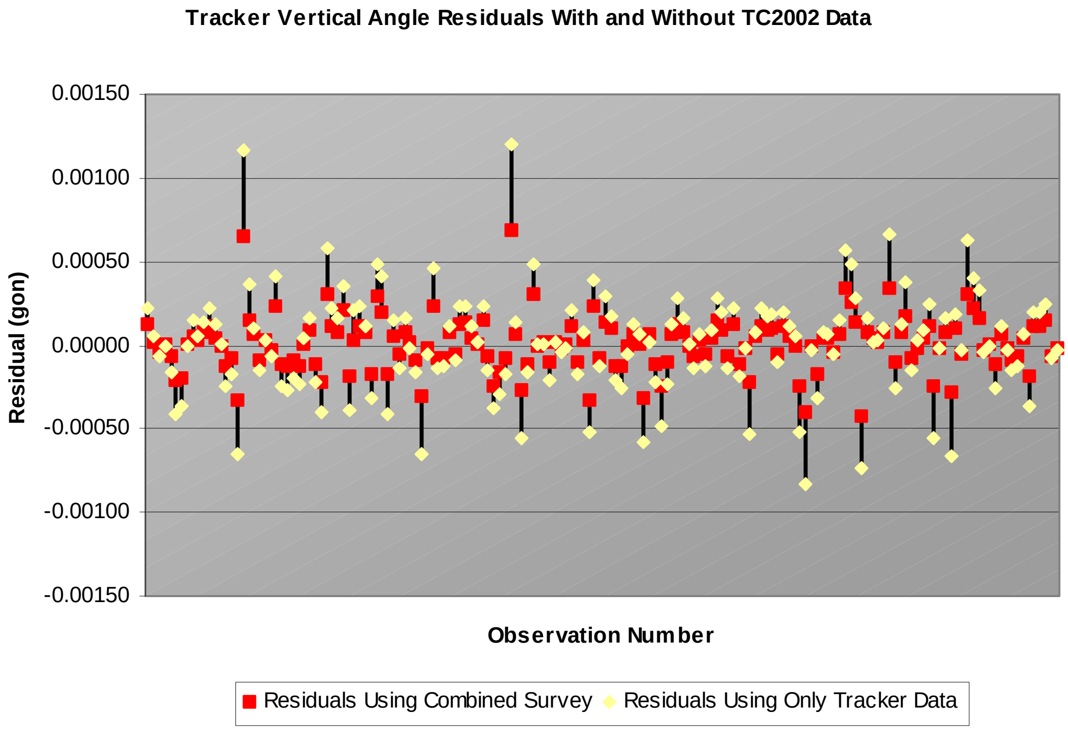


Figure 15. Graphs of Residuals

The graphs in figure 15 show how the residuals for horizontal and vertical angles actually do improve by adding total station data to the tracker survey. This is not the case for the distances because of the substantial discrepancy in measurement quality between the two instruments. Keep also in mind that these overlapping surveys have the same geometry and so the improvement is generally negligible for most observations. A complimentary survey that would take advantage of any weaknesses in the first survey would be more useful but then there is little justification as to using a less accurate instrument unless there is no choice.

4.3.3: Adding Scale and Offsets to the Adjustment

In Section II the line survey indicated that the discrepancy between the laser tracker and the total station was due to an offset associated with using an out-of-date prism constant. Through an R0 test, the correction to the SC4 offset values changed from 0.01845 meters (the Geonet value) to 0.0187 meters; this is a difference of 250 µm (see Section III). If the distances are not corrected by this amount, LEGO determines a constant offset of –218 µm ± 7 µm. These values remain the same whether using the total station level option or not.

Analyzing the same data but allowing only a scale to be determined (i.e., not allowing offset) produces a global scale value of 0.999971 ± 0.000001. This is a scale of about 29 ppm and is definitely significant but this does not mean it is real. To demonstrate why, the authors allow both scale and offset to be determined by LEGO. This gives a scale of 0.999994 ± 0.000002 (6 ppm) and an offset of –178 µm ± 0.2 which is only slightly less than the offset-only determination and is within the noise of the total station abilities.

Table 13. Offset and Scale before Correction

LEGO Setting	Scale (ppm)	Offset (µm)
Scale only	29	-
Allow both	6	178

Table 13 summarizes how an offset can appear as a scale of a significant size if only scale is assumed to contribute to the adjustment. This matches the combined survey in Section II with a much more rigorous network. With the distances corrected for offset, and allowing for both scale and offset in the adjustment, we find no remaining significant scale.

Table 14. Offset and Scale after Correction

LEGO Setting	Scale (ppm)	Offset (µm)
Offset only	-	32
Scale only	2.5	-
Allow both	-6.4	72

At this point, it should be noted that all the results have been obtained without the use of the fact the total stations were leveled. Adding this assumption changes the resulting statistics (especially of the vertical angles) but not by any significant amount. In particular, the determination of the offset and/or the scale factor remains unchanged (32 µm ± 7 µm and 2.5 ppm ± 1.0 ppm). The next step is to allow LEGO to determine offsets for each station instead of using just one global value. The results being less than about 60 µm illustrate how the corrected offsets used in this adjustment have had the global portion of the offset correctly removed. Some very small systematic and random effects do remain as normal for any survey such as this one.

This section presents a substantial case for why outdated prism constants could be confused with scale. If the distances between the instrument and the target points are all approximately the same (say 2 meters for example), one will find it impossible to distinguish between a scale factor and an offset. As shown, it is also very important to have good standard deviations that reflect the conditions of the survey being conducted, otherwise one can underestimate or overestimate the impact of either short or long distances resulting in confusion. Further study would be warranted using an actual field survey.

V. CONCLUSION AND RECOMENDATIONS

The SMX 4500 laser tracker provides an accurate set of measurements for both distances and angles. Through progressively more and more rigorous testing, an apparent scale problem thought to be inherent with the laser tracker was in fact an offset problem associated with using outdated prisms constants with the total station.

ACKNOWLEDGMENTS

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