Parity Violation in Graviton Non-gaussianity

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Abstract

We study parity violation in graviton non-gaussianity generated during inflation. We develop a useful formalism to calculate graviton non-gaussianity. Using this formalism, we explicitly calculate the parity violating part of the bispectrum for primordial gravitational waves in the exact de Sitter spacetime and prove that no parity violation appears in the non-gaussianity. We also extend the analysis to slow-roll inflation and find that the parity violation of the bispectrum is proportional to the slow-roll parameter. We argue that parity violating non-gaussianity can be tested by the CMB. Our results are also useful for calculating three-point function of the stress tensor in the non-conformal field theory through the gravity/field theory correspondence.

1 Introduction

The most promising candidate for unifying particle interactions is string theory. String theory predicts a parity-odd gravitational Chern-Simons interactions in the low energy limit. Thus, experimental or observational detections of parity violation in the gravity sector certainly provide us with direct information concerning the UV completion of general relativity or the ultimate unified theory.

A prospective and reliable route to this end is to explore parity violation in relics of inflation in the early universe. In particular, measurements of parity violation in the primordial gravitational waves produced during inflation are expected to bring us valuable information about the Planck scale physics. From this standpoint, it has been intensively discussed how to observe the parity violation in the power spectrum directly using the laser interferometer and indirectly using $\langle TB \rangle$ correlation in the cosmic microwave background (CMB). Remarkably, some speculative gravitational theories with a parity violating term produce significant parity violation in the power spectrum of primordial gravitational waves. Unfortunately, in the conventional inflationary scenario a parity violating term leads to only very small amount of circular polarization in the power spectrum of primordial gravitational waves (see, e.g. [1, 2] and references therein).

In principle, we can also seek parity violation in higher-order correlation functions [3, 4]. Since the power spectrum and the higher-order statistics are often sensitive to different kinds of parity violating interactions, they may have different correlations with other revealing statistical features. For example, in a recent paper of Maldacena and Pimentel, they discussed graviton non-gaussianity in the exact de Sitter spacetime and found that the pattern of parity violating bispectrum is severely constrained by the conformal invariance [5]. One might then be tempted to deduce that their result implies the unsuppressed parity-violation in the bispectrum. We argue that this is not immediately obvious since they have not calculated observable quantities explicitly. In this short article, we develop a new tool to analyze graviton correlation functions and show that parity violation does *not* show up in the case of the exact de Sitter spacetime. A more detailed discussion can be found in [6].

2 A new formalism for graviton correlators

With the aid of the helicity basis, we present a useful method to evaluate graviton non-gaussianity generated by a parity-violating Weyl cubic term. Tensor perturbations on the Friedmann-Lemaître-

Robertson-Walker (FLRW) universe are given by

$$\mathrm{d}s^2 = a^2(\eta)\left[-\mathrm{d}\eta^2 + (\delta_{ij} + h_{ij})\,\mathrm{d}x^i\mathrm{d}x^j\right], \qquad h_{ii} = \partial_i h_{ij} = 0\,. \tag{2.1}$$

The gravitational action and the corresponding equations of motion are given by

$$S = \frac{M_{\rm pl}^2}{8}\int \mathrm{d}\eta\mathrm{d}^3x a^2\left(h'_{ij}h'_{ij} - \partial_k h_{ij}\partial_k h_{ij}\right), \qquad h''_{ij} + 2\frac{a'}{a}h'_{ij} - \Delta h_{ij} = 0\,. \tag{2.2}$$

We have two physical degrees of freedom for tensor perturbations which are characterized by the symmetric polarization tensors. Since we are interested in the presence/absence of parity-violation, it is of use to expand in a helicity basis $e_{ij}^{(\pm)}(\mathbf{k})$ satisfying

$$e_{ii}^{(s)}(\mathbf{k}) = 0\,, \qquad k_j e_{ij}^{(s)}(\mathbf{k}) = 0\,, \qquad \epsilon_{ijl}\frac{\partial}{\partial x^l}\left[e_{mj}^{(s)}(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}\right] = ske_{im}^{(s)}(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}\,. \tag{2.3}$$

The index $s = \pm$ specifies two helicity states. We choose the normalization and the phase of polarization tensors in such a way that

$$e_{ij}^{(s)}(\mathbf{k})e_{ij}^{*(s')}(\mathbf{k}) = \delta_{ss'}\,, \qquad e_{ij}^{*(s)}(\mathbf{k}) = e_{ij}^{(-s)}(\mathbf{k}) = e_{ij}^{(s)}(-\mathbf{k})\,. \tag{2.4}$$

In this article we focus on the pure gravity parity-violating sector where the action is composed of the Weyl tensor $W_{\mu\nu\rho\sigma}$ and its Hodge dual. Since the quadratic term is topological, the leading term comes from the cubic interactions

$$S_{\rm PV} = -b\int \mathrm{d}\eta\mathrm{d}^3x\sqrt{-g}\epsilon^{\mu\nu\lambda\rho}W_{\mu\nu}{}^{\alpha\beta}W_{\alpha\beta}{}^{\gamma\delta}W_{\lambda\rho\gamma\delta}\,. \tag{2.5}$$

Since the FLRW universe is conformally flat, this term does not affect the equations of motion (2.2) and the power spectrum. They could contribute to the non-gaussianity, which we now turn to discuss.

In order to simplify computations, introduce a new variable γ_{ij} as $\gamma'_{ij} := ah'_{ij}$, in terms of which we notice a useful relation $W^\mu{}_{\nu\lambda\rho}(h) = a^{-1}W^\mu{}_{\nu\lambda\rho}(\gamma)\big|_{\rm Minkowski}$. We define

$$\gamma_{ij}^{\pm} := \frac{1}{2}\left(\gamma'_{ij} \mp \mathrm{i}\epsilon_{jkl}\partial_l\gamma_{ik}\right), \tag{2.6}$$

which is symmetric, transverse and traceless. We find that a simple relation $(W^\pm)^3 = 64a^{-9}(\gamma^\pm)^3$ holds irrespective of the expanding history of the universe. Here $W^\pm = W \pm \mathrm{i}\star W$ are the complexified self-dual and anti-self-dual Weyl tensors $\star W^\pm = \mp\mathrm{i}W^\pm$. In the flat spacetime, the solutions γ_{ij} with two helicity states of fixed $|\mathbf{k}|$ form a unitary representation of three-dimensional Euclid group [7]. This representation decomposes into two irreducible representations, which are nothing but helicity states described by $\gamma_{ij}^\pm$. The same is true for the de Sitter spacetime. In [5], another useful method–a spinor helicity formalism–was invented.

Utilizing above, we can write the parity-violating cubic action (2.5) in a general FLRW background in terms of these new perturbation variables $\gamma_{ij}^\pm$ as

$$S_{\rm PV} = 8ib\int \mathrm{d}\eta\mathrm{d}^3x a^{-5}\left[(\gamma_{ij}^+)'(\gamma_{jk}^+)'(\gamma_{ki}^+)' - (\gamma_{ij}^-)'(\gamma_{jk}^-)'(\gamma_{ki}^-)'\right]. \tag{2.7}$$

From this, we can read off the interaction Hamiltonian $H_{\rm PV}(\eta)$ as

$$\begin{aligned} H_{\rm PV} = &-\frac{8ib}{(2\pi)^6}a^{-5}\int \mathrm{d}^3k_1\mathrm{d}^3k_2\mathrm{d}^3k_3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) \\ &\times\left[(\gamma_{ij}^+(\eta,\mathbf{k}_1))'(\gamma_{jk}^+(\eta,\mathbf{k}_2))'(\gamma_{ki}^+(\eta,\mathbf{k}_3))' - (\gamma_{ij}^-(\eta,\mathbf{k}_1))'(\gamma_{jk}^-(\eta,\mathbf{k}_2))'(\gamma_{ki}^-(\eta,\mathbf{k}_3))'\right]. \end{aligned} \tag{2.8}$$

We should emphasize that $\gamma_{ij} = \gamma_{ij}^+ + \gamma_{ij}^-$ does not represent gravitational waves in Minkowski spacetime but an auxiliary field. From the definition $\gamma'_{ij} := ah'_{ij}$, we can give an explicit mode expansion expression for $\gamma_{ij}^{\pm}$ as:

$$(\gamma_{ij}^{\pm})' = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2} M_{\rm pl}\sqrt{2k}} \sum_{s=\pm} \Big[e_{ij}^{(s)}(\mathbf{k})\,(\partial_\eta \mp isk)\,(a u_k'(\eta))\, a_s(\mathbf{k}) + e_{ij}^{*(s)}(-\mathbf{k})\,(\partial_\eta \mp isk)\,(a u_k'(\eta))^*\, a_s^\dagger(-\mathbf{k}) \Big] e^{i\mathbf{k}\cdot\mathbf{x}} \,, \tag{2.9}$$

wher u_k is the mode function. The main utility of our formalism is that a concise expression (2.7) considerably simplifies calculations of bispectrum in a general expanding universe.

3 Parity violation

Using the formulation summarized in the previous section, we compute the graviton non-gaussianity. Let us first consider the simplest case in which the back ground spacetime is pure de Sitter. The mode function in a de Sitter background reads $u_k = Hk^{-1}(1+ik\eta)\,e^{-ik\eta}$, where H is a constant Hubble parameter. It is worth noting here the following property

$$\gamma_{ij}^{\pm}(\eta,\mathbf{k}) = \left(\gamma_{ij}^{\mp}(\eta,-\mathbf{k})\right)^\dagger \,. \tag{3.1}$$

This traces back to the property of helicity basis (2.4) and plays a key rôle in proving no parity violation in the exactly de Sitter universe.

Introducing the projection operators $\Pi^{\pm}_{ij,kl}(\mathbf{p}) = e_{ij}^{(\pm)}(\mathbf{p}) e_{kl}^{*(\pm)}(\mathbf{p})$, we can compute the bispectrum as

$$\begin{aligned} &\langle \gamma^{\pm}_{i_1j_1}(0,\mathbf{p}_1)\gamma^{\pm}_{i_2j_2}(0,\mathbf{p}_2)\gamma^{\pm}_{i_3j_3}(0,\mathbf{p}_3)\rangle \\ &= \pm 384 ib M_{\rm pl}^{-6} H^5 (2\pi)^3 p_1^2p_2^2p_3^2 \delta(\mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_3) \frac{5!}{(p_1+p_2+p_3)^6} \\ &\quad\times \Big[\Pi^{\pm}_{i_1j_1,kl}(\mathbf{p}_1)\Pi^{\pm}_{i_2j_2,lm}(\mathbf{p}_2)\Pi^{\pm}_{i_3j_3,mk}(\mathbf{p}_3) + \Pi^{\pm}_{i_1j_1,kl}(\mathbf{p}_1)\Pi^{\pm}_{i_2j_2,lm}(\mathbf{p}_2)\Pi^{\pm}_{i_3j_3,mk}(\mathbf{p}_3) \Big] \,. \end{aligned} \tag{3.2}$$

Note that the mixed terms $\langle\gamma^+\gamma^+\gamma^-\rangle$ and $\langle\gamma^-\gamma^-\gamma^+\rangle$ vanish as expected. These results are in agreement with those obtained by Maldacena and Pimentel [5].

Even if these correlators are nonvanishing, this does not immediately imply that the parity violation can be observed. This is due to the fact that neither $\langle(\gamma^+)^3\rangle$ nor $\langle(\gamma^-)^3\rangle$ themselves are direct observables. To see this more concretely, let us define the right-handed and left-handed circular polarizations by

$$h^R := h_{ij} e_{ij}^{*(+)} \,, \qquad h^L := h_{ij} e_{ij}^{*(-)} \,, \tag{3.3}$$

respectively. A possible observable quantity in which a parity violation is encoded is their difference $\langle(h^R)^3\rangle - \langle(h^L)^3\rangle$. Using the relation $h_{ij}(0,\mathbf{k}) = -Hk^{-2}[\gamma^+_{ij}(0,\mathbf{k}) + \gamma^-_{ij}(0,\mathbf{k})]$, It is straightforward to verify that

$$\langle h^R(0,\mathbf{p}_1)h^R(0,\mathbf{p}_2)h^R(0,\mathbf{p}_3)\rangle = 0 \,, \qquad \langle h^L(0,\mathbf{p}_1)h^L(0,\mathbf{p}_2)h^L(0,\mathbf{p}_3)\rangle = 0 \,. \tag{3.4}$$

It turns out that there exists no parity violation in a pure de Sitter universe.

Next, let us analyze the case in which the universe undergoes a slowly rolling inflationary expansion. Since the variable $\gamma_{ij}^{\pm}$ does not correspond to the helicity decomposition when the spacetime departs from de Sitter, we can expect the appearance of parity violation.

When we expand every term up to the first order in the slow roll parameter $\epsilon := -\dot{H}/H^2$ where H is now defined by $H := \dot{a}/a$, the possible sources of parity violation stem from the following three parts: (i) change in the asymptotic mode function, (ii) change of $\gamma_{ij}^{\pm}$ and (iii) change of the cosmic expansion. Putting all contributions together, we find that the contribution from (i) is higher-order. Hence the main cause of parity violation is due to (ii) and (iii). An elementary but lengthy calculation reveals that

$$\begin{aligned} h^L(0,\mathbf{p}_1)h^L(0,\mathbf{p}_2)h^L(0,\mathbf{p}_3)\rangle &= -\langle h^R(0,\mathbf{p}_1)h^R(0,\mathbf{p}_2)h^R(0,\mathbf{p}_3)\rangle \\ &= 6!(2\pi)^4 \epsilon (bH_*^2)(H_*/M_{\rm pl})^6 \delta^{(3)}(\mathbf{p}) \frac{(p_1+p_2-p_3)(p_2+p_3-p_1)(p_3+p_1-p_2)}{p^3(p_1p_2p_3)^2} \,, \end{aligned} \tag{3.5}$$

where $\mathbf{p} = \mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3$ is the total momentum. Hence parity violation shows up in the bispectrum and its magnitude is proportional to the slow roll parameter. It is interesting to observe that the bispectrum of curvature perturbations is also proportional to the slow-roll parameter in the conventional single inflationary scenario [3].

It should be stressed that this kind of parity violation can be observed in the CMB through the three-point correlators $\langle TTB \rangle$, etc. In the conventional slow-roll inflationary scenario, the amplitude might be too small to be detected in near future. However, in the non-conventional scenarios, we might have much larger parity violation.

4 Conclusions

We have developed a useful formalism to evaluate graviton correlation functions. The present formalism enables us to calculate higher-order parity violating correlation functions rather straightforwardly. As an illustrating application of this formalism, we studied parity violation in the early universe through non-gaussianity of gravitons and found that that no parity violation arises in the bispectrum for the exact de Sitter background. This does not mean that the $W^2 \star W$ term fails to produce any parity violating interaction. We have shown that the parity violation is not encoded in the graviton non-gaussianity. The situation is different when the spacetime departs from the exact de Sitter. In slow-roll inflationary case we have found parity violation in the graviton bispectrum proportional to the slow roll parameter. This is attributable to the fact that the boost generator in general FLRW spacetime is not the isometry of the background, thence both of the helicity components would mix except for de Sitter.

We also discussed that parity violation in the bispectrum can be observed e.g., in the $\langle TTB \rangle$ correlation in the CMB [8]. It might be also possible to detect the signature of parity violation through direct observations of primordial gravitational waves using a space interferometer observatory.

It is known that exact de Sitter correlation function is related to correlation function of stress tensor in the conformal field theory through analytic continuation [3]. Hence, the graviton non-gaussianity has an interesting application. Similarly, our results would be useful for calculating three-point functions of stress tensor in the non-conformal field theory using gravity/field theory correspondence [9].

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