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Hadron properties from lattice QCD

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Abstract. We discuss the status of current dynamical lattice QCD simulations in connection to the emerging results on the low-lying baryon spectrum, excited states of the nucleon and the investigation of the structure of scalar mesons.

1. Introduction

Simulations of lattice QCD are nowadays being performed with dynamical quarks with masses close to their physical ones. Such simulations with physical pions remove the need for a chiral extrapolation, thereby eliminating a significant source of a systematic uncertainty that has proved difficult to quantify in the past. Various fermion discretization schemes are being employed by various collaborations. MILC has recently presented results on the pseudoscalar decay constants using Highly Improved Staggered Quark (HISQ) ensembles with the strange and charm quarks fixed to their physical values and for a range of masses for the two light quarks ($N_f = 2 + 1 + 1$) including physical values of the light sea-quark masses (1). The BMW collaboration has produced results on the pion sector using $N_f = 2 + 1$ $\mathcal{O}(a)$ -improved Clover simulations employing HEX smeared links with light quark masses over a range of masses even below the physical pion mass for four lattice spacings (2). A number of other collaborations are using improved Wilson fermions to simulate with physical or near physical values of the two dynamical light quark masses, in some cases including a dynamical strange quark with mass fixed to its physical value. Clover gauge configurations have been produced by the QCDSF and PACS-CS collaborations and pion mass $m_\pi \sim 150$ MeV for $N_f = 2$ (3) and $N_f = 2 + 1$ with re-weighting to reach the physical pion value (4). The European Twisted Mass Collaboration (ETMC) has also generated $N_f = 2$ gauge configurations using twisted mass fermions including the clover term (5). Using these 'physical ensembles' one has now the possibility to study hadron properties directly.

2. Hadron spectrum

The first quantity that we would like to reproduce from lattice QCD are the masses of the low-lying hadrons. These are extracted from Euclidean correlation functions

$$G(\vec{q}, t_s) = \sum_{\vec{x}_s} e^{-i\vec{x}_s \cdot \vec{q}} \langle J(\vec{x}_s, t_s) J^\dagger(0) \rangle = \sum_{n=0, \dots, \infty} A_n e^{-E_n(\vec{q})t_s} \xrightarrow{t_s \rightarrow \infty} A_0 e^{-E_0(\vec{q})t_s} \quad (1)$$

in the large Euclidean time limit, after projecting to zero momentum, $\vec{q} = \vec{0}$.



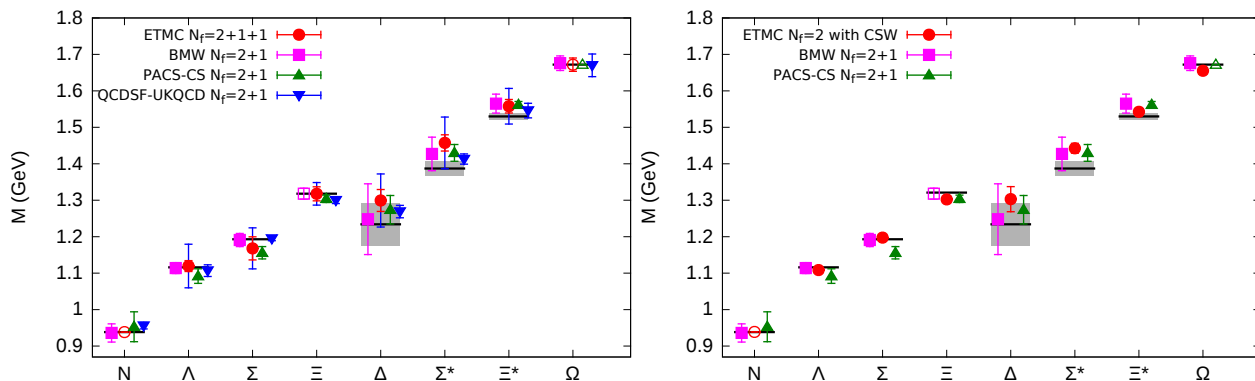


Figure 1. Lattice QCD results on the octet and decuplet baryon masses compared to the experimental values shown by the horizontal bands. Results by ETMC are shown in red circles. Left: using $N_f = 2+1+1$ ensembles after performing a chiral extrapolation (statistical errors are shown in red, whereas the blue error bar includes an estimate of the systematic errors due to the chiral extrapolation (6)). Right: for the physical ensemble (7). In both plots we also show results using clover fermions from BMW (8) (magenta squares), from PACS-CS (9) (green triangles), and from QCDSF-UKQCD collaborations (10) using $N_f = 2 + 1$ SLiNC configurations (blue inverted triangles). Open symbols show the baryon mass used as input to the calculations.

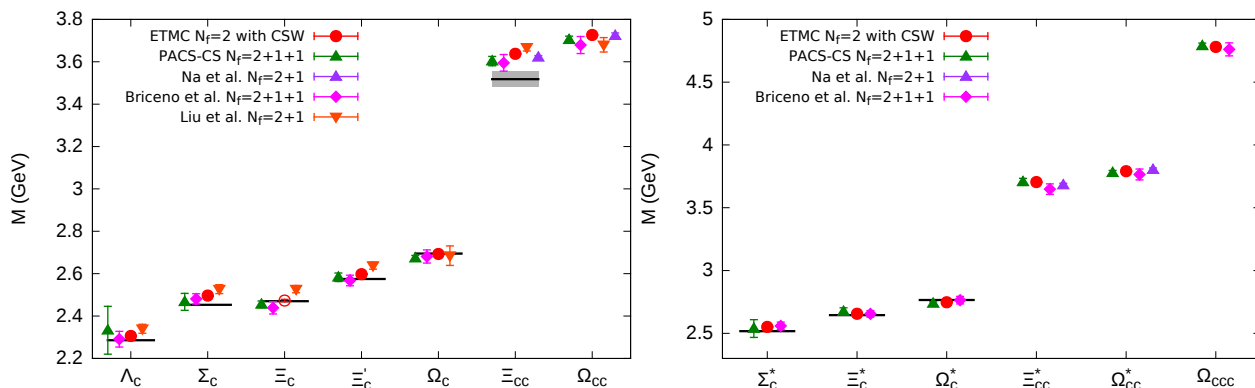


Figure 2. Results by ETMC are shown in red circles for the mass of the spin-1/2 (left) and spin-3/2 (right) charmed baryons for the physical ensemble. Included are results from various hybrid actions with staggered sea quarks from Refs. (11) (purple triangles), (12) (magenta diamonds) and (13) (orange inverted triangles). Results from PACS-CS (14) are shown in green triangles.

In Fig. 2 we show the spectrum of the octet and decuplet baryons. We show two sets of results using twisted mass fermions (TMF). One set is obtained with $N_f = 2 + 1 + 1$ gauge configurations at three lattice spacings, determined using the nucleon mass as $a = 0.094$ fm, 0.082 fm and 0.065 fm. Thus results can be extrapolated to the continuum limit. The continuum results are chirally extrapolated using heavy baryon chiral perturbation theory to leading and next to leading order. We take the difference between the two orders as an estimate of the systematic error due to the chiral extrapolation, which constitutes the biggest systematic error on the results as can be seen by the blue error bars (6). Results obtained at two different lattice volumes and showed no observable effect within our statistics and thus volume corrections were not performed. The other set of TMF results shown in Fig. 2 is obtained using simulations with physical values of the light quark masses (physical ensemble), thus requiring no chiral extrapolation, at one lattice spacing and volume. Both sets agree with experimental values,

indicating that finite lattice spacings effects for the physical ensemble are small (7). These results are now much more precise since the chiral extrapolation was not needed. In Fig. 2 we show the corresponding results for the mass of the charmed baryons for the physical ensemble of TMF. As can be seen, the known values of the masses of charmed baryons are reproduced and thus our computation provides a prediction for the yet unmeasured masses. Our preliminary results for the yet unmeasured mass of the Ξ_{cc}^* is 3.700(85) GeV, for the Ω_{cc}^+ is 3.726(42) GeV, for Ω_{cc}^{*+} 3.785(49) GeV and for Ω_{ccc}^{++} 4.778(33) GeV.

3. Nucleon excited states

Having successfully computed the mass of the nucleon and the other low-lying baryons from lattice QCD we turn to the question of the excited nucleon spectrum. We limit our discussion here to the two lowest states, namely the $P_{11}(1440)$ positive parity resonance known as the Roper and the negative parity state $S_{11}(1535)$. This is an example where the mass ordering of these states is contrary to the prediction of the constituent quark model where the negative parity state is expected to be lower in mass than the P_{11} state.

The method to study excited states is to use an enlarged basis of interpolating fields and construct a correlation matrix

$$G_{jk}(\vec{q}, t_s) = \sum_{\vec{x}_s} e^{-i\vec{x}_s \cdot \vec{q}} \langle J_j(\vec{x}_s, t_s) J_k^\dagger(0) \rangle, j, k = 1, \dots, N. \quad (2)$$

Solving the generalized eigenvalue problem (GEVP)

$$G(t)v_n(t; t_0) = \lambda_n(t; t_0)G(t_0)v_n(t; t_0) \rightarrow \lambda_n(t; t_0) = e^{-E_n(t-t_0)}. \quad (3)$$

yields asymptotically the N lowest eigenstates (15).

We have used a variational approach to study the excited states of the nucleon in the positive and negative parity channels (16). Our variational basis consisted of two types of nucleon interpolating fields with different levels of Gaussian smearings.

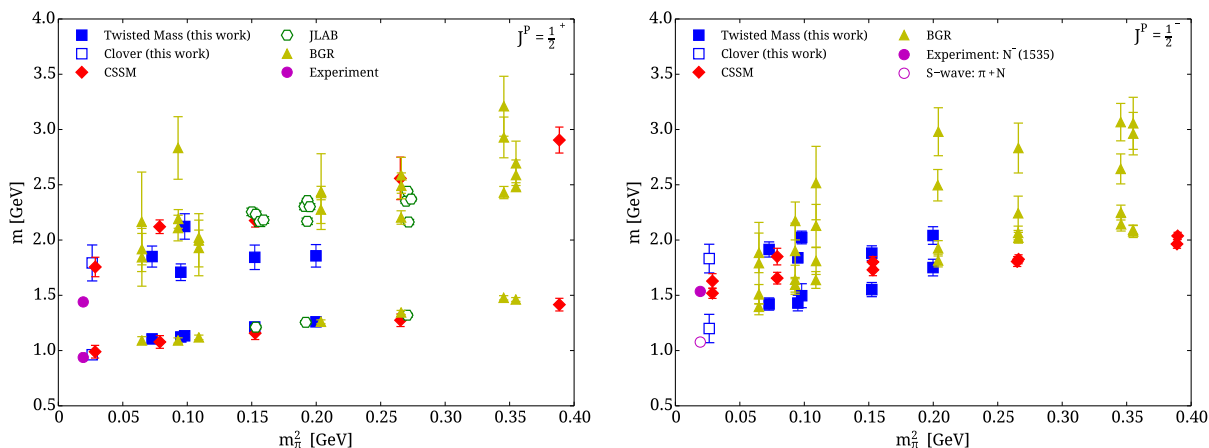


Figure 3. The positive (left) and negative (right) states of the nucleon. Filled and open squares are results using $N_f = 2$ twisted mass and clover fermions respectively. $N_f = 2 + 1$ Clover fermions by the CSSM collaboration (17) (red diamonds), by the Hadron Spectrum Collaboration (18) (open hexagons) using the Chirally Improved Dirac Operator by the Bern-Graz-Regensburg (BGR) collaboration (19) (yellow triangles) are also shown. The CSSM results on the negative parity states are from (20).

We analyzed a total of five ensembles of $N_f = 2$ twisted mass fermions with pion mass in the range of about 270 MeV to 450 MeV and lattice spacing $a = 0.089$ fm determined from the nucleon mass. For this value of the lattice spacing cut-off effects on the mass of the nucleon and hyperons were found to be smaller than the statistical errors. Therefore, we limit ourselves to studying only one lattice spacing. In addition, we analyzed an ensemble of $N_f = 2$ Clover fermions with pion mass $m_\pi \sim 160$ MeV and lattice spacing $a \simeq 0.073$ fm (3). Our results are shown in Fig. 3 for the positive and negative channels. Results from other lattice QCD computations are also included. The first observation is that all lattice results are in reasonable agreement for the ground state energies of both parity channels. The second major observation is that our data for the first excited state of the nucleon in the positive parity channel, although consistent at near physical pion mass with the other lattice calculation at similar pion mass, namely that from the CSSM Collaboration, yield a value that is still higher than the experimentally measured mass for the Roper. Given that our lattice volume is comparable to that of Ref. (17) volume effects can be responsible for the larger values. In the negative parity channel we can clearly see that for all pion masses considered the negative parity ground state is consistent with a πN state in an S-wave. To the statistical accuracy available to us, the first excited negative parity state appears to be converging to $N^-(1535)$, however the errors are too large to draw concrete conclusions. It is also apparent that the results on the higher states are much more spread and carry much larger errors, thus requiring further study.

4. Scalar mesons

Using the variational approach one can study scalar mesons. Our understanding of the light scalar ($J^P = 0^+$) meson sector below 1 GeV is still unclear. The observed mass ordering of the $f_0(980)$ and $a_0(980)$ states appears inverted from what would be naively expected from the conventional quark model. Instead, the interpretation as four-quark states provides an explanation of the inverted mass values of these scalars (21). In this four-quark scenario, the quark content will be given by

$$\begin{aligned} \sigma &= ud\bar{u}\bar{d}, & f_0 &= \frac{1}{\sqrt{2}}(us\bar{u}\bar{s} + ds\bar{d}\bar{s}), \\ a_0^- &= ds\bar{u}\bar{s}, & a_0^0 &= \frac{1}{\sqrt{2}}(us\bar{u}\bar{s} - ds\bar{d}\bar{s}), & a_0^+ &= us\bar{d}\bar{s} \\ \kappa^+ &= ud\bar{d}\bar{s}, & \kappa^0 &= ud\bar{u}\bar{s}, & \bar{\kappa}^0 &= us\bar{u}\bar{d}, & \kappa^- &= ds\bar{u}\bar{d} \quad . \end{aligned} \quad (4)$$

Within this interpretation, the mass degeneracy of $f_0(980)$ and $a_0(980)$ is natural and the mass ordering is understandable. The broad width of the σ and κ can be also easier explained since the decay channels to $\pi\pi$ and $K\pi$ respectively are OZI super-allowed.

A number of lattice QCD studies of scalar mesons have been undertaken (22–25). Our on-going investigation of the low-lying scalar nonet (26) focuses first on the study of the $a_0(980)$ in an effort to shed light on its structure from QCD. Our variational basis consists of the conventional two quark operator, as well as, different types of four quark operators, corresponding to two mesons or a diquark-antidiquark structure. Including the two quark operator inevitably means that there are diagrams which contain closed fermion loops, which makes the study even more challenging. Preliminary results are obtained using a 4 correlation matrix and $N_f = 2 + 1$ clover gauge configurations generated by the PACS-CS collaboration (27). The lattice size is $32^3 \times 64$ with lattice spacing $a \approx 0.09$ fm. Two ensembles are analyzed corresponding to $m_\pi \approx 300$ MeV, and $m_\pi \approx 150$ MeV. Results are still preliminary and currently we are improving our calculations. Including explicitly scattering states for the $\eta_{ss} + \pi$ and $K + \bar{K}$ type with explicit projection to zero momentum is expected to improve our ability to isolate the $a_0(980)$ state.

5. Conclusions

Simulations at the physical point are now becoming available enabling the computation of the masses of the low-lying hadron without the need of chiral extrapolation. This eliminates an up to now ill-determined systematic error from lattice QCD evaluations. Results on other benchmark quantities such as the nucleon axial charge g_A , and momentum fractions $\langle x \rangle_{u-d}$ are also being computed directly at the physical point (7; 28). We will need high statistics and careful cross-checks to finalize results on these quantities at the physical point. The study of excited states and resonances is under way but many challenges need still to be resolved.

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