Magnetic white dwarfs: Observations, theory and future prospects

Enrique García-Berro

*Departament de Física, Universitat Politècnica de Catalunya,
c/Esteve Terrades 5, 08860 Castelldefels, Spain*

*Institute for Space Studies of Catalonia, c/Gran Capità 2–4,
Edif. Nexus 201, 08034 Barcelona, Spain
enrique.garcia-berro@upc.edu*

Mukremin Kilic

*Homer L. Dodge Department of Physics and Astronomy,
440 W. Brooks Street, Norman, OK 73019, USA*

Souza Oliveira Kepler

*Instituto de Fisica, Universidade Federal do Rio Grande do Sul,
91501-900 Porto-Alegre, RS, Brazil*

Isolated magnetic white dwarfs have field strengths ranging from 10^3 G to 10^9 G, and constitute an interesting class of objects. The origin of the magnetic field is still the subject of a hot debate. Whether these fields are fossil, hence the remnants of original weak magnetic fields amplified during the course of the evolution of the progenitor of white dwarfs, or on the contrary, are the result of binary interactions or, finally, other physical mechanisms that could produce such large magnetic fields during the evolution of the white dwarf itself, remains to be elucidated. In this work, we review the current status and paradigms of magnetic fields in white dwarfs, from both the theoretical and observational points of view.

Keywords: Stars; stars: white dwarfs; magnetic fields.

1. Introduction

Isolated magnetic white dwarfs have field strengths ranging from 10^3 to 10^9 G, and are about 10% of the total population of single white dwarfs, although the precise percentage is still the subject of some debate. Specifically, the percentage of single magnetic white dwarfs in volume-limited surveys[1] is typically 15%, whereas in magnitude-limited samples[2] this percentage decreases to about 4%. The number of white dwarfs with well determined magnetic fields has increased noticeably with the advent of large scale surveys, of which the Sloan Digital Sky Survey (SDSS)[3] is the leading example. The pioneering detections of magnetic fields in single white dwarfs[4] were done in the mid 30s, and opened a new field of research. These early discoveries were followed by more studies, which allowed us to increase the sample of single white dwarfs with measured magnetic fields to about a few dozens. However, from an observational point of view, the largest breakthrough in the search for magnetic white dwarfs arrived with the advent of large, automatic, systematic surveys. In particular, the SDSS has allowed us to unveil a population of about 600 magnetic white dwarfs.[5]

Despite being this an interesting field of research, because of its many applications to other research areas — of which we mention the field of cataclysmic variables, to give just one example — the impressive advance in the observational side, has not been followed by theory, which remains one step behind. This lag is partially due to the intrinsic difficulty of modeling magnetic fields. Indeed, modeling magnetic fields is a tough endeavour, as in most cases it requires full three-dimensional (3D) simulations. Consequently, in many applications crude simplifications are done. However, this is not the only reason why we still do not have a comprehensive and complete picture of magnetic white dwarfs. In particular, we do not have a full evolutionary picture of the progenitors of magnetic white dwarfs. The two main hypothesis are the following ones. Either magnetic fields are inherited from a weak magnetic field of the progenitor star — the so-called fossil field hypothesis — or are originated by the evolution in a binary system. Both hypothesis have advantages and drawbacks, and no definite consensus about this issue has been reached so far. We discuss them in detail in Sec. 3 below. Additionally, there are other competing scenarios which challenge those two previously mentioned, which are also examined in the same section.

Here we briefly review the current status and paradigms of magnetic fields in white dwarfs, from both the theoretical and observational points of view. However, we first would like to draw the attention of our reader to the excellent and recent review of Ref. 6, where a very thorough and in depth examination of our current understanding of the research field was done. Our work is organized as follows. Section 2 is devoted to summarize the most relevant observational characteristics of the population of isolated magnetic white dwarfs. Specifically, in Sec. 2.1 we pay attention to the mass distribution of magnetic white dwarfs, while in Sec. 2.2 we discuss their rotational periods. In Sec. 3, we critically review the proposed scenarios to explain the presence of magnetic fields. Later, in Sec. 4, we elaborate on some of the practical applications of the field. Finally, in Sec. 5, we summarize the main results and propose some interesting future research lines. Before going into details we would like to state that the selection of papers for explicit citation may be somewhat incomplete, for several reasons. The first one is that the field is rapidly evolving, the second one is because of space limitations, and finally the last reason is that this selection of references is the product of the own special research trajectory of the authors. While we have tried to be as complete as possible, we are well aware that this has not been possible. Hence, we apologize in advance for any unintentionally missed references.

2. Observations of Magnetic White Dwarfs

Most magnetic white dwarfs belong to the so-called DA spectral type, that is, they have hydrogen-rich atmospheres. The reason for this is that more than 80% of normal white dwarfs also belong to the DA spectral class. Thus, the measurements of the magnetic field strength in the majority of the cases rely on the spectroscopic

determination of the Zeeman splitting of the Balmer series of hydrogen. It can
be easily proved that for sufficiently low magnetic field strengths the splitting of
these lines depends linearly on the magnetic field strength. This allows to place
upper limits on the existence of magnetic fields as small as 10^5 G. However, as
the field strength increases nonlinear terms become more and more important and
the determination of the field strength becomes more complicated. Specifically,
quadratic terms are important for field strengths of the order of ~ 1 MG, and for
larger magnetic field strengths the situation becomes even more complicated, as the
subcomponents of the spectroscopic lines intermix in wavelength. Nevertheless, as
of today we have a handful of isolated magnetic white dwarfs for which we have
reliable determinations of magnetic field strengths as large as 800 MG.[6] Another
technique frequently used, because its value in detecting very strong magnetic fields,
consists of measuring the continuum circular polarization.[7] However, this technique
is demanding observationally and is only useful for white dwarfs with magnetic fields
strengths exceeding 10^8 G.

Observations show that the population of isolated magnetic white dwarfs has
two significant general properties. The first of these is that apparently there is no
clear correlation between the magnetic field strength and the effective temperature.
This would mean that the field does not evolve appreciably along the white dwarf
cooling track. Nonetheless, this is still a controversial issue. In particular, it is
worth mentioning that recent observations[8] have demonstrated that the mean field
increases at the effective temperature at which the partially degenerate envelope
becomes convective. Whether this effect is significant deserves further scrutiny.
The fact that the field does not evolve along the cooling sequence (if indeed this
is the case) can be well explained by simply computing the ohmic timescale. This
timescale is defined as $t_{\mathrm{ohm}} \sim 4\pi\sigma L^2/c^2$, where L is the typical scale length for the
variation of the magnetic field inside the star, and σ is the electric conductivity.
Adopting $L \simeq R$ and typical values for σ, it can be shown[9] that the ohmic timescale
is indeed very long (of the order of 10^{11} yr). The second important general property
is that the topology of the magnetic field can be very complicated[10–12] in most
of the cases. However, in practice and for the sake of simplicity when no more
information is available, it is customary to simply assume that the field geometry
is dipolar.

There are as well some magnetic white dwarfs with hydrogen-deficient atmo-
spheres. A few of them have helium-rich atmospheres and show HeI lines in their
spectra. For these white dwarfs the field strength is thus determined using atomic
helium lines, a much more difficult task.[13] There is another group of white dwarfs
with hydrogen-deficient atmospheres with significant carbon abundances, the so-
called DQ white dwarfs. In some of these enigmatic white dwarfs field strengths
of the order of 100 MG have been measured using spectropolarimetry.[14] However,
this is not the only class of white dwarfs which show enhanced carbon abundances.
Besides this group of white dwarfs which are essentially cool, there is a recently

discovered[15] population of hot DQs, which have carbon-dominated atmospheres. It is intriguing that about half of them are magnetic white dwarfs. Finally, there is a distinct group of magnetic white dwarfs, known as magnetic DZ white dwarfs, with metals in their atmospheres.[16–18]

2.1. *Mass distribution of magnetic white dwarfs*

The determination of masses of magnetic white dwarfs is a tough task, because of the inherent difficulties in modeling the line profiles in the presence of a magnetic field. In particular, we lack theoretical models allowing to model accurately pressure broadening for large magnetic fields. According to this consideration, we only have reliable mass determinations for a reduced subset of all magnetic white dwarfs[19], whereas for most high-field white dwarfs the mass determination is somewhat uncertain. However, a characteristic trend emerges from observations. In particular, it turns out that the average mass of isolated high-field magnetic white dwarfs — namely, those with magnetic fields larger than $1\,\mathrm{MG}$ — is substantially larger than that of single field white dwarfs. Specifically, the population of high-field magnetic white dwarfs has a mean mass of $0.784 \pm 0.047\,M_\odot$,[20] whereas the average mass of single nonmagnetic white dwarfs is $0.643 \pm 0.136\,M_\odot$,[21] clearly pointing towards a different evolutionary channel for these white dwarfs.

2.2. *Rotational periods*

The vast majority of magnetic white dwarfs rotate slowly, as it occurs for nonmagnetic white dwarfs.[22] Specifically, the rotation periods of isolated magnetic white dwarfs encompass a wide interval,[23,24] with a lower limit of $\sim$700 s, whereas for some magnetic white dwarfs the measured rotation periods are actually much longer, on the order of about 100 yr. As a matter of fact, there is weak evidence for a bimodal distribution of rotation periods, with a handful of magnetic white dwarfs with periods clustered around hours, and a second (more numerous) subset of stars with periods much longer than this value, typically hundreds of years. We note that this is a crucial issue, since it would allow us to discern the progenitors of magnetic white dwarfs (see next section). However, until now the intrinsic difficulties of measuring accurate periods using photometry and polarimetric variability have hampered the efforts to provide a definite answer to this problem.

3. The Origin of the Magnetic Field

The search for the progenitors of magnetic white dwarfs is an active field of research, and unfortunately no consensus on this issue has been reached yet. Generally speaking, the evolutionary scenarios giving rise to the known population of magnetic white dwarfs should be able to explain three well established observational features of their ensemble properties. The first one is that high-field magnetic white dwarfs are usually more massive than their nonmagnetized counterparts — see Sec. 2.1.

The second important observational fact is that most magnetic white dwarfs are slow rotators — see Sec. 2.2. Finally, there is another interesting observational fact that deserves close attention. For nonmagnetic white dwarfs there exists a well known population of binaries in which one of the members of the pair is a main-sequence star, while the other one is a white dwarf.[25] Realistic population synthesis models are able to reproduce the most relevant properties of well characterized samples.[26,27] Thus, it is commonly assumed that the scenarios that produce this population are relatively well understood, although much work still remains to be done. However, magnetic white dwarfs are predominantly single stars.[28] Even more, it is found that, surprisingly, the white dwarf companion in cataclysmic variables is magnetic in about 25% of the systems. All this strongly suggests that binarity plays a key role in explaining the origin of at least some fraction of the presently observed population of magnetic white dwarfs. There are two competing scenarios which may eventually explain the formation of magnetic white dwarfs. These are the fossil field hypothesis and the binary scenario. In the following, we examine them separately.

3.1. *The fossil field hypothesis*

We start describing the fossil field hypothesis.[29,30] Within this evolutionary channel the magnetic field of white dwarfs is simply the consequence of the evolution of a single progenitor along all the standard stellar evolutionary phases. Specifically, within this scenario magnetic white dwarfs descend from rotating Ap and Bp stars, which are the only class of main-sequence stars known to have substantial magnetic fields, between 10^3 and 10^5 G. If the effects of mass loss are neglected and we further assume that magnetic flux is conserved it is easy to show that the field will be amplified by a factor of $\sim 10^4$ when the progenitor becomes a white dwarf. Even if the assumptions are relaxed, and a significant amount of magnetic flux is carried away by mass loss during advanced evolutionary stages before a white dwarf is formed, it is expected that the magnetic field of the resulting white dwarf would be comparable to those typically found in magnetic white dwarfs. However, this scenario faces a serious drawback. Specifically, it is not able to explain why there are not magnetic white dwarfs in post-common envelope binaries with a main-sequence companion of spectral type K or M.

3.2. *The binary hypothesis*

Within the second scenario, the so-called binary hypothesis, the magnetic field arises from the interaction of the future magnetic white dwarf with a companion during its previous evolution. This evolutionary channel has been the subject of much recent attention, and there are several variants of the scenario. For instance, it has been suggested[31,32] that strong magnetic fields are produced during a common envelope episode in a close binary system in which one of the components of the pair is degenerate. During this phase, spiral-in of the secondary induces differential rotation

in the extended convective envelope, resulting in a stellar dynamo that produces the magnetic field. However, it has also been shown[33] that the magnetic field produced in this way does not penetrate into the white dwarf, and it decays rapidly when the common envelope is ejected. It has been also recently demonstrated[34] that the hot, differentially rotating convective corona resulting from the merger of two degenerate cores[35] produces strong magnetic fields that are confined to the outer layers of the resulting remnant and do not decay for very long timescales. Indeed, detailed 3D numerical simulations[36] using state-of-the-art computer codes have shown that the a very small magnetic field is amplified during the merger episode and that the remnant of the merger is strongly magnetized. Hence, this evolutionary channel would explain some of the gross properties of the population of high-field magnetic white dwarfs. It might be argued that this scenario might be in conflict with the observational fact that most magnetic white dwarfs are slow rotators. However, it has been also recently shown[37] that coupling between the magnetosphere and the debris region resulting from the disruption of the secondary star during the merger episode can brake the magnetized white dwarf and bring the rotational periods to values comparable to those observationally found. Finally, we mention that there is additional observational evidence supporting the binary hypothesis. In particular, recent large-scale searches[38] for magnetic central stars of planetary nebula and hot subdwarfs have confirmed previous suggestions that the these stars are basically nonmagnetic.[39–41]

3.3. *Population synthesis studies*

Population synthesis studies are crucial to discern the origin of magnetic white dwarfs. It is important to do this because in the first case the number of mergers in the Solar neighborhood is an important piece of evidence in determining if this evolutionary channel might provide enough progenitors to explain the number of high-field magnetic white dwarfs in a volume limited sample, whereas for the second case the number Ap and Bp stars, due to its intrinsic scarcity, may not be sufficient to explain the fraction of single magnetic white dwarfs. According to these considerations, the predictions of such studies for both the binary scenario and the fossil field evolutionary channel have been compared to observations in recent years. Moreover, dedicated surveys have provided us with a number of binary systems which potentially will merge within a Hubble time, and this can be directly compared to the predictions of the theoretical models, allowing in this way to test our models for the binary scenario.

We start this section, by examining the statistics of the fossil field evolutionary channel. As a matter of fact, it was early recognized that the number of strongly magnetic Ap and Bp stars could be insufficient to explain the observed incidence of magnetism in white dwarfs. Recent studies[42] have argued that these stars could not be the only progenitors of magnetic white dwarfs, because the birth rate of such stars is not enough to explain the number of observed magnetic white dwarfs.

Nevertheless, more recent studies[30] have concluded that this problem can be easily overcome by taking into account that about 40% of late type B stars have undetectable magnetic fields. This fraction of magnetic main-sequence stars would be enough to reconcile theory and observations of magnetic Ap and Bp stars with distances smaller than 100 pc.[43] Moreover, it is observationally found that the incidence of magnetism in A and B stars increases with mass. All this precludes from discarding the fossil field evolutionary channel. Even more, it is quite plausible that at least some magnetic white dwarfs have this kind of progenitors.

We now turn our attention to the binary scenario. The most recent studies of this kind[34,44] agree in predicting that a sizable fraction of magnetic white dwarfs can be explained by this scenario, provided that several types of coalescences are considered. To better illustrate this, Fig. 1 shows the frequency distribution of remnant masses of the different merger channels for a sample of 10^3 mergers. In this frequency distribution all the remnants with masses larger than the Chandrasekhar limiting mass have been removed. As can be seen, the total mass distribution (open histogram) presents a first peak for masses smaller than $\sim 0.4\,M_\odot$, corresponding to mergers in which a helium white dwarf is produced, then sharply increases for increasing remnant masses and afterwards smoothly decreases for masses larger than $\sim 0.6\,M_\odot$. When the theoretical distribution is sampled for ~ 14 objects — the total number of magnetic white dwarfs within 20 pc, see below — fairly flat distributions are obtained for masses ranging from $0.8\,M_\odot$ to $1.4\,M_\odot$.

We now discuss the statistics of the local sample. Within 20 pc of the Sun there are 122 white dwarfs,[45] and several of them are magnetic.[42] This sample is 80%
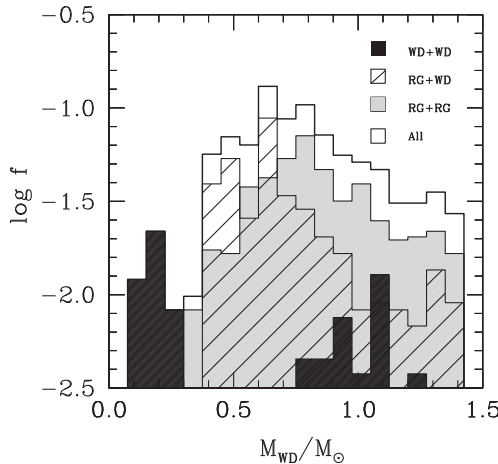

Fig. 1.   Mass distribution of the remnants of several merger channels in the Solar neighborhood, from Ref. 29. The different histograms show the frequency of the merger channels considered here. Specifically, the black histogram shows the masses of the remnants of the mergers of double white dwarf binaries, the dashed histogram that of the mergers of a binary system composed of a red giant and a white dwarf, the shaded histogram that of the mergers of two red giants, while the total mass distribution is shown using a solid line.

complete, but still suffers from poor statistics. However, it is useful because for it we have a reliable determination of the true incidence of magnetism in white dwarfs. Mass determinations are available for 121 of these white dwarfs, and there are 14 magnetic white dwarfs. Of these, 8 have magnetic fields larger than 10^7 G, and 3 have masses larger than $0.8 \, M_\odot$ — a value which is $\sim 2.5\sigma$ away from the average mass of field white dwarfs. The selection of this mass cut is somewhat arbitrary but, given the strong bias introduced by the initial mass function, it is expected that the vast majority of high-field magnetic white dwarfs more massive than $0.8 \, M_\odot$ would be the result of stellar mergers.

The population synthesis calculations predict that ~ 4 white dwarfs are the result of double degenerate mergers, and have masses larger than $0.8 \, M_\odot$, in good agreement with observations. This has to be compared with the fraction of white dwarfs more massive than $\sim 0.8 \, M_\odot$ resulting from single stellar evolution, which is $\sim 10\%$. Consequently, the expected number of massive white dwarfs in the local sample should be ~ 12. Instead, the local sample contains 20, pointing towards a considerable excess of massive white dwarfs, which could be the progeny of mergers. The rest of the population of magnetic white dwarfs (~ 5) could be the result of the evolution of single stars[46,47] — see above.

Finally, we mention that the number of coalescing binaries previously discussed compares well with the results obtained using very different population synthesis codes. In summary, we are confident that a substantial fraction of high-field magnetic white dwarfs should be the result of stellar mergers.

3.4. *Assessing the birth rates*

The rate of double degenerate mergers has been the subject of much attention recently, because of its implications on different areas of high energy astrophysics. Among them we mention explicitly the following ones. The coalescence of two white dwarfs is one of the possible scenarios to account for Type Ia supernovae outbursts.[48,49] It is thought as well that the merger of two degenerate cores could lead to the formation of magnetars.[50] Also, the hot and massive white dwarf members of the Galactic halo could be the result of the coalescence of a double white-dwarf binary system.[51,52] Additionally, hydrogen-deficient carbon and R Corona Borealis stars[53–55] are thought to be the consequence of the merging of two white dwarfs. Also, the relatively high photospheric metal abundances of some hydrogen-rich white dwarfs with circumstellar disks around them could also be explained by the merger of a carbon–oxygen and a helium white dwarf.[57] However, we note that not all massive white dwarfs with large metal abundances show significant infrared excesses, and thus it is unlikely that they harbor disks around them.[58] AM Canum Venaticorum systems are as well thought to be the consequence of a merger, as also are single subdwarf B/O stars.[56] Last but not least, the phase previous to the coalescence of a double white-dwarf close binary system has been shown to be a powerful source of gravitational waves that would be eventually detectable

by LISA.[59] In this section, we review the status of the field in the context of the scenarios leading to the formation of magnetized white dwarfs.

The two most significant efforts to find close binary systems in which both components of the pair are degenerate are the ESO supernovae Ia progenitor survey (SPY)[60,61] and the extremely low mass (ELM) white dwarf survey.[62–64] Both are dedicated surveys, and adopt different observing strategies. Specifically, the SPY survey is a magnitude limited survey aimed at searching for double-degenerates, whereas the primary aim of the ELM survey is to search for binary systems containing a low-mass white dwarf. Both surveys have provided us with an invaluable wealth of observational data, consisting of several dozens of new double-degenerate systems, which allows us to compare the results of the population synthesis models described before with the observed distributions. To the findings of these surveys the several double-degenerate systems found serendipitously in the SDSS must be added.

In synthesis, the main result of these observational efforts is that, as of today, none of the surveys has been able to find a progenitor system for Type Ia supernovae. That is, none of the surveys has found yet a double-degenerate system with a total mass larger than the Chandrasekhar limiting mass that will merge in less than a Hubble time. However, there are other interesting results that are more suitable for our interests. We focus primarily on the ELM survey, because it is the most recent one. The observed distribution of periods peaks at around half-a-day, and follows a lognormal distribution.[65] Also, the mass distribution of the low-mass companion peaks at $\sim 0.2\,M_\odot$ with a very narrow dispersion, whereas the mass distribution of the massive companion follows a Gaussian law, which peaks at $\sim 0.75\,M_\odot$, with a relatively large dispersion of about $\sim 0.25\,M_\odot$. The estimated birth rate of these systems, once corrected for the observational biases and selection effects, is $\sim 4.0 \times 10^{-3}\,\mathrm{yr}^{-1}$. However, the birth rate of systems that will end up their evolution as R Coronae Borealis stars is $\sim 3.0 \times 10^{-3}\,\mathrm{yr}^{-1}$,[66,67] those giving rise to AM Canum Venaticorum systems is $\sim 1.0 \times 10^{-4}\,\mathrm{yr}^{-1}$,[68] and that of systems that will produce underluminous supernovae is $\sim 1.0 \times 10^{-4}\,\mathrm{yr}^{-1}$.[69] Thus, the birth rate of systems that would eventually produce magnetic white dwarfs is considerably smaller, $\sim 8.0 \times 10^{-4}\,\mathrm{yr}^{-1}$. These estimates are consistent with the theoretical expectations described in the previous section, and with the merger rates derived by analyzing the SDSS.[70]

A tantalizing possibility is that some R Coronae Borealis stars turn into magnetic white dwarfs. Whether this is possible remains to be assessed. However, if true, the contribution of mergers to the birth rate of magnetic white dwarfs may be higher than previously thought, and certainly larger than the current estimates, $8.0 \times 10^{-4}\,\mathrm{yr}^{-1}$. Given the $4.0 \times 10^{-3}\,\mathrm{yr}^{-1}$ merger rate from the ELM Survey, it can argued that there are enough mergers in the Solar neighborhood to explain magnetic white dwarfs.

3.5. *A new scenario*

Finally, we mention that a new evolutionary scenario is progressively emerging[71]. The basic assumption within this scenario hinges on the observational fact that the fraction of magnetic white dwarfs seems to increase for decreasing luminosities.[72] Thus, it could be well possible that the magnetic fields of low-magnetized white dwarfs could be originated by an internal physical process. Specifically, the number of single magnetic white dwarfs increases abruptly for luminosities $\log(L/L_\odot) \lesssim -3.5$. Interestingly, for an otherwise typical white dwarf of mass $\sim 0.6\,M_\odot$ this luminosity corresponds to a core temperature of $\sim 10^6\,\mathrm{K}$, which is the temperature at which crystallization sets in.[73,74] This strongly suggests that the convective mantle[75,76] that results from carbon–oxygen phase separation[77,78] upon crystallization would produce a stellar dynamo resembling closely that occurring in the interior of Solar system planets.[79] Actually, it can be shown that the energy involved in the Rayleigh–Taylor unstable region is the same as what is needed to explain low-field magnetic white dwarfs, namely those with magnetic field strengths smaller than 0.1 MG. Nonetheless, this is still a preliminary model that should be further developed and their predictions should be compared with observations.

4. Applications

In addition to their obvious and numerous applications to astrophysical phenomena occurring in cataclysmic variables, the theory of magnetic white dwarfs also has many interesting applications that deserve to be mentioned. In the following, we detail some of them.

4.1. *Anomalous X-ray pulsars*

One of the possible applications of these types of studies is that high-field magnetic white dwarfs could explain the properties of at least a fraction of anomalous X-ray pulsars. This class of pulsars shares some similarities with short gamma-ray repeaters, which radiate short ($\approx 100\,\mathrm{ms}$), repeating bursts of soft γ- and X-rays at irregular intervals. In particular, their rotation periods cluster between 2 and 12 s, have large magnetic fields, and have quiescent X-ray luminosities of the order of $10^{35}\,\mathrm{erg\,s^{-1}}$. The traditional explanation for this enigmatic class of pulsars is that they are magnetars.[80,81] Nevertheless, there are alternative models that challenge this picture. In particular, following early suggestions[82,83] it has been recently proposed[84] that these sources can be better explained assuming that the object is a rapidly rotating highly magnetized white dwarf. Recent calculations[85] have shown that this model can explain the properties of the anomalous X-ray pulsar 4U 0142+61, thus making this a very suggestive formation scenario for these objects.

4.2. *Millisecond pulsars*

Millisecond pulsars are a distinct subset of the known population of pulsars. They have magnetic fields with strengths ranging from 10^8 to 10^9 G, somewhat smaller than the rest of radio pulsars, which have magnetic fields up to 10^{13} G. Moreover, they are frequently found in binary systems. Actually, $\sim75\%$ of them have companions.[86]

It is generally accepted that millisecond pulsars are neutron stars that were originated in a core-collapse supernova event in a binary system. Within this evolutionary scenario the massive star that will eventually yield the newborn neutron star accretes material from the companion, and the system is detected as a low-mass X-ray binary.[87] In this case the magnetic field is originated by the standard recycling hypothesis. That is, the field is constrained deep in the superconducting core of the neutron star. However, the large prevalence of low-field millisecond pulsars in binary system has brought into question the standard evolutionary scenario. One of the possibilities is that these pulsars are formed by accretion-induced collapse of an oxygen–neon white dwarf[88]. In this evolutionary route an accreting massive white dwarf, with a core made of oxygen and neon, reaches the threshold density to enable electron captures on ^{24}Mg and ^{24}Na first, and later on ^{20}Ne and ^{20}F, to finally ignite Ne and O explosively at central densities higher than $\sim2 \times 10^{10}\,\mathrm{g\,cm^3}$. At these very high central densities, fast electron captures occurring on the nuclear statistical equilibrium material would rapidly drive the Chandrasekhar mass below the actual mass of the degenerate core and, consequently, gravitational collapse would ensue, leading to the formation of a neutron star.[89,90] Within this formation route white dwarfs with initially small magnetic fields (of about 10^4 G) can explain naturally the observed properties of these pulsars, by simply assuming that the magnetic field is amplified by flux conservation. The observational counterparts in this scenario would also be low-mass X-ray binaries.

Recent population synthesis studies[86] have examined both possibilities and have concluded that the birth rates of binary millisecond pulsars formed through accretion-induced collapse are comparable to and can exceed those for core collapse, but these types of studies are not yet conclusive, so clearly more efforts need to be pursued.

4.3. *Magnetic double degenerates*

Magnetic double degenerates are rare systems, especially if the two degenerate stars form a wide pair. However, these binary systems have the advantage of allowing us to study the origin of the magnetic field. In these systems the components of the binary are sufficiently separated to have evolved independently, so the age of the system and the distance can be evaluated studying the nonmagnetic companion.[91] However, very few systems of this type are known. Among them we mention the following ones. RE J0317-853[92], was discovered by ROSAT and is relatively close to us,[19] thus allowing for accurate measurements. The pair is composed by a mas-

sive white dwarf of mass $\sim$0.85 $M_\odot$ and an ultramassive white dwarf of unknown mass which has a magnetic field of $\sim$450 MG.[93] Another example is the pair formed by PG 1258+593 and SDSS J130033.48+590407.0. In this case both white dwarfs have nearly equal normal masses, $\sim$0.54 $M_\odot$. The magnetic component has a field strength of 6 MG, and is the cool component of the system. To them we add two recent new discoveries: SDSS J092646.88+132134.5 + J092647.00+132138.4 and SDSS J150746.48+521002.1 + J150746.80+520958.0. The white dwarfs in these systems are more massive than usual in field white dwarfs. All these binary systems are common proper motion pairs. However, there are also systems for which the components are not well resolved. This is the case of LB 11146 which we know is a close binary system,[94,95] a characteristic shared with similar systems, like RE J1439+75,[96] and G62–46.[97] With these very few systems it is difficult to reach definite conclusions, but this is a promising line of future research.

4.4. *Magnetic white dwarfs and type Ia supernovae*

Type Ia supernovae are one of the most energetic explosive events in the cosmos. Since there is a relationship linking its intrinsic brightness and the shape of their light curves and they can be detected at very large distances they can be used as standardizable cosmological candles. This has opened a new era in cosmology, and has enabled us to discover the acceleration of the universe,[98,99] and to determine the cosmological parameters.

Despite their importance, we still do not know the nature of the progenitors of Type Ia supernovae, which remains a long-standing mistery. We do know that the outburst is powered by the explosion of a carbon–oxygen white dwarf in a binary system, but we do not know the precise mechanism that destabilizes the white dwarf, and several hypothesis have been put forward. In the so-called single-degenerate channel, accretion from a nondegenerate companion onto the primary companion leads to the formation and explosion of Chandrasekhar-mass white dwarfs. However, recent observational evidence suggests that a diversity of progenitors exists, including a significant population of sub-Chandrasekhar and super-Chandrasekhar mass systems.[100,101] Therefore, alternatives have been proposed. The most widely accepted competing model consists of the merger of a binary white dwarf system.[48,49] This is known as the double-degenerate channel. However, there are other alternative scenarios. These include the core-degenerate channel[102–106], and the white-dwarf collisional scenario.[107–111] Here, for obvious reasons, we focus on the double-degenerate channel.

The double-degenerate channel offers natural explanations to a variety of observational facts, including the absence of Hα in the nebular phase,[112] and the delay time distribution.[113,114] However, this scenario also has several major shortcomings that need to be addressed. The most recent theoretical works have paid attention primarily to the violent merger mechanism.[115–119] This mechanism is based on the behavior found in extensive numerical simulations of the final phases of the coales-

cence. During these phases the secondary star is tidally disrupted and is rapidly accreted onto the primary in a few dynamical timescales. In contrast, the primary star remains almost intact. However, not all the mass of the disrupted secondary is accreted onto the primary. In fact, all simulations predict that a hot, virialized accretion disk surrounding the primary, with a mass of about half of the mass of the secondary, is formed,[35,120–122] while the remaining mass is indeed accreted and forms a hot, convective corona.[34] This region is prone to magneto-rotational instability. The early suggestions that this mechanism could give rise to powerful magnetic fields has been recently confirmed using full 3D magneto-hydrodynamic calculations,[36,123,124] but unfortunately the only simulations done so far do not encompass massive enough white dwarfs, a requisite to produce a powerful detonation.[125]

Magnetic fields most likely play a crucial role in explaining some properties of Type Ia supernovae. However, despite this potentially important interest, very few studies have addressed this issue, and much work still remains to be done. For instance, the characteristics of some overluminous supernovae, with nickel masses larger than $1.0\,M_\odot$ like SN 2003fg, SN 2006gz, SN 2007if and SN 2009dc, might be explained if a sufficiently large magnetic field is present.[126,127] However, modeling these super-Chandrasekhar explosions requires taking into account not only the effects of the magnetic pressure, but also dealing with general-relativistic corrections. A full treatment of these issues has only recently been done,[128,129] and although this research line is promising more theoretical calculations are needed to confirm the results obtained so far. Finally, we mention that another possibility has arisen recently.[130] Namely, the post-merger evolution of the coalescences with a total mass larger than the Chandrasekhar limit could be dominated by the magnetic and accretion torques. Thus, a delayed explosion of the central spinning white dwarf would be possible. This detonation would be caused by magnetic braking.

5. Summary and Outlook

In this paper, we have reviewed our current understanding of magnetic white dwarfs. This class of objects, is interesting not only "*per se*", but also for its many and interesting applications in other areas of contemporary astrophysics. Some of them have also been reviewed here. The current observational sample comprises about ∼250 objects for which we have reliable determinations of the magnetic field strength, and for several of them we also have relatively accurate mass determinations. However, it is worth emphasizing that probably there are more magnetic white dwarfs with low-field strengths for which the current limitations of the observational techniques have not allowed us to determine the strength of the magnetic field, and thus there is quite likely a hidden population of magnetic white dwarfs with very low magnetic field strengths. Nevertheless, the existing wealth of observational data — primarily provided by the recent, advanced large-scale surveys, like the SDSSS — is nowa-

days being analyzed. This includes not only studying the properties of individual objects, but also deriving the ensemble properties of the population of magnetic white dwarfs. This last analysis has allowed us to unveil two sub-groups of stars. First, there is a group of magnetic white dwarfs with moderately low magnetic fields which have masses close to the average of their nonmagnetic analogs. The second sub-group consists of a distinct set of massive white dwarfs with very high magnetic fields, typically of the order of 10^9 G. These observational advances have yielded some insight on the origin of the magnetic fields, but still there is much work to be done in this respect, and clearly theoretical models need to be improved to match observations. In particular, we stress that there are two competing theoretical scenarios for the formation of magnetic white dwarfs, and there is not yet enough concluding evidence favoring one of them. However, it is also true that the field has advanced in several distinct ways since the discovery of the first magnetic white dwarf.

Certainly, the next decade will see a dramatic increase in the number of known magnetic white dwarfs. Future releases of large-scale surveys, like that of the astrometric satellite Gaia[131] or that of the large synoptic survey telescope (LSST) project,[132] will definitely allow us to find many variable white dwarfs, including magnetic white dwarfs with spots. However, in the case of Gaia, ground-based follow-up spectroscopy of these objects will be crucial to getting the most out of the Gaia observations. With a geometric parallax accuracy of 1 milli-arcsecond and very deep exposures, the LSST parallax survey will match the faint-end precision of Gaia, providing a nearly complete catalog (including accurate parallaxes) of white dwarfs up to $M_v = 15$ in selected regions of the southern sky. Moreover, it is foreseen that a significant fraction of them will be magnetic. By analyzing these samples with model spectra we expect to have a much clearer picture of the population of magnetic white dwarfs. More than anything else, these enhanced samples will undoubtedly constitute important tools for unraveling the origin and evolution of magnetic fields in stars.

Acknowledgments

References

1. E. M. Sion *et al.*, *Astron. J.* **147** (2014) 129.
2. J. Liebert, P. Bergeron, J. B. and Holberg, *Astron. J.* **125** (2003) 348.
3. D. G. York *et al.*, *Astron. J.* **120** (2000) 1579.
4. G. P. Kuiper, *Pub. Astron. Soc. Pac.* **46** (1934) 287.

5. S. O. Kepler, I. Pelisoli, D. Koester, G. Ourique, S. J. Kleinman, A. D. Romero, A. Nitta, D. J. Eisenstein, J. E. S. Costa, B. Külebi, S. Jordan, P. Dufour, P. Giommi, and A. Rebassa-Mansergas, *Mon. Not. R. Astron. Soc.* **446** (2015) 4078.
6. L. Ferrario, D. de Martino and B. T. Gänsicke, *Space Sci. Rev.* **27** (2015) 111.
7. J. C. Kemp, *Astrophys. J.* **162** (1970) 169.
8. S. O. Kepler *et al.*, *Mon. Not. R. Astron. Soc.* **429** (2013) 2934.
9. A. Cumming, *Mon. Not. R. Astron. Soc.* **333** (2002) 589.
10. A. Putney and S. Jordan, *Astrophys. J.* **449** (1995) 863.
11. F. Euchner, K. Reinsch, S. Jordan, K. Beuermann and B. T. Gänsicke, *Astron. Astrophys.* **442** (2005) 651.
12. F. Euchner, S. Jordan, K. Beuermann, K. Reinsch and B. T. Gänsicke, *Astron. Astrophys.* **451** (2006) 671.
13. S. Jordan, P. Schmelcher, W. Becken and W. Schweizer, *Astron. Astrophys.* **336** (1998) L33.
14. J. Liebert, J. R. P. Angel, H. S. Stockman and E. A. Beaver, *Astrophys. J.* **225** (1978) 181.
15. P. Dufour, J. Liebert, G. Fontaine and N. Behara, *Nature* **450** (2007) 522.
16. M. A. Hollands, B. T. Gänsicke and D. Koester, *Mon. Not. R. Astron. Soc.* **450** (2015) 681.
17. S. O. Kepler *et al.*, in *19th European Workshop on White Dwarfs, ASP Conf. Ser.* Vol. 493 (June 2015), p. 449.
18. S. O. Kepler *et al.*, *Mon. Not. R. Astron. Soc.* in press, arXiv:1510.08409.
19. B. Külebi, S. Jordan, E. Nelan, U. Bastian and M. Altmann, *Astron. Astrophys.* **524** (2010) A36.
20. N. M. Silvestri *et al.*, *Astron. J.* **134** (2007) 741.
21. P.-E. Tremblay, H.-G. Ludwig, M. Steffen and B. Freytag, *Astron. Astrophys.* **559** (2013) A104.
22. G. Fontaine and P. Brassard, *Pub. Astron. Soc. Pac.* **120** (2008) 1043.
23. S. Jordan and S. Friedrich, *Astron. Astrophys.* **383** (2002) 519.
24. C. S. Brinkworth, M. R. Burleigh, K. Lawrie, T. R. Marsh and C. Knigge, *Astrophys. J.* **773** (2013) 47.
25. A. Rebassa–Mansergas, C. Agurto–Gangas, M. R. Schreiber, B. T. Gänsicke and D. Koester, *Mon. Not. R. Astron. Soc.* **433** (2013) 3398.
26. M. Zorotovic *et al.*, *Astron. Astrophys.* **568** (2014) A68.
27. J. Camacho *et al.*, *Astron. Astrophys.* **566** (2014) A86.
28. J. Liebert, L. Ferrario, D. T. Wickramasinghe and P. S. Smith, *Astrophys. J.* **804** (2015) 93.
29. J. R. P. Angel, E. F. Borra and J. D. Landstreet, *Astrophys. J. Suppl. Ser.* **45** (1981) 457.
30. D. T. Wickramasinghe and L. Ferrario, *Mon. Not. R. Astron. Soc.* **356** (2005) 1576.
31. C. A. Tout, D. T. Wickramasinghe, J. Liebert, L. Ferrario and J. E. Pringle, *Mon. Not. R. Astron. Soc.* **387** (2008) 897.
32. J. Nordhaus, S. Wellons, D. S. Spiegel, B. D. Metzger and E. G. Blackman, *Proc. Nat. Acad. Sci.* **108** (2011) 3135.
33. A. T. Potter and C. A. Tout, *Mon. Not. R. Astron. Soc.* **402** (2010) 1072.
34. E. García–Berro *et al.*, *Astrophys. J.* **749** (2012) 25.
35. P. Lorén–Aguilar, J. Isern and E. García–Berro, *Astron. Astrophys.* **500** (2009) 1193.
36. S. Ji *et al.*, *Astrophys. J.* **773** (2013) 136.
37. B. Külebi, K. Y. Ekşi, P. Lorén–Aguilar, J. Isern and E. García–Berro, *Mon. Not. R. Astron. Soc.* **431** (2013) 2778.

38. A. Asensio Ramos, M. J. Martínez González, R. Manso Sainz, R. L. M. Corradi and F. Leone, *Astrophys. J.* **787** (2014) 111.
39. G. Mathys *et al.*, *Astron. Nachr.* **333** (2012) 30.
40. J. D. Landstreet, S. Bagnulo, L. Fossati, S. Jordan and S. J. O'Toole, *Astron. Astrophys.* **541** (2012) A100.
41. S. Jordan, S. Bagnulo, K. Werner and S. J. O'Toole, *Astron. Astrophys.* **542** (2012) A64.
42. A. Kawka, S. Vennes, G. D. Schmidt, D. T. Wickramasinghe and R. Koch, *Astrophys. J.* **654** (2007) 499.
43. J. Power, G. A. Wade, M. Aurière, J. Silvester and D. Hanes, *Contrib. Astron. Obs. S.* **38** (2008) 443.
44. G. P. Briggs, L. Ferrario, C. A. Tout, D. T. Wickramasinghe and J. R. Hurley, *Mon. Not. R. Astron. Soc.* **447** (2015) 1713.
45. J. B. Holberg *et al.*, *Astron. J.* **135** (2008) 1225.
46. R. Aznar Cuadrado *et al.*, *Astron. Astrophys.* **423** (2004) 1081.
47. E. G. Blackman, A. Frank, J. A. Markiel, J. H. Thomas and H. M. Van Horn, *Nature* **409** (2001) 485.
48. R. F. Webbink, *Astrophys. J.* **277** (1984) 355.
49. I. Jr. Iben and A. V. Tutukov, *Astrophys. J.* **54** (1984) 335.
50. A. R. King, J. E. Pringle and D. T. Wickramasinghe, *Mon. Not. R. Astron. Soc.* **320** (2001) L45.
51. G. D. Schmidt, P. Bergeron, J. Liebert and R. A. *Astrophys. J.* **394** (1992) 603.
52. L. Segretain, G. Chabrier and R. Mochkovitch, *Astrophys. J.* **481** (1997) 355.
53. R. G. Izzard, C. S. Jeffery and J. Lattanzio, *Astron. Astrophys.* **470** (2007) 661.
54. G. C. Clayton, T. R. Geballe, F. Herwig, C. Fryer and M. Asplund, *Astrophys. J.* **662** (2007) 1220.
55. R. Longland *et al.*, *Astrophys. J.* **737** (2012) L34.
56. L. Bildsten, K. J. Shen, N. N. Weinberg and G. Nelemans, *Astrophys. J.* **662** (2007) L95.
57. E. García–Berro *et al. Astrophys. J.* **661** (2007) L179.
58. B. M. S. Hansen, S. Kulkarni and S. Wiktorowicz, *Astron. J.* **131** (2006) 1106.
59. P. Lorén–Aguilar, J. Guerrero, J. Isern, J. A. Lobo and E. García–Berro, *Mon. Not. R. Astron. Soc.* **356** (2005) 627.
60. R. Napiwotzki *et al.*, *Astron. Nachr.* **322** (2001) 411.
61. R. Napiwotzki *et al.*, *The Messenger* **112** (2003) 25.
62. M. Kilic, W. R. Brown, C. Allende Prieto, S. J. Kenyon and J. A. Panei, *Astrophys. J.* **716** (2010) 122.
63. W. R. Brown, M. Kilic, C. Allende Prieto and S. J. Kenyon, *Astrophys. J.* **723** (2010) 1072.
64. A. Gianninas, M. Kilic, W. R. Brown, P. Canton and S. J. Kenyon, *Astrophys. J.* in press arXiv:1509.07134.
65. J. J. Andrews, A. M. Price-Whelan and M. A. Agüeros, *Astrophys. J.* **797** (2014) L32.
66. M. Kilic *et al.*, *Mon. Not. R. Astron. Soc.* **438** (2014) L26.
67. X. Zhang, C. S. Jeffery, X. Chen and Z. Han, *Mon. Not. R. Astron. Soc.* **445** (2014) 660.
68. P. J. Carter *et al.*, *Mon. Not. R. Astron. Soc.* **429** (2013) 2143.
69. R. J. Foley *et al.*, *Astron. J.* **138** (2009) 376.
70. C. Badenes and D. Maoz, *Astrophys. J.* **749** (2012) L11.
71. J. Isern, E. García–Berro and B. Külebi, in preparation (2015).

72. N. Giammichele, P. Bergeron and P. Dufour, *Astrophys. J. Suppl. Ser.* **199** (2012) 29.
73. L. G. Althaus, A. H. Córsico, J. Isern and E. García–Berro, *Astron. Astrophys. Rev.* **18** (2010) 471.
74. I. Renedo *et al.*, *Astrophys. J.* **717** (2010) 183.
75. J. Isern, R. Mochkovitch, E. García–Berro and M. Hernanz, *Astrophys. J.* **485** (1997) 308.
76. J. Isern, E. García–Berro, M. Hernanz and G. Chabrier, *Astrophys. J.* **528** (2000) 397.
77. E. García–Berro, M. Hernanz, R. Mochkovitch and J. Isern, *Astron. Astrophys.* **193** (1988) 141.
78. E. García–Berro, M. Hernanz, J. Isern and R. Mochkovitch, *Nature* **333** (1988) 642.
79. U. R. Christensen, V. Holzwarth and A. Reiners, *Nature* **457** (2009) 167.
80. R. C. Duncan and C. Thompson, *Astrophys. J.* **392** (1992) L9.
81. C. Thompson and R. C. Duncan, *Mon. Not. R. Astron. Soc.* **275** (1995) 255.
82. M. Morini, N. R. Robba, A. Smith and M. van der Klis, *Astrophys. J.* **333** (1988) 777.
83. B. Paczynski, *Astrophys. J.* **365** (1990) L9.
84. M. Malheiro, J. A. Rueda and R. Ruffini, *Pub. Astron. Soc. Jpn.* **64** (2012) 56.
85. J. A. Rueda *et al.*, *Astrophys. J.* **772** (2013) L24.
86. J. R. Hurley, C. A. Tout, D. T. Wickramasinghe, L. Ferrario and P. D. Kiel, *Mon. Not. R. Astron. Soc.* **402** (2010) 1437.
87. D. Bhattacharya and E. P. J. van den Heuvel, *Phys. Rep.* **203** (1991) 1.
88. F. C. Michel, *Nature* **329** (1987) 310.
89. J. Gutiérrez *et al.*, *Astrophys. J.* **459** (1996) 701.
90. J. Gutiérrez, R. Canal and E. García–Berro, *Astron. Astrophys.* **435** (2005) 231.
91. J. Girven *et al.*, *Mon. Not. R. Astron. Soc.* **404** (2010) 159.
92. M. A. Barstow *et al.*, *Mon. Not. R. Astron. Soc.* **277** (1995) 971.
93. M. R. Burleigh, S. Jordan and W. Schweizer, *Astrophys. J.* **510** (1999) L37.
94. J. Liebert, P. Bergeron, G. D. Schmidt and R. A. Saffer, *Astrophys. J.* **418** (1993) 426.
95. E. P. Nelan, *Astron. J.* **134** (2007) 1934.
96. S. Vennes, L. Ferrario and D. T. Wickramasinghe, *Mon. Not. R. Astron. Soc.* **302** (1999) L49.
97. P. Bergeron, M. T. Ruiz and S. K. Leggett, *Astrophys. J.* **407** (1993) 733.
98. A. G. Riess *et al.*, *Astron. J.* **116** (1998) 1009.
99. S. Perlmutter *et al. Astrophys. J.* **517** (1999) 565.
100. A. J. Ruiter *et al.*, *Mon. Not. R. Astron. Soc.* **417** (2011) 408.
101. R. A. Scalzo, A. J. Ruiter and S. A. Sim, *Mon. Not. R. Astron. Soc.* **445** (2014) 2535.
102. W. M. Sparks and T. P. Stecher, *Astrophys. J.* **188** (1974) 149.
103. M. Livio and A. G. Riess, *Astrophys. J.* **594** (2003) L93.
104. A. Kashi and N. Soker, *Mon. Not. R. Astron. Soc.* **417** (2011) 1466.
105. M. Ilkov and N. Soker, *Mon. Not. R. Astron. Soc.* **428** (2013) 579.
106. G. Aznar–Siguán, E. García–Berro, P. Lorén–Aguilar, N. Soker and A. Kashi, *Mon. Not. R. Astron. Soc.* **450** (2015) 2948.
107. C. Raskin, F. X. Timmes, E. Scannapieco, S. Diehl and C. Fryer, *Mon. Not. Roy. Astron. Soc.* **399** (2009) L156.
108. S. Rosswog, D. Kasen, J. Guillochon and E. Ramirez-Ruiz, *Astrophys. J.* **705** (2009) L128.

109. T. A. Thompson, *Astrophys. J.* **741** (2011) 82.
110. D. Kushnir, B. Katz, S. Dong, E. Livne and R. Fernández, *Astrophys. J.* **778** (2013) L37.
111. G. Aznar–Siguán, E. García–Berro, P. Lorén–Aguilar, J. José and J. Isern, *Mon. Not. R. Astron. Soc.* **434** (2013) 2539.
112. D. C. Leonard, in *Supernova 1987A: 20 Years After: Supernovae and GammaRay Bursters, AIP Conf. Ser.* Vol. 937 (2007), p. 311.
113. D. Maoz and C. Badenes, *Mon. Not. R. Astron. Soc.* **407** (2010) 1314.
114. D. Maoz, K. Sharon and A. Gal-Yam, *Astrophys. J.* **722** (2010) 1879.
115. R. Pakmor *et al.*, *Nature* **463** (2010) 61.
116. R. Pakmor, S. Hachinger, F. K. Röpke and W. Hillebrandt, *Astron. Astrophys.* **528** (2011) A117.
117. R. Pakmor *et al.*, *Astrophys. J.* **747** (2012) L10.
118. C. Raskin, E. Scannapieco, C. Fryer, G. Rockefeller and F. X. Timmes, *Astrophys. J.* **746** (2012) 62.
119. M. Dan, S. Rosswog, J. Guillochon and E. Ramirez-Ruiz, *Mon. Not. R. Astron. Soc.* **422** (2012) 2417.
120. J. Schwab, K. J. Shen, E. Quataert, M. Dan and S. Rosswog, *Mon. Not. R. Astron. Soc.* **427** (2012) 190.
121. C. Zhu, P. Chang, M. H. van Kerkwijk and J. Wadsley, *Astrophys. J.* **767** (2013) 164.
122. M. Dan, S. Rosswog, M. Brüggen and P. Podsiadlowski, *Mon. Not. R. Astron. Soc.* **438** (2014) 14.
123. R. Kashyap, R. Fisher and E. García–Berro, *Astrophys. J.* **800** (2015) L7.
124. C. Zhu, R. Pakmor, M. H. van Kerkwijk and P. Chang, *Astrophys. J.* **806** (2015) L1.
125. I. R. Seitenzahl, C. A. Meakin, D. Q. Lamb and J. W. Truran, *Astrophys. J.* **700** (2009) 642.
126. U. Das and B. Mukhopadhyay, *Phys. Rev. D* **86** (2012) 042001.
127. U. Das, B. Mukhopadhyay and A. R. Rao, *Astrophys. J.* **767** (2013) L14.
128. U. Das and B. Mukhopadhyay, *J. Cosmol Astropart. Phys.* **6** (2015) 050.
129. U. Das and B. Mukhopadhyay, *J. Cosmol Astropart. Phys.* **5** (2015) 045.
130. L. Becerra, J. Rueda, P. Lorén–Aguilar and E. García–Berro, in preparation (2015).
131. M. A. Barstow *et al.*, arXiv:1407.6163.
132. LSST Science Collab. (P. A. Abell *et al.*), arXiv:0912.0201.