Fermilab

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PARAMETRIZATION OF THE LONGITUDINAL AND TRANSVERSE DEVELOPMENT OF THE HADRONIC CASCADE IN AN ABSORBING MEDIUM

This note is a status report of our simulation work on the response of the CDF calorimeter (electromagnetic+hadronic) as a jet detector.

We aimed first of all at simulating the development of a hadronic cascade in an omogeneous medium by representing it both longitudinally and transversally with a distribution function, giving the energy deposited in the medium. The distribution function can be used to get quick informations on the hadronic cascade without the need of following in detail all particles in the cascade. The characteristics and the limits of validity of this function are discussed in comparison with the existing experimental data and computer simulations.

In section 1 and 2 we describe the longitudinal and transverse parametrization respectively, in section 3 some remarks are given on the use of the parametrization function in computer simulations. In section 4 we report some results from a preliminary study of high p_t jets in the CDF calorimeter.

1. Parametrization in the Longitudinal Direction

A longitudinal parametrization of a hadronic cascade has been recently published (1). In ref.(1) the developing cascade is described by an energy distribution function, which is given as a sum of two terms. The first one takes into account the e.m. contribution (mainly photons from

π° decay) and the second one the contribution of the hadrons. The function is supposed to describe the shape of the cascade starting with the primary interation taken as the origin. No energy deposition before the origin is taken into account. The function is as follows:

$$\frac{dE}{dz} = K\left[w\, s^{a-1} e^{-bs} + (1-w)\, t^{c-1} e^{-d.t}\right] \tag{1}$$

where z is the longitudinal coordinate along the cascade and the other parameters are defined as follows:

s = distance from the origin in radiation length = z/X_o

t = distance from the origin in absorption length = z/λ

a, b, c, d = shape parameters to be fitted to the data

w, 1-w = relative weights of the e.m. and hadronic contribution respectively

K = normalization constant

The shape parameters have been fitted in ref.(1) allowing for a logarithmic energy dependence ($a=a_1+a_2 \ell nE, b=b_1+b_2 \ell nE$..etc). The fit to some experimental data at $E \gtrsim 15$ GeV gives the following values:

$a = .62+.31\ \ell nE$

$b = .22$

$c = a$ (imposed)

$d = .91-.024\ \ell nE$

$w = .46$

where E is the energy of the incident hadron in GeV.

We shall need in the following an expression for the constant K, which can be obtained by imposing the normalization condition:

$$E = K\left[X_0 w \int_0^\infty s^{a-1} e^{-bs} ds + \lambda (1-w) \int_0^\infty t^{a-1} e^{-d.t} dt\right]$$

the integrals can be put in the form:

$$\int_0^\infty s^{a-1} e^{-bs} ds = \Gamma(a)/b^a ; \quad \int_0^\infty t^{a-1} e^{-d.t} dt = \Gamma(a)/d^a$$

such that we obtain for K the expression:

$$K = \frac{E/\Gamma(a)}{w X_0/b^a + (1-w)\lambda/d^a} \qquad (2)$$

The energy dependence of K for iron (X_0=1.76cm;λ=17.1cm) is shown in Fig.1.

In Fig.2a the parametrization function is compared with WA1 iron calorimeter data (ref.1). In Fig.2b the function is compared with the experimental data from Barish et al.(2). As a matter of fact, from a detailed comparison of the available data with the prediction of the parametrization function, we find an increasing discrepancy as the energy of the incident hadron decreases (see for instance the 5 GeV points in Fig.2b). We remark that below 3.5 GeV the parametrization is bound to fail, since the exponent a-1 becomes negative. Since we are interested in covering the energy range down to 2-3 GeV (see in Fig.3 the pion energy spectrum from high p_t jets in $p\bar{p}$ collision), we felt that this difficulty had to be cured. For this purpose we have studied the hadronic cascades in the range $3 \leq E_h \leq 50$ GeV using both the available data and the predictions by a well tested Montecarlo (3). We find that the main discrepancy is due to the fact that formula (1) does not reproduce the rapid absorption of the cascades at z≳25 cm, which is indicated by the available data at E≲10 GeV and also clearly predicted by the Montecarlo. We have introduced a correction in formula (1) to account to a rough approximation for this effect. The corrected formula to be used for E≲10 GeV is as follows:

$$\frac{dE}{dz} = H\left[w\, s^{.036\cdot E} e^{-bs} + (1-w)\, t^{.036\cdot E} e^{-d[1+.3(1-p)t]t}\right] \qquad (3)$$

where $$p = \left(\frac{E-1}{9}\right)^4$$

the normalization constant H is different from expression (2). By evaluating the integral of formula (3) one obtains:

$$H = K\,(4 - p^2)/3$$

(see in Fig.1 the energy behavior of H).

Function (3) differs from formula (1) in two ways. The energy dependence of the a-1 exponent is changed from logarithmic into linear. The singularity at 3.5 GeV is thus removed (see Fig.4). A term linearly dependent on z has been added in the exponent of the hadronic part. The energy dependence of this term (.3(1-p)) is shown in Fig.4. It has the effect of making the tail of the cascade to drop quickly with energy.

The above corrections are such that function (3) departs gradually from function (1) below E=10 GeV (see Fig.5), and reproduces better the strong absorption of the low energy cascades that is seen in the data.

The rather unphysical quick change of the normalization factor at

10 GeV (see Fig.1) has no influence on the practical use of the model.

2. Parametrization in the Transverse Direction

It has been shown experimentally (4) that the transverse diffusion of a hadronic cascade has a conical shape, the transverse width of the cascade being approximately linear versus depth in the absorbing medium. This result for 10 GeV pions is shown in Fig.6, where the FWHM of the transverse distribution is plotted versus the depth in iron. The dependence is invariant for different absorbing media, provided that the distance from the vertex is expressed in gr/cm^2.

In our transverse parametrization we adopt this linear law, assuming it to be true at all energies. As a further simplification we assume that the transverse distribution can be described by a single exponential (actually a better fit would be obtained with two exponentials (5)).

We thus describe the transverse distribution with the following expression:

$$\frac{d^2E}{dx\,dz} = \frac{e^{-\frac{x}{g(E,z)}}}{g(E,z)} \cdot \frac{dE}{dz} \qquad (4)$$

being

$$g(E,z) = \alpha(E) + \beta(E)\cdot z$$

and where x is the transverse radial coordinate.

In order to study the energy dependece of the coefficients α and β we report in Fig.7 the data of ref.(4) at 10 GeV and the data of ref.(6) at 50 and 140 GeV. The data of ref.(6) have been elaborated in order to obtain the exponential length starting from the experimental distributions in the transverse strip counters.

One sees that g(E,z) is approximately energy independent.

A global fit to all data gives:

$$\alpha = .82 \quad \text{and} \quad \beta = .07$$

the line in Fig.7 corresponds to these values of α and β. The prediction by Montecarlo simulation at 3,5 and 10 GeV are also shown in Fig.7 and they are found to be represented fairly well by the fitted line.

3. The Use of the Parametrization Function in a Simulating Program

In principle by expression (4) one can get quickly the amount of energy released by a showering hadron in an arbitrary volume of the calorimeter. In practice this task would require an integration of expression (4) that could be hard to do depending on the volume shape. In front of this difficulty we have found the following way out. We have evaluated the density of energy deposition at regular steps of length Δz; at each step the corresponding energy is spread out in a plane normal to the shower axis according to the transverse distribution.

Of course the planes where the shower energy is considered to be released will in general not coincide with the real sensitive planes of the calorimeters. That is not a problem since at this stage we only want to sample the total energy released within a given volume. However, if one wanted to evaluate the amount of energy released in the real sensitive planes (in order to study light collection problems, counter uniformity or other detector performances), one could need to know the energy released in the real sensitive planes. In view of such possible future needs, we have also developed an algorithm that solves this problem.

Attention has been paid in choosing the proper length Δz of the longitudinal step, since an energy dependent systematic error can be introduced if showers of different length were sampled by steps of fixed length. Let us consider the sum that approximates the total energy in our sampling procedure:

$$K \sum_{i=1}^{\infty} \left(\frac{dE}{dz}\right)_i \cdot \Delta z \qquad (5)$$

Expression (5) gives exactly the hadron energy at the limit $z \to 0$. We plot in Fig.8 the expression (5) divided by E versus Δz, for different hadron energies. One sees that a step of 1-2 cm would be a safe choice, resulting in systematic errors of a few percents. Such small steps, however, would imply very lengthy computations. Also, one could use larger steps, e.g. Δz = 10 cm, and then apply an energy dependent correction factor based on the curves of Fig.8. Another possibility could be to use an energy dependent step. Experience gained in practice will tell us which is the most practical way to follow.

4. Preliminary Study of Jets in the CDF Calorimeters

In this section we describe a first attempt of using the distribution function to evaluate the diffusion of high p_t jets into the CDF calorimeters. We have used a sample of $p\bar{p}$ events at $\sqrt{s}$ = 2 TeV, where the elastic $q\bar{q}$ scattering produces two high p_t jets ($p_t \geq 50$ GeV/c) and two target jets due to fragmentation of the spectator quarks. The particles are followed in their trajectories through the solenoidal CDF magnetic field (15 KGauss), and the impact points of all particles into the calorimeters are memorized. An approximation is made by attributing the full photon energy to the e.m. cell that is hit (no leakage between towers), with a fluctuation of $\sigma/E = .15/\sqrt{E}$. On the other hand, the hadrons divide their energy between the cells of the hadronic and e.m. calorimeters, according to distribution function.

We have sampled longitudinally the distribution function in steps 1 and 5 cm wide in the e.m. and hadronic calorimeter respectively. Hadron with energy less than 1 GeV were ignored. The cascade origin was generated according the exponential distribution $e^{-z/\lambda}$, where λ is assumed equal to be 18 cm both for lead and for iron. The hadron energy was let to fluctuate with a gaussian distribution ($\sigma/E = .65/\sqrt{E}$). We have fluctuated also the cascade length and width according to a gaussian distribution of the parameters x_0, λ and g(z).

In Fig.9a) and b) we show as an example the number of photons and hadrons hitting the calorimeter cells in a particular event. In Fig.10a) and b) the energy response of the e.m. and hadronic calorimeters for the same event is reported. The energy of two jets was 57 and 78 GeV respectively.

Many events of this type have been generated, and all results look plausible. We are now proceeding to try to reconstruct the jets, and we anticipate for the longer term future the generation and reconstruction of more complex physical events.

REFERENCES

(1) - R. K. Bock, T. H. Hansl-Kozanecka and T. P. Shah "Parametrization of the longitudinal development of the hadronic showers in sampling calorimeters" CERN-EP/80-206,13/11/1980.

(2) - B. C. Barish et al. "Calibration of a sampling total absorption detector designed for neutrino experiments" NIM 130 (1975), 49-60.

(3) - A. Baroncelli "Study of total absorption counters for very high energy particles" NIM 118 (1974),445-451.

(4) - B. Friend et al. "Measurements of the energy flow distribution of 10 GeV/c hadronic showers in iron and in aluminium" NIM 136 (1976), 505-510.

(5) - F. Binon et al. "A hodoscope calorimeter for high energy hadrons" CERN-EP 80-15 7/2/1980.

(6) - M. Holder et al. "Performance of a magnetized total absorption calorimeter between 15 GeV and 100 GeV" NIM 151 (1978), 69-80.

FIGURE CAPTIONS

(1) - Energy dependence of the normalization constants H(E<10 GeV) and K(E>10 GeV) of the longitudinal distribution functions (1) and (3) respectively (see text).

(2) - a) Measured (data points of ref(1)) and computed longitudinal energy distribution of hadron cascade in iron.
b) Measured (data points of ref(2)) and computed longitudinal energy distribution of hadron cascade in iron.

(3) - Energy distribution of secondaries in jets with $p_t \gtrsim 50$ GeV/c, according to FFF (smooth curve drown through the spectrum).

(4) - Energy dependent shape parameters in the parametrization function.

(5) - Distribution function (1), (full line), and (3), (dashed line), in the low energy range.

(6) - Width dependence of the hadronic cascade on the distance from vertex for 10 GeV pions. (Ref(4)).

(7) - Width dependence of the hadronic cascade on the distance from vertex. Experimental data from ref(6) and ref(4). Computed widths using the Montecarlo simulation of ref(3).

(8) - The ratio of the expression (5) divided the hadron energy E versus the step length Δz at different hadron energies.

(9) - a) Photon number in e.m. calorimeter. The marked cells indicate the jet direction.
b) Hadron number in hadronic calorimeter with energy E>1 GeV.

(I0) - a) Energy collected in the e.m. calorimeter.
b) Energy collected in the hadronic calorimeter.

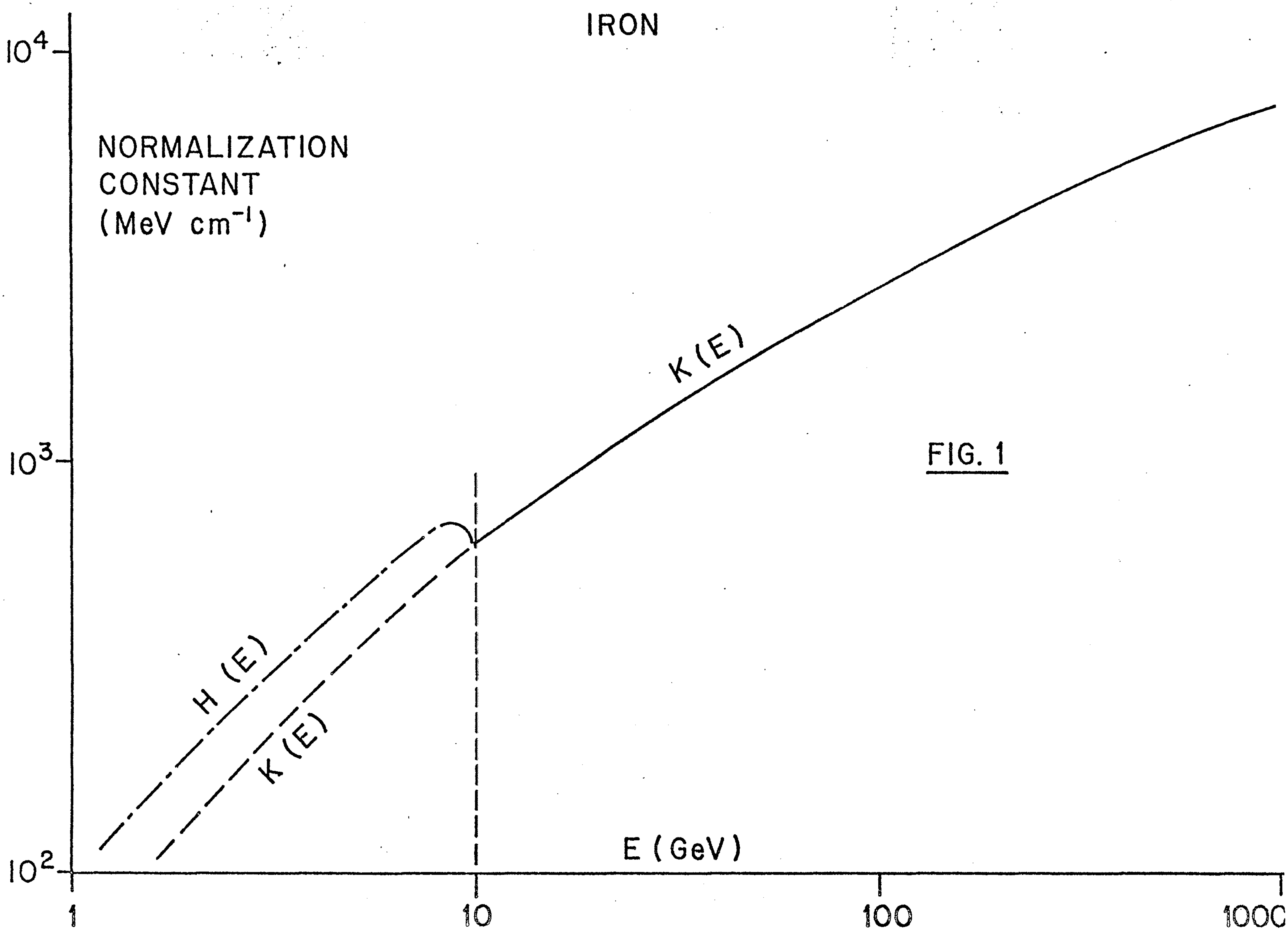

IRON
NORMALIZATION
CONSTANT
(MeV cm⁻¹)
10^4
10^3
10^2
K (E)
H (E)
K (E)
E (GeV)
1
10
100
1000

FIG. 1

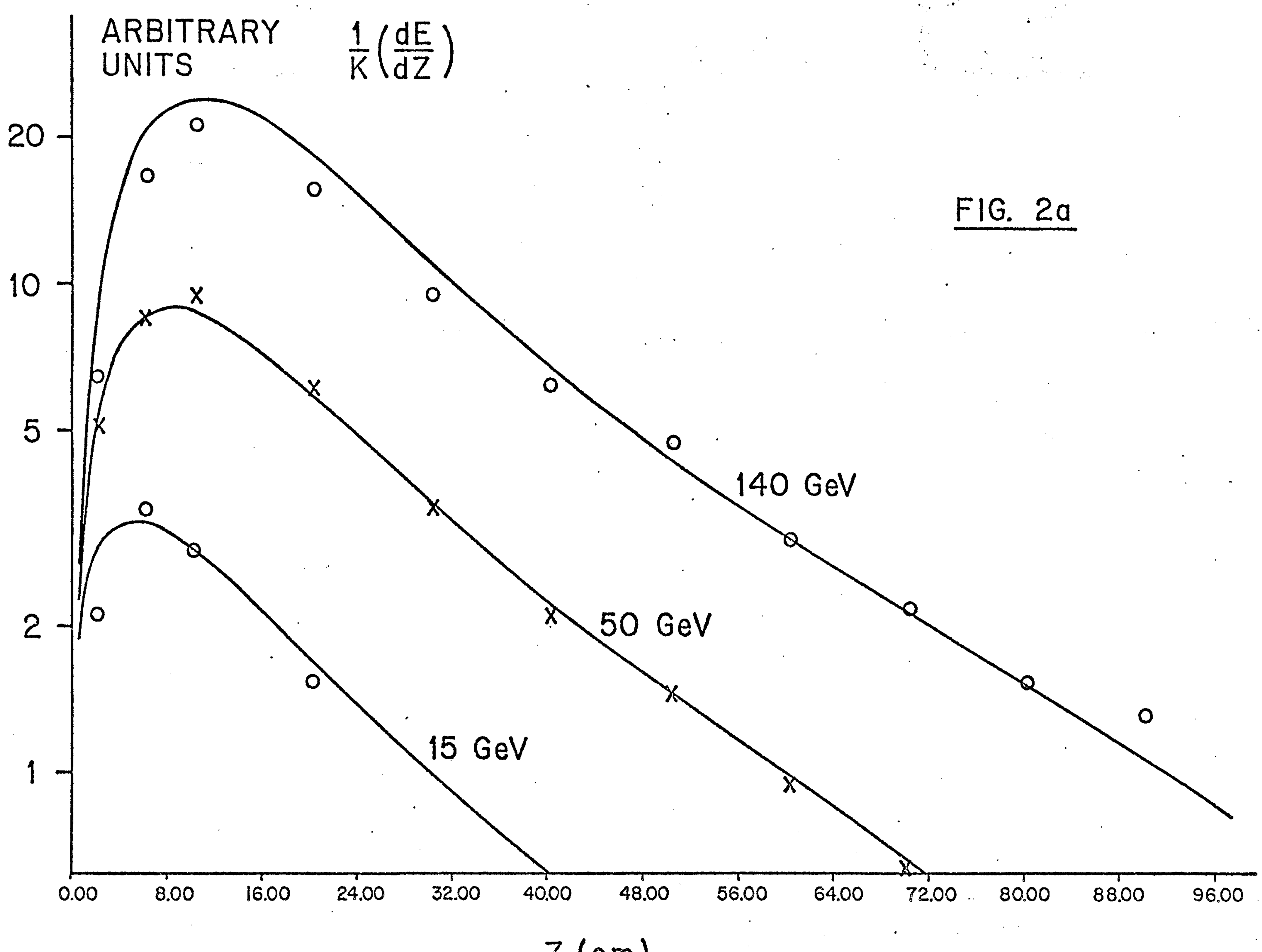

ARBITRARY UNITS
$\frac{1}{K}\left(\frac{dE}{dZ}\right)$
20
10
5
2
1
140 GeV
50 GeV
15 GeV
0.00
8.00
16.00
24.00
32.00
40.00
48.00
56.00
64.00
72.00
80.00
88.00
96.00

FIG. 2a

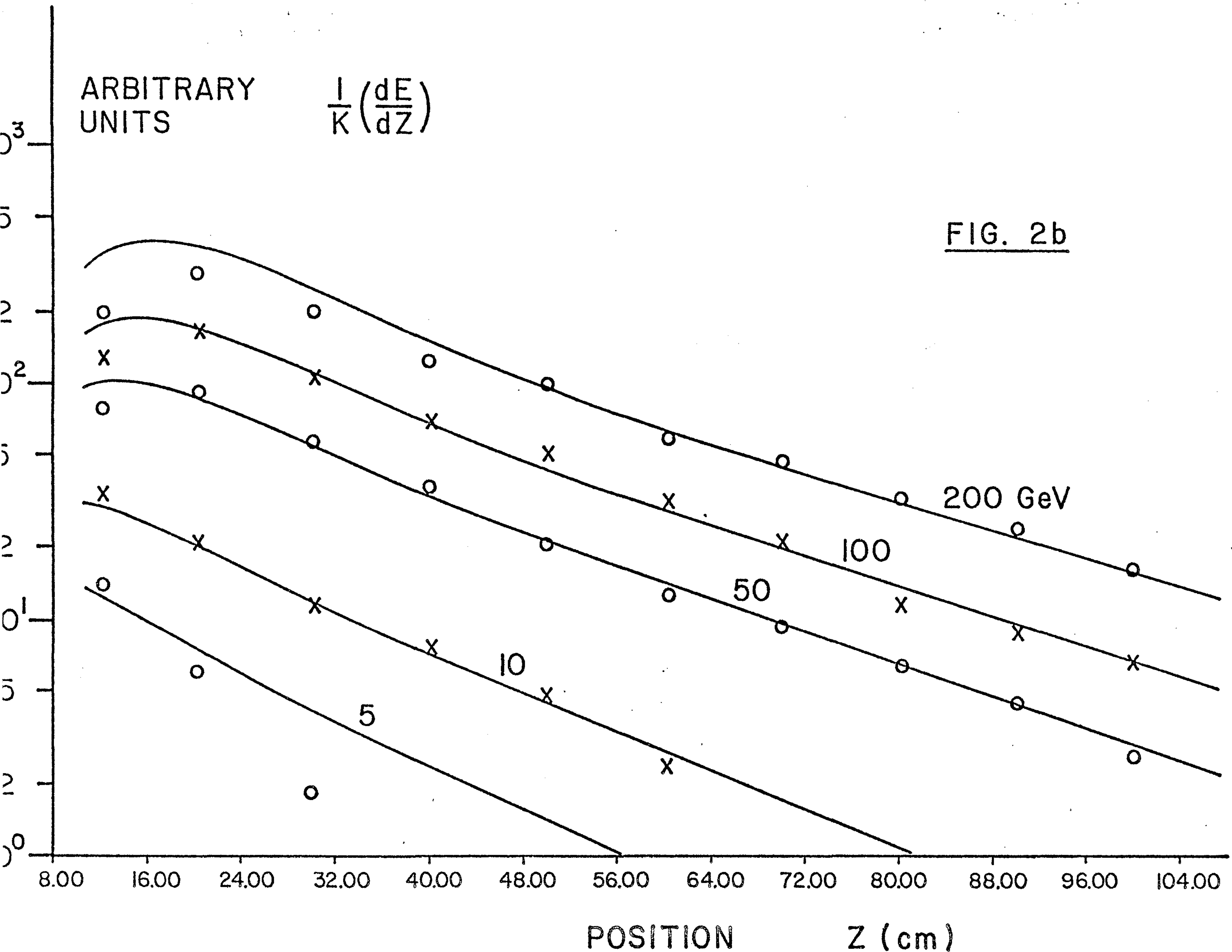

ARBITRARY UNITS
$\frac{1}{K}\left(\frac{dE}{dZ}\right)$
200 GeV
100
50
10
5
8.00
16.00
24.00
32.00
40.00
48.00
56.00
64.00
72.00
80.00
88.00
96.00
104.00
POSITION Z (cm)

FIG. 2b

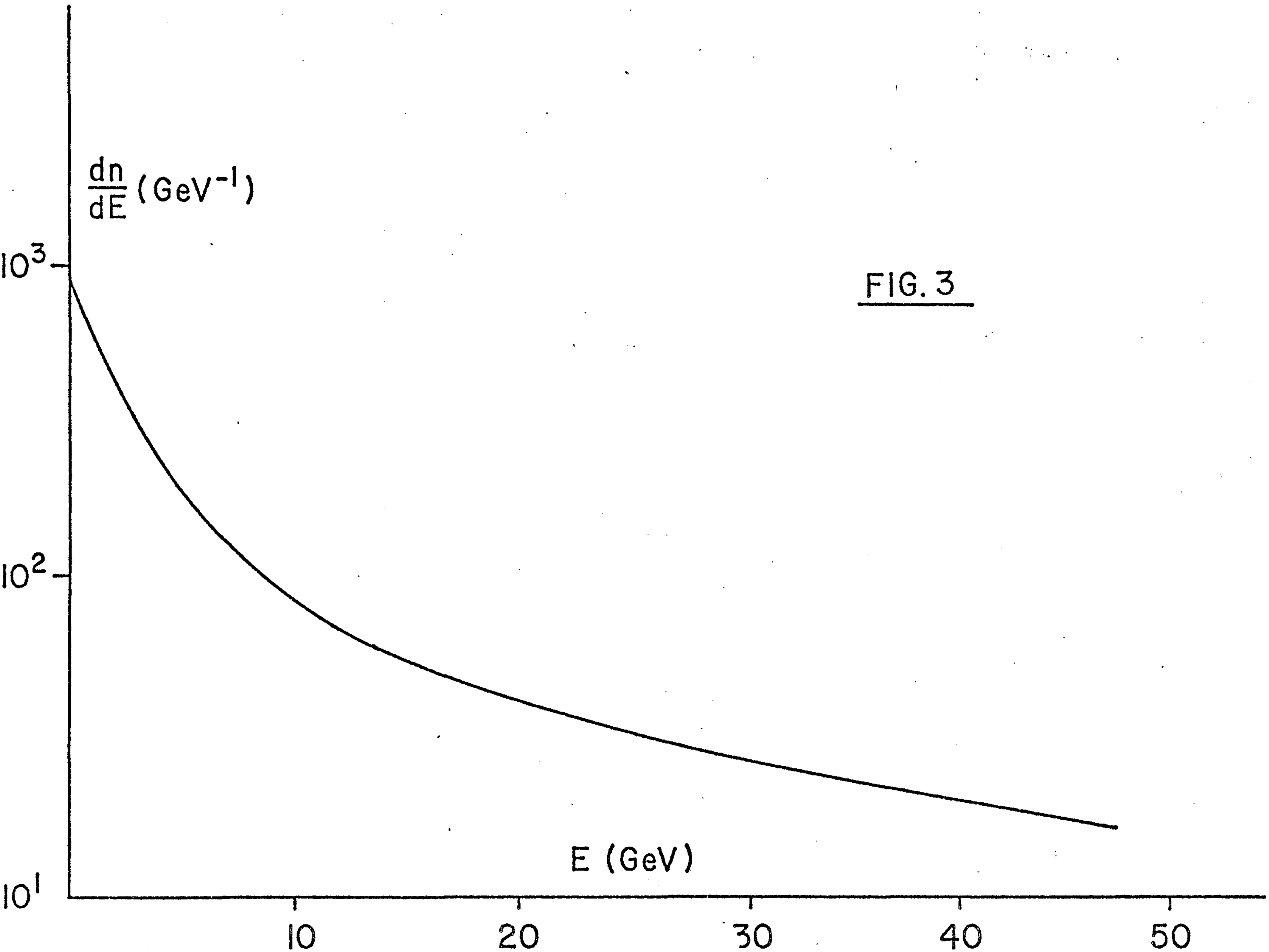


FIG. 3

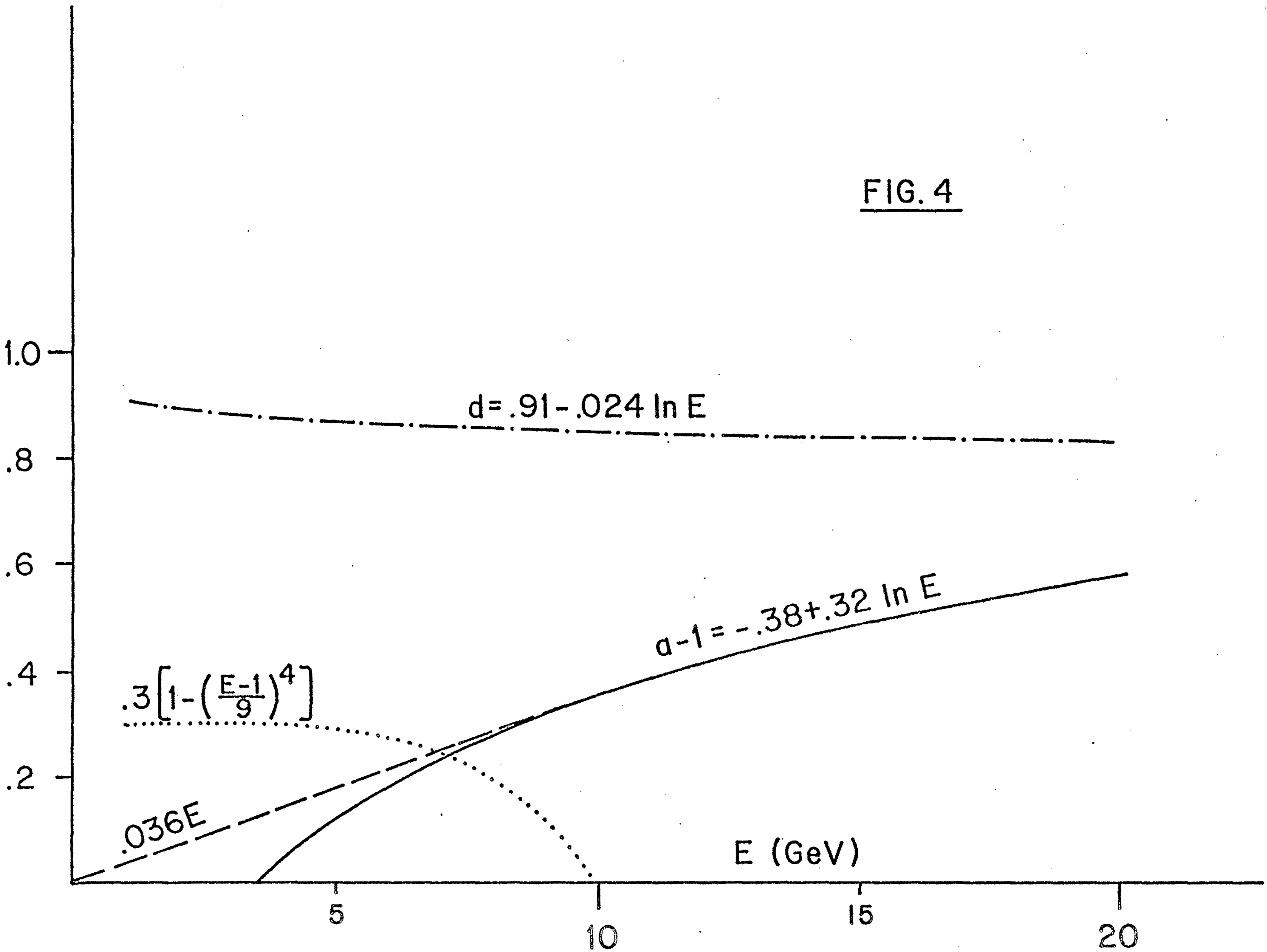
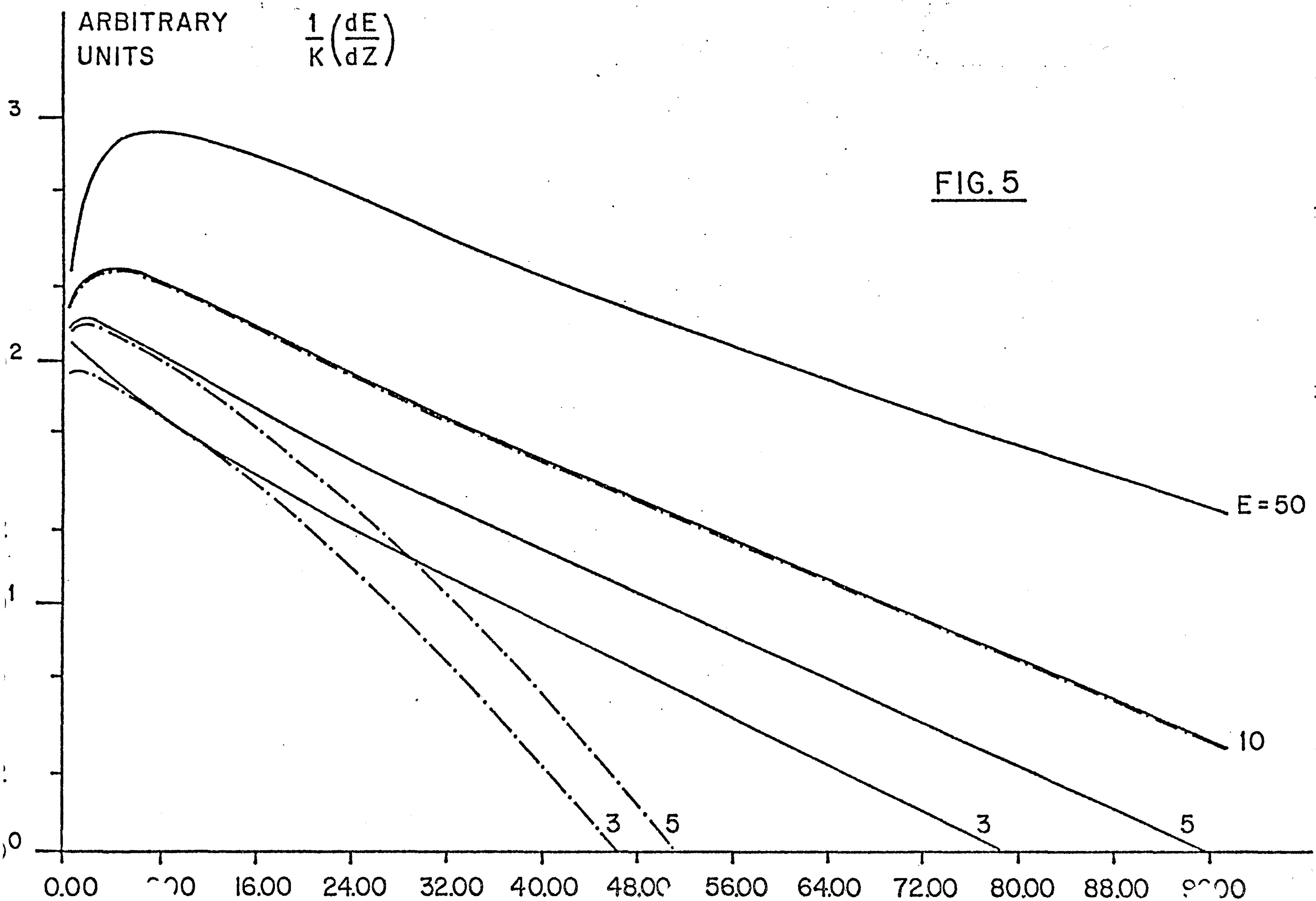


FIG. 4



ARBITRARY UNITS
$\frac{1}{K}\left(\frac{dE}{dZ}\right)$
3
2
1
0
E = 50
10
3
5
3
5
0.00
16.00
24.00
32.00
40.00
56.00
64.00
72.00
80.00
88.00

FIG. 5

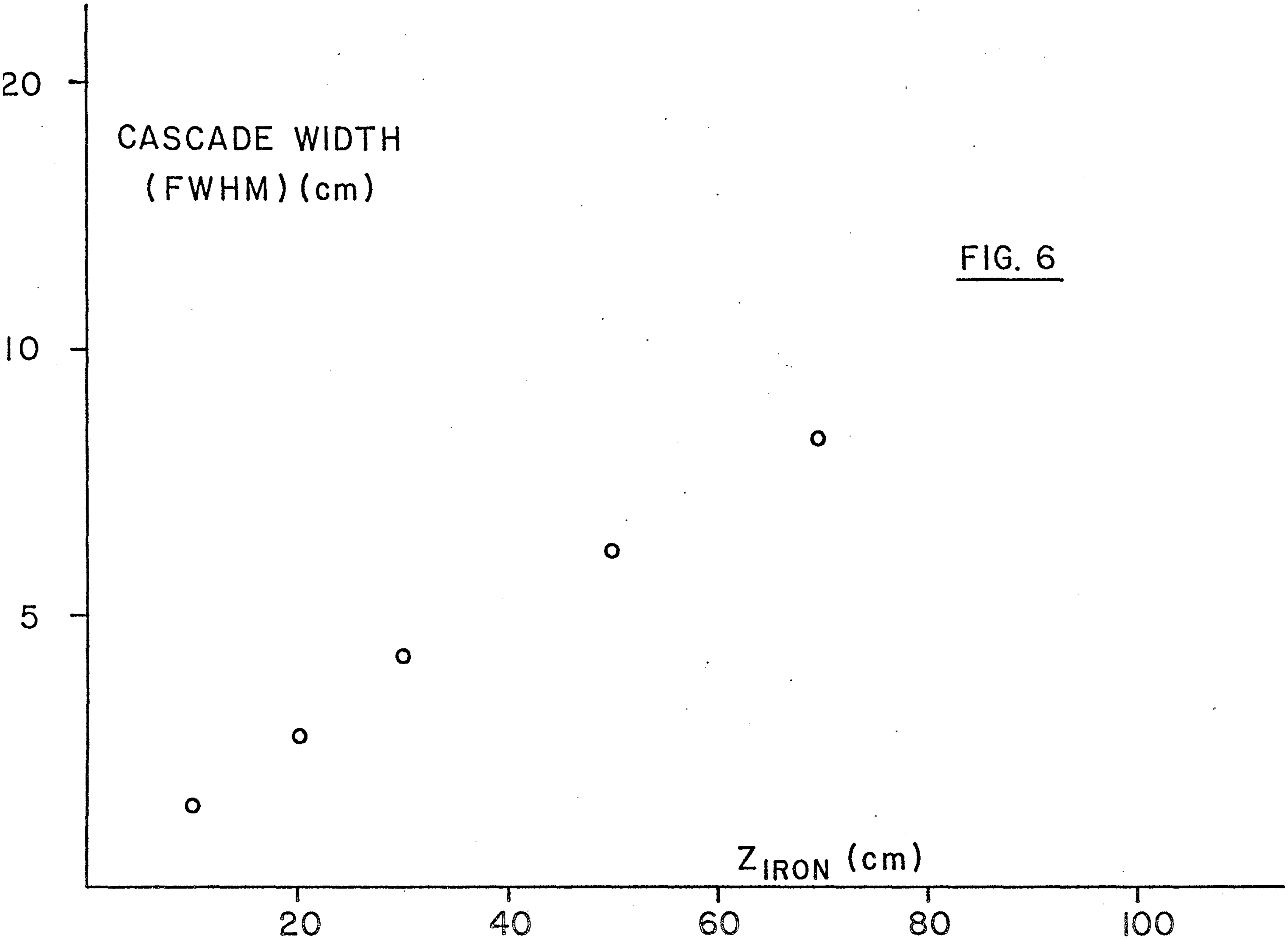
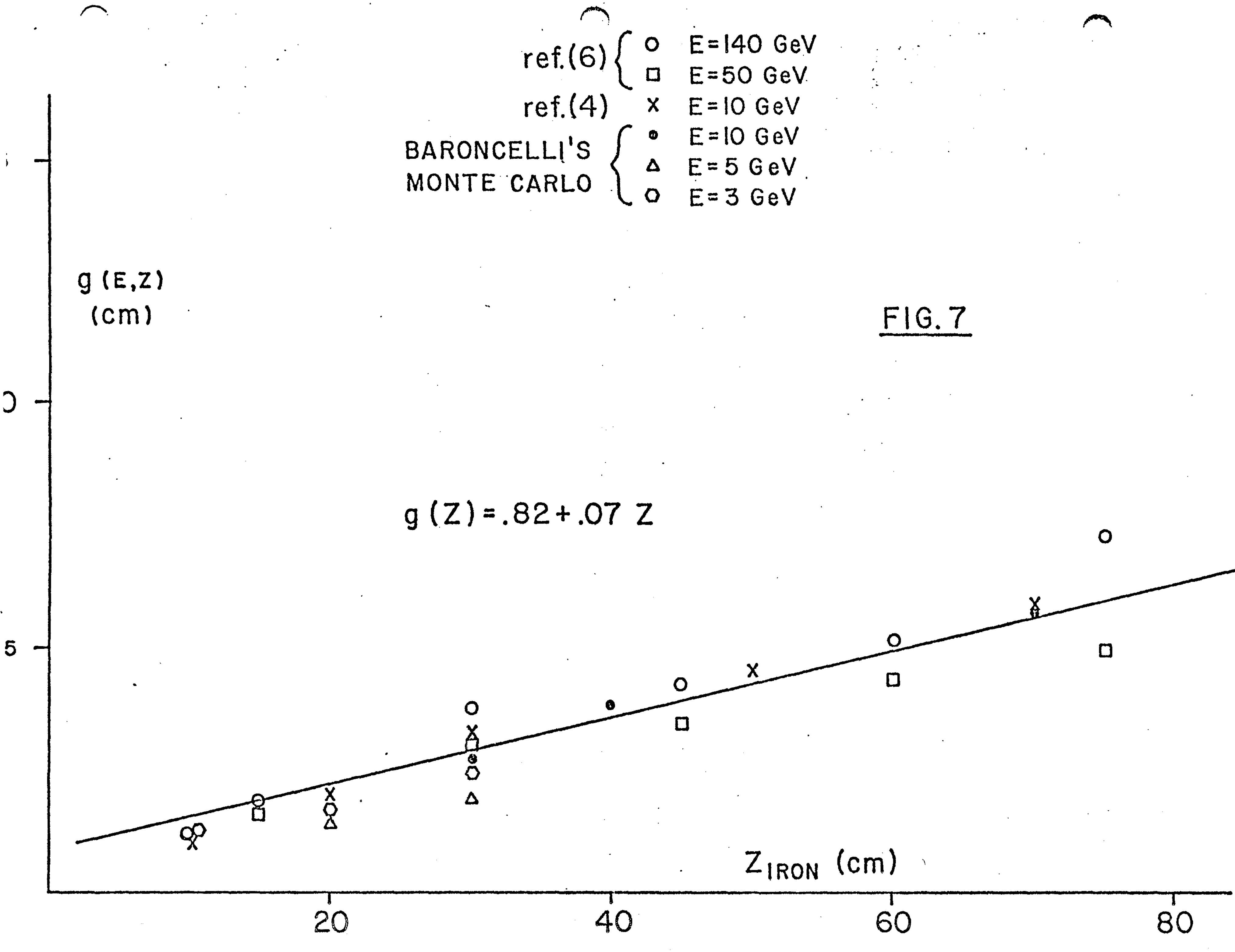


FIG. 6



ref.(6) ○ E=140 GeV
□ E=50 GeV
ref.(4) x E=10 GeV
BARONCELLI'S MONTE CARLO
• E=10 GeV
△ E=5 GeV
⬡ E=3 GeV
g (E,Z) (cm)
g (Z)=.82+.07 Z
Z IRON (cm)
20
40
60
80
5

FIG. 7

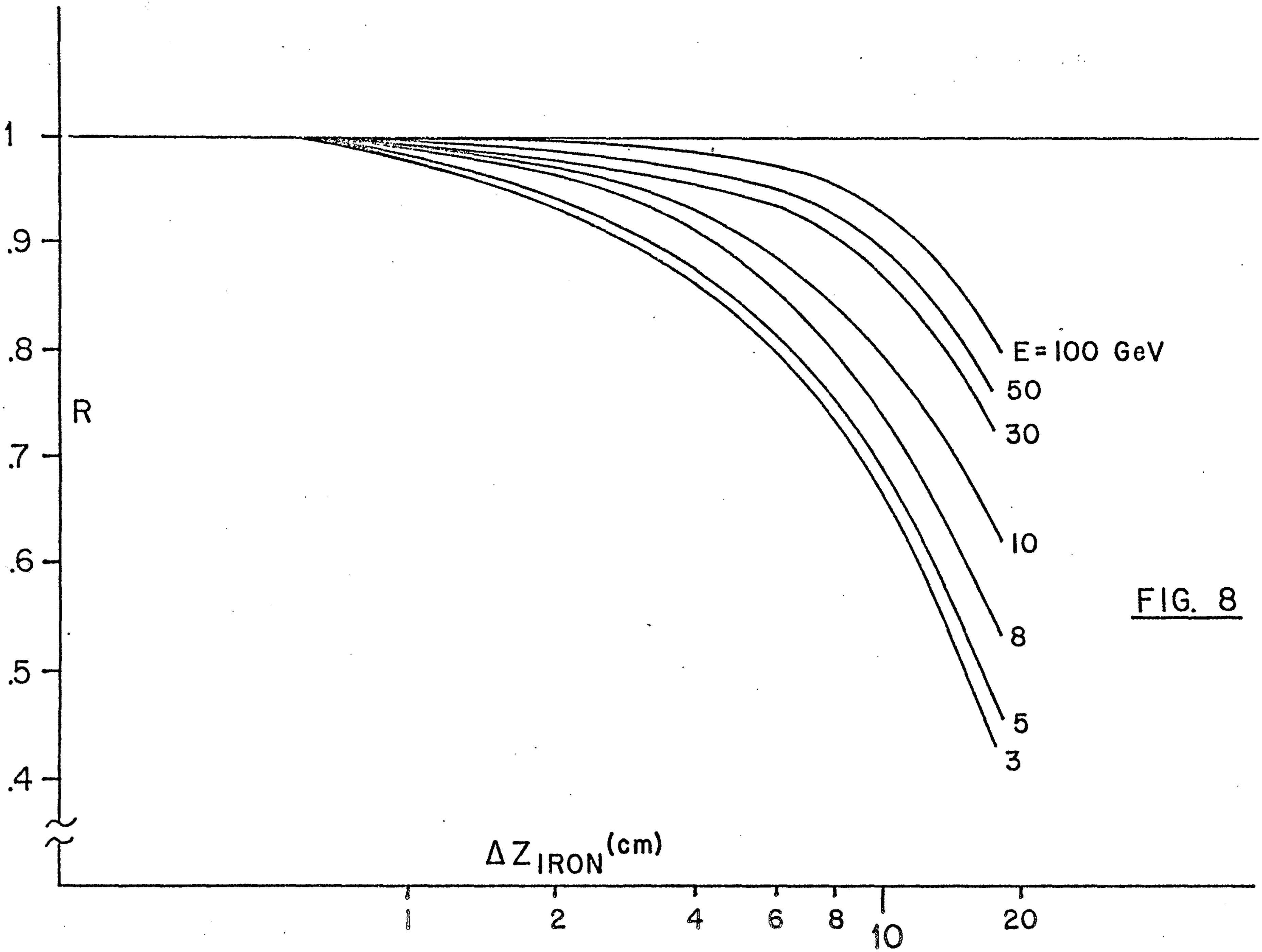


FIG. 8

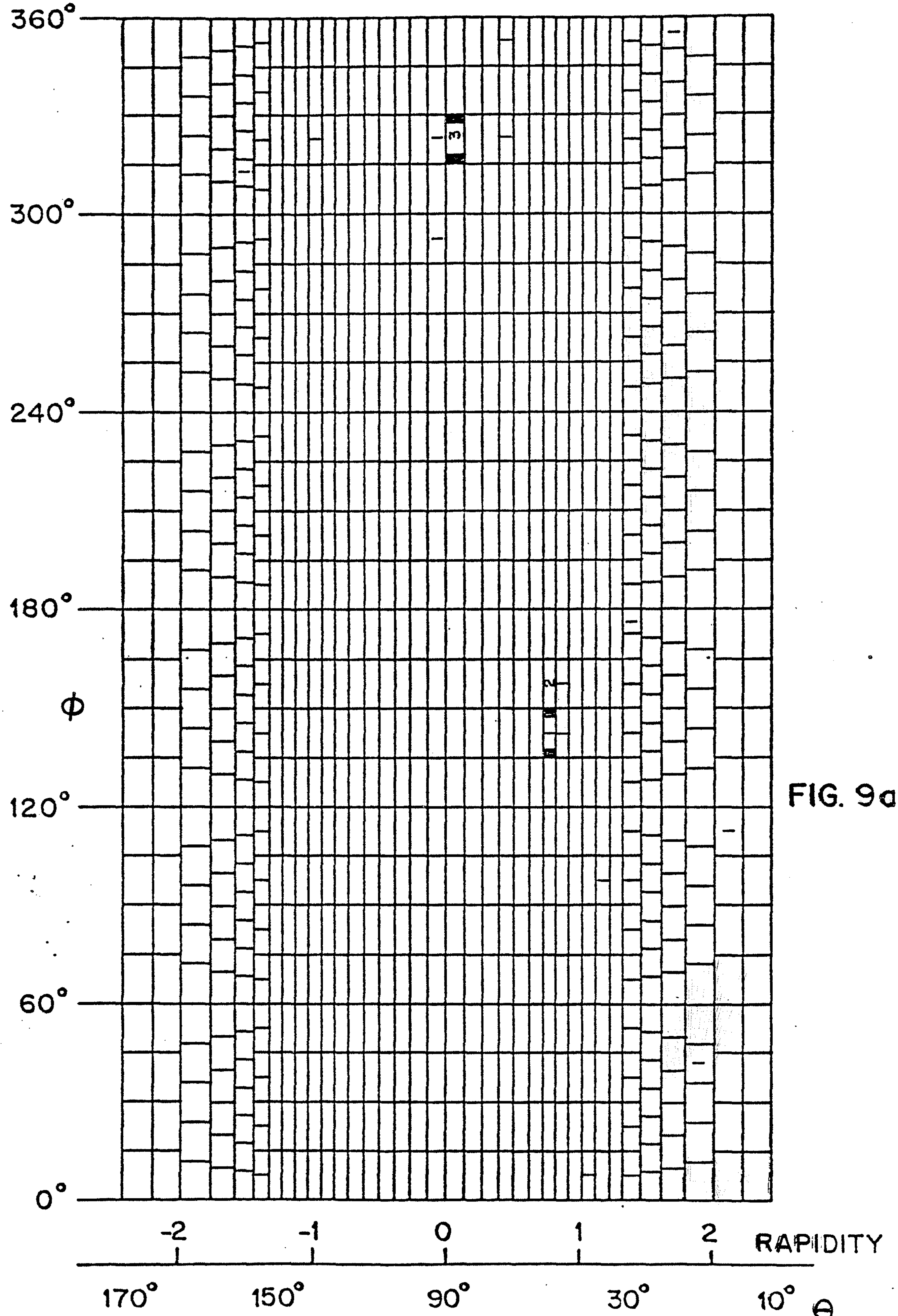


FIG. 9a

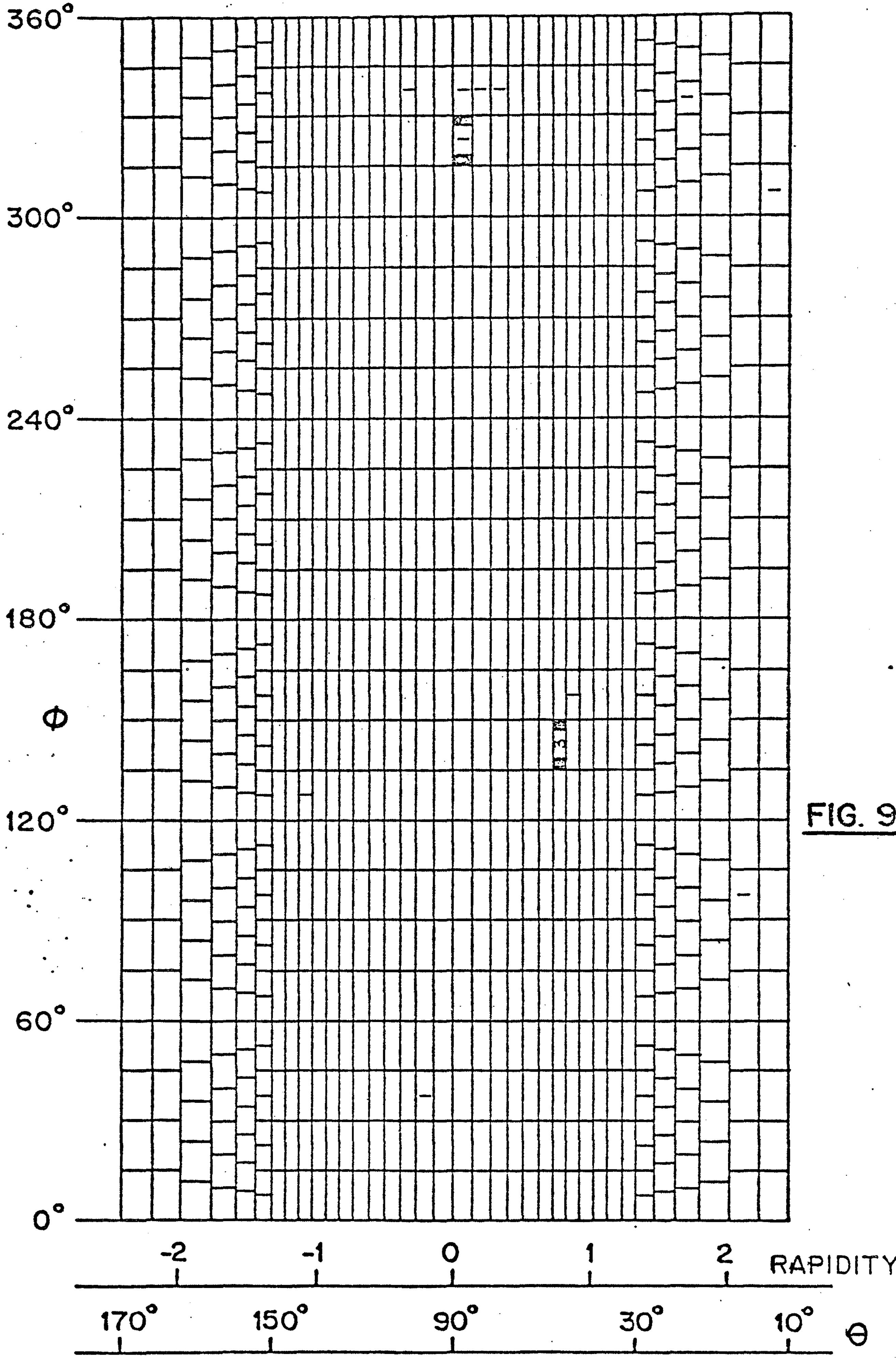


FIG. 9b

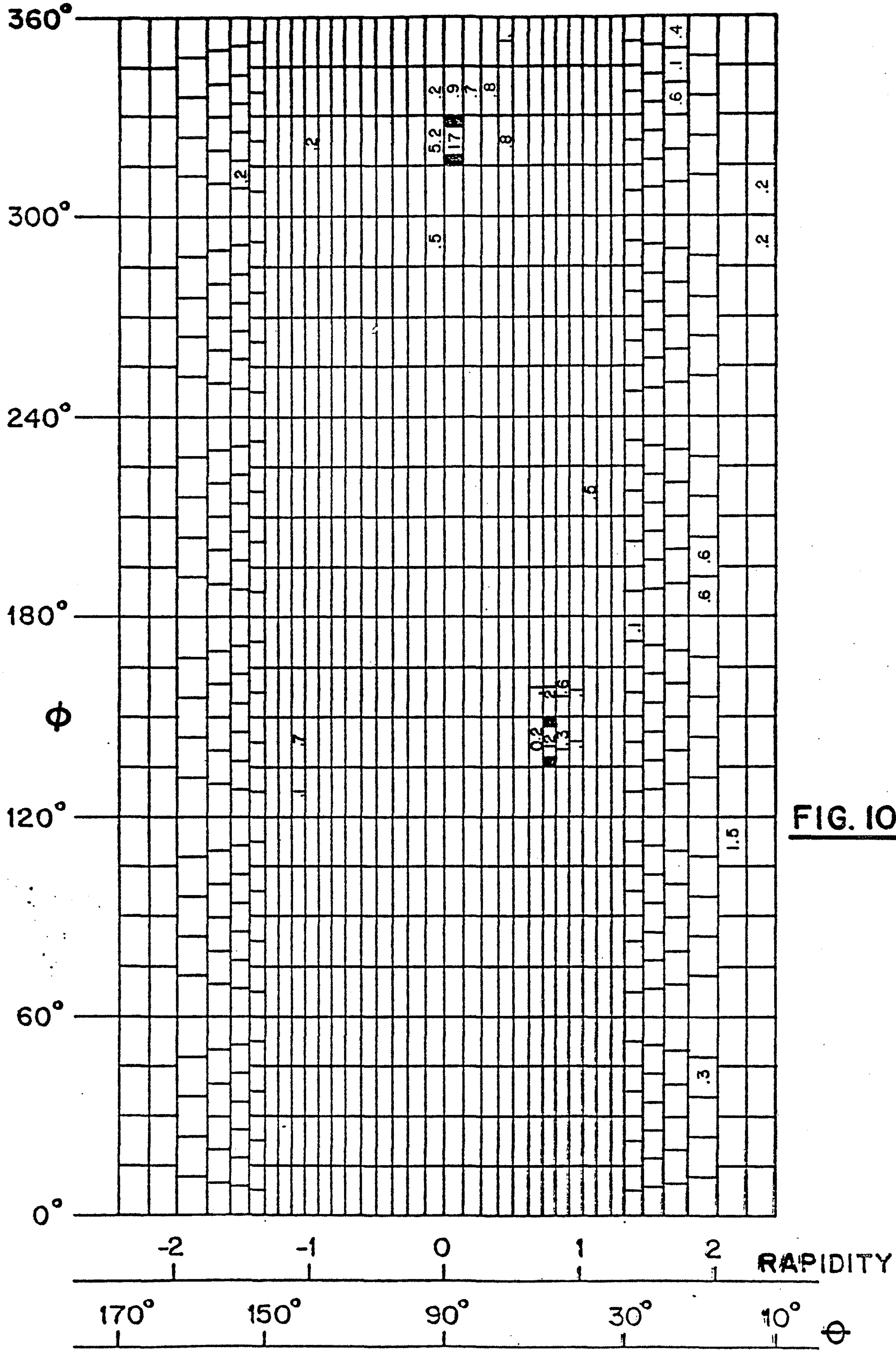


FIG. 10a

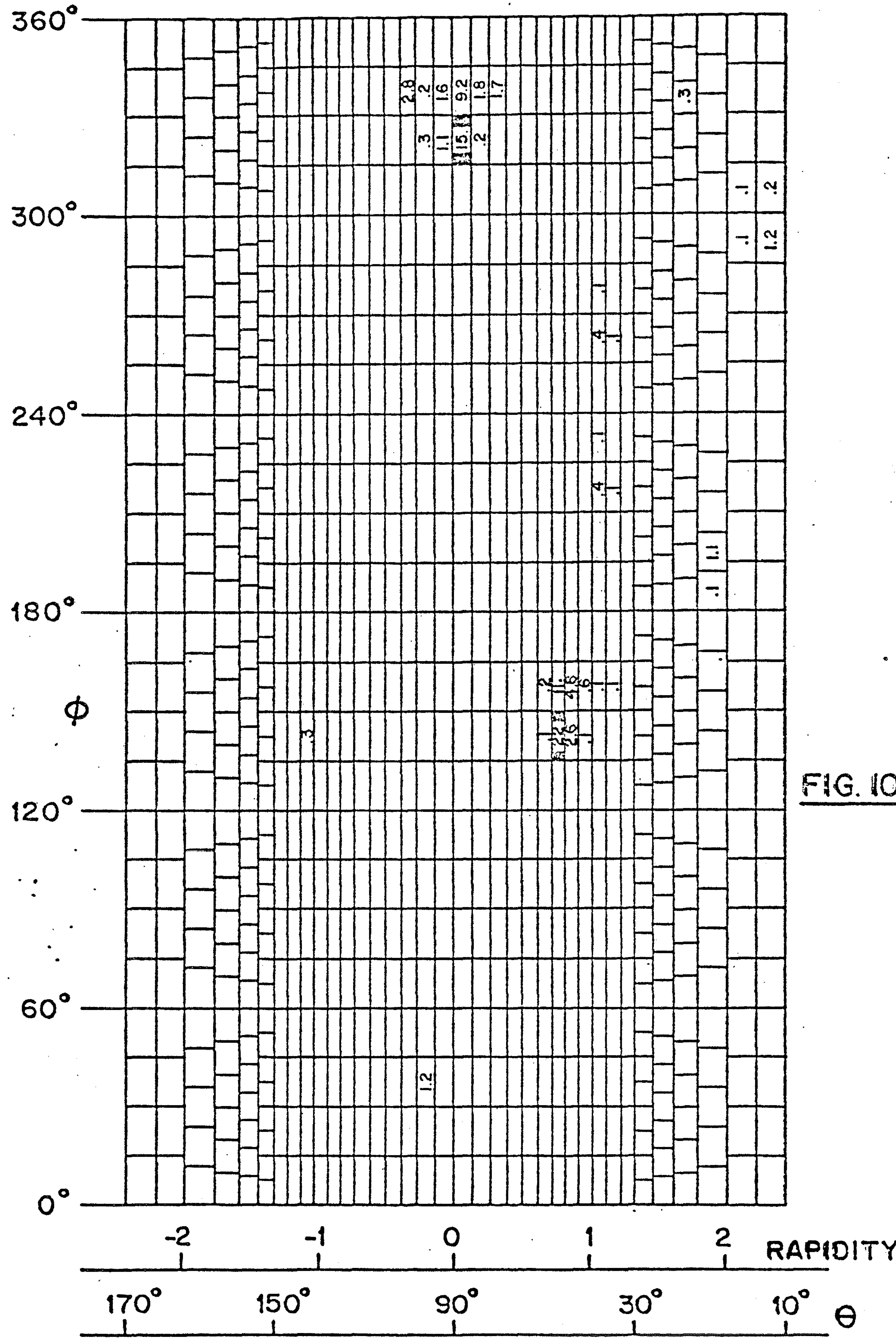


FIG. 10b