An Algorithm to Obtain the Probability Oscillation of the Neutrino from the Atmosphere with a Layer Model of the Earth

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Abstract. The neutrino oscillation probability behavior traveling from the source in the atmosphere, its propagation through the Earth and its detection are shown using Earth Layers model by atmospheric neutrino oscillation analysis. Neutrino oscillation plots are generated from a developed computacional code, considering the influence or not of the atmosphere.

1. Introduction

Neutrino oscillation is a quantum phenomenon proposed by Bruno Pontecorvo in 1957 [1, 2] in analogy to the oscillation between Kaons. Neutrino oscillation is a phenomenon of flavor change when it propagates through space, a detailed description can be found in [3]. Brief description of the neutrino oscillation model is given, where the Greek subscripts α and β denoted flavor states ($|\nu_\alpha\rangle$) and the Latin subscripts j and k denoted mass states ($|\nu_k\rangle$). The flavor and mass states are an orthonormal basis respectively and satisfy (1),

$$\langle\nu_k|\nu_j\rangle = \delta_{kj} \qquad \langle\nu_\alpha|\nu_\beta\rangle = \delta_{\alpha\beta} \tag{1}$$

where δ_{kj} and $\delta_{\alpha\beta}$ are the Kronecker delta. So the neutrino flavor state is a combinations of the neutrino mass states and vice versa,

$$|\nu_\alpha\rangle = \sum_k U^*_{\alpha k}\,|\nu_k\rangle \qquad |\nu_k\rangle = \sum_\alpha U_{\alpha k}\,|\nu_\alpha\rangle \tag{2}$$

where $U_{\alpha k}$ are complex numbers that are elements of the mixing matrix U and $U^*_{\alpha k}$ is the complex conjugate of $U_{\alpha k}$. In a vacuum the massive neutrino states $|\nu_k\rangle$ are eigenstates of the Hamiltonian $\hat{\mathrm{H}}$ of the Schrödinger equation

$$\hat{\mathrm{H}}|\nu_k\rangle = E_k|\nu_k\rangle, \qquad E_k = \sqrt{\vec{p}^{\,2} + m_k^2} \tag{3}$$

where the Schrödinger equation in vacuum is wriiten with natural units ($c = \hbar = 1$)

$$i\frac{\partial}{\partial t}|\nu_k(t)\rangle = \hat{\mathrm{H}}|\nu_k(t)\rangle \tag{4}$$

Massive neutrino states evolve in time as plane waves

$$|\nu_k(t)\rangle = e^{-iE_k t}|\nu_k\rangle \tag{5}$$

using (2) and (5) we obtain $|\nu_\alpha(t)\rangle$ is a combination of flavor states $|\nu_\beta\rangle$,

$$|\nu_\alpha(t)\rangle = \sum_{k,j} U^*_{\alpha k} U_{\beta j} e^{-iE_k L}|\nu_\beta\rangle; \quad L = ct \tag{6}$$

The probability that a neutrino originally of flavor α will later be observed as having flavor β is

$$\mathrm{P}_{\alpha\beta} = \sum_{k,j} U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j} e^{-i(E_k - E_j)L} \tag{7}$$

Also, considering that the antineutrino flavor state $|\bar{\nu}_\alpha\rangle$ is described for combinations of antineutrino mass states $|\bar{\nu}_k\rangle$ and vice versa

$$|\bar{\nu}_\alpha\rangle = \sum_k U_{\alpha k}\,|\bar{\nu}_k\rangle \qquad |\bar{\nu}_k\rangle = \sum_\alpha U^*_{\alpha k}\,|\bar{\nu}_\alpha\rangle \tag{8}$$

Similarly, the probability that an antineutrino originally of flavor α is later observed as having flavor β is

$$\mathrm{P}_{\bar{\alpha}\bar{\beta}} = \sum_{k,j} U_{\alpha k} U^*_{\beta k} U^*_{\alpha j} U_{\beta j} e^{-i(E_k - E_j)L} \tag{9}$$

In neutrino oscillation experiments, the travel time is not measured, but it is known, which is the distance L between the source and the detector.

For this work, three flavors of neutrino (electron, muon and tau) and the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) mixing matrix were considered. The most recent data on the parameters for neutrino oscillation are taken from reference [4].

Neutrinos undergo interactions when they propagate through matter, this effect was first proposed by S. P. Mikheyev, A. Yu. Smirnov [5] and L. Wolfenstein [6]; which is known as the MSW effect. Electron neutrinos propagating through matter are subjected to a potential A due to the interactions of the charged current of the electrons in the matter.

$$A = \pm\sqrt{2} G_F N_e \tag{10}$$

where G_F is the Fermi constant, N_e is electronic density and the sign is for neutrinos (+) or antineutrinos (-).

Assuming constant electron density N_e in the matter where the neutrino propagates, the representation of potential mass states basis $\hat{\mathrm{V}}_m$ needs to be calculated by

$$\hat{\mathrm{V}}_m = \mathrm{U}^{-1}\hat{\mathrm{V}}_\mathrm{f}\mathrm{U} \qquad \hat{\mathrm{V}}_f = A\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{11}$$

where $\hat{\mathrm{V}}_f$ is the potential flavor states basis and U the PMNS mixing matrix. The unitary transformation that leads from the initial state of arbitrary flavor $\nu_f(0) = \psi_e(0)|\nu_e\rangle + \psi_\mu(0)|\nu_\mu\rangle + \psi_\tau(0)|\nu_\tau\rangle$ at the time $t = 0$ of neutrino production to the state of the neutrino that is detected at time $t > 0$, is given by

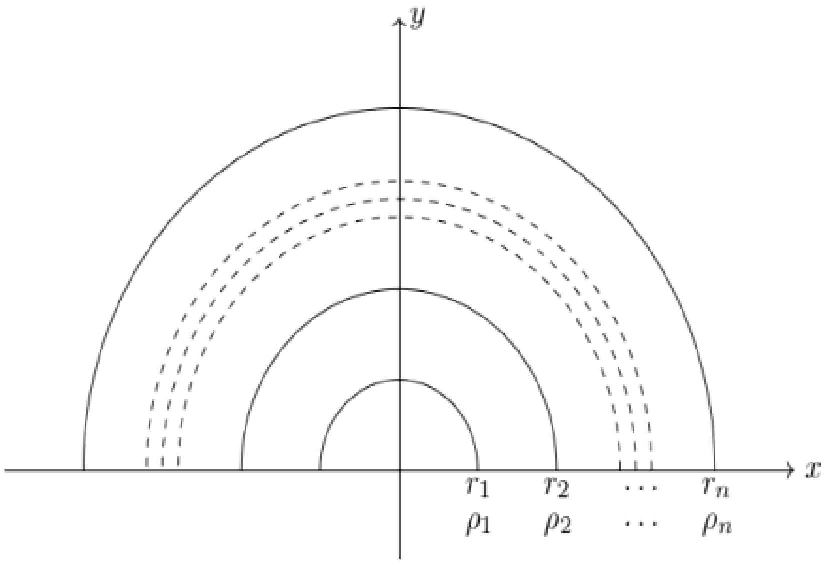

Figure 1. Representation of the layers of the Earth

$$\hat{\mathrm{U}}_f(L) = e^{i\hat{\mathrm{H}}_f L}, \qquad \hat{\mathrm{U}}_f(L) := \hat{\mathrm{U}}_f(L, 0) \tag{12}$$

where $\hat{\mathrm{U}}_f(L_2, L_1)$ is the flavor evolution operator from time t_1 to t_2 and this operator can be rewritten as $\hat{\mathrm{U}}_f(L) = \mathrm{U}e^{i\hat{\mathrm{H}}_m L}\mathrm{U}^{-1}$.

In the case that neutrinos propagate through matter, the Hamiltonian flavor states basis is not diagonal (11) and the flavor evolution operator $\hat{\mathrm{U}}_f(L)$ is calculated adding the potential $\hat{\mathrm{V}}_m = \mathrm{U}^{-1}\hat{\mathrm{V}}_\mathrm{f}\mathrm{U}$

$$\hat{\mathrm{H}}_f = \mathrm{U}(\hat{\mathrm{H}}_m + \mathrm{U}^{-1}\hat{\mathrm{V}}_\mathrm{m}\mathrm{U})\mathrm{U}^{-1} \tag{13}$$

A more detailed analysis of the procedure to calculate the probability of oscillation in matter can be seen [7].

If having a series of homogeneous layers of matter with density ρ_i, potential A_i, width L_i, with $i = 1, \ldots, n$. The flavor evolution operator is given by

$$\hat{\mathrm{U}}_f(L) = \prod_{i=1}^{n} \hat{\mathrm{U}}_f(L_i) = \hat{\mathrm{U}}_f(L_n) \cdots \hat{\mathrm{U}}_f(L_1), \qquad L = \sum_{i=1}^{n} L_i \tag{14}$$

where $\hat{\mathrm{U}}_f(L_i)$ is calculated with the potential A_i.

Finally, if the Earth is considered as a model of concentrically different spheres of homogeneous density, as shown in Fig. 1, each with radius r_i and density ρ_i, then (14) is used to calculate the oscillation probability of neutrinos traveling through from Earth where L is the total length that a neutrino travels from the source to its detection.

2. Methodology

A computational code is developed to calculate the probability of oscillation of neutrino in matter, where the model of the Earth is implemented according to the values of Table 1 taken from [8].

Table 1. Data

Radius [km]	Density [g/cm^3]	
$r_1 = 1220$	$^*\rho_1(r) = 13.0$	$r_0 \leq r < r_1$
$r_2 = 3480$	$\rho_2(r) = 11.3$	$r_1 \leq r < r_2$
$r_3 = 5701$	$\rho_3(r) = 5.0$	$r_2 \leq r < r_3$
$r_4 = 6378$	$\rho_4(r) = 3.3$	$r_3 \leq r < r_4$

* $r_0 = 0$ km

For the case in which the atmosphere is considered, the approximation considering that the neutrino travels in a vacuum.

The probability is calculated based on the length from its creation to its detection, the incident energy of the neutrino and the detector is considered to be on the Earth's surface. Also the total length L depends on the height h where the neutrino is created in the atmosphere, it is calculated by the expression

$$L = \sqrt{R^2 \cos^2\theta + h(2R + h)} - R\cos\theta \tag{15}$$

where R is the radius of the Earth, θ is the zenith angle is measured in a system anchored to the detector and h height from the normal to the Earth's surface. An equation similar to (15) can be obtained using the nadir angle as the authors Ohlsson and Snellman in [7].

3. Results

Fig. 2 shows the plots of neutrino oscillation probability where four Earth layers are considered in the absence the atmosphere, that is, it is considered that neutrinos are generated on the Earth's surface. Fig. 3 shows the plots probability considering four layers of Earth and the atmosphere where a height from the normal to the Earth's surface of $h = 15$ km where neutrinos are generated is considered.

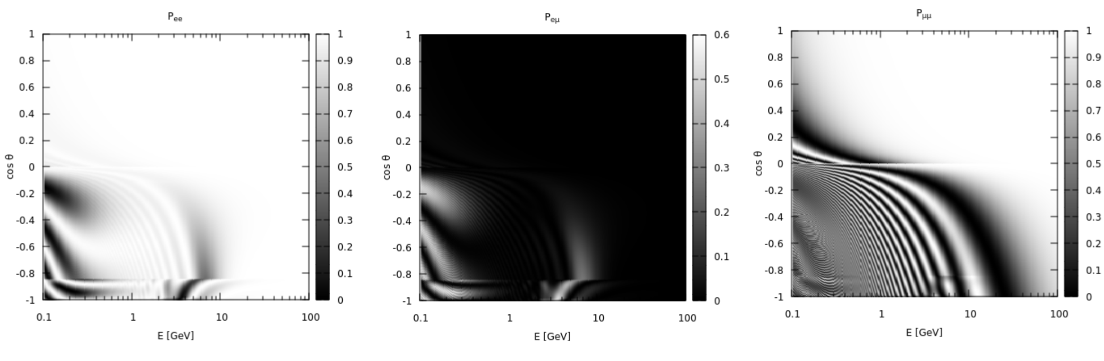

Figure 2. Neutrino oscillation probability in the absence the atmosphere

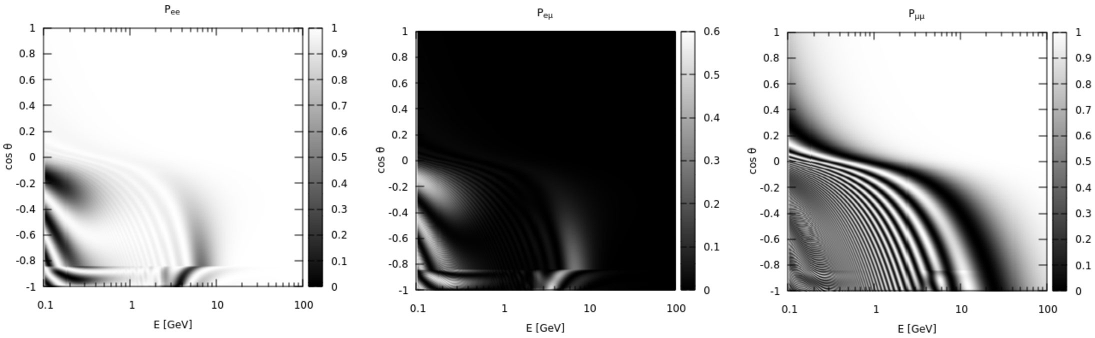

Figure 3. Neutrino oscillation probability considering the atmosphere

4. Discussion

When comparing the plots of Fig. 2 with the plotss of Fig. 3, a discontinuity can be noticed in the plotss of Fig. 2 for $\theta = \pi/2$, this effect is due to the fact that the plotss of Fig. 2 do not consider the layer of the atmosphere. Consult [8] to see the neutrino oscillation probability plots of the Super-Kamiokande collaboration, when considering the layer of the atmosphere the discontinuity shown in Fig. 2 disappears. Like this work in [8] consider a four-layer model of the Earth.

On the other hand, the discontinuity in the probability for $\cos\theta = 0.8$, this is due to the neutrinos passing through the Earth's core between r_2 and r_3, see Table 1, where the density is doubled. If you want to smooth out this discontinuity, you have to consider a model with more layers in this area.

A smooth plot in the core can be obtained if more layers of the Earth are taken in this area, but more computation time is needed for the calculation of the neutrino oscillation probability.

5. Conclusions

In this work, a study of the effects that matter has on neutrino oscillations was carried out, taking as a particular case, the propagation of neutrinos on Earth with a model of four Earth layers. In addition, the comparison of the probability of neutrino oscillation was made when considering the atmospheric layer with a thickness of 15 km, finding that by not considering the atmosphere, a discontinuity is formed in the behavior of the probability, which disappears when it is considered. It was found that the analysis of four layers of the Earth considering the atmosphere, Fig. 3, are similar to those found in [8] and the discontinuity disappears when the atmosphere shown in Fig. 2 is not considered.

The developed code can be implemented for the oscillation of neutrinos produced in an accelerator, since it only passes through one layer of the Earth, then it is only necessary to calculate the flavor evolution operator for one layer, where an average density is taken that is representative for the experiment. The present study becomes important, since it is a key piece for the analysis of atmospheric and accelerator neutrinos.

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