Coulomb scattering in the nonlocal framework

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Introduction

Recent studies [1] have renewed interest in the nonlocal nature of nucleon-nucleus interaction by demonstrating its importance in predicting scattering and reaction observables. Incorporation of nonlocality transforms conventional Schrödinger equation into an integro-differential equation of the form:

$$\left[\frac{\hbar^2}{2\mu}\nabla^2 - V_{\rm L}(\mathbf{r}) + E\right]\Psi(\mathbf{r}) = \int V_{\rm NL}(\mathbf{r},\mathbf{r}')\Psi(\mathbf{r}')d\mathbf{r}' \,, \quad (1)$$

where $V_{\rm L}$ is some local interaction, while $V_{\rm NL}$ is the nonlocal interaction.

Apparently, it is difficult to solve an integro-differential equation analytically or numerically. In our recent work [2] we have developed a very efficient and highly precise technique using Iterative Perturbative Approach (IPA) to solve this equation. This method is successfully used to study neutron scattering off different nuclei across the periodic table in the low energy region. In this paper we use this method to study Coulomb scattering, focusing on the proton scattering in particular.

Formalism

After performing partial wave expansion of $V_{\rm NL}(\mathbf{r},\mathbf{r}')$ and $\Psi(\mathbf{r})$ in Eq.(1), the radial form of the integro-differential for the proton scattering off the spin-zero nucleus is written as:

$$\hat{\mathcal{L}}\, u_{jl}(r) = \frac{2\mu}{\hbar^2}\int_0^\infty g_l(r,r')u_{jl}(r')dr' \,, \quad (2)$$

with, $\hat{\mathcal{L}} \equiv \left[\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} + \frac{2\mu}{\hbar^2}(E - V_{\rm L}(r))\right]$,

$$g_l(r,r') = 2\pi r r' \int_{-1}^{1} V_{\rm NL}(\mathbf{r},\mathbf{r}')\, P_l(\cos\theta)\, d(\cos\theta) \,.$$

The local interaction, $V_{\rm L}(r)$, comprises of Coulomb and spin-orbit interactions, written as

$$V_{\rm L}(r) = V_c(r) - U_{\rm SO}(r) f_{jl} \,, \quad (3)$$

with $f_{jl} = [j(j+1) - l(l+1) - (3/4)]/2$. A separable form is used for $V_{\rm NL}(\mathbf{r},\mathbf{r}')$ [3] resulting in

$$V_{\rm NL}(\mathbf{r},\mathbf{r}') = H(|\mathbf{r}-\mathbf{r}'|)\, U\left(\frac{|\mathbf{r}+\mathbf{r}'|}{2}\right) . \quad (4)$$

The function $H(|\mathbf{r}-\mathbf{r}'|)$ is chosen to be a Gaussian with the range β and is normalized to unity. For different potentials, namely, proton-nucleus (U), Coulomb (V_c) and the spin-orbit ($U_{\rm SO}$) we use the parameterized form given by Tian *et al.* [4].

The solution of Eq.(2) is obtained by using IPA [2], which is expressed as

$$u_{jl}(r) = u^0_{jl}(r) + \sum_{k=1}^{\infty} u^k_{jl}(r) \,, \quad (5)$$

where $u^0_{jl}(r)$ is the zeroth-order solution and $u^k_{jl}(r)$ is the higher-order correction quantifying the deviation from the exact solution. The zeroth-order solution is obtained by solving: $\hat{\mathcal{O}} u_{jl}(r) = 0$, where

$$\hat{\mathcal{O}} \equiv \frac{d^2}{dr^2} - X_l(r)\frac{d}{dr} + W_l(r)\left[-\frac{l(l+1)}{r^2} + \frac{2\mu}{\hbar^2}(E - V_{\rm L}(r)) - I_{l0}(r)\right], \quad (6)$$

$$X_l(r) = \frac{I_{l1}(r)}{(1 - I_{l2}(r))} \,, \quad W_l(r) = \frac{1}{(1 - I_{l2}(r))} \,,$$

$$I_{ln}(r) = \frac{2\mu}{\hbar^2}\int_0^\infty \frac{(r'-r)^n}{n!} g_l(r,r')dr' \,.$$

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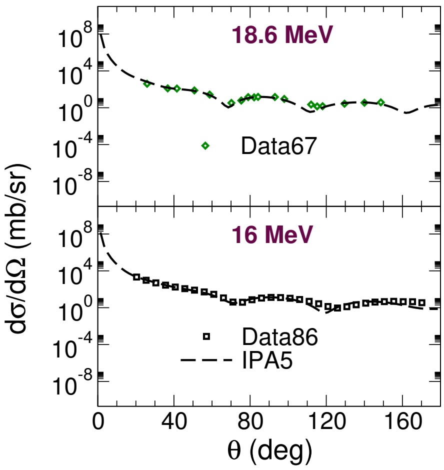
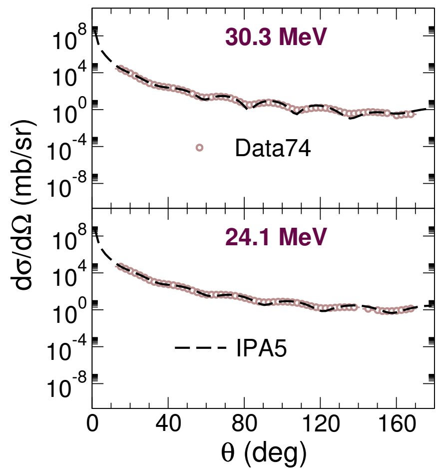


(a) p-^{56}Fe scattering



(b) p-^{208}Pb scattering

FIG. 1: Calculated angular distributions along with the data sets for proton scattering off ^{56}Fe (Data86: [5] and Data67: [6]) and ^{208}Pb (Data74: [7]).

The higher-order correction terms are obtained with the help of following iterative scheme:

$$\hat{\mathcal{O}} u_{jl}^{i+1}(r) - W_l(r)\,\xi_{jl}^i(r) \;=\; 0\,, \quad (i \geq 0) \qquad (7)$$

where,

$$\xi_{jl}^i(r) \;=\; \frac{2\mu}{\hbar^2}\int_0^\infty g_l(r,r')u_{jl}^i(r')dr' \;-\; \hat{\mathcal{G}}\,u_{jl}^i(r)\,,$$

$$\hat{\mathcal{G}} \;\equiv\; I_{l0}(r) + I_{l1}(r)\frac{d}{dr} + I_{l2}(r)\frac{d^2}{dr^2}$$

The wave function, $u_{jl}(r)$, thus obtained, is then matched with the free-state wave function to calculate the S-matrix, which in turn is employed in computation of observables.

Results and Conclusions

In Fig.(1) we compare the angular cross sections calculated after 5 iterations (labelled as IPA5) with the experiments. As an illustration we show results for proton scattering off ^{56}Fe and ^{208}Pb. The calculated results are found to be in good agreement with the experiments [5–7]. Thus, it can be concluded that the IPA can be used to study the Coulomb scattering also.

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