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MICROWAVE IMPEDANCE MATCHING OF FEED WAVEGUIDES TO THE DISK-LOADED ACCELERATOR STRUCTURE OPERATING IN THE $2\pi/3$ MODE

I. INTRODUCTION AND HISTORICAL REVIEW

An interesting problem in microwave impedance matching arises when an ordinary rectangular waveguide is to be connected to the traveling wave periodic structure used in a microwave linear electron accelerator.

The periodic structure is derived by starting from uniform circular waveguide operating in the TM_{01} mode. There is a longitudinal component of electric field in this mode which would be useful for accelerating bunches of electrons, if the phase velocity of the wave could be reduced to the velocity of light (or lower, for use in the short initial process of capturing and bunching the electrons and accelerating them to very close to the velocity of light).

By adding inductive loading disks with a center aperture, the phase velocity of the wave can be reduced as desired. The aperture size and disk spacing can be used to control the strength of the electric field on the axis, for a given amount of power available. With equally spaced disks, a periodic traveling wave structure is created, which can also be looked upon as a series of cascaded coupled resonant cavities or two-port networks, forming a bandpass filter, with an infinite number of passbands and stop bands.[1] Only the lowest frequency passband is of interest here.

The combination of aperture size and disk spacing was studied for the purpose of obtaining the highest accelerating field on the axis of the structure, compatible with the lowest stored energy and filling time. It was found that the best spacing of disks would be 3-1/2 disks per wavelength.[2] In early accelerators at Stanford, the four disk structure was chosen because the measurement of its microwave properties was simplified for this particular disk spacing.[3]

The "mode" of such a periodic structure is customarily defined as the phase shift per cavity, in radians. The definition can be applied in two senses.

First is the normal operating condition, in which the phase velocity is equal to the velocity of light, and the frequency is fixed to assure this. In this case the guide wavelength is equal to the free space wavelength, and the phase shift per cavity, or the mode, is simply $(2\pi)/n$, where n is the number of disks per free space wavelength at the predetermined fixed operating frequency. Thus the operating mode of a four disk (per wavelength) structure is $(2\pi)/4$, or $\pi/2$, and for three disks, it is $(2\pi)/3$.

The second use of the definition is in studying the properties of the periodic structure and in doing development work such as microwave measurements and impedance matching. The guide wavelength becomes infinite at the low frequency end of the passband, thus the mode is 0 (zero) because the phase is constant everywhere and there is zero phase shift per cavity. At the high frequency end of the passband, the guide wavelength becomes twice the disk spacing (for any disk spacing whatever) and the mode is $(2\pi)/2$, or π. By varying the frequency over the passband range, the "mode" actually varies from 0 to π radians phase shift per cavity, although only at the design frequency does the phase velocity coincide with the velocity of light, and only then if the disks are spaced no further apart than two disks per free space wavelength.

Further experimental work at Stanford showed an appreciable advantage in accelerating field, for a given group velocity, for the three disks per wavelength spacing over that of the four disks, and this new spacing was chosen for use in several commercial accelerators and the Stanford two-mile machine.

II. THE PROBLEM AND A TECHNIQUE FOR MEETING IT

The peak power required for adequate accelerating field strength is great enough to require high power sources of microwave energy. The transmission line between a microwave source and the periodic structure is customarily a standard rectangular waveguide, operating in the TE_{10} mode, and is either pressurized or evacuated.

Matching the junction between two different transmission lines is a problem for which standard techniques exist.[4] One approach is to make microwave impedance measurements in the standard transmission line, while moving the position of a total reflection (usually a short circuit) by known discrete amounts in the nonstandard but uniform transmission line. This technique would apparently be directly usable for matching the junction between rectangular waveguide and the accelerator periodic structure (Fig. 1), even though the periodic structure is not a uniform transmission line. In a periodic structure, the design phase shift per period is known, and by detuning successive cavities, it would appear that the effect equivalent to moving a short known amounts on a uniform line would be obtainable.

However, in actual practice it is often desired to match an unfinished coupler which has been manufactured integrally attached to its periodic structure, while the periodic structure is also unfinished. The latter is unfinished in the sense that the cavity diameter, the most critical dimension with respect to phase shift, is manufactured slightly oversize because of the practical impossibility of holding its mechanical tolerance as perfectly as desired. (An oversize cavity can be adjusted electrically by denting, squeezing, or otherwise pushing in on the outside of the cavity wall.) But this lack of exact knowledge of the unfinished cavities' phase shifts is just what causes the failure of the matching technique referred to above. In the procedure to be given in Section III, none of the cavities after the first one need to be resonated.

The nature of the situation is such that the problem can still be attacked and workable modifications of the technique described above can be made relatively easily for a four disk (or $\pi/2$) structure,[5] less readily for a three disk (or $2\pi/3$) one. Matching a coupler to a $\pi/2$ mode structure has been done using a detuning plunger and a dielectric sliver partially coated with resistive material. These are positioned in the first two or more cavities (including the one with the matching junction) while impedance data is taken in the input waveguide, from which it is possible to determine what steps to take to adjust the junction for proper coupling and tuning.

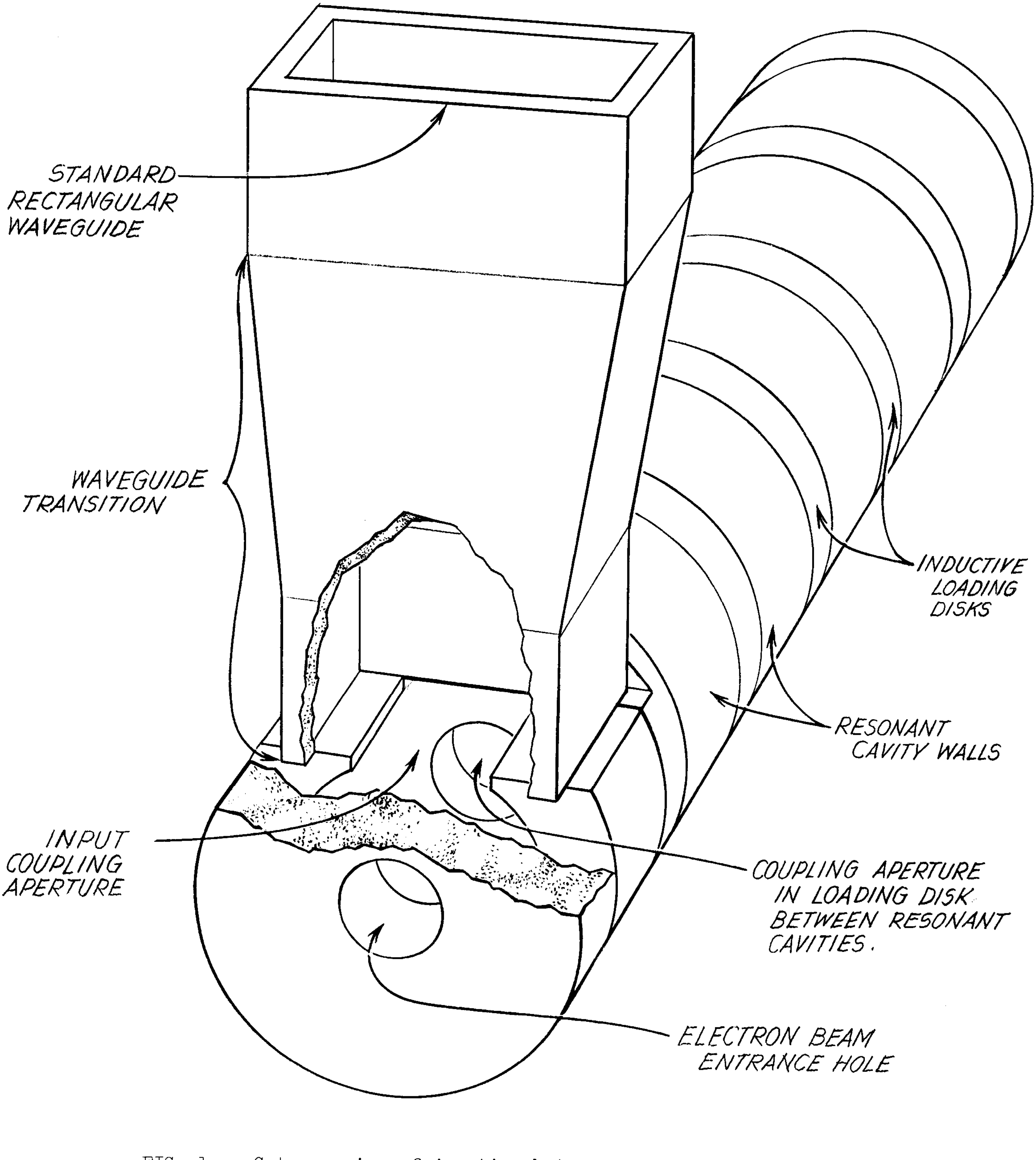


FIG. 1 -- Cutaway view of junction between rectangular waveguide and the periodic structure.

It should be realized that the determination of the dimensions of any particular junction for proper coupling should have to be determined only once, after which the couplers for succeeding accelerating sections can be manufactured to the finished dimensions with sufficient accuracy. The need to tune the first cavity remains, but this is done in the same way as for the other cavities.

III. MATCHING AND TUNING PROCEDURE (Method of R. L. Kyhl)

The following procedure for matching and tuning an undercoupled waveguide junction to a $2\pi/3$ mode structure eliminates the need to have exactly tuned cavities. The input coupling cavity is the only cavity tuned at any time during this procedure.

1. Detune the input cavity with the usual metallic plunger, and determine the "detuned short" positions in the input waveguide for each of three frequencies:[6]
   a. The frequency for the $2\pi/3$ mode (operating frequency),
   b. The frequency for the $\pi/2$ mode in the structure (as determined from resonances in a short section of exactly tuned test cavities),
   c. The arithmetic mean of the above frequencies in "a" and "b", above.
2. Retract the detuning plunger so as to detune the following cavity.
3. At the average frequency given in step 1 (c), mechanically deform the first cavity to bring the input impedance point one-quarter of a guide wavelength away from the "detuned short" position for this frequency.
4. Leaving the detuning plunger in position, take impedance data at the $\pi/2$ mode and the $2\pi/3$ mode frequencies. If the coupling were correct, the data points would fall on a Smith chart plot as shown in Fig. 2. Since it is assumed that the cavity is undercoupled (in order to be able to remove metal at the coupling aperture), the data points will actually fall as in Fig. 3.

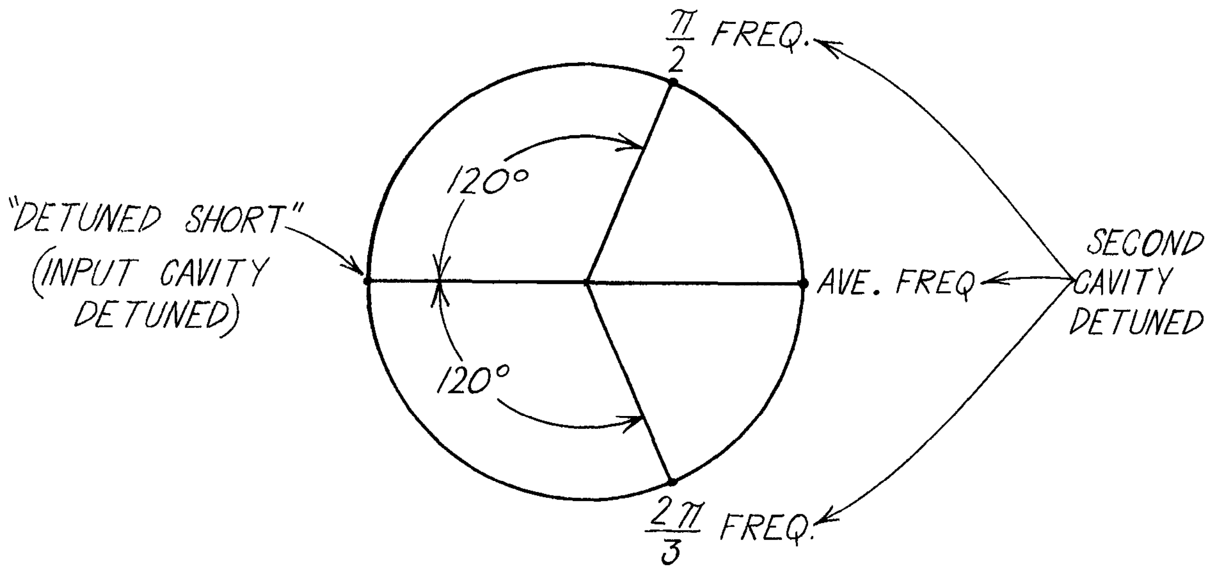


FIG. 2--Smith chart plot, first cavity properly coupled and tuned.

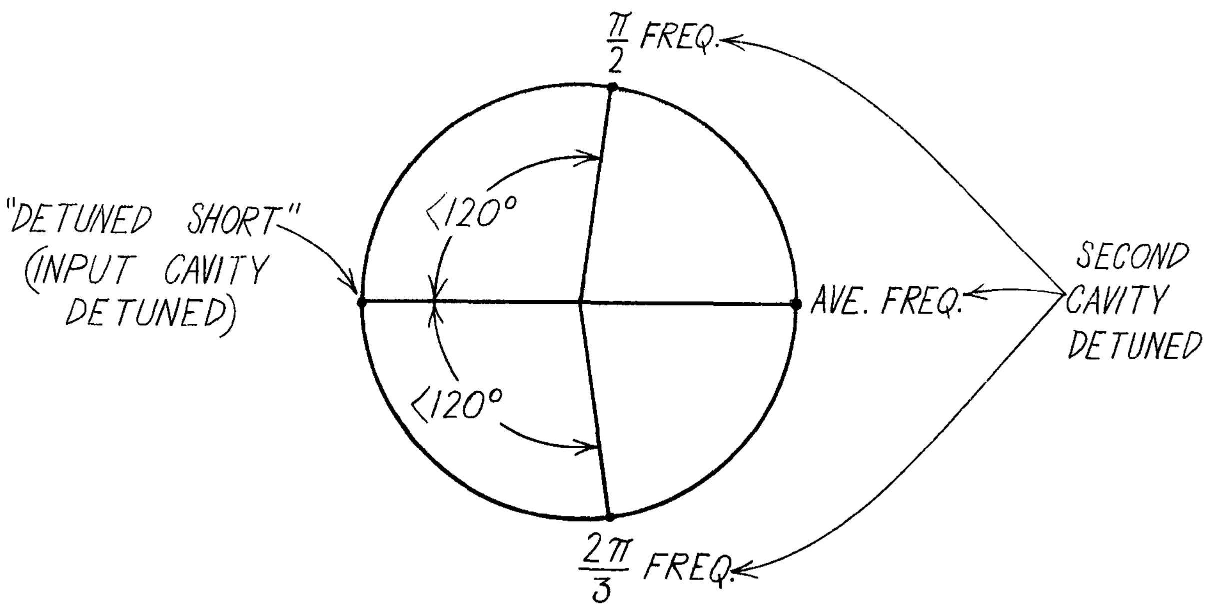


FIG. 3--Smith chart plot, first cavity tuned but undercoupled.

5. The aperture should be opened a small amount by machining, and the new dimensions measured and recorded. This will detune the first cavity slightly, in addition to increasing the coupling.
6. Repeat steps 3, 4, and 5, until, at some repetition of step 4, the coupling is found to be correct. The coupler is now both tuned and matched. Measure the input coupling aperture dimensions and record for future use.

IV. BASIS FOR THE FOREGOING PROCEDURE

The electrical properties of the periodic structure operating in the lowest passband can be determined from the study of an equivalent lumped circuit bandpass filter network representation.[7,8,9,10]

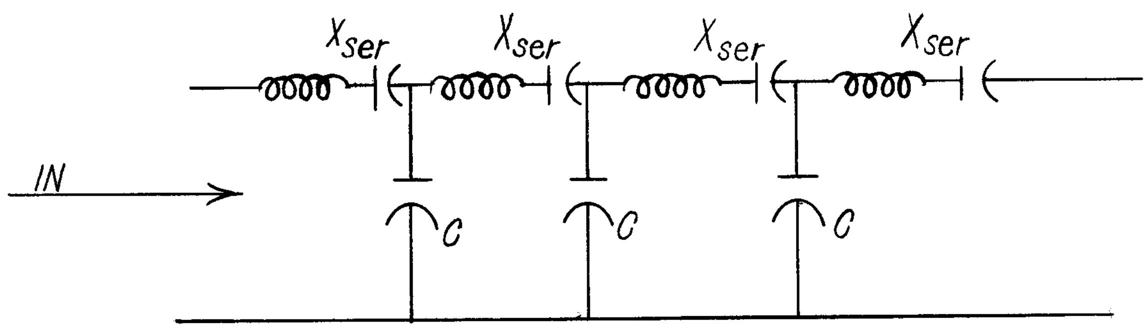


FIG. 4--Equivalent circuit of microwave linear accelerator periodic structure.

In Fig. 4, X_{ser} is the total reactance of the series arm, and is assumed to be linear with frequency over the passband.[11,12] C is the coupling capacitance between cavities. Its susceptance is assumed to be constant over the passband, and is evaluated at the $\pi/2$ mode frequency for definiteness. The first cavity is assumed to be essentially identical to the other cavities. The coupling between cavities is assumed to be correct, as the disk aperture dimensions are held to sufficient accuracy in manufacturing.

Including a means of coupling, in this case represented by a transformer because of magnetic field coupling between the waveguide and the first cavity[13] the circuit becomes as shown in Fig. 5.

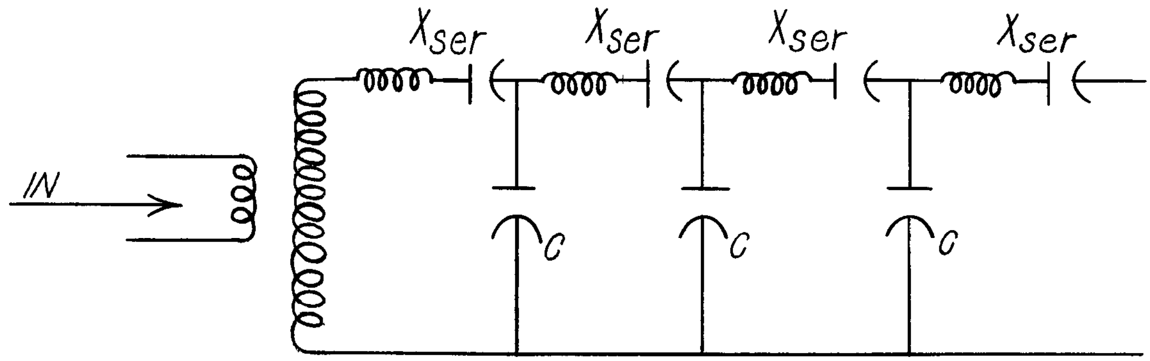


FIG. 5--Input coupling transformer added to Fig. 4.

If the coupler cavity is detuned, and to represent the modifying effects of fringing fields at the coupling aperture and at the entrance hole for the electron beam, frequency insensitive reactances can be added to the network as in Fig. 6.

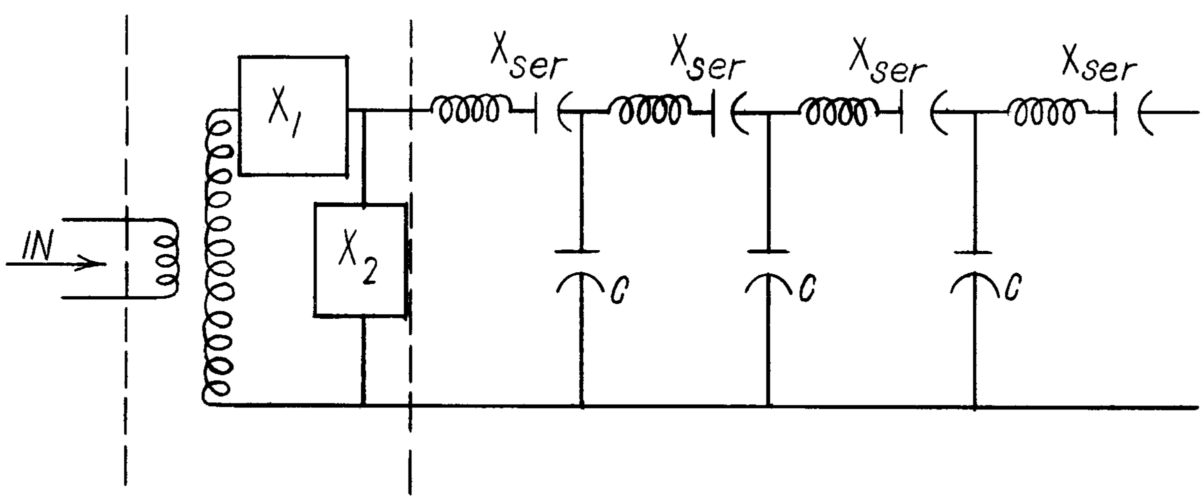


FIG. 6--Periodic structure with arbitrary input coupling network.

The input network shown between vertical dashed lines in Fig. 6 has three arbitrary elements, and is therefore capable of representing any actual frequency insensitive coupling system used with the periodic structure.

The next step is to modify the circuit for mathematical convenience by choosing to view the periodic structure halfway through a coupling capacitor C placed at the network input. At the same time the arbitrary input coupling network is modified accordingly, and is represented by a general two-port network in Fig. 7.

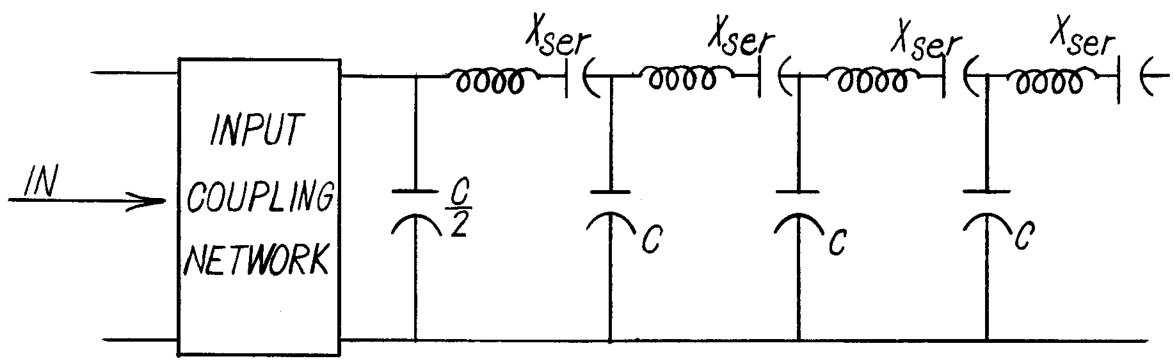


FIG. 7--Periodic structure and input network, suitable for mathematical analysis.

The circuit of the periodic structure can now be considered as suitable for analysis as a standard bandpass filter, viewed at the mid-shunt terminals.[14]

It is perhaps more basic to start the analysis by considering the structure and input network when operating in the $\pi/2$ mode.

The mid-shunt characteristic admittance Y_o of the periodic network is given by [15]

$$Y_o = \sqrt{y_1 y_2} \sqrt{1 + \frac{y_2}{4y_1}}$$

Y_o is a pure conductance over the passband, varying from infinity at the low end to zero at the high frequency end.

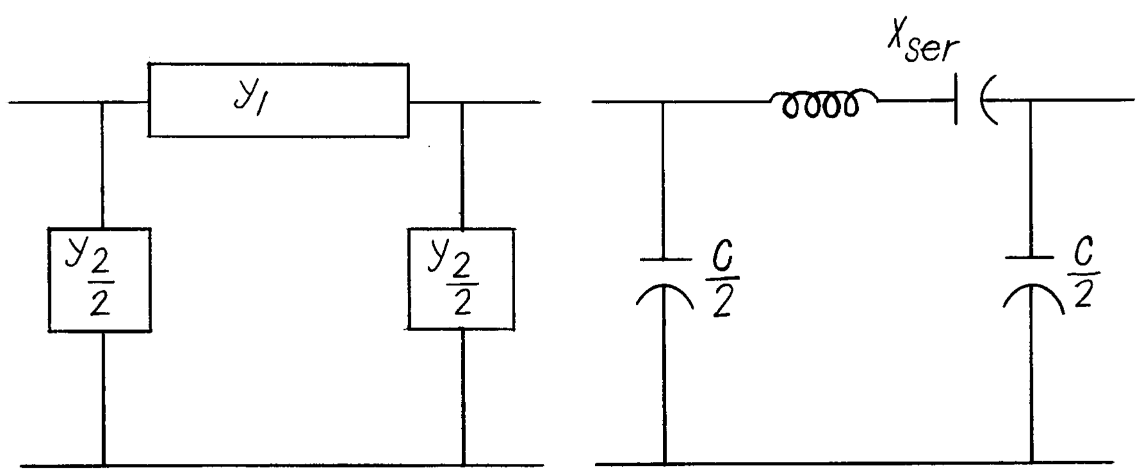
$X_{ser}(0)$ is zero at the low frequency end of the passband (0-mode), representing series resonance of the frequency sensitive elements of the cavity. The high frequency end of the passband (π-mode) occurs when the mid-shunt network section (Fig. 8) becomes series resonant, that is, when

$$jX_{ser} - j4X_C = 0$$

or

$$X_{ser}(\pi) = 4X_C = \frac{4}{Y_C}$$

where it should be recalled that $X_C = 1/Y_C$ is constant over the passband.



a. General mid-shunt section

b. Mid-shunt section of accelerator periodic structure

FIG. 8

At the $\frac{\pi}{2}$ frequency,

$$X_{ser}\left(\frac{\pi}{2}\right) = 2X_C = \frac{2}{Y_C}$$

as it will be recalled that X_{ser} varies linearly with frequency over the passband. The $\pi/2$ mode frequency is halfway across the passband.

$$Y_o\left(\frac{\pi}{2}\right) = \sqrt{\frac{jY_C}{jX_{ser}\left(\frac{\pi}{2}\right)}} \sqrt{1 + \frac{jY_C}{4/jX_{ser}\left(\frac{\pi}{2}\right)}}$$

$$= \sqrt{\frac{Y_C^2}{2}} \sqrt{1 - \frac{1}{2}} = \frac{Y_C}{2}$$

Properly placing a detuning plunger in a cavity of the periodic structure has the effect in the equivalent circuit of disconnecting the series arm from the preceding section of the network. See Figs. 9 and 10.

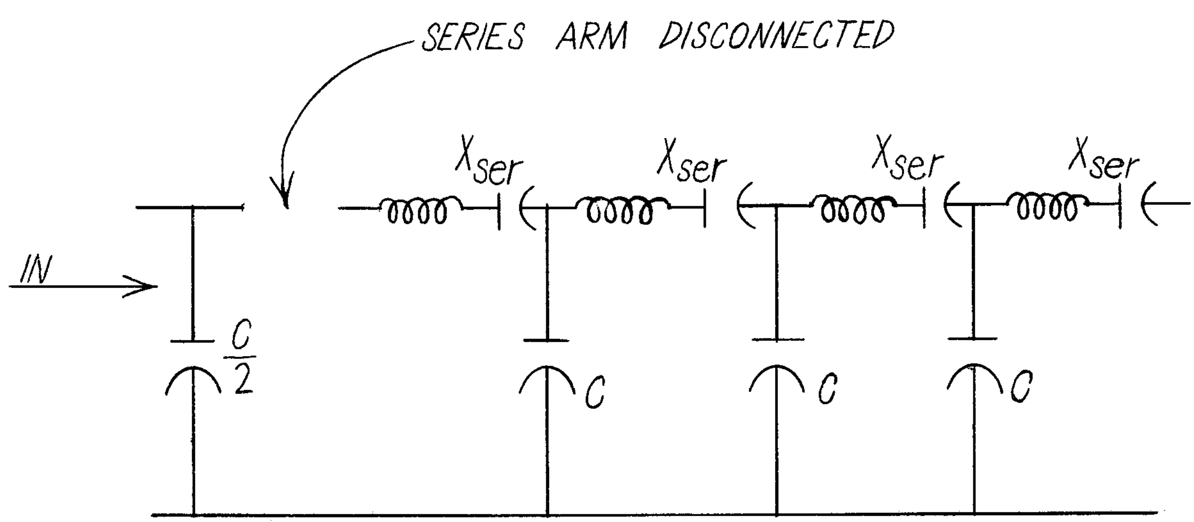


FIG. 9--Periodic structure with coupler cavity detuned.

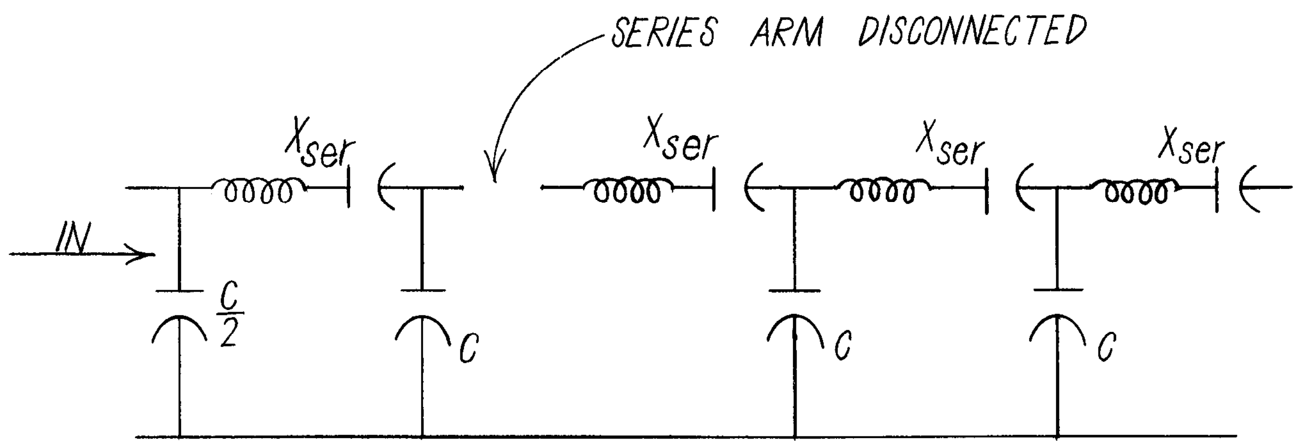


FIG. 10--Periodic structure with second cavity detuned.

The input admittance in Fig. 9 is evidently given by

$$y_{in} = j \frac{Y_C}{2} .$$

In Fig. 10,

$$y_{in} = - j \frac{Y_C}{2} ,$$

by definition of the mode of the network, and verified by calculation.

We now have data points which can be plotted on a Smith Chart. We will normalize admittances to the characteristic admittance of the network, $Y_o = Y_C/2$, and view the system at the input to the coupling network, maintaining normalized admittances in the input transmission line. Thus we obtain Fig. 12, considering the Smith Chart to be the unit circle in the complex reflection coefficient plane (Fig. 11), with admittance contours of pure conductance and pure susceptance overlaid on the circle.

We are now prepared to ascertain the change in position of the points in Fig. 12 if we wish to perform the same impedance measurements on a structure designed, tuned, and matched for operation in the $2\pi/3$ mode.

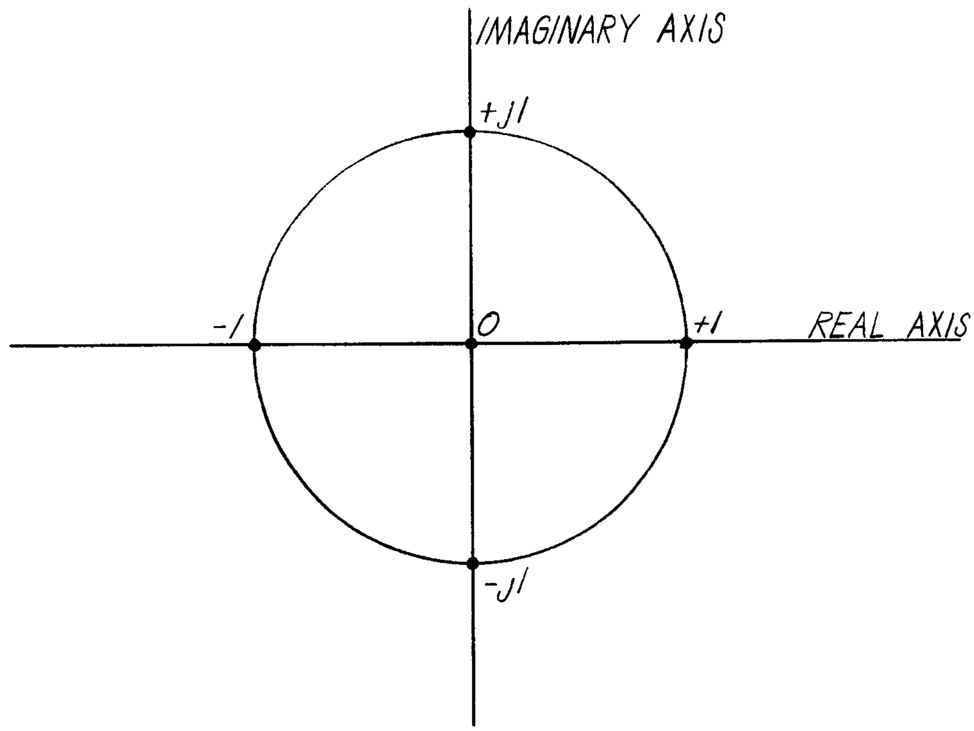


Fig. 11--Unit circle of reflection coefficients in the complex plane. Basis of Smith chart.

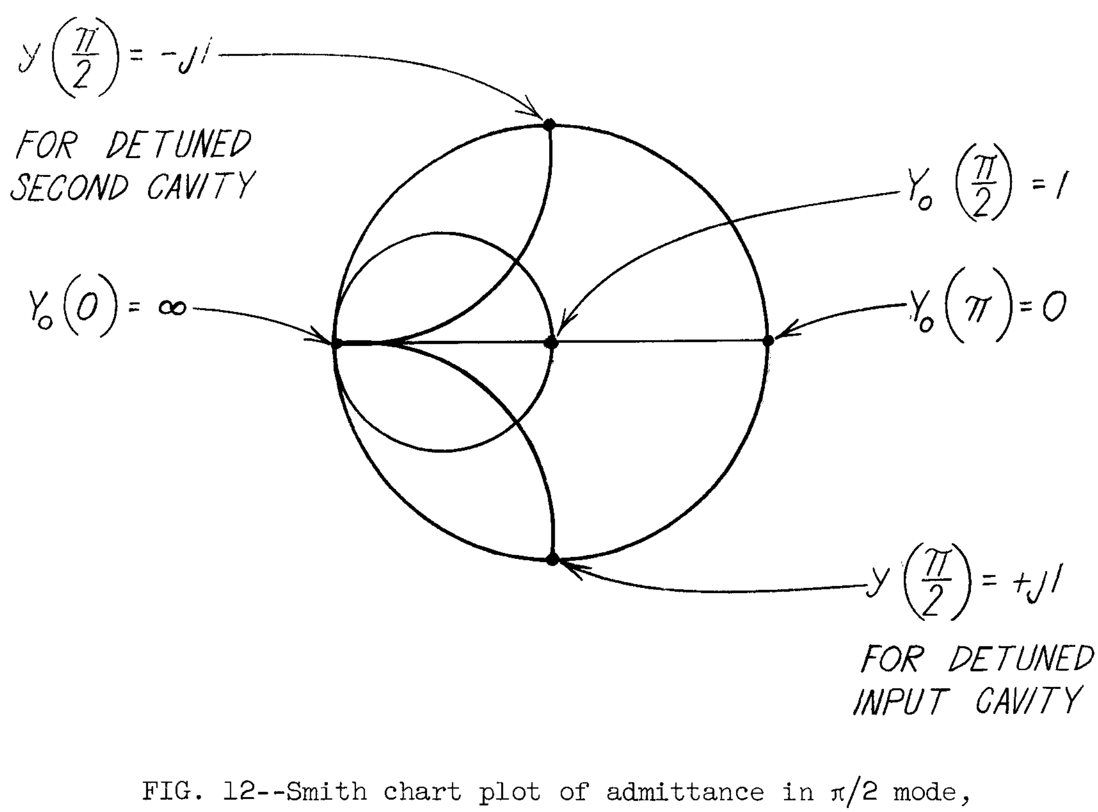


FIG. 12--Smith chart plot of admittance in $\pi/2$ mode, and characteristic admittance across passband.

We have first to calculate the ratio of characteristic admittances in the $\pi/2$ and in the $2\pi/3$ modes. The usual design of accelerator structures, with low group velocity and reasonably thin disks widely spaced, has a Brillouin or ω - β diagram which is very nearly sinusoidal in shape and can be represented by

$$f \quad = f\left(\frac{\pi}{2}\right) - \left[f\left(\frac{\pi}{2}\right) - f(0)\right] \cos \beta d$$

where

f	is the frequency
$f\left(\frac{\pi}{2}\right)$	is the frequency for the $\pi/2$ mode,
$f(0)$	is the frequency for the 0 mode,
β	is the phase shift in radians per unit length of the periodic structure,
d	is the length of the periodic spacing.

Then for $\beta d = \frac{2\pi}{3}$, $f\left(\frac{2\pi}{3}\right) = f(0) + \frac{3}{4}\left[f(\pi) - f(0)\right]$,

$$X_{ser}\left(\frac{2\pi}{3}\right) = \frac{3}{4} X_{ser}(\pi) = \frac{3}{Y_C} ,$$

$$Y_o\left(\frac{2\pi}{3}\right) = \frac{1}{\sqrt{3}} \frac{Y_C}{2} = \frac{1}{\sqrt{3}} Y_o\left(\frac{\pi}{2}\right)$$

We also note that in Fig. 10

$$y_{in}\left(\frac{2\pi}{3}\right) = 0.$$

Now the Smith Chart plot of Fig. 12 can be renormalized to represent the conditions existing when the input network is adjusted for a match in the $2\pi/3$ mode of operation. To accomplish this normalization, all that is necessary is to multiply all admittances on the chart of Fig. 12

by $\sqrt{3}$, which will bring $Y_o(2\pi/3)$ to the center of the chart, Fig. 13.

When Fig. 2 is rotated 60° counter-clockwise, the data points will be seen to coincide with those of Fig. 13.

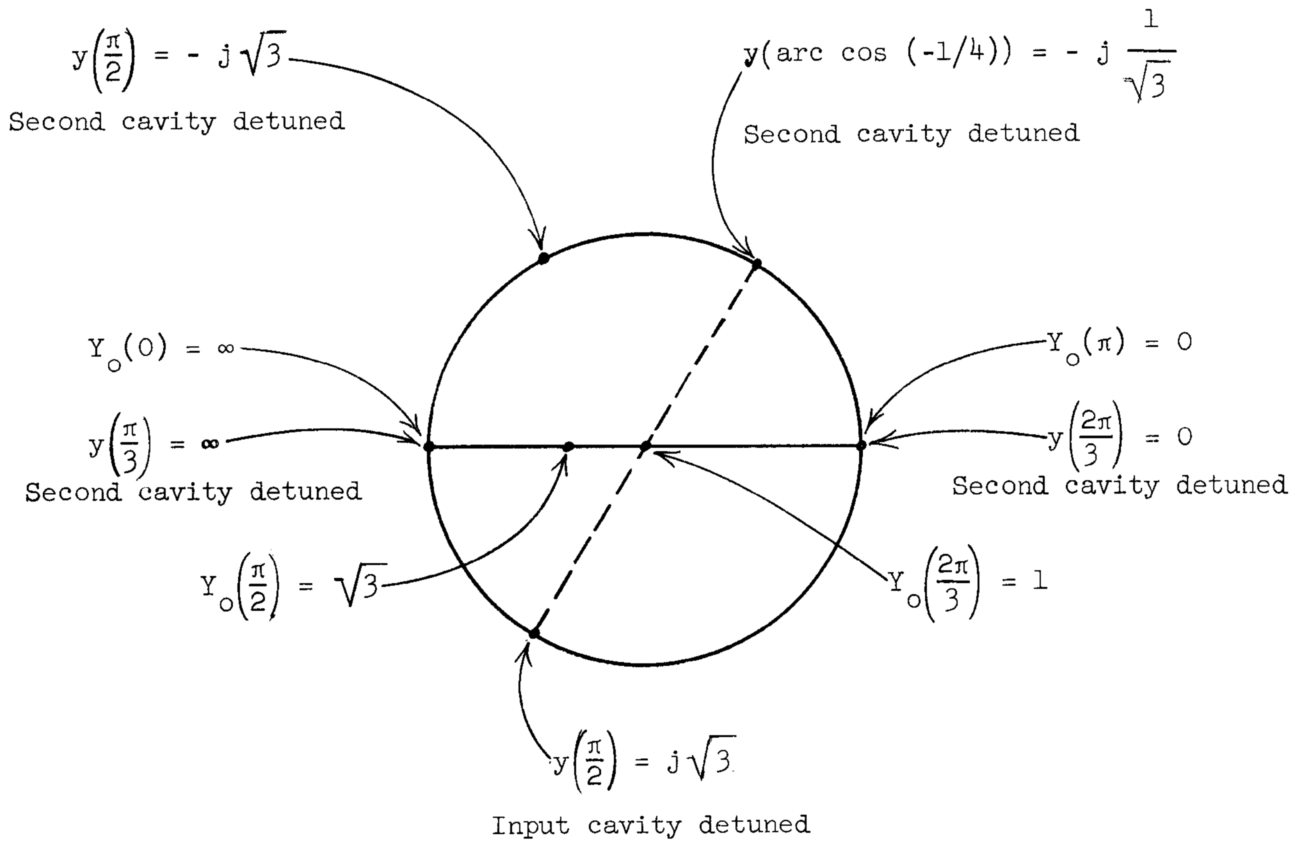


FIG. 13--Admittance plot for structure matched in the $2\pi/3$ mode. Arc cos $(-1/4)$ is the mode for the mean frequency between the $\pi/2$ mode and the $2\pi/3$ mode frequencies.

If the input coupler is undercoupled, the points of Fig. 13 are redistributed by a new mismatch multiplication factor. Consider the conformal and symmetry properties of the transformation. Then, if the input cavity is tuned, at the average frequency between the $\pi/2$ and $2\pi/3$ frequencies, to produce the input admittance point diametrically opposite to the (shifted) detuned input cavity admittance point, the detuned second cavity admittance points for the $\pi/2$ and $2\pi/3$ frequencies will remain symmetrically disposed with respect to this diameter. This verifies Fig. 3 and completes the proof of the matching procedure.

V. ADDITIONAL DISCUSSION

It was mentioned above that it is impractical to manufacture a complete periodic structure with the cavity diameter held to its final tuned dimension. However, it should be possible to produce a set of loose cavity walls and loading disks of such accuracy that they yield the design value of phase shift per cavity when resonated in a multi-cavity test arrangement made by clamping the pieces together in a suitable jig. Then the coupler to be matched could be clamped to the set of cavities and the impedance data taken by detuning successive cavities and interpreting the data as previously mentioned.

Also, it should be possible to adapt the Kyhl method to matching a coupler in the $\pi/2$ mode as well as in the $2\pi/3$ mode, or in fact to any mode in which the coupling between cavities remains small enough to avoid interaction between cavities when detuning one of them.

It would be preferable to select a pair of experimentally determined frequencies (for matching a $\pi/2$ structure) corresponding to modes obtainable from a relatively short length of resonant test section. Inspection of Fig. 13 shows that these modes should be between the $\pi/3$ and $2\pi/3$ modes, as the closer the chosen modes are to the $\pi/2$ mode, the more sensitive the phase of the input admittance is to deviation from proper coupling. This would call for a very long test section for modes close to $\pi/2$.

As a practical example, a 12 cavity test section could be set up. It will have, among others, resonances at the $5\pi/12$ and $7\pi/12$ modes. The respective normalized input admittances at the corresponding frequencies will be -j3.146 and -j0.318 when the second cavity is detuned and the first cavity is properly coupled. These admittance points will lie respectively counter-clockwise and clockwise by 0.0760 guide wavelength from the -j1.0 susceptance point seen at the input when the second cavity is detuned and the input admittance is measured at the $\pi/2$ mode frequency.

VI. ACKNOWLEDGMENT

The basic approach to the matching procedure for the $2\pi/3$ structure was worked out by Dr. R. L. Kyhl while the author was employed by High Voltage Engineering Corporation.

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