PROCEEDINGS OF SCIENCE

Celestial amplitudes for Goldstone bosons and soft theorems

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The S-matrix for a QFT in 4D Minkowski space is an inherently holographic object, i.e. defined at the (conformal) boundary of spacetime. A section of this boundary is the celestial 2-sphere and Lorentz group acts on it by conformal transformations. I will briefly review scattering, when translated from the basis of plane waves (translation eigenstates) to the conformal basis (dilatation eigenstates). The resulting object is called a celestial amplitude and the change of basis is implemented for massless particles by a Mellin transform. I will apply this formalism to amplitudes of Goldstone bosons with an emphasis on their soft theorems. The illustrative example will be the U(1) (non)-linear sigma model.

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1. Introduction and motivation

The ultimate dream of the celestial holography program [1, 2] is to provide a holographic description of quantum gravity (in the spirit of the AdS/CFT correspondence) for asymptotically flat spacetimes.

However, this goal is still a long way ahead, and for the time being there is a more pragmatic approach. The more modest point of view is that celestial amplitudes are S-matrix elements in a new basis of scattering states that make some properties more manifest. In particular, it was shown [1, 2] that soft theorems for scattering amplitudes can be written as Ward identities for asymptotic symmetries. This reformulation quickly led to the discovery of a new soft theorem for gravity [3].

This connection works so far well for gauge theories (e.g. QED, Yang–Mills theory or gravity), where asymptotic symmetries can be interpreted as "large" gauge transformations (i.e. those that do not vanish at null or timelike infinity of spacetime).

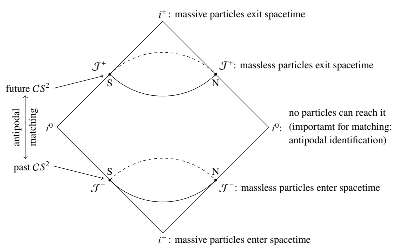
Yet, there is another class of theories, where soft theorems play a fundamental role. It is thus interesting to investigate their fate within the celestial holography program. These are theories of scalar particles with a spontaneously broken continuous global symmetry. Goldstone theorem then implies the presence of massless modes in the spectrum, whose low energy effective field theory (EFT) – a non-linear sigma model (NLSM) – implements soft theorems. The simplest of such models is a single charged (i.e. complex) scalar field with a spontaneously broken $\mathrm{U}(1)$ symmetry.

However, Ward identities for the broken currents that produce (the gradients of) the Goldstone modes from the vacuum cannot be straightforwardly interpreted as soft theorems for celestial amplitudes. This is due to the fact that they are currents of global symmetries, in contrast to local asymptotic symmetries for gauge theories.

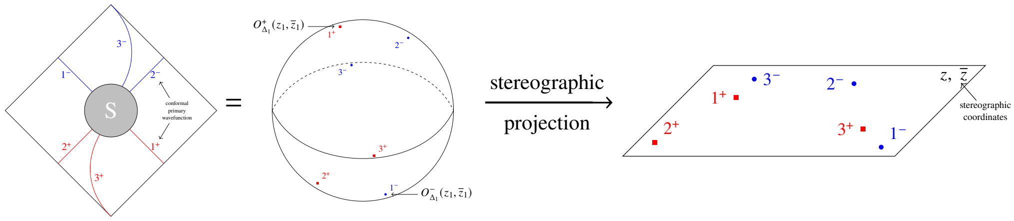
Plan of the note. I will provide only a very basic motivation for the change of basis for the S-matrix elements, leading thus to *celestial amplitudes*. Then I will summarize the non-technical results of the exploration of celestial amplitudes of Goldstone bosons. Many more details and explicit computations of celestial amplitudes of various Goldstone boson EFTs can be found in [4].

2. Symmetry considerations

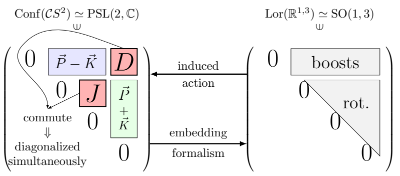
The celestial conformal field theory (CCFT) lives on the celestial sphere CS^2, a section of null infinity in the Penrose diagram of Minkowski space,



that is parametrized via a stereographic projection by a complex coordinate z.



The 2D stereographic plane associated to CS^2 can be embedded into the 4D projective Minkowski lightcone (embedding formalism for 2D CFT), which induces an isomorphism between the 2D conformal group $\mathrm{PSL}(2,\mathbb{C})$ and 4D Lorentz group $\mathrm{SO}(1,3)$



$$\begin{matrix} \mathrm{PSL}(2,\mathbb{C}) & \simeq & \mathrm{SO}(1,3) \\ \Downarrow & & \Downarrow \\ M & \mapsto & \Lambda(M) \end{matrix} \; :$$

to a point x in Minkowski spacetime associate a hermitian matrix $\hat{x} = x^\mu \sigma_\mu$; then the isomorphism between M and Λ takes the form: $M\hat{x}M^\dagger \leftrightarrow \Lambda(M)\cdot x$

This isomorphism provides the basic setup for the following step.

3. Motivating the change of basis for 1-particle states (massless particles)

For simplicity, I will sketch the *conformal basis construction* for massless particles only, as they are helicity s eigenstates. For massless particles it is convenient to work in the spinor-helicity formalism, where the momentum of a particle gets decomposed into a Kronecker product of a positive and negative chirality Weyl spinor, $p_{\alpha\dot{\alpha}} = \lambda_\alpha \widetilde{\lambda}_{\dot{\alpha}}$.

Assume, such a particle moves in direction-3. Being a helicity eigenstate with eigenvalue s means $\lambda \mapsto e^{i\frac{s}{2}}\lambda$, $\widetilde{\lambda} \mapsto e^{-i\frac{s}{2}}\widetilde{\lambda}$ for a rotation about the 3-axis. This leaves the ratio $z := \frac{\lambda_1}{\lambda_2}$ invariant, which thus gets identified with the direction of movement. Hence the rotation about the 3-axis (i.e. the J generator above) is already diagonalized for a massless particle.

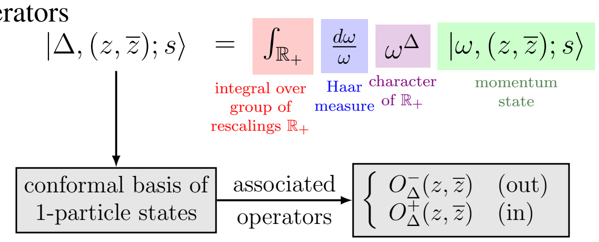
In a (celestial) CFT under construction, a fundamental role is played by the dilatation generator D (i.e. a boost around the 3-axis). It is desirable to adapt the basis of 1-particle states to this generator and thus diagonalize it. A key observation from the isomorphism in the previous section is that the generators J and D commute and hence can be diagonalized simultaneously. This implies that the conformal basis of 1-particle states will be labelled by the helicity s (eigenvalue of J) and a yet to be specified scaling dimension Δ (eigenvalue of D).

A conformal transformation $M \in \mathrm{PSL}(2,\mathbb{C})$ acts on the Weyl spinors and by direct substitution also on the null momentum as

$$\begin{pmatrix}\lambda_1\\ \lambda_2\end{pmatrix} \mapsto \begin{pmatrix} a & b\\ c & d\end{pmatrix}\begin{pmatrix}\lambda_1\\ \lambda_2\end{pmatrix} \implies z = \frac{\lambda_1}{\lambda_2} \mapsto \frac{a\lambda_1 + b\lambda_2}{c\lambda_1 + d\lambda_2} = \frac{az+b}{cz+d}, \text{ hence } \boxed{p_{\alpha\dot{\alpha}} = \omega \begin{pmatrix} z\\ 1\end{pmatrix}\begin{pmatrix}\bar{z} & 1\end{pmatrix}},$$

where $\omega = \lambda_2\widetilde{\lambda}_2$ is the Rindler energy given by $E = \omega\,(1 + z\bar{z})$. Now the action of a dilatation (boost about the 3-axis) on the null momentum can be easily deduced. It rescales the momentum by a positive constant $\xi \in \mathbb{R}_+$, $p_{\alpha\dot{\alpha}} \mapsto \xi p_{\alpha\dot{\alpha}}$. This operation leaves $(z,\bar{z})$ invariant and rescales ω as $\omega \mapsto \xi\omega$.

So far, the momentum 1-particle states are parametrized as $|\vec{p};s\rangle := |\omega,(z,\bar{z});s\rangle$. To pass from the momentum basis to the conformal basis of 1-particle states, one needs to diagonalize the scaling transformation of ω. In analogy to the Fourier transform that diagonalizes the group of translations $\mathbb{R}$, the group of rescalings $\mathbb{R}_+$ is diagonalized by a Mellin transform providing a definition of the *conformal basis of 1-particle states* and by the operator state correspondence also associated primary operators



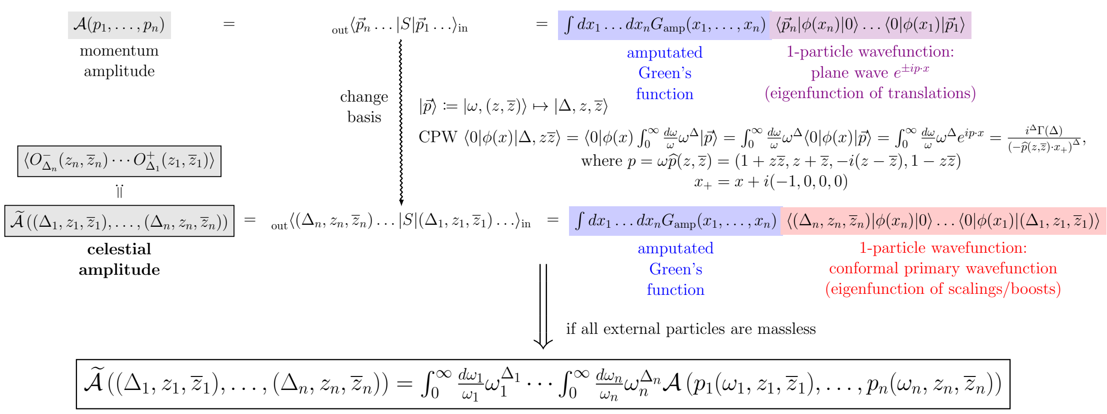
4. LSZ reduction: from conformal primary wavefunctions to celestial amplitudes

Finally, let me motivate the following result [5] – by the change of basis for 1-particle states from eigenstates of *translations* to *boost/dilatation* eigenstates, the S-matrix in the new basis can be written as a conformal correlator on CS^2, i.e. a *celestial amplitude*

$$\langle \text{out}|S|\text{in}\rangle_{\text{boost}} := \langle O^{\pm}_{\Delta_1}(z_1,\bar{z}_1)\dots O^{\pm}_{\Delta_N}(z_N,\bar{z}_N)\rangle_{\text{CCFT}} = \left[\prod_{j:\text{massive}} \int_{H_3} d\hat{p}_j K_{\Delta_j}\left(\hat{p}_j|(z_j,\bar{z}_j)\right)\right] \times \left[\prod_{i:\text{massless}} \int_0^\infty \frac{d\omega_i}{\omega_i}\omega_i^{\Delta_i}\right] \langle \text{out}|S|\text{in}\rangle_{\text{transl.}},$$

where each external leg is acted on by a different integral transform, based on whether it is massive/massless. Massive legs are transformed by a more complicated integral kernel K called a bulk to boundary propagator on the mass-shell hyperboloid H_3 and $\langle \text{out}|S|\text{in}\rangle_{\text{transl.}}$ is the standard momentum space scattering amplitude.

This is achieved by a LSZ reduction of correlation (Green's) functions to S-matrix elements with respect to the two different basis of 1-particle states defined above. Performing the reduction for momentum states, one obtains momentum scattering amplitudes, while for the conformal basis one obtains celestial amplitudes. The main logic is summarized in the following diagram



5. Conclusions

- for the charged scalar model with spontaneously broken U(1) global symmetry, low point *celestial amplitudes of Goldstone bosons* can be explicitly computed and their *soft theorems verified* [4] (this holds also for a large variety of other models of Goldstone bosons considered in that paper)
- however, unlike for gauge theories, where soft theorems were equivalent to Ward identities for asymptotic symmetries (large gauge transformations), in this setting there is no natural candidate for an asymptotic symmetry, so the connection between soft theorems and Ward identities remains unclear
- for the celestial amplitude to be well defined, there is tension between IR and UV behavior of the momentum amplitude – at least one of them should be sufficiently soft (technically, a fundamental strip of holomorphy is associated with a Mellin transform from which the function can be analytically continued elsewhere; when these conditions are not satisfied the strip shrinks and the celestial amplitude is at best defined in a distributional sense as a function of the scaling dimensions)

Acknowledgments

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