

RECEIVED: November 11, 2023

ACCEPTED: November 14, 2023

PUBLISHED: December 14, 2023

Erratum: In-medium loop corrections and longitudinally polarized gauge bosons in high-energy showers

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ERRATUM TO: [JHEP12\(2018\)120](#)

ARXIV EPRINT: [1806.08796](#)

Eqs. (A.17) and (A.18) for $F_{\text{seq}}(x_e, y_e, \Delta t)$ and $\mathcal{A}_{\text{seq}}^{\text{pole}}(x_e, y_e)$ were not properly written to handle front-end transformations, such as in eq. (A.47) for evaluation of the (o+p+q) subset of $\gamma \rightarrow e\bar{e}$ virtual diagrams. In the case of F_{seq} , the Ω 's should have been $\bar{\Omega} = \Omega \text{sgn}(M)$, though this makes no difference where $\text{sgn}(M)$'s are squared. Overall, eq. (A.17) should be replaced by

$$F_{\text{seq}}(x_e, y_e, \Delta t) = \frac{\alpha_{\text{EM}}^2 P_{e \rightarrow e}(x_e) P_{\gamma \rightarrow e}(\frac{y_e}{1-x_e})}{4\pi^2(1-x_e)} \left[\text{Re}(i(\Omega \text{sgn } M)_i) \text{Re}(\Delta t (\Omega_f^{\text{seq}})^2 \csc^2(\Omega_f^{\text{seq}} \Delta t)) + \text{Re}(i(\Omega \text{sgn } M)_f^{\text{seq}}) \text{Re}(\Delta t \Omega_i^2 \csc^2(\Omega_i \Delta t)) \right], \quad (\text{A.17})$$

To fix eq. (A.18) for $\mathcal{A}_{\text{seq}}^{\text{pole}}(x_e, y_e)$, see appendix A of ref. [1] below for a correct derivation in the context of our later application to gluon splittings in QCD. The translation of that result to the case at hand is that our eq. (A.18) should be replaced by

$$\mathcal{A}_{\text{seq}}^{\text{pole}}(x_e, y_e) = -\frac{\alpha_{\text{EM}}^2 P_{e \rightarrow e}(x_e) P_{\gamma \rightarrow e}(\frac{y_e}{1-x_e})}{4\pi^2(1-x_e)} \text{Re} \left[i(\Omega \text{sgn } M)_i (1 + \frac{i\pi}{2} \text{sgn } M_f^{\text{seq}}) + i(\Omega \text{sgn } M)_f^{\text{seq}} (1 + \frac{i\pi}{2} \text{sgn } M_i) \right] \quad (\text{A.18a})$$

or equivalently

$$\begin{aligned} \mathcal{A}_{\text{seq}}^{\text{pole}}(x_e, y_e) &= -\frac{\alpha_{\text{EM}}^2 P_{e \rightarrow e}(x_e) P_{\gamma \rightarrow e}(\frac{y_e}{1-x_e})}{4\pi^2(1-x_e)} \text{Re}[i(\Omega \text{sgn } M)_{\text{i}} + i(\Omega \text{sgn } M)_{\text{f}}^{\text{seq}}] \\ &\times (1 - \frac{\pi}{2} \text{sgn } M_{\text{i}} \text{sgn } M_{\text{f}}^{\text{seq}}). \end{aligned} \quad (\text{A.18b})$$

[Note that we've also fixed the minor notational inconsistency that the arguments of $A_{\text{seq}}^{\text{pole}}(x, y)$ in the original publication should have been written $A_{\text{seq}}^{\text{pole}}(x_e, y_e)$.]

Also in order to make front-end transformation work correctly, the factor of $(x_e y_e z_e)^{3/2}$ appearing in the denominator of (A.30) and (E.27) should be $|x_e y_e z_e|^{3/2}$. This is for the same reasons as discussed in footnote 42 except that the last sentence of the footnote was inadequate: though it's true that the front-end transformation rule (4.5) does not change the sign of the product $x_e y_e z_e$, the transformation rules (A.47) and (A.48) to get NLO corrections to $\gamma \rightarrow e\bar{e}$ do change that sign.

In the last line of both eqs. (A.43) and (F.40), the factor $\frac{\hat{x}_2 \hat{x}_3}{\hat{x}_1 \hat{x}_2} \mathcal{D}_2^{(\mathbb{I})}$ should read $\frac{\hat{x}_2 \hat{x}_3}{\hat{x}_1 \hat{x}_4} \mathcal{D}_2^{(\mathbb{I})}$. Though this corrects a misprint in the presentation of one of our final analytic formulas (A.43), our numerical results in figure 12 were implemented with the correct formula and so are unchanged.

In figures 33 and 34, there are a total of three diagrams that contain two black spectator lines. For those diagrams, the bras and the kets were accidentally swapped in the specification of the δH matrix elements. Specifically,

$$\langle \mathbf{P}' | -i \delta H | \mathbf{P}_{32}, \mathbf{P}_{41} \rangle, \langle \mathbf{B}' | -i \delta H | \mathbf{B}_{32}, \mathbf{B}_{41} \rangle, \langle \mathbf{P}' | +i \overline{\delta H} | \mathbf{P}_{32}, \mathbf{P}_{41} \rangle, \langle \mathbf{B}' | +i \overline{\delta H} | \mathbf{B}_{32}, \mathbf{B}_{41} \rangle$$

should be respectively replaced by

$$\langle \mathbf{P}_{32}, \mathbf{P}_{41} | -i \delta H | \mathbf{P}' \rangle, \langle \mathbf{B}_{32}, \mathbf{B}_{41} | -i \delta H | \mathbf{B}' \rangle, \langle \mathbf{P}_{32}, \mathbf{P}_{41} | +i \overline{\delta H} | \mathbf{P}' \rangle, \langle \mathbf{B}_{32}, \mathbf{B}_{41} | +i \overline{\delta H} | \mathbf{B}' \rangle$$

in figure 33. Similarly

$$\langle \mathbf{P}_{32}, \mathbf{P}_{41} | -i \delta H | \mathbf{P}' \rangle, \langle \mathbf{B}_{32}, \mathbf{B}_{41} | -i \delta H | \mathbf{B}' \rangle$$

should be replaced by

$$\langle \mathbf{P}' | -i \delta H | \mathbf{P}_{32}, \mathbf{P}_{41} \rangle, \langle \mathbf{B}' | -i \delta H | \mathbf{B}_{32}, \mathbf{B}_{41} \rangle$$

in figure 34.

In the text following equation (D.2d), the definition of \mathbf{e}_{\pm} should read $\mathbf{e}_{\pm} \equiv \mathbf{e}_x \pm i\mathbf{e}_y$.

In the last line of figure 36, $\delta^{(2)}(\mathbf{B}_{23}) \delta^{(2)}(\mathbf{B}_{41})$ should be $\delta^{(2)}(\mathbf{B}_{34}) \delta^{(2)}(\mathbf{B}_{12})$.

The overall sign of the right-hand sides of eqs. (F.30), (F.33), (F.39) and (F.42) should be negated compared to the original publication. That is, for eqs. (F.30), (F.39), and (F.42), the overall minus sign should be changed to +, whereas for (F.33) an overall minus sign should be inserted in front of the right-hand side.

The variables x and y were accidentally interchanged on the right-hand side of (H.14), and also the real part should be taken of the right-hand side. The correct version is

$$2 \text{Re} \left[\frac{\Delta d\Gamma}{dx dy} \right]_{x\bar{y}y\bar{x}} = 2(1-x)^{-\epsilon} \text{Re} \left\{ \left[\frac{\Delta d\Gamma}{dx dy} \right]_{xy\bar{y}\bar{x}} \text{with } (x, y, E) \rightarrow \left(\frac{-x}{1-x}, \frac{y}{1-x}, (1-x)E \right) \right\}. \quad (\text{H.14})$$

None of the above corrections affected our final results for electron-initiated overlapping splitting rates except for our misprint in (A.43), already correctly implemented in our numerics. As a result, the numerical results of the later work of ref. [16] (in the original publication’s bibliography) are unchanged.

Acknowledgments

We are grateful to Omar Elgedawy for helping us double check these corrections.

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References

- [1] P. Arnold, T. Gorda and S. Iqbal, *The LPM effect in sequential bremsstrahlung: analytic results for sub-leading (single) logarithms*, *JHEP* **04** (2022) 085 [[arXiv:2112.05161](#)] [[INSPIRE](#)].