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BEAM LOADING COMPENSATION IN CHARGE-VARYING SCENARIOS WITH RF-TRACK

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Abstract

High intensity linacs based on compact accelerating RF structures suffer from beam loading effects, which result into a bunch-to-bunch energy loss as a consequence of the beam-induced excitation of the fundamental accelerating mode. To track charged particles under this effect, the code RF-Track implemented a beam loading module in version 2.2.2. For ultrarelativistic scenarios in travelling-wave structures, the simulation tool was limited to trains of bunches with equal charge per bunch. In this work, we present the latest update of the beam loading module in version 2.3.0, extending its capabilities to account for this effect in trains with different charges per bunch and allowing the performance of beam loading compensation studies in these scenarios.

INTRODUCTION

The Beam Loading (BL) effect is the gradient reduction that occurs in accelerating radiofrequency (RF) structures, when the particles traveling through it excite the fundamental mode, which exhibits a decelerating longitudinal electric field. Energy conservation requires that bunches lose energy in order to excite the mentioned mode [1]. In addition, given that standard operation discourages the damping of the fundamental mode, each bunch-induced excitation will last in the cavity for a long time. For this reason, consecutive bunches can also lose energy when they are affected by the beam induced field left by earlier ones.

A self-consistent module to simulate BL effects upon particle tracking was implemented in RF-Track's version 2.2.2 [2]. It is based on the power-diffusive model derived in Ref [3], which, for travelling-wave (TW) structures, gives the following gradient reduction equation:

$$-\frac{\partial G}{\partial t} = v_g \frac{\partial G}{\partial z} + \left(\frac{\partial v_g}{\partial z} - \frac{v_g}{\frac{r}{Q}} \frac{\partial \frac{r}{Q}}{\partial z} + \frac{\omega}{Q} \right) \frac{G}{2} + \frac{\omega \frac{r}{Q} \mathcal{T} \tilde{I}}{2} , \quad (1)$$

where G is the gradient, ω the angular frequency, v_g the group velocity of the cavity, Q its unloaded quality factor, r/Q the normalized shunt impedance per unit length, $\mathcal{T}$ the time-transit factor and $\tilde{I}$ is defined as:

$$\tilde{I} = \beta_z \frac{q_{\text{bunch}}}{T} F, \quad (2)$$

with F being the form-factor of the bunch, q_{bunch} its total charge, β_z its average longitudinal velocity (normalized by the speed of light c) and T the RF period.

Version 2.2.2 calculated beam loading effects based on the resolution of Eq. (1) prior to tracking, as bunches with equal charge and velocity c were assumed. This limits the simulation of scenarios where particles are lost along the accelerator, or where trains with different charge per bunch are injected. For this reason, a more flexible algorithm has been implemented in version 2.3.0, whose major upgrades are presented in this conference proceedings at [4].

FLEXIBLE BEAM LOADING MODEL

The longitudinal electric field (E_z) of the fundamental mode of a lossless TW structure can be written, by means of the Floquet theorem, as the superposition of several spatial harmonics ($l \in \mathbb{Z}$) as:

$$E_z(z,t) = \text{Re}\left[\sum_{l=-\infty}^{\infty} E_l \exp\left(-j(k_l z + \omega t)\right) \right], \quad (3)$$

where j is the imaginary unit, $E_l \in \mathbb{R}$ is the amplitude of each harmonic and k_l its wave number, which satisfies the following property:

$$k_l - k_{l-1} = \frac{2\pi}{L} . \quad (4)$$

where L stands for length of each periodic cell.

Usually, TW structures are designed in a way that the phase velocity of one harmonic, l^*, is c and $k_{l^*} = \omega/c$. In this case, for an ultrarelativistic particle, the gradient reads as:

$$\begin{aligned} G(z) &= \frac{1}{L} \int_0^L \mathrm{d}z E_z(z, \frac{z}{c}) = \\ &= \frac{1}{L} \sum_{l=-\infty}^{+\infty} E_l \int_0^L \cos\left(\frac{2\pi(l-l^*)z}{L} \right) \mathrm{d}z \\ &= \sum_{l=-\infty}^{+\infty} E_l \left[\frac{\sin\left(2\pi(l-l^*)\right)}{2\pi(l-l^*)} \right] = E_{l^*} . \end{aligned} \quad (5)$$

Equation (5) provides a physical meaning to the accelerating gradient: It is the amplitude of the spatial harmonic whose phase velocity synchronizes with the beam velocity.

Therefore, solving Eq. (1) for a single particle (G_{single}), one can compute the longitudinal electric field excited by this particle as $E_z(z,t) = \text{Re}\left[G_{\text{single}}(z,t) \exp\left(-j\omega(z/c - t)\right)\right]$. With this, the longitudinal wakefield (w_l) caused by this particle upon a test particle located at a distance s can be defined as [5]:

$$w_l(s) = \frac{c}{q_{\text{single}}} \int_{-\infty}^{\infty} \mathrm{d}t E_z(ct - s, t) . \quad (6)$$

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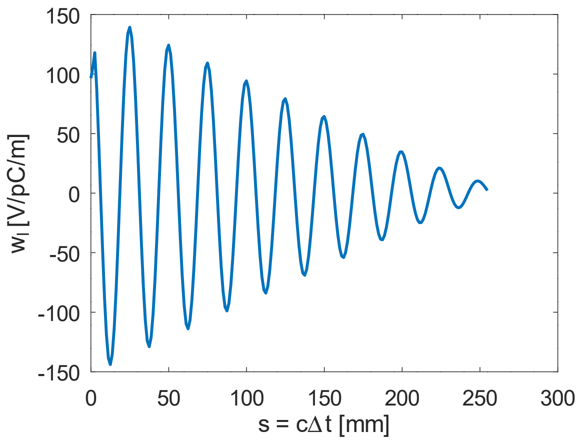

Figure 1: RF-Track calculation of the longitudinal wakefield per unit length for an X-band TW structure with $Q = 7200$, $v_g = 0.453\,c$ and $r/Q = 2300\,\Omega/\mathrm{m}$.

An example of the longitudinal wake function calculated for a constant impedance TW structure is shown in Fig. 1.

Therefore, for a beam with linear charge density λ_q in an RF structure with length L_total, the BL kick is defined as:

$$F_{z,\text{BL}}(z) = -\frac{1}{L_\text{total}} \int_{-\infty}^{\infty} \mathrm{d}s\, \lambda_q(s) w_l(z-s)\ . \tag{7}$$

Computation of BL Forces in RF-Track

For a train of N_bunches bunches with total charge-per-bunch $q^{(i)}_\text{bunch}$ ($i = 1, .., N_\text{bunches}$), the linear charge density can be expressed as the sum of the linear charge densities of each bunch $\lambda_q^{(i)}$ as:

$$\lambda_q(z) = \sum_{i=1}^{N_\text{bunches}} \lambda_q^{(i)}(z)\ . \tag{8}$$

To compute the BL force upon a particle that belongs to the j-th bunch ($j \leq N_\text{bunches}$), we substitute Eq. (8) in Eq. (7) and separate the terms as follows:

$$\begin{aligned} F_{z,\text{BL}}(z) &= -\sum_{i=1}^{N_\text{bunches}} \frac{1}{L_\text{total}} \int_{-\infty}^{\infty} \mathrm{d}s\, \lambda_q(s)^{(i)} w_l(z-s) \\ &= \underbrace{-\frac{1}{L_\text{total}} \int_{-\infty}^{\infty} \mathrm{d}s\, \lambda_q(s)^{(j)} w_l(z-s)}_{\text{Short-range Force}} \\ &\quad \underbrace{-\sum_{i\neq j} \frac{1}{L_\text{total}} \int_{-\infty}^{\infty} \mathrm{d}s\, \lambda_q(s)^{(i)} w_l(z-s)}_{\text{Long-range Force}}\ . \end{aligned} \tag{9}$$

To calculate the short-range force term in Eq. (9), the procedure is inherited from RF-Track's solid wakefield solver routine, which uses the Fast-Fourier-Transform (FFT) algorithm from the GSL Scientific Library [6].

To calculate the long-range force term in Eq. (9), we model far bunches as point-like particles. In this case, $\lambda_q^{(i)}(z) = q^{(i)}_\text{bunch}\delta(z - z_i)$, with z_i the mean position of the i-th bunch.

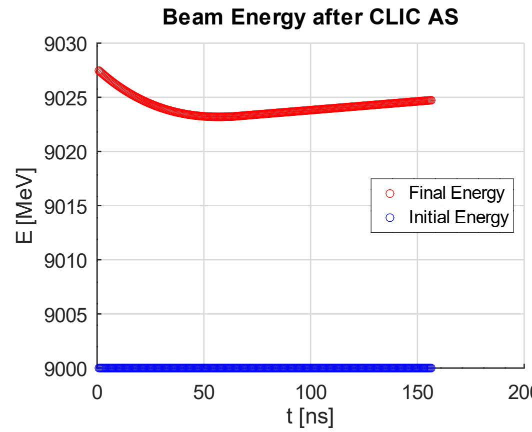

Figure 2: RF-Track calculation of the longitudinal phase-space of a charge-varying beam after CLIC AS according to Table 1.

This leads to the following simplified expression:

$$F^\text{long-range}_{z,\text{BL}}(z) = \frac{-1}{L_\text{total}} \sum_{i\neq j} q^{(i)}_\text{bunch}\, w_l(z - z_i)\ . \tag{10}$$

Equation (10) allows an efficient implementation of the long-range BL effects because the single particle wakefield w_l can be computed prior to tracking at the so-called initialization phase. Therefore, the long-range beam-induced kick is calculated on the fly by evaluating Eq. (10) by cubic interpolation.

BEAM LOADING IN CHARGE-VARYING SCENARIOS

As a test scenario, a train of 312 electron bunches with charge varying linearly from −600 to −300 pC/bunch has been tracked along an accelerating structure from the Compact Linear Collider (CLIC) [7] main linac. The structure and simulation parameters are shown in Table 1.

Table 1: CLIC AS Structure and Beam Parameters [7]

Quantity	**Units**	**Value**
Frequency	GHz	11.99
Number of cells	-	27
Group velocity	%c	(1.65, 0.83)
Quality factor	-	(5536, 5738)
Norm. shunt impedance (p.u.l)	kΩ/m	(14.59, 17.95)
Input power	MW	61.30
Number of bunches	-	312
Charge per bunch	pC	(-600,-300)
Bunch spacing	ns	0.50
Initial energy	GeV	9.0

The longitudinal phase space of the beam after the TW structure is shown in Fig. 2, where a start-to-end energy spread of 4.26 MeV is observed.

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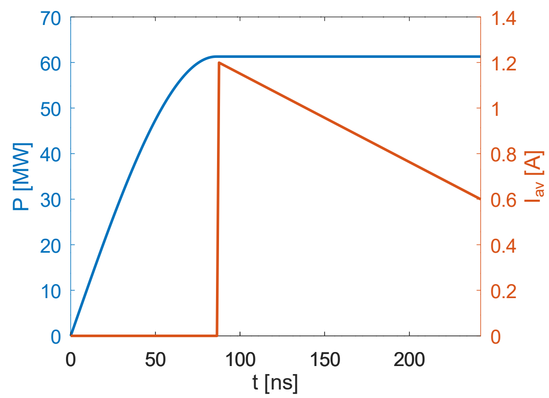

Figure 3: Optimal input power profile for BL compensation and charge-varying intensity profile.

Even though the train length is larger than the filling time (66 ns for this structure), there is no stabilization of the energy along the bunches. This illustrates a crucial feature of beam-loading scenarios without equal charge per bunch: the absence of a steady state. Time-dependent beam intensities do not allow for net power balance in the structure, and therefore, all beam-induced dynamics in these circumstances are transient.

BL Compensation

A beam loading compensation routine was implemented in RF-Track's version 2.2.3, as presented in Ref [8]. For TW structures, the strategy is based on the resolution of Eq. (1) for the unloaded case, where a suitable choice on boundary and initial conditions allows for the compensation of this effect.

In practice, if the beam is injected at a given time, t_{inj}, prior to the unloaded filling of the structure, and the input power P_{input} pulse is optimally tuned, the BL-induced energy spread can be mitigated.

To do so, we benefit from RF-Track's interface with Octave to use the "Nelder-Mead simplex method" [9], which is a single objective optimisation method in a multi-dimensional space and is named "fminsearch" in Octave. The merit function to minimize the long-range BL contribution was defined as:

$$f = (\langle E_1 \rangle - \langle E_{\text{end}} \rangle)^2 \,, \tag{11}$$

where $\langle E_1 \rangle$ is the mean energy of the first bunch and $\langle E_{\text{end}} \rangle$ the mean energy of the last one.

The degrees of freedom to minimize Eq. (11) were t_{inj} and the coefficients a_1 and a_2 that define the input power spline as:

$$P_{\text{input}}(t) = \begin{cases} a_1 t^3 + a_2 t^2 + a_3 t & \text{if } t < t_{\text{inj}} \\ P_{\text{input, max}} & \text{if } t \geq t_{\text{inj}} \end{cases} \tag{12}$$

Here, a_3 was chosen to ensure continuity of Eq. (12) at $t = t_{\text{inj}}$.

The optimal parameters that result from the minimization are shown in Table 2, and the corresponding input power curve can be seen in Fig. 3.

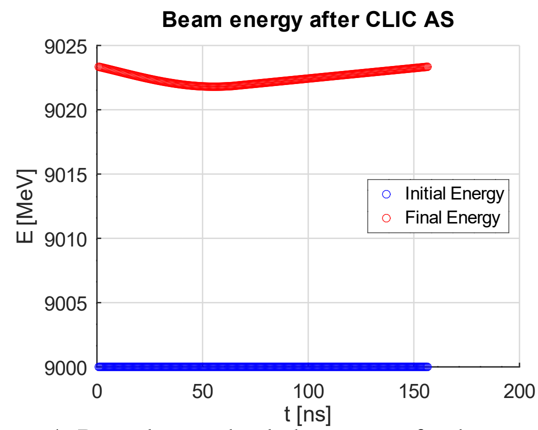

Figure 4: Beam longitudinal phase-space for the power and injection scheme in Fig. 3.

Figure 4 illustrates the beam longitudinal phase-space for the compensated scheme, showing that the energy spread is reduced from 4.23 MeV in Fig. 2 to 1.55 MeV.

Table 2: Optimal Input Power Parameters

Parameter	**Unit**	**Value**
a_1	MW/ns^3	−48.61
a_2	MW/ns^2	122.3
a_3	MW/ns	150.6×10^3
t_{inj}	ns	86.57

CONCLUSIONS

This work presents the latest update of the Beam Loading tool implemented in RF-Track's version 2.3.0, which allows the reproduction of beam loaded scenarios for trains whose bunches do not have the same charge.

The new algorithm calculates the longitudinal wakefield associated to the excitation of the fundamental mode in the considered RF structure, and computes both the short- and long- range forces associated to it.

The developed tool was used for tracking a linearly-decreasing charge-per-bunch train of electrons along CLIC's main linac TW structure, demonstrating that steady-state is not achieved in charge-varying scenarios. Furthermore, a compensation strategy was proposed by optimization of the input power shape and the injection time, achieving a beam-induced energy spread reduction of 63.4 %.

REFERENCES

[1] J. E. Leiss. "Beam Loading and transient behavior in travelling wave electron linear accelerators", in *Linear Accelerators*, Lapostolle, Amsterdam, Holland: A. L. North Holland Publishing Company, 1970, pp. 147–172.

[2] A. Latina. *RF-Track Reference Manual*. CERN, Geneva, Switzerland, June 2020. doi:10.5281/zenodo.3887085.

ISBN: 978-3-95450-219-6 ISSN: 2226-0366 doi: 10.18429/JACoW-LINAC2024-MOPB041

[3] J. Olivares Herrador, A. Latina, A. Aksoy, N. Fuster-Martinez, B. Gimeno, D. Esperante. "Implementation of the beam-loading effect in the tracking code RF-track based on a power-diffusive model", *Front. Phys.*, vol. 12, 2024. doi:10.3389/fphy.2024.1348042.

[4] A. Latina, "Studies of single and multi-bunch instabilities in linacs using RF-Track ", presented at LINAC'24, Chicago, IL, August 2024, paper MOPB039, this conference.

[5] T. Wangler. *RF linear accelerators*. Amsterdam, Holland: Wiley, 2008. doi:10.1002/9783527623426

[6] Fast Fourier Transform GSL documentation, https://www.gnu.org/software/gsl/doc/html/fft.html

[7] M. Aicheler *et al.*, "A Multi-TeV linear collider based on CLIC technology: CLIC Conceptual Design Report", in *CERN Yellow Reports: Monographs*, CERN, Geneva, Switzerland, 2012. https://cds.cern.ch/record/1500095

[8] J. Olivares Herrador *et al.*, "Simulation of beam loading compensation with RF-Track", in *Proc. IPAC'24*, Nashville, TN, May 2024, pp. 3136–3139. doi:10.18429/JACoW-IPAC2024-THPC56

[9] J. Nelder and R. Mead, "A Simplex Method for Function Minimization", *Comput. J.*, vol. 7, no. 4, pp. 308-–313, 1965. doi:10.1093/comjnl/7.4.308