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INELASTIC SCATTERING OF POLARIZED LEPTONS FROM POLARIZED NUCLEONS

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ABSTRACT

A previously derived sum rule, based on $U(6) \otimes U(6)$ equal-time commutation relations for the space-components of the electromagnetic current, implies mean polarization asymmetries of greater than 20% throughout most of the inelastic continuum.

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I. INTRODUCTION

Sometime ago, a high energy sum rule involving electromagnetic scattering of longitudinally polarized leptons from polarized protons and neutrons was derived[1] and then dismissed as "worthless." However, it turns out to be interesting to reconsider that negative conclusion in the light of the present experimental and theoretical situation.[2,3] We find that given "naive" quark-model equal-time commutation relations for the space-components of the electromagnetic current[4] and reasonable estimates for the convergence of the related sum rule, there must be parallel-antiparallel asymmetry effects of greater than 20% over a large region of the "deep inelastic" continuum. It appears that the relevant experiments with electrons — or even muons — may be feasible.

In Section II we review the kinematics of polarized lepton scattering from a polarized target[5] and make contact with the sum rule previously derived. Our main result is Eq. (2.10) and its consequences. In Section III, we estimate the magnitude of the asymmetry effects, given the present data. The Appendix provides more details of the kinematics and a simplified derivation of the sum rule.

II. KINEMATICS

The differential cross section for electroproduction of a hadron system Γ from a left-handed incident lepton[5] can be written at high Q^2 and ν (specifically $\nu \gg M$; $\nu^2 \gg Q^2$) as follows:

$$\frac{d\sigma_L}{dQ^2 d\nu\, d\Gamma} = \frac{\pi}{EE'} \frac{d\sigma_L}{d\Omega dE'} = \frac{4\pi\alpha^2}{Q^4} \frac{E'}{E} \sum_{n \text{ in } d\Gamma} \left| \langle n | j^{\text{lept.}} \cdot J | Ps \rangle \right|^2 (2\pi)^3 \delta^4(P_n - P - q) \tag{2.1}$$

where

E = energy of incident lepton

E' = energy of scattered lepton

θ = angle of scattered lepton relative to incident lepton

q = four-momentum of incident virtual photon

$Q^2 = -q^2 = 4EE' \sin^2 \theta/2$

$\nu = E - E'$

P, s = proton four-momentum and spin

The polarization of the virtual photon is determined in terms of the lepton current, which can be computed explicitly. For $\nu \gg M$, $\nu^2 \gg Q^2$ it is especially simple:

$$j^{\mu}_{\text{lept}} \cong \frac{\sqrt{Q^2}}{\nu}\left[\epsilon^{\mu}_{s} + \sqrt{\frac{E}{2E'}}\,\epsilon^{\mu}_{L} + \sqrt{\frac{E'}{2E}}\,\epsilon^{\mu}_{R}\right] \tag{2.2}$$

The $\epsilon_i \left(\epsilon^2_S = +1,\ \epsilon^2_R = \epsilon^2_L = -1\right)$ are normalized polarization vectors for longitudinal (S) right-handed (R), and left-handed (L) virtual photon helicity states.

If the final hadron system which is detected is rotated rigidly about the direction of $\underset{\sim}{q}$ by angle ϕ (in laboratory-frame) and if the initial hadron is polarized along the direction of $\underset{\sim}{q}$, the cross section is modified only by the replacement in (2.2)

$$\begin{aligned} \epsilon^{\mu}_{R,L} &\rightarrow \epsilon^{\mu}_{R,L}\, e^{\pm i\phi} \\ \epsilon^{\mu}_{s} &\rightarrow \epsilon^{\mu}_{s} \end{aligned} \tag{2.3}$$

Upon averaging over ϕ, the interference terms between amplitudes of differing helicity vanish[6] and the cross section becomes

$$\int \frac{d\phi}{2\pi}\, \frac{d\sigma_L}{dQ^2 d\nu\, d\Gamma} \cong \frac{\alpha}{\pi Q^2 \nu}\, \frac{E'}{E}\left(1 - \frac{Q^2}{2M\nu}\right)\left[\frac{d\sigma_S}{d\Gamma} + \frac{E}{2E'}\,\frac{d\sigma_L}{d\Gamma} + \frac{E'}{2E'}\,\frac{d\sigma_R}{d\Gamma}\right] \tag{2.4}$$

where we use the Hand-Berkelman[7] notation

$$\frac{d\sigma_i}{d\Gamma} = \frac{4\pi^2\alpha}{\nu - \frac{Q^2}{2M}} \sum_{n\,\mathrm{in}\,d\Gamma} \left|\langle n|\epsilon_i \cdot J|Ps\rangle\right|^2 (2\pi)^3 \delta^4(P_n - P - q) \tag{2.5}$$

For the present application, we sum over all Γ but do not average over the initial polarization s. For fixed ν and Q^2 and for $E \to \infty$, polarization effects vanish and

$$\lim_{E\to\infty} \frac{d\sigma_L}{dQ^2 d\nu} = \frac{4\pi\alpha^2}{Q^4} W_2(Q^2,\nu) \tag{2.6}$$

Comparison with Eq. (2.4) yields (for $\nu^2 \gg Q^2$)

$$W_2 = \frac{Q^2}{4\pi^2\alpha\nu} \left(1 - \frac{Q^2}{2M\nu}\right)\left(\sigma_T + \sigma_S\right) \tag{2.7}$$

with

$$2\sigma_T = \sigma_L + \sigma_R \tag{2.8}$$

and we can rewrite the integrated version of (2.4) as

$$\frac{d\sigma_L}{dQ^2 d\nu} = \frac{4\pi\alpha^2}{Q^4} \frac{E'}{E} W_2(Q^2,\nu)\left[1 + \frac{\nu}{E'} \frac{\sigma_L}{\sigma_L+\sigma_R+2\sigma_S} - \frac{\nu}{E} \frac{\sigma_R}{\sigma_L+\sigma_R+2\sigma_S}\right] \quad \nu \gg M;\ \nu^2 \gg Q^2 \tag{2.9}$$

With this form for the cross section, we can make contact with the sum rule derived in Ref. 1. Using subscripts P and A (instead of R and L) to denote parallel and antiparallel configurations of virtual-photon spin with respect to nucleon spin, we see that the sum rule (6.16) of Ref. 1 may be written

$$\lim_{Q^2\to\infty} \int_0^\infty d\nu\, W_2(\nu, Q^2)\left(\frac{\sigma_A - \sigma_P}{\sigma_A + \sigma_P + 2\sigma_S}\right) = Z = \lim_{P_z\to\infty} \frac{i}{2}\int d^3x \langle P|\left[J_x(\underset{\sim}{x},0), J_y(0)\right]|P\rangle$$
$$= \begin{cases} \overline{Z} + \frac{1}{6}\left|\frac{g_A}{g_V}\right| & \text{quark algebra; proton target} \\ \overline{Z} - \frac{1}{6}\left|\frac{g_A}{g_V}\right| & \text{quark algebra; neutron target} \end{cases} \tag{2.10}$$

Here $\left|g_A/g_V\right| \approx 1.2$ is the ratio of β-decay coupling constants and $\overline{Z}$ is an isotopic-scalar contribution which depends upon the model of the nucleon. Equation (2.10) is our principal result. In Appendix I, we produce a simplified derivation of the result, along with more general kinematical considerations.

III. A LOWER BOUND FOR POLARIZATION EFFECTS

Most theoretical models[8-12] anticipate a near equality between W_{2n} and W_{2p}. If they are significantly different, this in itself would most likely imply a "parton" interpretation[8,9] of the deep-inelastic experiments and consequently nonvanishing polarization effects of the type exhibited in (2.10). If it turns out experimentally that $W_{2n} \approx W_{2p}$, it will be harder to decide between the two classes of theories – those based on a parton interpretation and those based on a diffraction mechanism (Pomeranchuk exchange,[10,11] vector dominance,[12] etc.) – on the basis of unpolarized data alone. Depending upon the sign of $\overline{Z}$, the magnitude of the right-hand side of (2.10) must be greater than 0.2 for either proton or neutron target. Because the integral over W_{2p} experimentally is rather small, the polarization effects must be large. Let us suppose that the sum rule (2.10) converges at some value $M\nu_0/Q^2 \equiv W_0$. Then, using the premise[2] $\sigma_T \gg \sigma_S$ and $\lim_{\nu\to\infty} \nu W_2(\nu, Q^2) = 0.33$ we find from the data[2]

$$\int_0^{Q^2 W_0} d\nu \; W_{2p}(\nu, Q^2) \approx 0.33 \left[\log W_0 - 0.2\right] \tag{3.1}$$

$$(W_0 > 2)$$

We define the mean asymmetry $\bar{\epsilon}_p$ as

$$\bar{\epsilon}_p = \frac{\displaystyle\int_0^{Q^2 W_0} d\nu\, W_{2p}(\nu, Q^2)\, \frac{\sigma_A - \sigma_P}{\sigma_A + \sigma_P + 2\sigma_S}}{\displaystyle\int_0^{Q^2 W_0} d\nu\, W_{2p}(\nu, Q^2)} \tag{3.2}$$

$$= \frac{\bar{Z} + \frac{1}{6}\left|g_A/g_V\right|}{0.33\left[\log W_0 - 0.2\right]} \cong Z' + \frac{0.6}{\log W_0 - 0.2}$$

Thus either $\left|\bar{\epsilon}_p\right|$ or $\left|\bar{\epsilon}_n\right|$ must be greater than $0.6\left[\log W_0 - 0.2\right]^{-1}$. The predicted value of $\bar{\epsilon}$, assuming $Z' = 0$, is plotted in Fig. 1.

Notice that for given E, E', θ (and $\nu^2 \gg Q^2$) the experimental asymmetry Δ is

$$\Delta = \frac{(d\sigma_A/d\nu\, dQ^2) - (d\sigma_P/d\nu\, dQ^2)}{(d\sigma_A/d\nu\, dQ^2) + (d\sigma_P/d\nu\, dQ^2)} \tag{3.3}$$

$$\cong \frac{\nu(E+E')}{2EE'}\left(\frac{\sigma_A - \sigma_P}{\sigma_A + \sigma_P + 2\sigma_S}\right)\left[1 + \frac{\nu^2}{2EE'}\left(\frac{\sigma_T}{\sigma_T + \sigma_S}\right)\right]^{-1}$$

The most advantageous case for observing an experimental asymmetry occurs for large scattering angles, for which $E' \ll E$. Then, from (3.3), Δ becomes

$$\Delta \approx \frac{\sigma_A - \sigma_P}{\sigma_A + \sigma_P} \tag{3.4}$$

and from (3.2) and Fig. 1, for either proton or neutron

$$\bar{\Delta} \geq \bar{\epsilon} \geq 0.2 \tag{3.5}$$

provided $W_0 \leq 30$.

That large scattering angles are most favorable follows simply from the fact that this situation corresponds to backward scattering in the center-of-mass frame of electron and nucleon. Under this circumstance the virtual photon helicity must be the same as that of the incident lepton.

It appears to be possible to produce electron[13] or muon polarized beams which have nearly 100% longitudinal polarization. Polarized targets of $\sim 4\%$ polarization per nucleon are at present in use.[14] Therefore, nearly 1% raw asymmetries are predicted; this may well be within range of muon-scattering as well as electron-scattering experiments in the future.

The use of "naive" commutation relations of space-components of currents has been criticized.[15-18] It has been shown that, given the validity of the perturbation expansion of a renormalizable field theory, such equal-time commutators are modified from their naive (canonical) values by the effects of the interactions. It can also be argued that, since the perturbation expansion gives unreliable results for asymptotic behavior of matrix elements of currents (e.g. elastic form factors), this may also be the case for the commutators. But in any case, the commutator $[J_x, J_y]$ in question is an observable and this polarization experiment <u>measures</u> its matrix element between nucleons. Any reasonable nonvanishing value should be detectable experimentally.

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APPENDIX

KINEMATICAL DETAILS AND THE SUM RULE

The approximation $\nu^2 \gg Q^2$ used in the text is valid provided $\nu \gg M$, because from kinematics

$$\nu \geq \frac{Q^2}{2M} \tag{A1}$$

which implies

$$\frac{\nu^2}{Q^2} \geq \frac{\nu}{2M} \tag{A2}$$

Likewise, because the sum rule (2. 10) is valid only as $Q^2 \to \infty$, the inequality $\nu \gg M$ will be satisfied in that limit as a consequence of (A1). Therefore, in principle the formulae quoted in the text should suffice. However, in practice the neglected terms may contribute and we here present the correct formulae, which are somewhat more opaque. Instead of (2. 2), the correct expression (neglecting only lepton mass) is

$$j_\mu^{lept} = \sqrt{\frac{Q^2}{\nu^2+Q^2}} \left\{ \sqrt{1 - \frac{Q^2}{4EE'}}\, \epsilon_\mu^S + \frac{(E+E')}{\sqrt{8EE'}} \left(\epsilon_\mu^L + \epsilon_\mu^R\right) + \sqrt{\frac{\nu^2+Q^2}{8EE'}} \left(\epsilon_\mu^L - \epsilon_\mu^R\right) \right\} \tag{A3}$$

Equations (2. 4)-(2. 6) remain correct, but (2. 7) is replaced by

$$W_2 = \frac{Q^2}{4\pi^2 \alpha \nu} \left(1 + \frac{Q^2}{\nu^2}\right)^{-1} \left(1 - \frac{Q^2}{2M\nu}\right) \left(\sigma_T + \sigma_S\right) \tag{A4}$$

Also the important equation (2. 9) becomes

$$\frac{d\bar{\sigma}_L}{dQ^2 d\nu} = \frac{4\pi\alpha^2}{Q^4} \frac{E'}{E} W_2(Q^2, \nu) \left[1 - \frac{Q^2}{4EE'} + \frac{\nu^2+Q^2}{2EE'} \left(\frac{\sigma_L + \sigma_R}{\sigma_L + \sigma_R + 2\sigma_S}\right) + \frac{\sqrt{\nu^2+Q^2}(E+E')}{2EE'} \left(\frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R + 2\sigma_S}\right) \right] \tag{A5}$$

The experimental asymmetry Δ defined in (3.3) becomes

$$\Delta = \frac{\sqrt{\nu^2+Q^2}(E+E')}{2EE'}\left(\frac{\sigma_A-\sigma_P}{\sigma_A+\sigma_P+2\sigma_S}\right)\left[1-\frac{Q^2}{4EE'}+\frac{\nu^2+Q^2}{2EE'}\left(\frac{\sigma_T}{\sigma_T+\sigma_S}\right)\right]^{-1} \tag{A6}$$

To derive the sum rule we observe that

$$\begin{aligned} J_{xy} &\equiv \sum_n \left|\langle n|\epsilon_L\cdot J|Ps\rangle\right|^2(2\pi)^3\delta^4(P_n-P-q)-\sum_n \left|\langle n|\epsilon_R\cdot J|Ps\rangle\right|^2(2\pi)^3\delta^4(P_n-P-q) \\ &= \frac{i}{2\pi}\int d^4x\, e^{iq\cdot x}\langle Ps|\left[J_x(x),J_y(0)\right]-\left[J_y(x),J_x(0)\right]|Ps\rangle \\ &= \frac{i}{\pi}\int d^4x\, e^{iq\cdot x}\langle Ps|\left[J_x(x),J_y(0)\right]|Ps\rangle \end{aligned} \tag{A7}$$

where in the last line, a 90° rotation about the z-axis was used to relate the two commutators. The z-axis is taken along $\underset{\sim}{q}$, which is also the direction of $\pm\, \underset{\sim}{s}$.

From (2.5) and (2.7), and in the laboratory frame,

$$J_{xy} = \left(\nu-\frac{Q^2}{2M}\right)\frac{(\sigma_L-\sigma_R)}{4\pi^2\alpha} \approx \frac{2\nu^2}{Q^2}W_2(\nu,Q^2)\left(\frac{\sigma_L-\sigma_R}{\sigma_L+\sigma_R+2\sigma_S}\right) \tag{A8}$$

Under a Lorentz-transformation in the z-direction, $\frac{P_0}{M}J_{xy}$ remains invariant. Let $\underset{\sim}{q}=0$, $P_z\to\infty$, $q_0\to\infty$, $P_0/q_0=-\omega=-M\nu/Q^2$ fixed. In that limit,[19] letting $\tau=P_0t$

$$\begin{aligned} &\lim_{P_z\to\infty}\frac{i}{\pi}\int d^3x\,dt\, e^{-i\omega\tau}\langle Ps|\left[J_x\left(\underset{\sim}{x},\frac{\tau}{P_0}\right),J_y(0)\right]|Ps\rangle \\ &= \lim_{\substack{P_z\to\infty\\ q_0\to\infty\\ \omega\ \text{fixed}}}\frac{2M\nu^2}{Q^2}W_2(\nu,Q^2)\left(\frac{\sigma_L-\sigma_R}{\sigma_L+\sigma_R+2\sigma_S}\right) \end{aligned} \tag{A9}$$

As in Ref. 19, νW_2 (times the cross section ratio in parentheses) becomes a function of ω alone. Integrating over all ω, we get

$$\lim_{Q^2 \to -\infty} 2\int_{-\infty}^{\infty} \frac{d\omega}{\omega} \nu W_2 \left(\frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R + 2\sigma_S}\right) = \lim_{Q^2 \to -\infty} 4\int_0^{\infty} d\nu \; W_2(\nu, Q^2) \left(\frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R + 2\sigma_S}\right)$$

$$= \lim_{P_z \to \infty} 2i \int d^3x < Ps \left| \left[J_x(\underset{\sim}{x}, 0), \; J_y(0) \right] \right| Ps > \equiv 4Z$$

This reproduces the sum rule (6.16) of Ref. 1. The "justification" of the equality of the limit for timelike and spacelike $Q^2 \to \infty$ can be done along the lines used in deriving general asymptotic sum rules.[19] But it must be said that the derivation used in Ref. 1 remains the most reliable.

REFERENCES

1. J. Bjorken, Phys. Rev. 148, 1467 (1966); Eq. (6.16).
2. E. Bloom, D. Coward, H. DeStaebler, J. Drees, G. Miller, L. Mo, R. Taylor, M. Breidenbach, J. Friedman, G. Hartmann, and H. Kendall, Report No. SLAC-PUB-642, and references quoted therein, Stanford Linear Accelerator Center, Stanford University, Stanford, California.
3. J. Bjorken, Report No. SLAC-PUB-571, Stanford Linear Accelerator Center, Stanford University, Stanford, California.
4. R. P. Feynman, M. Gell-Mann, and G. Zweig, Phys. Rev. Letters 13, 678 (1964).
5. We follow the approach of N. Christ and T. D. Lee, Phys. Rev. 143, 1310 (1966). See also N. Dombey, Rev. Mod. Phys. 41, 236 (1969).
6. If $\underset{\sim}{s}$ is not parallel to $\underset{\sim}{q}'$, this azimuthal average cannot be made. There then exists an additional interference term between the longitudinal amplitude and transverse amplitudes. This term is most likely small because of the observed smallness of σ_S; in any case it can be independently determined by varying E' and θ at fixed Q^2 and ν. A superconvergence sum rule for this interference term has been discussed by H. Burkhardt and W. Cottingham (University of Birmingham preprint); see also reference 5.
7. L. Hand, Proceedings of the 1967 Symposium on Electron and Photon Interactions.
8. R. P. Feynman (unpublished); J. Bjorken and E. Paschos, Report No. SLAC-PUB-572, Stanford Linear Accelerator Center, Stanford University, Stanford, California.
9. S. Drell, D. Levy, and T. Yan, Phys. Rev. Letters 22, 744 (1969).
10. H. Harari, Phys. Rev. Letters 22, 1078 (1969).
11. H. Abarbanel, M. Goldberger, and S. Treiman, Phys. Rev. Letters 22, 500 (1969).
12. J. Sakurai, Phys. Rev. Letters 22, 978 (1969).

13. V. Hughes, M. Lubell, M. Posner, and W. Raith, Proceedings of the 6th International Conference on High Energy Accelerators, Cambridge Electron Accelerator Report, CEAL-2000 (1967).

14. O. Chamberlain (private communication); M. Borghini, S. Mango, O. Runoffsson, and J. Vermeulen, Proceedings of the International Conference on Polarized Targets and Ion Sources, Saclay, France (1966).

15. A. Vainstein and B. Ioffe, Soviet Physics: JETP Letters 6, 341 (1967).

16. S. Adler and W. Tung, Phys. Rev. Letters 22, 978 (1969).

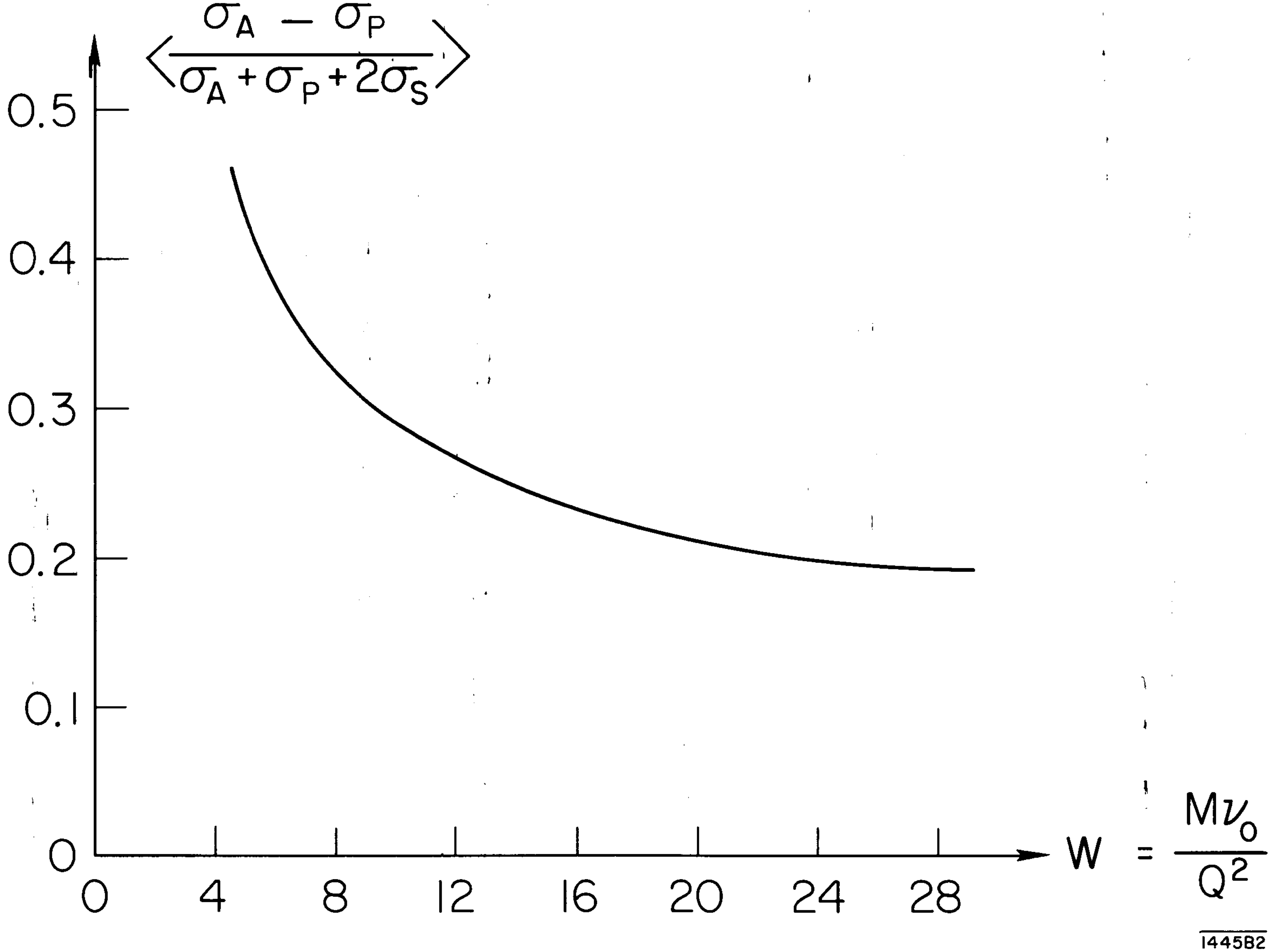
17. R. Jackiw and G. Preparata, Phys. Rev. (to be published).

18. R. Jackiw, CERN-TH-1065.

19. J. Bjorken, Phys. Rev. 179, 1547 (1969).

FIGURE CAPTION

1. Mean asymmetry $\bar{\epsilon}$ for polarized electron-nucleon scattering as function of $W_0 = M\nu_0/Q^2$, where ν_0 is the energy at which the sum rule (2.10) converges. $\bar{\epsilon}$ is given by (3.2) with $Z' = 0$.



$\left\langle \frac{\sigma_A - \sigma_P}{\sigma_A + \sigma_P + 2\sigma_S} \right\rangle$
0.5
0.4
0.3
0.2
0.1
0
0
4
8
12
16
20
24
28
$W = \frac{M\nu_0}{Q^2}$
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Fig. 1