Some Theoretical Aspects of Searches for Heavy Neutrino Emission in Kaon Decays

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Abstract.
Searches for heavy neutrino emission in $K^+ \to e^+\nu_e$ and $K^+ \to \mu^+\nu_\mu$ decays have provided very powerful constraints on sterile neutrinos and their mixings. In this talk at Kaon-2022 we discuss theoretical aspects of these searches and remark on recent experimental progress.

1. Introduction

Neutrino masses and mixing are of fundamental importance in particle physics. One of the key questions in this area concerns the spectrum of neutrino mass eigenstates and their mixing to form interaction eigenstates. In 1980, a new class of tests to search for massive neutrino emission in nuclear and particle decays was proposed and was applied to existing data to derive upper limits on such emission [1, 2, 3, 4]. These tests were subsequently applied in dedicated experiments on $\pi^+ \to \ell^+\nu_\ell$ and $K^+ \to \ell^+\nu_\ell$ decays (denoted $\pi^+_{\ell 2}$ and $K^+_{\ell 2}$, respectively), where $\ell = e, \mu$. The first searches in $K^+_{\mu 2}$ and K^+_{e2} decays were performed at KEK [5, 6, 7]. Recent searches for heavy neutrino emission in $K^+_{\mu 2}$ decays were performed by E939 at BNL [8] and OKA at Serpukhov [9], and in $K^+_{\mu 2}$ and K^+_{e2} decays by the NA62 experiment at CERN [10]-[13]. Several generations of peak search experiments with π^+_{e2} and $\pi^+_{\mu 2}$ decays were carried out at TRIUMF from 1983 [14] to the recent PIENU experiment [15], and at SIN/PSI, dating from the early search with $\pi^+_{\mu 2}$ decays [16]. The most stringent limits from peak searches are from the PIENU, BNL 949, and NA62 experiments. With D. Bryman, we have analyzed current data and obtained some new bounds in [17, 18].

2. Peak Search Test

Let us denote the charged weak current as $J_\lambda = \bar{\ell}_L \gamma_\lambda \nu_{\ell,L}$, where $\ell = e,\ \mu,\ \tau$, with $\nu_\ell = \sum_{i=1}^{3+n_s} U_{\ell i}\nu_i$, where ν_i are neutrino mass eigenstates. Three of these, ν_i, $i = 1, 2, 3$, comprise the main components of the active neutrino interaction (flavor) eigenstates ν_e, ν_μ, and ν_τ, and there could be some number n_s of other ν_i, which would be the main mass eigenstates in electroweak-singlet (sterile) neutrino interaction eigenstates ν_{s_1}, ..., $\nu_{s_{n_s}}$. We focus on the simplest case $n_s = 1$ here; it is straightforward to generalize this to $n_s \geq 2$.

A particularly sensitive test for sterile neutrinos makes use of the two-body leptonic decays of charged pseudoscalar mesons M^+, including π^+ and K^+, as well as heavy-quark mesons D^+, D^+_s, B^+. We mainly focus on $K^+_{\ell 2}$ here, but use the general notation $M^+ \to \ell^+\nu_\ell$ with $\ell = e,\ \mu$

(denoted as $M^+_{\ell 2}$). The signature of heavy neutrino emission is a monochromatic peak in the energy spectrum of the final-state ℓ^+ recoiling opposite the massive neutrino ν_4, with energy (in the M^+ rest frame) $E_\ell = (m_M^2 + m_\ell^2 - m_{\nu_4}^2)/(2m_M)$. If an experiment were to observe such an additional peak in the $d\Gamma/dE_\ell$ spectrum, one could determine the value of m_{ν_4} from E_ℓ. From an experimental upper bound on $BR(M^+ \to \ell^+\nu_4)$, one can obtain an upper bound on $|U_{\ell 4}|$ for a given m_{ν_4}.

In the Standard Model (SM)

$$\Gamma(M^+ \to \ell^+\nu_\ell)_{SM} = \frac{G_F^2|V_{ab}|^2 f_M^2 m_M m_\ell^2}{8\pi}\left(1 - \frac{m_\ell^2}{m_M^2}\right)^2 ,$$

where $V_{ab} = V_{ud}$ for $M^+ = \pi^+$, $V_{ab} = V_{us}$ for $M^+ = K^+$, etc. and $f_\pi = 130$ MeV, $f_K = 160$ MeV. With a heavy ν_4,

$$\Gamma(M^+ \to \ell^+\nu_\ell) = \frac{G_F^2|V_{ab}|^2 f_M^2 m_M^3}{8\pi}\left[\sum_{i=1}^{3}|U_{\ell i}|^2\,\delta_\ell^{(M)}\left(1-\delta_\ell^{(M)}\right)^2 + |U_{\ell 4}|^2\,\rho(\delta_\ell^{(M)},\delta_{\nu_4}^{(M)})\theta(m_M - m_\ell - m_{\nu_4})\right],$$

where $\delta_\ell^{(M)} = m_\ell^2/m_M^2$, $\delta_{\nu_4}^{(M)} = m_{\nu_4}^2/m_M^2$; the kinematic rate factor is $\rho(x,y) = f_{\mathcal{M}}(x,y)[\lambda(1,x,y)]^{1/2}$, where $f_{\mathcal{M}}(x,y) = x + y - (x-y)^2$ arises from the square of the matrix element and $[\lambda(1,x,y)]^{1/2}$ arises from the final-state phase space, with $\lambda(z,x,y) = x^2 + y^2 + z^2 - 2(xy + yz + zx)$; and $\theta(\xi)$ is the Heaviside theta function: $\theta(\xi) = 1$ if $\xi > 0$ and $\theta(\xi) = 0$ if $\xi \le 0$. For decays into ν_i, $i = 1,2,3$ of negligibly small mass, $\rho(\delta_\ell^{(M)},\delta_{\nu_i}^{(M)}) = \rho(\delta_\ell^{(M)},0) = \delta_\ell^{(M)}(1-\delta_\ell^{(M)})^2$. The sum $\sum_{i=1}^3 |U_{\ell i}|^2 = 1 - |U_{\ell 4}|^2$, so the SM decay term is reduced by this factor.

It is convenient to define the ratio $\bar\rho(x,y) \equiv \rho(x,y)/\rho(x,0) = \rho(x,y)/[x(1-x)^2]$. For the $M^+ \to e^+\nu_4$ decays, the ratio $\bar\rho(\delta_e^{(M)},\delta_{\nu_4}^{(M)})$ increases very rapidly as $\delta_{\nu_4}^{(M)}$ increases from 0 and reaches a maximum of $\sim 4/[27\delta_e^{(M)}]$ at $m_{\nu_4} = m_M/\sqrt{3}$. In K^+_{e2} decay, this maximum is $(\bar\rho)_{max} = 1.4 \times 10^5$, occurring at $m_{\nu_4} = 285$ MeV. This reflects the removal of the helicity suppression of the K^+_{e2} decay to neutrinos of negligibly small masses. Now

$$\frac{\Gamma(M^+ \to \ell^+\nu_4)}{\Gamma(M^+ \to \ell^+\nu_\ell)_{SM}} = \frac{|U_{\ell 4}|^2\bar\rho(\delta_\ell^{(M)},\delta_{\nu_4}^{(M)})}{1 - |U_{\ell 4}|^2} \simeq |U_{\ell 4}|^2\bar\rho(\delta_\ell^{(M)},\delta_{\nu_4}^{(M)}) ,$$

since $|U_{\ell 4}|^2 \ll 1$. Hence, denoting $\Gamma(M^+ \to \ell^+\nu_4)_{ul}$ as the upper limit on $\Gamma(M^+ \to \ell^+\nu_4)$ from experiment, one derives the resultant upper limit on $|U_{\ell 4}|^2$:

$$|U_{\ell 4}|^2 < \frac{\frac{\Gamma(M^+\to\ell^+\nu_4)_{ul}}{\Gamma(M^+\to\ell^+\nu_\ell)_{SM}}}{\bar\rho(\delta_\ell^{(M)},\delta_{\nu_4}^{(M)})} .$$

Using this peak search, the KEK experiments obtained the upper limit $|U_{\mu 4}|^2 \lesssim 10^{-6}$ for 150 MeV $< m_{\nu_4} <$ 300 MeV and a similar upper limit on $|U_{e4}|^2$ [6, 7]. The BNL E949 [8] and CERN NA62 [13] experiments on $K^+_{\mu 2}$ have set the upper limits $|U_{\mu 4}|^2 \lesssim 10^{-8}$ for 220 MeV $< m_{\nu_4} <$ 380 MeV, while the NA62 peak search with K^+_{e2} has set the upper limit $|U_{e4}|^2 \lesssim 10^{-9}$ for 150 MeV $< m_{\nu_4} <$ 400 MeV [12, 13].

3. Constraints from $R^{(M)}_{e/\mu}$

In addition to producing an anomalous peak in $d\Gamma/dE_\ell$, the emission of a heavy neutrino in $M^+_{\ell 2}$ decays would cause a deviation in the observed ratio of decay rates or branching ratios,

$R^{(M)}_{e/\mu} = BR(M^+ \to e^+\nu_e)/BR(M^+ \to \mu^+\nu_\mu)$ from its SM value [1, 2]. Thus, the consistency of the measured ratios $R^{(\pi)}_{e/\mu}$ and $R^{(K)}_{e/\mu}$ with the SM predictions yields further constraints on heavy neutrino effects. In the SM,

$$R^{(M)}_{e/\mu,SM} = \frac{\rho(\delta^{(M)}_e, 0)}{\rho(\delta^{(M)}_\mu, 0)}(1+\delta_{RC}) = \frac{m_e^2}{m_\mu^2}\left[\frac{1-\frac{m_e^2}{m_M^2}}{1-\frac{m_\mu^2}{m_M^2}}\right]^2(1+\delta_{RC})$$

where δ_{RC} is the radiative correction term. Let us define $\bar{R}^{(M)}_{e/\mu} = R^{(M)}_{e/\mu}/R^{(M)}_{e/\mu,SM}$. With a heavy ν_4,

$$\bar{R}^{(M)}_{e/\mu} = \frac{1-|U_{e4}|^2 + |U_{e4}|^2\bar{\rho}(\delta^{(M)}_e, \delta^{(M)}_{\nu_4})}{1-|U_{\mu4}|^2 + |U_{\mu4}|^2\bar{\rho}(\delta^{(M)}_\mu, \delta^{(M)}_{\nu_4})} ,$$

where the $\theta(m_M - m_\ell - m_{\nu_4})$ is implicit. With a given M, there are three different intervals for m_{ν_4}:

(i) $I^{(M)}_1$: $m_{\nu_4} < m_M - m_\mu$, where both the $M^+ \to e^+\nu_4$ and $M^+ \to \mu^+\nu_4$ decays can occur;

(ii) $I^{(M)}_2$: $m_M - m_\mu < m_{\nu_4} < m_M - m_e$, where only the $M^+ \to e^+\nu_4$ decay is kinematically allowed by phase space;

(iii) $I^{(M)}_3$: $m_{\nu_4} > m_M - m_e$, where both decays $M^+ \to e^+\nu_4$ and $M^+ \to \mu^+\nu_4$ are forbidden.

For example, for $m_{\nu_4} \in I^{(M)}_2$, $\bar{R}^{(M)}_{e/\mu} = [1-|U_{e4}|^2+|U_{e4}|^2\bar{\rho}(\delta^{(M)}_e, \delta^{(M)}_{\nu_4})]/(1-|U_{\mu4}|^2)$. If, for a given m_{ν_4}, one knows, e.g., from peak-search experiments, that $|U_{\mu4}|^2$ is sufficiently small that the denominator can be approximated well by 1, then an upper bound on the deviation of $[\bar{R}^{(\pi)}_{e/\mu}]_{exp}$ from 1 yields an upper bound on $|U_{e4}|^2$:

$$|U_{e4}|^2 < \frac{\bar{R}^{(\pi)}_{e/\mu} - 1}{\bar{\rho}(\delta^{(\pi)}_e, \delta^{(\pi)}_{\nu_4}) - 1} .$$

As with the peak search method, this gives a very stringent upper limits on $|U_{e4}|^2$, because $\bar{\rho}(\delta^{(M}_e, \delta^{(M)}_{\nu_4}) \gg 1$ over much of the kinematic region in m_{ν_4}. The best current measurement of $R^{(\pi)}_{e/\mu}$ is from PIENU experiment [19], $R^{(\pi)}_{e/\mu} = (1.2344 \pm 0.0023_{stat} \pm 0.0019_{syst}) \times 10^{-4}$. The PDG average is $R^{(\pi)}_{e/\mu} = (1.2327 \pm 0.0023) \times 10^{-4}$. In comparison with the SM prediction including radiative corrections [20, 21], $R^{(\pi)}_{e/\mu,SM} = 1.23524(15) \times 10^{-4}$, this yields $\bar{R}^{(\pi)}_{e/\mu} = 0.9980 \pm 0.0019$, consistent with the SM. New upper bounds on $|U_{e4}|^2$ were derived in [17, 18] using this constraint. These upper bounds extend from $\sim 10^{-4}$ to $\sim 10^{-7}$ for m_{ν_4} from a few MeV to 50 MeV.

The uncertainty in the PIENU measurement of $R^{(\pi)}_{e/\mu}$ is $\simeq 2 \times 10^{-3}$, while the estimated uncertainty in the theoretical calculation of $R^{(\pi)}_{e/\mu}$ is 1.2×10^{-4}. There are plans for a new experiment called PIONEER at PSI, which will measure $BR(\pi^+ \to e^+\nu_e)$ to an accuracy of $\sim 10^{-4}$, matching the accuracy of theoretical calculation [22, 23]. This will enable a more stringent test for heavy neutrino effects, as well as testing $e-\mu$ universality to higher precision.

For $R^{(K)}_{e/\mu}$, the SM prediction with radiative corrections is $R^{(K)}_{e/\mu,SM} = (2.477 \pm 0.001) \times 10^{-5}$ and the best measurement is from NA62, $R^{(K)}_{e/\mu} = (2.488 \pm 0.010) \times 10^{-5}$ [24]. The PDG average is $R^{(K)}_{e/\mu} = (2.488 \pm 0.009) \times 10^{-5}$, yielding $\bar{R}^{(K)}_{e/\mu} = 1.0044 \pm 0.0037$, consistent with SM. the This was also used in [17, 18] to derive upper bounds on $|U_{e4}|^2$.

In addition to [17, 18], some recent studies and reviews of bounds on heavy neutrino mixings include [25]-[30], which contain references to the extensive literature. Although our discussion here is phenomenological, we note that there are models that suggest the possibility of heavy neutrinos in the mass range of interest for searches with K decays, e.g., [31, 32], among others.

4. Other Constraints

4.1. Bounds from Neutrino Production and Decay

Further bounds on heavy neutrinos in the mass range relevant for K decays arise from searches for production of heavy neutrinos from meson decays, followed by the neutrino decays in several experiments such as CHARM, PS191, NuTeV, T2K, Belle, etc. Relevant neutrino decays include both charged-current (CC) and neutral-current (NC) contributions. The condition for the neutral weak leptonic current to be diagonal in mass eigenstates is that all leptons of a given charge and chirality must have the same weak T and T_3 [33]. The presence of electroweak-singlet neutrinos violates this condition, so this shows that in the presence of a ν_4, the neutral weak leptonic current contains terms that are nondiagonal in mass eigenstates. Hence, a ν_4 in the interesting mass range for K decays has, among its dominant decays, tree-level decays in which ν_4 makes a NC transition to ν_ℓ, $\ell = e,\ \mu, \tau$, and a virtual Z, which then materializes to $f\bar{f}$, where f is a SM fermion, e.g, $f = \nu_\ell, e,\ \mu$ or quark. These include the invisible NC decays $\nu_4 \to \nu_\ell \bar{\nu}_{\ell'} \nu_{\ell'}$, and other 3-body NC leptonic decays such as $\nu_4 \to \nu_\ell e^+ e^-$. There are also 3-body CC leptonic decays such as $\nu_4 \to e^- e^+ \nu_e$, etc. For $m_{\nu_4} > m_{\pi^0} = 135$ MeV, the NC two-body decay $\nu_4 \to \nu_\ell \pi^0$ has a large branching ratio. Its rate is proportional to $|U_{\ell 4}|^2$, so $\sum_{\ell=e,\mu,\tau} \Gamma(\nu_4 \to \nu_\ell \pi^0) \propto |U_{e4}|^2 + |U_{\mu 4}|^2 + |U_{\tau 4}|^2$. Importantly, a lower bound on this sum does not constrain an individual term such as $|U_{e4}|^2$ or $|U_{\mu 4}|^2$.

4.2. BBN Bounds

An upper bound on a heavy neutrino lifetime τ_{ν_4}, and hence a lower bound on lepton mixings, arises from the constraint that the ν_4 should not upset the successful predictions of primordial (big bang) nucleosynthesis (BBN) (e.g. [25, 34, 35] and references therein). In assessing plots with BBN curves showing lower bounds on the individual quantities $|U_{\ell 4}|^2$, it is necessary to take into account that the BBN bounds are on sums of $|U_{\ell 4}|^2$, not on individual terms. For example, as is indicated in the plot of bounds on $|U_{e4}|^2$ in Fig. 26 in [29], the BBN curve assumes that $U_{\mu 4} = 0$ and $U_{\tau 4} = 0$ so that only $|U_{e4}|^2$ is constrained, and in the plot of bounds on $|U_{\mu 4}|^2$ in Fig. 27 in [29], the BBN curve assumes that $U_{e4} = 0$ and $U_{\tau 4} = 0$, so that only $|U_{\mu 4}|^2$ is constrained. Hence, these BBN exclusion curves do not apply directly in the generic case where U_{e4}, $U_{\mu 4}$, and $U_{\tau 4}$ are all nonzero.

4.3. Neutrinoless Double Beta Decay

The upper bounds on heavy neutrino mixings $|U_{e4}|^2$ and $|U_{\mu 4}|^2$ from experiments performing peak searches and measuring $R^{(M)}_{e/\mu}$ are independent of whether ν_4 is a Dirac or Majorana neutrino. However, if neutrinos are Majorana fermions, then a ν_4 in the O(100) MeV mass range could give a significant contribution to neutrinoless double beta decay ($0\nu 2\beta$). For $m_{\nu_i} \lesssim 200$ MeV, the effective mass constrained by $0\nu 2\beta$ searches is $m_{\beta\beta} = |\sum_i U_{ei}^2 m_{\nu_i}|$. Limits on $0\nu 2\beta$ decays from KamLAND-Zen, EXO-200, GERDA, and Majorana Demonstrator are $m_{\beta\beta} \lesssim 0.1$ eV. Assuming that the $U_{e4}^2 m_{\nu_4}$ term dominates the sum in $m_{\beta\beta}$, this yields the rough upper limit $|U_{e4}|^2 \lesssim (0.1 \text{ eV})/m_{\nu_4}$, and thus $|U_{e4}|^2 \lesssim (5 \times 10^{-9})(200 \text{ MeV}/m_{\nu_4})$. This is comparable to the upper limit on $|U_{e4}|^2$ set by NA62 in this mass range.

5. Conclusions

Searches for heavy neutrino emission in two-body leptonic decays of pseudoscalar mesons such as $\pi^+_{\ell 2}$ and $K^+_{\ell 2}$ are very powerful and robust methods to obtain bounds on possible heavy neutrinos, owing to the monochromatic nature of the signal and the removal of helicity suppression for $M^+ \to e^+\nu_e$ decays over a large interval of m_{ν_4}. After the pioneering searches in $K^+_{\mu 2}$ and K^+_{e2} decays at KEK, the BNL E949 experiment and the CERN NA62 experiment have set very stringent upper limits on $|U_{\mu 4}|^2$, and NA62 has set extremely stringent upper limits on $|U_{e4}|^2$. At lower masses, both the peak search and measurement of $R^{(\pi)}_{e/\mu}$ by PIENU have yielded excellent upper bounds on $|U_{\ell 4}|^2$. This search for heavy neutrinos will continue, achieving greater sensitivity in the future, with experiments including NA62, PIONEER, the DUNE near detector, as well as others probing higher masses. Proposed experiments such as SHiP (Search for Hidden Particles) at CERN and others can also extend these searches.

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