

BENCHMARKING EQUILIBRIUM EMITTANCE SIMULATION TOOLS FOR THE FUTURE CIRCULAR COLLIDER *

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Abstract

The determination of equilibrium emittance stands as a critical factor in optimizing the luminosity of the Future Circular Collider (FCC). In order to have accurate simulations and understanding of the emittance, multiple effects have to be taken into consideration including errors in the machine, solenoid effects, synchrotron radiation, and beam-beam effects. The novel Xsuite software aims to cover many of these effects. In this paper, we present benchmark studies and first results for determining equilibrium emittances using Xsuite and other simulation codes.

INTRODUCTION AND BACKGROUND

One of the most important features governing the luminosity of the Future Circular electron-positron Collider (FCC-ee) is the equilibrium emittance caused by the balance of quantum excitations and radiation damping. The horizontal emittance is largely impacted by the horizontal dispersion caused by the bending magnets in the arcs, causing quantum excitations. However, the vertical emittance is significantly smaller due to there being no dispersion by design and greatly depends on the alignment of the machine and compensation methods. An accurate simulation of the equilibrium emittance under realistic conditions is therefore of the utmost importance.

For the simulations to be as accurate as possible, it has to include all major emittance generating effects and allow the study of these effects to be interplaying simultaneously. The effects include alignment and magnetic errors that produce vertical dispersion and coupling, tilted solenoid effects as well as beam-beam effects. To allow for such comprehensive simulations major efforts have been directed to the development and testing of such tools. Matrix methods can give a quick first indication of the emittance of the machine, whilst tracking methods are more accurate, especially in the presence of coupling and more versatile as they allow the inclusion of other effects such as beam-beam. Both methods fulfil different purposes and have to be equally well tested.

The novel Xsuite [1] software package allows for such studies using both matrix and tracking methods, and as part of the EPFL FCC-ee software framework, we aim to benchmark the relevant features of this code and facilitate comprehensive studies that explore the interplay of these effects. On top of this, we aim to use these tools to study realistic commissioning and tuning strategies to get an accurate picture of the emittance and hence the machine performance. In

this study we aim to test the new radiation features in Xsuite by benchmarking them against other well-established codes, in particular MADX [2] and SAD [3]. A comprehensive comparison between MADX and SAD has been done in the past for the FCC-ee study and can be found in [4].

The error-free horizontal emittance and the vertical emittance produced with a local, coupling free vertical wiggler have been benchmarked in Xsuite and shown to produce reliable results [5]. Similarly, the emittance blow due to beam-beam has been studied using the beam-beam module in Xsuite [6]. In particular this beam-beam study sets up lattices with different vertical equilibrium emittances produced by a vertical wiggler and studies the vertical emittance with beam-beam, comparing it to results obtained from SAD.

In this paper we focus on vertical emittance produced from magnet misalignments that are obtained using matrix and tracking methods. A comparison between the matrix and tracking methods can also be helpful, in order to gauge the effectiveness of the matrix methods as an indicator. We also assess a first case where equilibrium emittances are computed for the FCC-ee lattice with relaxed optics, showing how these tools can be used for optimisation and tuning studies.

EMITTANCE WITH LATTICE ERRORS

Method

To study the impact of misalignment errors, identical misalignments must be applied to the lattices in all three codes. Whilst misalignment errors from MADX can be imported into Xsuite through the built-in converter, this is not the case for SAD. Moreover, to obtain enough statistics for an accurate comparison, it is important to sample multiple error seeds.

To do this, the scripts in FCC-ee Xample Error Tracking repository [7] were used, these scripts allow for the creation of random errors in Python that are then converted to Xsuite. To allow reproducibility, the error seed can be set, and the individual errors of each magnet are stored. The script was modified to also export the errors in a format that can be read by SAD.

To test that the errors are accurately converted in all three cases, errors from a single seed were applied to two magnets, and the orbit and dispersion were compared. This was then repeated for a single seed, but applying vertical and horizontal displacement errors to all quadrupoles and sextupoles.

For the complete study, the lattice for the $\bar{t}\bar{t}$ operation mode with a beam energy of 182.5 GeV was used and vertical misalignments were applied to all quadrupoles and sextupoles. These misalignments were generated using a Gaussian with a standard deviation of 2 nm, truncated at

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2.5σ . This was done for 500 error seeds. The generated lattices and errors were loaded into MADX, SAD and Xsuite for further simulations. The results from MADX and SAD were stored in a text file for further processing in python, whilst the studies in Xsuite were run natively in python. The emittances were computed using the matrix methods built into all three codes, whilst SAD and Xsuite also obtained the equilibrium emittance by tracking a test particle for 5000 turns, more than 100 damping times.

Since the high beam energy results in significant synchrotron radiation losses, the strength of each magnet has to be adjusted to the local average beam energy, this is known as tapering. All three codes have an independently implemented tapering procedure and the native procedures were used for each case, making this study also an adequate way of testing the different tapering implementations. In the past, the matrix module in MADX has been shown to give incorrect results for tapered lattices, while working out the emittance for an effectively tapered lattice with 1 GeV beam energy and no tapering and scaling the obtained emittance up with energy has proven to be reliable [8]. Therefore, the MADX procedure also included a simulation that ran at 1 GeV and scaled the results.

Tracking Results

The emittance from tracking was computed from the tracking data of a test particle in both codes by truncating the data from the first turns, corresponding to one damping time and then computing the emittance as $\sqrt{\langle y^2 \rangle \langle y'^2 \rangle - \langle yy' \rangle^2}$. The results of the 500 seeds were binned into histograms and are shown in Fig. 1, showing an overall good agreement. To further highlight this agreement, the logarithm of the emittance can also be binned and plotted in a histogram as shown in Fig. 2. The histograms of the logarithm of the emittances appear to follow a Gaussian distribution and computing the mean and standard deviation for both cases yields -35.36 ± 1.07 and -35.33 ± 1.06 , showing very good agreement.

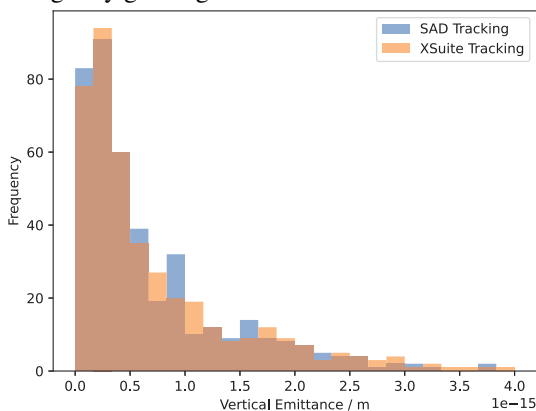


Figure 1: Emittance from tracking in 500 lattices with vertical alignment errors in SAD and Xsuite.

The distribution of the ratio between the results obtained by the two codes for individual seeds can also be computed

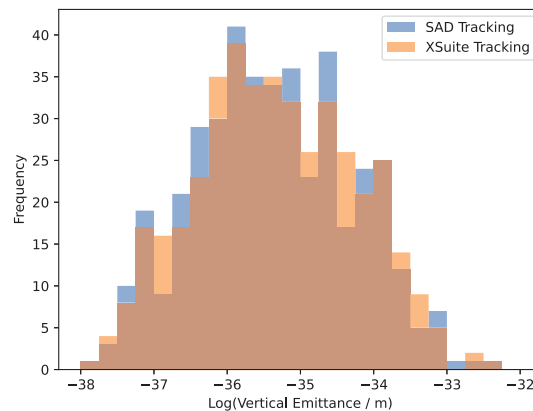


Figure 2: Logarithm of emittance from tracking in 500 lattices with vertical alignment errors in SAD and Xsuite.

and is shown in Fig. 3. The disagreement between the codes is most likely due to the random process of the quantum excitations which will have slightly different pseudo random number generators in the two codes and should tend to unity with more particles or turns. This was confirmed by repeating the study with a lower number of turns and obtaining a significantly larger spread.

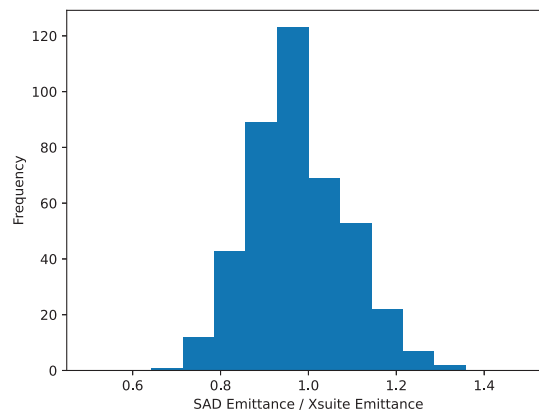


Figure 3: Ratio between emittances computed from tracking in 500 lattices with vertical alignment errors in SAD and Xsuite.

Matrix Methods

The results of the matrix methods for the three codes were obtained and are plotted together with the tracking results and are shown in Fig. 4. Figure 4 does not show the MADX results at full energy but only the scaled results, as the spread for the full energy was too large and there was no correlation with other data. The mean and standard error for the three codes were calculated as -35.86 ± 1.01 for SAD, -35.86 ± 0.99 for MADX, and -34.47 ± 0.90 for Xsuite computed using the fast self consistent method [9]. These numbers and the plots in Fig. 4 show that the SAD and MADX results in general provide a slight underestimate, whilst the Xsuite results provide an overestimate that disagrees with the tracking results to a greater extent than the matrix methods of the other two codes. The reason for

this different behaviour and lower accuracy compared to the other two codes is under investigation.

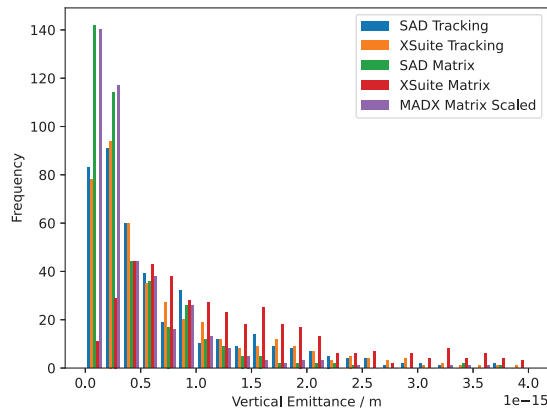


Figure 4: Emittance from tracking in 500 lattices with vertical alignment errors.

For seed-by-seed comparison, the results from each seed were normalised by dividing them by the tracking result obtained from Xsuite and this result is shown in Fig. 5 using a logarithmic scale for clarity. The results show a near-perfect agreement between the SAD and MADX results. Whilst the SAD and MADX results are consistently lower than the tracking results, the Xsuite results are mostly larger than the tracking results; this is not as consistent as for the other two codes. Overall this shows that the Matrix results are a strong, albeit not perfect, indicator for the deprecation of the equilibrium emittance with vertical misalignment errors.

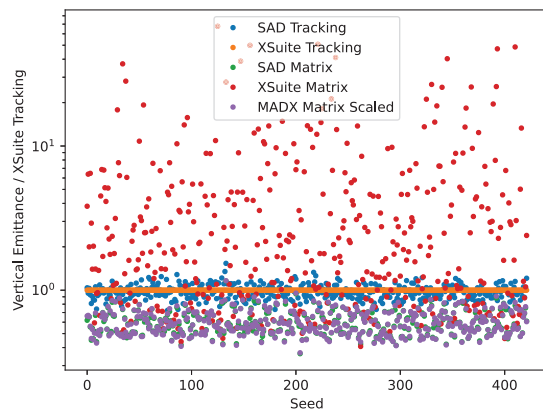


Figure 5: Emittance from tracking in 500 lattices with vertical alignment errors divided by Xsuite tracking results.

RELAXED OPTICS

There are a large number of studies that can be performed with these tools, now benchmarked. As an example we present here the results from a study that uses Xsuite tracking in misaligned lattices to understand whether a relaxed insertion region optics as described in [10]. The motivation for this study was shown in [8], where misalignment of insertion region magnets results in a significantly higher impact

on the vertical emittance compared to those in the arcs. It was therefore proposed that a relaxed insertion region optics, that is with a larger β^* and thereby lower β in the IR magnets would show a smaller increase of the vertical emittance for the same misalignment. In addition a relaxed, IR optics will represent the first steps of a possible commissioning plan for such collider.

To test this, 100 random error seeds were applied to baseline FCC-ee lattices and lattices with relaxed optics. In both cases, the same errors were applied to the individual magnets. The emittances were computed using the matrix methods in SAD and the results are shown in Fig. 6. The mean and standard error of the emittance for the baseline optics is -36.3 ± 4.4 , compared to -36.5 ± 4.4 for the relaxed case, indicating no significant improvement.

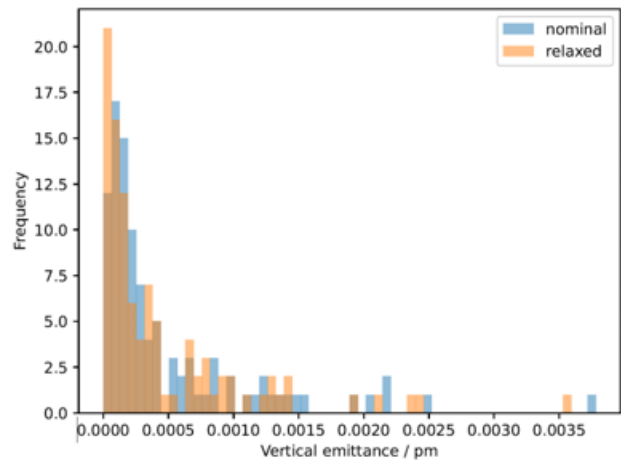


Figure 6: Emittance from tracking in 100 lattices with vertical alignment errors with baseline and relaxed optics.

CONCLUSION AND OUTLOOK

We have compared the emittance simulations with magnetic alignment errors using two methods and three simulation tools, showing good agreement between the tracking results from SAD and Xsuite, highlighting that they are ready to be used for precise predictions of emittances. The matrix methods have been shown to be reliable indicators of emittances but slightly disagree with tracking results. This disagreement is especially big for the Xsuite matrix method, which also disagrees with the matrix methods of the other two codes and therefore needs to be further understood.

A first applied study with relaxed optics, shows that, contrary to expectations, a relaxed optics does not reduce the vertical emittance from misalignment errors. The next step of this work is to bring multiple effects together to reliably determine the equilibrium emittance, this includes weak-strong beam-beam effects, various approximations of the tilted experimental solenoid and more detailed lattice errors and tapering schemes. Apart from emittance, these studies will also have to monitor other performance indicators like the dynamic aperture, lifetimes and polarisation and will require comprehensive correction and strategies.

REFERENCES

- [1] G. Iadarola *et al.*, “Xsuite: An Integrated Beam Physics Simulation Framework,” in *Proc. HB’23*, Geneva, Switzerland, Oct. 2023, pp. 73–80. doi:10.18429/JACoW-HB2023-TUA2I1
- [2] L. Deniau, H. Grote, G. Roy, F. Schmidt, “Methodical Accelerator Design - X,” Geneva, Switzerland. <http://cern.ch/madx>.
- [3] KEK Accelerator Laboratory, “Strategic Accelerator Design,” Tsukuba, Japan. <https://acc-physics.kek.jp/SAD/>.
- [4] L. van Riesen-Haupt, H. Burkhardt, R. Tomás, T. Persson, “Comparison of Accelerator Codes for Simulation of Lepton Colliders,” in *Proc. IPAC’21*, Campinas, Brazil, May 2021, pp. 1334–1337. doi:10.18429/JACoW-IPAC2021-TUPAB004
- [5] G. Iadarola, Overview of X-suite and its use for FCC-ee, FCC IS 2023 WP2 Workshop, Rome, Italy, 13-15 Nov. 2023. <https://indico.cern.ch/event/1326738/contributions/5650145/>
- [6] P. Kicsiny, Beam-beam Code Progress, FCC Week 2023, London, United Kingdom, 5-9 Jun. 2023. <https://indico.cern.ch/event/1202105/contributions/5390946/>
- [7] M. Hofer, *fccee_xample_error_tracking*, https://gitlab.cern.ch/mihofer/fccee_xample_error_tracking
- [8] T. K. Charles, B. Holzer, R. Tomas, K. Oide, L. van Riesen-Haupt, and F. Zimmermann, “Alignment & stability challenges for FCC-ee,” *EPJ Tech. Instrum.*, vol. 10, no. 1, Mar. 2023. doi:10.1140/epjti/s40485-023-00096-3
- [9] G. Iadarola, R. De Maria, K. Paraschou, L. van Riesen-Haupt, Computation of equilibrium beam sizes from synchrotron radiation in Xsuite, FCC-ee Optics Design Meeting, Geneva, Switzerland, 5 Oct 2023. <https://indico.cern.ch/event/1331315/>
- [10] L. van Riesen-Haupt, M. Hofer, M. Seidel, R. Tomas, and T. Pieloni, “Relaxed insertion region optics and linear tuning knobs for the Future Circular Collider,” presented at the IPAC’24, Nashville, TN, USA, May 2024, paper WEPR04, this conference.