

# TAPERED MODULAR QUADRUPOLE MAGNET TO REDUCE HIGHER-ORDER OPTICAL ABERRATIONS

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## Abstract

At UCLA's SAMURAI Laboratory, there will be a need for beam optics to accommodate operation over a range of beam energies. We present a modular quadrupole design that, in addition to satisfying this requirement, incorporates interchangeable tapered end-pieces for mitigation of higher-order aberrations [1]. The design progresses in an iterative fashion, whereby the tapered shapes, generated algorithmically, are fed into a field solver, and then the aberrations of the resulting particle trajectories are calculated and minimized.

## INTRODUCTION

Although focusing magnets are relatively simple elements of beam optics in theory, the fact that their lengths are finite, in reality, leads to the necessity to consider their fringe fields. According to R. Baartman, the higher-order optical aberration coefficients (larger than 3rd order) of any beam focusing element is directly related to the "hardness" of the fringe field.

Traditional focusing magnets all have flush ends, resulting in fields that vary dramatically when the charged particles travel through the magnet's length. The "softer" one can make the fringe field, the lower these aberration coefficients become.

In other words, a focusing magnet where the focusing strength decreases in the same manner as in its fringe field will have the softest fringe field and hence, the lowest aberration coefficients. Baartman shows that a taper to the pole tips in the beam direction can achieve, to some degree, such effects. However, one should be aware that this change to the design will require the magnet to run at higher current density or smaller aperture to achieve the same focusing power [1].

With the idea of designing and implementing such tapered quadrupole magnets for SAMURAI at PBPL, UCLA, we simulate beam optics at 30 MeV in order to compare the new magnet to traditional magnets.

## DESIGN

In his discussion for the taper function, Baartman uses  $\text{sech}^2$  function as his taper as it simulates the fringe field the best [1]. Since the idea is that any field "softer" than the traditional quadrupole should yield smaller higher-order aberration coefficients, we choose a more arbitrary taper function and hypothesize that this should reduce, to some degree, the higher-order aberrations.

Using the python library Open Cascade Technology, we are able to generate our surface of the tapered pole tips using Eq. (1). In our case of the quadrupole magnet for SAMURAI,  $z_0 = 0$  mm,  $z_1 = 125$  mm,  $b = 62.5$  mm, and  $h = 25$  mm. The distance between  $z_0$  and  $z_1$  constitutes the full length of the magnet, which is  $2b = 125$  mm. The height  $h$  is determined by the dimension of the magnet and the desired radius of aperture.

$$f(z) = h \left( \frac{1}{1 + \tanh \frac{z-z_0}{b}} + \frac{1}{1 - \tanh \frac{z-z_1}{b}} \right). \quad (1)$$

With the taper function defined, we move on to the usual function to generate pole tips for focusing magnets. In Eq. (2),  $R$  is the distance from the pole tip to the center of the magnet,  $\phi \in [-\pi/2N, \pi/2N]$  is the angle in the  $x-y$  plane (assuming  $z$ -direction is the beam direction), and  $N = 2$  for quadrupole. ( $N = 3$  for sextupole and 4 for octupole) Then a simple extrusion by rotation will generate the tapered surface of our pole tips.

$$R = f(z) \cos(N\phi)^{1/N}. \quad (2)$$

After the surface of the pole tips are generated, they are imported into SOLIDWORKS. A simple Boolean operation between the surface and a brick with the same dimensions as truncated pole tips suffices to produce our tapered pole tips, as in Fig. 1. Finally, we generate the quadrupole magnet using VBA Macro script for truncated quadrupole and replace the traditional pole tips with the tapered ones. The final 3-D model can be seen in Fig. 2.

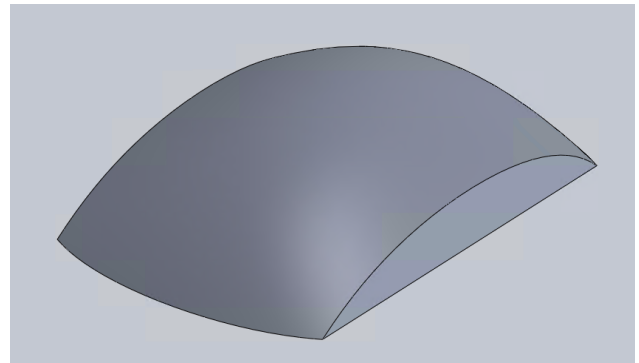


Figure 1: 3-D Model of Tapered Pole Tips. The surface of the pole tips is generated via Open Cascade Technology. The model itself is a simple Boolean operation between the brick of the same dimensions and the surface aforementioned.

In our final assembly of the focusing elements, the tapered quadrupole magnet will presents itself in the form of interchangeable pole tips, truncated at one end but tapering at

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the other. The tips will then replace the tips at two ends of a series of truncated quadrupole magnets. They will serve as a tapered extension to the existing traditional focusing series, achieving the lower aberration coefficients while not losing too much focusing power due to the flared out aperture. Such is our approach to try to incorporate Baartman's quadrupole shape in experimental reality.

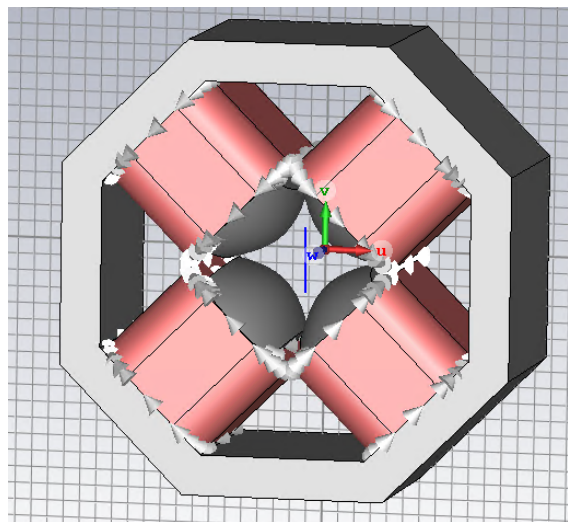


Figure 2: 3-D Model of Tapered Quadrupole Magnet.

## RESULTS

The beam simulation is ran with CST Studio Particle Tracking Simulation and exported for numerical analysis. To reduce the complexity required for such endeavor, we restrict our test beam of 30 MeV to displace only in the focusing direction and with no initial momentum in the  $x$  or  $y$  direction. The linear focus point is calculated using the particle closest to the ideal beam line. Then image on this plane is interpolated. Finally, curve fit between the image and the initial displacement is performed, resulting in aberration coefficients up to the order of 7 (Figs. 3 and 4).

In order to compare the tapered quadrupole magnets to the traditional ones, we wanted to do this in the same manner as Baartman. To him, a “fair” comparison is constrained to equal integrated gradient across the beam axis. In other words, the comparison is fair only when the trajectory analysis yields (almost) the same third-order aberration coefficients.

In our attempts to try to do the same, we realized that this “fair” comparison comes at the cost of much higher current density in the coils than the traditional quadrupole magnet. For SAMURAI, the 30 MeV test beam focuses linearly at approximately 0.15 mm outside the magnet with coils of  $1.5 \text{ A/mm}^2$  current density. However, the same test beam requires nearly triple the current density to bring down the third order aberrations to the same order of magnitude and the resulting point of focus will be inside the magnet itself, rendering the trajectory analysis unusable.

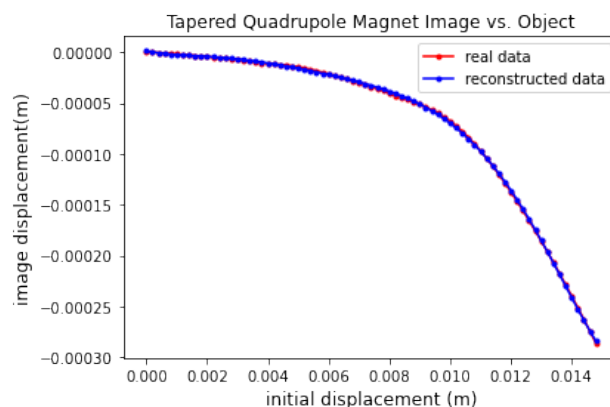


Figure 3: Image Displacement vs. Initial Displacement for the Tapered Quadrupole Magnet. The real data is exported from CST Studio particle tracking simulation and the reconstructed data is generated from the curve fit (polyfit).

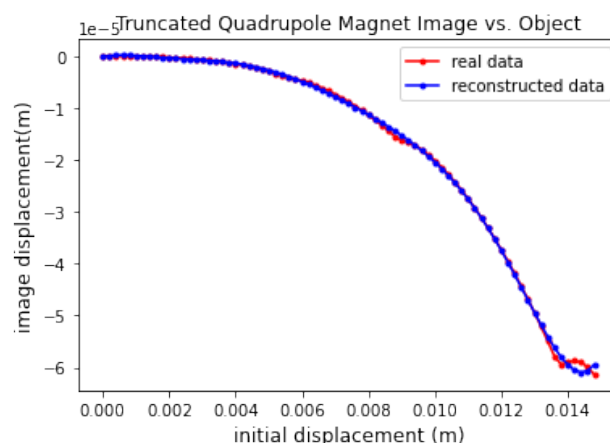


Figure 4: Image Displacement vs. Initial Displacement for the Truncated Quadrupole Magnet. The real data is exported from CST Studio particle tracking simulation and the reconstructed data is generated from the curve fit (polyfit).

We opted for a “middle ground” between the “fair” comparison and unreasonably high current density. In the region of acceptable current density, we find a point where the third-order aberration is still an order of magnitude higher than the traditional quadrupole, but the higher-order aberrations are significantly lower (Table 1).

In Fig. 5, the difference in third-order aberration coefficients is dwarfed by the magnitude of the higher-order aberrations but we should note that there is still an order of magnitude between the two.

## CONCLUSION

Baartman's quadrupole shapes are certainly viable in some cases and our numerical analysis of the trajectories verifies that tapered quadrupole of the same integrated strength will have much lower aberrations of high orders. In our attempt to make the quadrupole modular in a series, we limit

Table 1: Aberration Coefficients

| Order | Tapered<br>Quadrupole | Truncated<br>Quadrupole |
|-------|-----------------------|-------------------------|
| 7     | $1.10630431e+10$      | $3.55491274e+10$        |
| 6     | $-1.14325613e+08$     | $-1.58931878e+09$       |
| 5     | $-6.70700085e+06$     | $2.74976062e+07$        |
| 4     | $1.45148528e+05$      | $-2.32570096e+05$       |
| 3     | $-1.13521047e+03$     | $9.78240358e+02$        |
| 2     | $3.50759999e+00$      | $-2.04130849e+00$       |
| 1     | $-6.39624187e-03$     | $1.53989848e-03$        |
| 0     | $1.39838871e-06$      | $-2.25155495e-07$       |

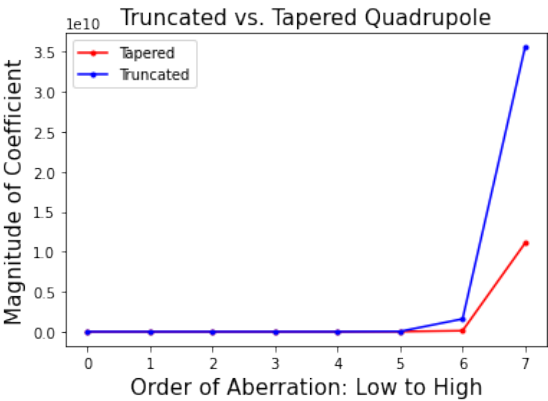


Figure 5: Comparison of Aberration Coefficients between Tapered and Traditional/Truncated Quadrupole Magnets.

ourselves to the same radius of aperture as the traditional quadrupole magnets in SAMURAI.

If we are to shrink the radius of aperture, we will be able to achieve the same integrated focusing strength without reaching some unreasonable current density. But such a structure will have little flexibility outside the project, rendering it impractical to implement in an experimental setup. The best way to apply the taper, in our opinion, is to construct interchangeable pole pieces for existing traditional magnets.

In the mean time, more work remains to be completed, as we have yet simulated the  $\text{sech}^2$  tapered quadrupole magnet as proposed by Baartman. We might find ourselves with more room towards lower current densities and lower aberrations.

REFERENCES

[1] R. Baartman, “Quadrupole Shapes”, *Physical Review Special Topics - Accelerators and Beams*, vol. 15, no. 7, p. 074002, Jul. 2012. doi:10.1103/PhysRevSTAB.15.074002