Back reaction in Einstein universe at finite temperature

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Abstract

We study the thermal and curvature effect to spontaneous symmetry breaking in ϕ^4 theory. The effective potential is evaluated in D-dimensional static universe with positive curvature $R \otimes S^{D-1}$ or negative curvature $R \otimes H^{D-1}$. It is shown that temperature and positive curvature suppress the symmetry breaking, while negative curvature enhances it. To consider the back-reaction we numerically solve the gap equation and the Einstein equation simultaneously. The solution gives the relationship between the temperature and the scale factor.

1 Introduction

Spontaneous symmetry breaking is one of the important concept in elementary particle physics. In early universe, especially the grand unification era ($\sim 10^{-34}$s), the strong curvature and high temperature state is realized. We can not neglect the thermal and curvature effects to the cosmic evolution. We need to treat the matter as quantized fields. However, we assume that it is adequate to consider the gravity as a classical field in GUT era. The thermal and curvature effects to spontaneous symmetry breaking has been studied by many authors [1, 2, 3, 4]. The curvature effect to the symmetry breaking comes from a scale factor dependence of the covariant derivative and a coupling between the scalar field and the gravitational field. One-loop and two-loop corrections to scalar field theories in the linear curvature approximation were found in Refs. [5] and [6], respectively. The relationship between the scale factor and the temperature for a free scalar field was calculated by solving the back reaction problem in Refs. [7, 8].

In the present article, we extend the work in Refs. [7, 8] to the ϕ^4 theory and investigate the structure of early universe. For this purpose we study the thermal and curvature effects to the spontaneous symmetry breaking in a D-dimensional static universe. To obtain finite results we confine ourselves to the spacetime dimensions $2 \leq D < 5$. As a simple example, we consider the scalar ϕ^4 theory where the Z_2 symmetry is spontaneously broken. The explicit expression of the 1-loop effective potential of ϕ^4 theory is derived on a positive curvature spacetime $R \otimes S^{D-1}$ or a negative curvature spacetime $R \otimes H^{D-1}$ [9]. We find the phase structure of the model by observing the minimum of the effective potential with varying temperature and scale factor in our previous work [10].

The expectation value of the stress-tensor, the right-hand side of the Einstein equation, is also calculated. We numerically solve the gap equation and the Einstein equation simultaneously on $R \otimes S^{D-1}$ and $R \otimes H^{D-1}$ at finite temperature. The solution gives the relationship between the temperature and the scale factor of the universe.

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2 Spontaneous symmetry breaking in $R\otimes S^{D-1}$ and $R\otimes H^{D-1}$ at finite temperature

In this section, we briefly review the analysis by the effective potential in curved spacetime at finite temperature and observe the property of phase transition with varying temperature and scale factor.

We introduce the constant curvature space $R\otimes S^{D-1}$ and $R\otimes H^{D-1}$ as Euclidean analog of the static universe. The manifold $R\otimes S^{D-1}$ is represented by the metric

$$ds^2 = dr^2 + a^2(d\theta^2 + \sin^2\theta d\Omega_{D-2}). \tag{1}$$

where $d\Omega_{D-2}$ is the metric on a unit sphere S^{D-2} and a is the scale factor. Similarly, the manifold $R\otimes H^{D-1}$ is defined by the metric

$$ds^2 = dr^2 + a^2(d\theta^2 + \sinh^2\theta d\Omega_{D-2}). \tag{2}$$

The Lagrangian of the scalar ϕ^4 theory is written as

$$\mathrm{L}(\phi) = -\frac{1}{2}\phi(\nabla + \xi_0 R)\phi + \frac{\mu_0^2}{2}\phi^2 - \frac{\lambda_0}{4!}\phi^4, \tag{3}$$

where ϕ is a real scalar field, $i\mu_0$ corresponds to the bare mass of the scalar field, λ_0 the bare coupling constant for the scalar self-interaction, ξ_0 the bare coupling constant between the scalar field and the gravitational field.

According to Ref. [10], in Euclidean spacetime the effective potential $V(\phi)$ can be written as

$$V(\phi) = \frac{\xi_0}{2}R\phi^2 + \frac{\lambda_0}{4!}\phi^4 + \frac{\hbar\lambda_0}{4}\int_0^{\phi^2} dm^2 G(x,x;m) + O(\hbar^{3/2}). \tag{4}$$

In this article we neglect $O(\hbar^{3/2})$ term. The two-point function $G(x,y)$ of the real scalar field at finite temperature on the manifolds $R\otimes S^{D-1}$ and on the manifolds $R\otimes H^{D-1}$ are given by Ref. [9].

The effective potential $V(\phi)$ obtained in the previous section is divergent at D=2 or D=4. We introduce the renormalization procedure by imposing the renormalization conditions

$$\left.\frac{\partial^2 V_0}{\partial\phi^2}\right|_{\phi=0} \equiv -\mu_r^2 - \xi_r R, \quad \left.\frac{\partial^4 V_0}{\partial\phi^4}\right|_{\phi=M} \equiv \lambda_r, \tag{5}$$

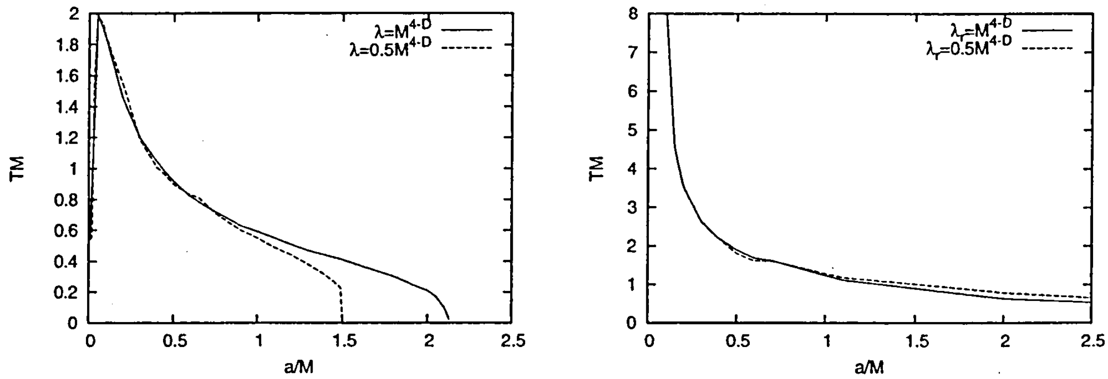
where M is the renormalization scale and $V_0(\phi)$ is the effective potential in flat spacetime at $T=0$. The renormalized effective potential is given by replacing the bare parameters μ_0 and λ_0 with μ_r and λ_r in the tree level terms in $V(\phi)$.

The behavior of the effective potential is illustrated in our previous paper [10]. The curvature effect suppresses the symmetry breaking in a positive curvature space $R\otimes S^{D-1}$, while it enhances the symmetry breaking in a negative curvature space $R\otimes H^{D-1}$.

3 Back reaction on $R\otimes S^{D-1}$ and $R\otimes H^{D-1}$ at finite temperature

In a quantum field theory the Einstein's field equation is given by

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi\langle T_{\mu\nu}\rangle, \tag{6}$$



(a) $R\otimes S^{D-1}$, $\mu^2=0.1M^2$, $\lambda=M^{4-D}, 0.5M^{4-D}$ (b) $R\otimes H^{D-1}$,$\mu^2=0.1M^2$, $\lambda=M^{4-D}, 0.5M^{4-D}$

Figure 1: The $T-a$ relationship for a conformally coupled scalar field $\xi=(D-2)/(4D-4)$.

where $\langle T_{\mu\nu}\rangle$ is the expectation value of the stress tensor in the ground state which is determined by observing the minimum of the effective potential. Units have been chosen so that $G=c=\hbar=k=1$ in this section.

To include the back reaction we solve the gap equation

$$\left.\frac{\delta V(\phi)}{\delta\phi}\right|_{\phi\to\langle\phi\rangle}=0. \tag{7}$$

and the 44 component of (6) simultaneously. Since $R_{44}=0$ in $R\otimes S^{D-1}$ and $R\otimes H^{D-1}$, the 44 component of the Einstein equation reduces to

$$-\frac{1}{2}R=8\pi\langle T_{44}\rangle. \tag{8}$$

By the variation over the metric, we obtain $\langle T_{44}\rangle$ on $R\otimes S^{D-1}$

$$\begin{aligned}\langle T_{44}\rangle &= \frac{1}{2}(-\mu_0^2+\xi_0 R)\langle\phi\rangle^2+\frac{\lambda_0}{4!}\langle\phi\rangle^2-\frac{1}{(4\pi)^{\frac{D-1}{2}}}\Gamma\left(\frac{3-D}{2}\right)Ta^{3-D}\\ &\quad\times\int_0^\infty\sum_{n=-\infty}^{n=\infty}\omega_n^2\frac{\Gamma\left(\frac{D-2}{2}+i\alpha_S\right)\Gamma\left(\frac{D-2}{2}-i\alpha_S\right)}{\Gamma\left(\frac{1}{2}+i\alpha_S\right)\Gamma\left(\frac{1}{2}-i\alpha_S\right)},\end{aligned} \tag{9}$$

and on $R\otimes H^{D-1}$,

$$\begin{aligned}\langle T_{44}\rangle &= \frac{1}{2}(-\mu_0^2+\xi_0 R)\langle\phi\rangle^2+\frac{\lambda_0}{4!}\langle\phi\rangle^2-\frac{1}{(4\pi)^{\frac{D-1}{2}}}\Gamma\left(\frac{3-D}{2}\right)Ta^{3-D}\\ &\quad\times\int_0^\infty\sum_{n=-\infty}^{n=\infty}\omega_n^2\frac{\Gamma\left(\frac{D-2}{2}+\alpha_H\right)}{\Gamma\left(\frac{4-D}{2}+\alpha_H\right)}.\end{aligned} \tag{10}$$

We numerically solve the gap equation (7) and the 44 component of the Einstein equation (8). The solution gives the relationship between temperature and radius of the universe. We show for the self-interacting fields, $\lambda_r=M^{4-D}$ and $\lambda_r=0.5M^{4-D}$ in Fig.1. The scalar self-interaction increases the scale factor a on $R\otimes S^{D-1}$, while it decreases a on $R\otimes H^{D-1}$.

4 Concluding remarks

We have investigated the thermal and curvature effect to spontaneous symmetry breaking at finite temperature and curvature in arbitrary dimensions ($2 \leq D < 5$).

In a positive curvature space $R \otimes S^{D-1}$ the broken symmetry is restored as the curvature increases. For a negative curvature space $R \otimes H^{D-1}$ we see that the symmetry breaking is enhanced by the curvature effect. Solving the gap equation and the Einstein equation, we show the relationship between the temperature and the scale factor. By the scalar self-interaction, the scale factor is increased on $R \otimes S^{D-1}$, whereas it is decreased on $R \otimes H^{D-1}$.

It is important to extend our analysis to the time dependent back ground. Thus it is important to consider the model in the state out of equilibrium. We remained it for future works.

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