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Dimension-8 SMEFT matching conditions for the low-energy effective field theory

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Abstract: In particle physics, the modern view is to categorize things in terms of effective field theories (EFTs). Above the weak scale, we have the SMEFT, formed when the heavy new physics (NP) is integrated out, and for which the Standard Model (SM) is the leading part. Below M_W, we have the LEFT (low-energy EFT), formed when the heavy SM particles ($W^{\pm}$, Z^0, H, t) are also integrated out. In order to determine how low-energy measurements depend on the underlying NP, it is necessary to compute the matching conditions of LEFT operators to SMEFT operators. These matching conditions have been worked out for all LEFT operators up to dimension 6 in terms of SMEFT operators up to dimension 6 at the one-loop level. However, this is not sufficient for all low-energy observables. In this paper we present the momentum-independent matching conditions of all such LEFT operators to SMEFT operators up to dimension 8 at tree level.

Keywords: Effective Field Theories, SMEFT, Specific BSM Phenomenology

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1 Introduction

Despite its enormous success in accounting for almost all experimental data to date, the Standard Model (SM) of particle physics still has no explanation for a number of other key observations, such as neutrino masses, the baryon asymmetry of the universe, dark matter, etc. For this reason, it is widely believed that there must exist physics beyond the SM. And since the LHC has not discovered any new particles up to a scale of $O(\text{TeV})$, this new physics (NP) is likely to be very massive.

When the NP is integrated out, one obtains an effective field theory (EFT), of which it is now generally believed that the SM is simply the leading part. This EFT must obey the SM gauge symmetry $\text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y$. Since the discovery of the Higgs boson, the default assumption is that this symmetry is realized linearly, i.e., the symmetry is broken via the Higgs mechanism, resulting in the Standard Model EFT, or SMEFT (see, e.g., refs. [1][1]). The SMEFT has been studied extensively: a complete and non-redundant list of dimension-6 operators is given in ref. [3], the dimension-7 operators can be found in refs. [4, 5], and the dimension-8 operators are tabulated in refs. [6, 7].

The LEFT (low-energy effective field theory) describes the physics below the W mass, and is produced when the heavy SM particles (W, Z, t, H) are also integrated out. (This is also called the WET (weak effective field theory).) In ref. [8], Jenkins, Manohar and Stoffer (JMS) present a complete and non-redundant basis of LEFT operators up to dimension 6, including those that violate B and L, and also give the matching to dimension-6 SMEFT operators at tree level.

At one loop, there are two additional types of contributions to the matching. First, there are the renormalization-group running effects which are enhanced by $\log(\mu_{\text{NP}}/\mu_{\text{EW}})$. Second, we have the threshold corrections, or one-loop matching at the electroweak scale, which are a part of the next-to-leading-log effects. Naively, the former contributions appear to dominate. However, for certain processes, the latter can be comparable [9]. The anomalous-dimension matrices for the renormalization-group running and the one-loop matching effects for the full SMEFT and LEFT bases have been computed in refs. [10]

[1] For a review see ref. [2].

and [11], respectively. (Note that, in order it to consistently include the threshold corrections at the electroweak scale, it is essential to have two-loop anomalous dimensions of the LEFT operators [12, 13].)

With this information, if a discrepancy with the SM is observed in a process that uses a particular LEFT operator, we will know which dimension-6 SMEFT operators are involved.

However, this is not always sufficient. Information about the contributions from higher-dimension operators may be important if the process in question is suppressed in the SM and/or is very precisely measured. Examples of observables for which such contributions must be taken into account include electroweak precision data from LEP [14], lepton-flavour-violating processes [15, 16], meson-antimeson mixing ($\Delta F = 2$) [9, 17], and electric dipole moments [18]. (Dimension-8 SMEFT operators have also been discussed in the context of high-energy processes, see refs. [19–26].)

We have the matching conditions of dimension-6 LEFT operators to dimension-6 SMEFT operators. A first step is therefore to extend this matching to include the (subdominant) dimension-8 SMEFT operators. But there is a complication: dimension-8 SMEFT operators will also produce dimension-8 LEFT operators. (A complete set of dimension-8 LEFT operators can be found in ref. [27].) Thus, additional LEFT operators must in principle also be considered.

Note that a distinction can be made between the various contributions. Consider four-fermion operators. These dimension-6 LEFT operators have no derivatives, and are therefore momentum-independent (MI). On the other hand, the dimension-8 extensions of four-fermion operators do contain derivatives, i.e., they are momentum-dependent (MD). As a consequence, their tree-level matching conditions to dimension-8 SMEFT operators are also MD.[2] Of course, the MD contributions can be at the same level in power counting as the MI contributions. Depending on the scale of the NP (Λ) and the masses of the fermions involved in the process under consideration, they can be numerically comparable to, or even larger than, the MI contributions. Thus, a full computation of the contributions from higher-dimension operators to low-energy processes must include both dimension-6 and dimension-8 LEFT operators and their MI and MD matching conditions to dimension-8 SMEFT operators. This is an enormous undertaking.

Fortunately, the MI and MD tree-level matching conditions can be separated. In the present paper, we focus only on the MI matching conditions. MD matching conditions will be presented elsewhere. Note that a complete analysis of the relationship between LEFT operators and dimension-8 SMEFT operators must also take into account the renormalization-group running of SMEFT operators from the NP scale down to low energy, as well as the threshold corrections at the electroweak scale. For bosonic SMEFT operators up to dimension 8, the anomalous dimensions have been calculated in refs. [29, 30].

In our analysis, we follow closely the approach of ref. [8], and extend it to include dimension-8 SMEFT operators. Below, we often refer to this paper simply by the initials of its authors, as JMS.

[2] In a similar vein, one can find MD contributions to dimension-7 LEFT operators due to dimension-6 and 7 SMEFT operators, see ref. [28].

We begin in section 2 with some preliminary remarks comparing our analysis with that of JMS, and discuss in general terms how matching conditions are computed. In section 3, we present the setup, showing how the presence of dimension-8 SMEFT operators affects the symmetry breaking, the generation of masses, and the couplings of the gauge and Higgs bosons to fermions. The computations required to derive the complete matching conditions are described in section 4. Although we do not compute the MD matching conditions, the various sources of such contributions are outlined here. We conclude in section 5. The results are presented in appendix D. Appendices A, B, C give a variety of information relevant to the details of the analysis.

2 Preliminaries

In ref. [8], JMS compute the tree-level SMEFT matching conditions for the LEFT operators. The matching conditions for operators that conserve both B and L involve only even-dimension SMEFT operators, and are given up to dimension 6. For operators that violate B and/or L, the matching conditions can involve even- or odd-dimension SMEFT operators (but not both), depending on the operator, and are computed to dimension 6 or dimension 5. In the present paper, we extend this analysis: we compute these matching conditions up to dimension 8 (dimension 7) if even-dimension (odd-dimension) SMEFT operators are involved. (In this paper, when we refer to "computing the matching conditions up to dimension 8," both of these possibilities are understood.) If one eliminates the dimension-8 or dimension-7 contributions, the results of JMS are reproduced. This makes it easy to compare the results. Also, we present the elements of our analysis in much the same order as JMS.

In the LEFT Lagrangian, we consider only operators up to dimension 6 (like JMS):

$$\mathcal{L}_{\rm LEFT} = \mathcal{L}_{\rm LEFT}^{\rm Neutrino\ mass} + \mathcal{L}_{\rm QCD+QED} + \sum_{n=5}^{6} \sum_{\mathcal{O}\in {\rm dim}\, n} \frac{C_{\mathcal{O}}}{\Lambda^{n-4}}\, \mathcal{O}\,. \tag{2.1}$$

For the SMEFT, all operators up to dimension 8 are included:

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_{n=5}^{8} \sum_{Q\in {\rm dim}\, n} \frac{C_Q}{\Lambda^{n-4}}\, Q\,. \tag{2.2}$$

(Note that using the same suppression scale Λ for both LEFT and SMEFT is just a matter of convention.)

Still, there are two differences in our notation:

- Our convention is to have dimensionless Wilson coefficients (WCs). For instance, for the dimension-6 SMEFT lagrangian, we write

$$\mathcal{L}_{\rm SMEFT}^{(6)} = \sum_{Q\in {\rm dim}\, 6} \frac{C_Q}{\Lambda^2}\, Q\,. \tag{2.3}$$

  This convention is different from that of JMS, which uses dimensionful WCs.

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- In the unbroken phase, the SM lagrangian is

$$\begin{aligned}\mathcal{L}_{\rm SM} =& -\frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}W^I_{\mu\nu}W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\ &+ \sum_{\psi=q,u,d,l,e} \overline{\psi} i \not{D} \psi + (D_\mu H)^\dagger (D^\mu H) - \lambda \left(H^\dagger H - \frac{1}{2}v^2\right)^2 \\ &- \left[\bar{l}_p e_r (Y_e)_{pr} H + \overline{q}_p u_r (Y_u)_{pr} \tilde{H} + \overline{q}_p d_r (Y_d)_{pr} H + \text{h.c.}\right] \\ &+ \frac{\theta_3 g_s^2}{32\pi^2} G^A_{\mu\nu}\tilde{G}^{A\mu\nu} + \frac{\theta_2 g^2}{32\pi^2} W^I_{\mu\nu}\tilde{W}^{I\mu\nu} + \frac{\theta_1 g'^2}{32\pi^2} B_{\mu\nu}\tilde{B}^{\mu\nu} \,. \end{aligned} \tag{2.4}$$

  This uses the same notation as JMS, with one exception: our Yukawa matrices (the Ys) are the hermitian conjugates of those of JMS.

In eq. (2.4), the fields q_r and l_r are (left-handed) $\mathrm{SU}(2)_L$ doublets, while u_r, d_r and e_r are (right-handed) $\mathrm{SU}(2)_L$ singlets, where $r = 1,\ 2,\ 3$ is a generation (weak-eigenstate) index. The physical (mass-eigenstate) states are the same for the charged leptons, the left- and right-handed u-type quarks, and the right-handed d-type quarks. For the left-handed d-type quarks, the relation between the weak and mass eigenstates is

$$d_{Lr} = V_{rd} d_L + V_{rs} s_L + V_{rb} b_L \equiv V_{rx} d_{Lx} \,, \tag{2.5}$$

where the left-hand side is a weak eigenstate, and the right-hand side is a linear combination of mass eigenstates. The V_{rx} are elements of the unitary mixing matrix, which is the Cabibbo-Kobayashi-Maskawa (CKM) matrix in the SM. Note: as in JMS, our LEFT matching conditions are given in the weak eigenstate basis. They can be written in terms of the physical states by using the above relation.

In our analysis, we make reference to several different sets of operators. The LEFT operators are taken from JMS [8], the dimension-6 SMEFT operators are found in ref. [3], and we use ref. [7] for the dimension-8 SMEFT operators. In all cases, we use the same notation for the operators and their WCs as is used in the references. For the dimension-7 SMEFT operators, we use a basis that is equivalent that of ref. [4], but with a different notation. For convenience, in the appendices, we present tables of all the operators used in this paper. These include LEFT operators (appendix A), along with dimension-5 to 8 SMEFT operators (appendix B).

It is useful to give an example that illustrates the various issues involved in deriving matching conditions. Consider the charged-current four-fermion operator

$$\mathcal{O}^{V,LL}_{\nu edu} = (\bar{\nu}_{Lp}\gamma^\mu e_{Lr})(\bar{d}_{Ls}\gamma_\mu u_{Lt}) + \text{h.c.}, \qquad \text{coefficient}: \frac{1}{\Lambda^2} C^{V,LL}_{\substack{\nu edu \\ prst}} \,. \tag{2.6}$$

We begin by examining the matching to the SM. That is, $\mathcal{O}^{V,LL}_{\nu edu}$ is taken to be an operator of the Fermi theory, whose coefficient has magnitude $4G_F/\sqrt{2}$. The SM Lagrangian consists only of operators of at most dimension 4. This four-fermion operator can be generated in the SM when a W is exchanged between the two fermion currents, and the W is integrated out. The SM matching condition is then

$$\frac{1}{\Lambda^2} C^{V,LL}_{\substack{\nu edu \\ prst}} = -\frac{g^2}{2M_W^2} [W_l]_{pr} [W_q]^*_{ts} \,. \tag{2.7}$$

Here, W_l and W_q are the couplings of the W to the lepton and quark pair, respectively. In the weak-eigenstate basis of the SM, $[W_l]_{pr} = \delta_{pr}$ and $[W_q]_{ts} = \delta_{ts}$. Knowing that the coefficient has magnitude $4G_F/\sqrt{2}$, this leads to the well-known relation

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}\,. \tag{2.8}$$

The matching to SMEFT at dimension 6 was computed by JMS. It is

$$\frac{1}{\Lambda^2} C^{V,LL}_{\substack{\nu edu \\ prst}} + \text{h.c.} = \frac{2}{\Lambda^2} C^{(3)}_{\substack{lq \\ prst}} - \frac{\bar{g}^2}{2M_W^2} [W_l]^{\text{eff}}_{pr} [W_q]^{\text{eff}*}_{ts} + \text{c.c.} \tag{2.9}$$

Since the SMEFT includes dimension-6 terms, it contains the four-fermion operator. That is, there is a *direct* contribution to the matching conditions, $C^{(3)}_{\substack{lq \\ prst}}$. As was the case in the SM, $C^{V,LL}_{\substack{\nu edu \\ prst}}$ can also be generated by the exchange of a W between the two fermion currents. This is represented by the second term above. Although this resembles the term in eq. (2.7), there are several differences:

1. In the presence of dimension-6 SMEFT operators, the coupling constant is modified: $g \to \bar{g}$. This is due to the fact that, when one adds dimension-6 corrections to the kinetic terms of the gauge bosons, these fields and the coupling constants must be redefined in order to ensure that the kinetic term is properly normalized.

2. In the SM, the W coupling to fermions is fixed by the fermion kinetic term, $\bar{\psi} \not{D} \psi$. In SMEFT, there are dimension-6 corrections, such as $H^\dagger i D_\mu H \bar{\psi} \gamma^\mu \psi$. These will change the magnitudes of the couplings, and permit inter-generational couplings, hence the 'eff' superscript on W_l and W_q.

The bottom line is that many dimension-6 SMEFT operators are implicitly present in the second term of eq. (2.9) above. Collectively, these operators form the *indirect* contributions. They must be carefully taken into account in the matching conditions. (Note that, if one expands the effective parameters appearing in the matching conditions, many terms will appear; those that are of higher order than dimension 8 are to be ignored.)

3 Setup

The Lagrangian for the SM in the unbroken phase is given in eq. (2.4). When the Higgs field acquires a vacuum expectation value (vev), given by the minimum of the Higgs potential, the symmetry is broken, and masses are generated for the $W^\pm$, the Z^0 and the fermions. One can easily compute the masses of the physical gauge bosons, as well as their couplings to the physical fermions, in terms of the parameters of $\mathcal{L}_{\text{SM}}$, in particular g, g' and v.

When one includes higher-order SMEFT operators of dimension 6, 8, etc., this whole process must be recalculated in order to take into account these new operators. One must make field redefinitions so that the kinetic terms are properly normalized, the minimum of the Higgs potential (i.e., the Higgs vev) must be recomputed, corrections to $\sin\theta_W$ must be taken into account, etc. One sees the effects of these additional operators in the redefinitions of the coupling constants, the couplings of gauge bosons to fermions, and other quantities that appear in both the direct and indirect contributions to the matching conditions.

In this section, we present the main effects of including SMEFT operators up to dimension 8. We emphasize those results that are important for the matching conditions. These results are in agreement with the predictions of the geometric formulation of the SMEFT [31].

3.1 Higgs sector

After the Higgs acquires a vev, we redefine the Higgs field as follows:

$$H \to \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ [1 + c_{H,\text{kin}}]h + v_T \end{pmatrix} . \tag{3.1}$$

Here, v_T and $c_{H,\text{kin}}$ are respectively determined by minimizing the Higgs potential and by normalizing the Higgs kinetic term.

3.1.1 Higgs vev

In the presence of SMEFT operators up to dimension 8, the Higgs potential is

$$V(H) = \lambda \left(H^2 - \frac{1}{2} v^2 \right)^2 - \frac{1}{\Lambda^2} C_H H^6 - \frac{1}{\Lambda^4} C_{H^8} H^8 , \tag{3.2}$$

where only the real part of the second component of the Higgs doublet, H, is taken to be nonzero. We define the physical Higgs vev, v_T, to be $v_T \equiv \sqrt{2}\, H^{\text{min}}$, where H^{min} minimizes the Higgs potential. This implies that

$$v_T = v \left(1 + \frac{3v^2}{8\lambda\Lambda^2} C_H + \frac{v^4}{4\lambda\Lambda^4} \left[\frac{63}{32\lambda} [C_H]^2 + C_{H^8} \right] \right) . \tag{3.3}$$

v_T is the physical parameter that appears in the matching relations, and whose value can in principle be determined by a fit to the data.

3.1.2 Higgs kinetic term

Including SMEFT contributions up to dimension 8, the Higgs kinetic term is

$$\mathcal{L}_{\text{SMEFT}}^{\text{Higgs kinetic}} = \frac{1}{2} \left[1 + \frac{2v_T^2}{\Lambda^2} \left(\frac{1}{4} C_{HD} - C_{H\Box} \right) + \frac{v_T^4}{4\Lambda^4} \left(C_{H^6}^{(1)} + C_{H^6}^{(2)} \right) \right] (1 + c_{H,\text{kin}})^2 (\partial_\mu h)(\partial^\mu h) . \tag{3.4}$$

In order for this term to be properly normalized, one must have

$$c_{H,\text{kin}} = \frac{v_T^2}{\Lambda^2} \left(C_{H\Box} - \frac{1}{4} C_{HD} \right) - \frac{v_T^4}{8\Lambda^4} \left(C_{H^6}^{(1)} + C_{H^6}^{(2)} \right) + \frac{3v_T^4}{2\Lambda^4} \left(C_{H\Box} - \frac{1}{4} C_{HD} \right)^2 . \tag{3.5}$$

This is essentially a redefinition of the normalization of the Higgs field.

3.1.3 Higgs mass

Taking into account the SMEFT contributions up to dimension 8, the Higgs boson mass term is

$$\mathcal{L}_{\text{SMEFT}}^{\text{Higgs mass}} = \frac{1}{2}\left[\lambda v^2 - 3\lambda v_T^2 + \frac{15 v_T^4}{4\Lambda^2}\, C_H + \frac{7 v_T^6}{2\Lambda^4}\, C_{H^8}\right](1 + c_{H,\text{kin}})^2 h^2\,. \tag{3.6}$$

This gives the following expression for the Higgs boson mass:

$$m_h{}^2 = (1 + c_{H,\text{kin}})^2 v_T^2 \left[2\lambda - \frac{3 v_T^2}{\Lambda^2}\, C_H - \frac{3 v_T^4}{\Lambda^4}\, C_{H^8}\right]. \tag{3.7}$$

3.2 Fermion mass matrices & Yukawa couplings

Before symmetry breaking, the SMEFT Lagrangian up to dimension 8 contains the following terms for charged leptons and quarks:

$$-\,(Y_\psi)_{pr}\,\bar{\chi}_p \psi_r\,\overline{H} + \frac{1}{\Lambda^2} \underset{pr}{C_{\psi H}}\,\bar{\chi}_p \psi_r\,\overline{H}(H^\dagger H) + \frac{1}{\Lambda^4} \underset{pr}{C_{\chi\psi H^5}}\,\bar{\chi}_p \psi_r\,\overline{H}(H^\dagger H)^2 + \text{h.c.}\,, \tag{3.8}$$

where $\psi \in \{e, u, d\}$ (right-handed $\text{SU}(2)_L$ singlets), $\chi \in \{l, q\}$ (left-handed $\text{SU}(2)_L$ doublets), $\overline{H} = H$ if $\psi \in \{e, d\}$ and $\overline{H} = \tilde{H}$ if $\psi \in \{u\}$. Here, the first term (dimension 4) belongs to the SM and the last two terms are SMEFT operators (respectively dimension 6 and 8). Lepton-number-violating terms are also present:

$$\frac{1}{\Lambda} \underset{pr}{C_5}\,\epsilon^{ij}\epsilon^{kl}(l_{ip}^T C l_{kr})\,H_j H_l + \frac{1}{\Lambda^3} \underset{pr}{C_{l^2H^4}}\,\epsilon^{ij}\epsilon^{kl}(l_{ip}^T C l_{kr})\,H_j H_l (H^\dagger H) + \text{h.c.}\,. \tag{3.9}$$

When the Higgs gets a vev, both mass matrices and Yukawa coupling terms are generated.

3.2.1 Fermion mass matrices

The SMEFT mass terms for charged leptons and quarks up to dimension 8 are

$$\begin{aligned}
\mathcal{L}_{\text{SM}}^{\text{Fermion mass}} &= -\frac{v_T}{\sqrt{2}}\left[(Y_e)_{pr}\overline{e}_{Lp} e_{Rr} + (Y_u)_{pr}\overline{u}_{Lp} u_{Rr} + (Y_d)_{pr}\overline{d}_{Lp} d_{Rr}\right] + \text{h.c.},\\
\mathcal{L}_{\text{SMEFT,6}}^{\text{Fermion mass}} &= \frac{v_T^3}{2\sqrt{2}\,\Lambda^2}\left[\underset{pr}{C_{eH}}\overline{e}_{Lp} e_{Rr} + \underset{pr}{C_{uH}}\overline{u}_{Lp} u_{Rr} + \underset{pr}{C_{dH}}\overline{d}_{Lp} d_{Rr}\right] + \text{h.c.},\\
\mathcal{L}_{\text{SMEFT,8}}^{\text{Fermion mass}} &= \frac{v_T^5}{4\sqrt{2}\,\Lambda^4}\left[\underset{pr}{C_{leH^5}}\overline{e}_{Lp} e_{Rr} + \underset{pr}{C_{quH^5}}\overline{u}_{Lp} u_{Rr} + \underset{pr}{C_{qdH^5}}\overline{d}_{Lp} d_{Rr}\right] + \text{h.c.}
\end{aligned} \tag{3.10}$$

This gives the following mass matrices:

$$[M_\psi]_{pr} = \frac{v_T}{\sqrt{2}}\left[(Y_\psi)_{pr} - \frac{v_T^2}{2\Lambda^2}\,\underset{pr}{C_{\psi H}} - \frac{v_T^4}{4\Lambda^4}\,\underset{pr}{C_{\chi\psi H^5}}\right]. \tag{3.11}$$

The SMEFT neutrino mass terms are

$$\mathcal{L}_{\text{SMEFT}}^{\text{Neutrino mass}} = \frac{{v_T}^2}{2\Lambda}\left[\underset{pr}{C_5} + \frac{{v_T}^2}{2\Lambda^2} \underset{pr}{C_{l^2H^4}}\right]\overline{\nu}_{Lp}^T C \nu_{Lr} + \text{h.c.}\,. \tag{3.12}$$

This gives the following mass matrices:

$$[M_\nu]_{pr} = -\frac{{v_T}^2}{\Lambda}\left[\underset{pr}{C_5} + \frac{{v_T}^2}{2\Lambda^2} \underset{pr}{C_{l^2H^4}}\right]. \tag{3.13}$$

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3.2.2 Yukawa couplings

The SM, dimension-6 and dimension-8 SMEFT Yukawa coupling terms for charged leptons and quarks are

$$\begin{aligned}
\mathcal{L}^{\text{Yukawa}}_{\text{SM}} &= -\frac{(1+c_{H,\text{kin}})}{\sqrt{2}}\Big[(Y_e)_{pr}\overline{e}_{Lp}e_{Rr}h + (Y_u)_{pr}\overline{u}_{Lp}u_{Rr}h + (Y_d)_{pr}\overline{d}_{Lp}d_{Rr}h\Big] + \text{h.c.},\\
\mathcal{L}^{\text{Yukawa}}_{\text{SMEFT},6} &= \frac{3(1+c_{H,\text{kin}})v_T^2}{2\sqrt{2}\,\Lambda^2}\left[\underset{pr}{C_{eH}}\overline{e}_{Lp}e_{Rr}h + \underset{pr}{C_{uH}}\overline{u}_{Lp}u_{Rr}h + \underset{pr}{C_{dH}}\overline{d}_{Lp}d_{Rr}h\right] + \text{h.c.}, \quad (3.14)\\
\mathcal{L}^{\text{Yukawa}}_{\text{SMEFT},8} &= \frac{5(1+c_{H,\text{kin}})v_T^4}{4\sqrt{2}\,\Lambda^4}\left[\underset{pr}{C_{leH^5}}\overline{e}_{Lp}e_{Rr}h + \underset{pr}{C_{quH^5}}\overline{u}_{Lp}u_{Rr}h + \underset{pr}{C_{qdH^5}}\overline{d}_{Lp}d_{Rr}h\right] + \text{h.c.}
\end{aligned}$$

This gives the following Yukawa couplings (up to dimension 8):

$$(Y_\psi)^{\text{eff}}_{pr} = \frac{1+c_{H,\text{kin}}}{\sqrt{2}}\left[\frac{\sqrt{2}}{v_T}\,[M_\psi]_{pr} - \frac{v_T^2}{\Lambda^2}\,\underset{pr}{C_{\psi H}} - \frac{v_T^4}{\Lambda^4}\,\underset{pr}{C_{\chi\psi H^5}}\right]. \tag{3.15}$$

There are also momentum-dependent Yukawa couplings (i.e., ordinary Yukawa couplings with additional derivatives acting on the Higgs field) occurring at dimension 6 in SMEFT. However, as discussed in the introduction, MD contributions to the matching conditions are not included in the present work (though they are briefly discussed in section 4.3).

The SMEFT Yukawa coupling terms for neutrinos are

$$\mathcal{L}^{\text{Neutrino Yukawa}}_{\text{SMEFT}} = \frac{v_T}{\Lambda}(1+c_{H,\text{kin}})\left[\underset{pr}{C_5} + \frac{{v_T}^2}{\Lambda^2}\underset{pr}{C_{l^2H^4}}\right]h\,\overline{\nu}^T_{Lp}C\nu_{Lr} + \text{h.c.}\,. \tag{3.16}$$

This gives the following Yukawa couplings:

$$(Y_\nu)^{\text{eff}}_{pr} = (1+c_{H,\text{kin}})\left[\frac{1}{v_T}\,[M_\nu]_{pr} - \frac{{v_T}^3}{2\Lambda^3}\underset{pr}{C_{l^2H^4}}\right]. \tag{3.17}$$

These Yukawa couplings enter the matching conditions of certain four-fermion operators in LEFT.

3.3 Electroweak gauge boson masses & mixing and coupling constants

3.3.1 Kinetic terms

Including the SMEFT contributions up to dimension 8, the kinetic terms of the electroweak gauge bosons after symmetry breaking are

$$\mathcal{L}^{\text{Electroweak kin}}_{\text{SMEFT}} = -\frac{1}{4}\left\{\begin{aligned}
&\left[1 - \frac{2v_T^2}{\Lambda^2}\,C_{HW} - \frac{v_T^4}{\Lambda^4}\,C^{(1)}_{W^2H^4}\right]W^I_{\mu\nu}W^{I\mu\nu}\\
&- \frac{v_T^4}{\Lambda^4}\,C^{(3)}_{W^2H^4}\,W^3_{\mu\nu}W^{3\mu\nu}\\
&+ \left[1 - \frac{2v_T^2}{\Lambda^2}\,C_{HB} - \frac{v_T^4}{\Lambda^4}\,C^{(1)}_{B^2H^4}\right]B_{\mu\nu}B^{\mu\nu}\\
&+ \left[\frac{2v_T^2}{\Lambda^2}\,C_{HWB} + \frac{v_T^4}{\Lambda^4}\,C^{(1)}_{WBH^4}\right]W^3_{\mu\nu}B^{\mu\nu}
\end{aligned}\right\}. \tag{3.18}$$

Here there are two issues that must be resolved. First, the kinetic terms must be properly normalized. Second, the $W^3_{\mu\nu}B^{\mu\nu}$ mixing term must be removed.

Proper normalization of the kinetic terms can be achieved by redefining the coupling constants and the normalization of the gauge fields:

$$\begin{aligned}
\overline{g} &= \left[1 + \frac{v_T^2}{\Lambda^2} C_{HW} + \frac{v_T^4}{2\Lambda^4} C^{(1)}_{W^2H^4} + \frac{3v_T^4}{2\Lambda^4} \left[C_{HW}\right]^2\right] g\,, \\
\overline{g}' &= \left[1 + \frac{v_T^2}{\Lambda^2} C_{HB} + \frac{v_T^4}{2\Lambda^4} C^{(1)}_{B^2H^4} + \frac{3v_T^4}{2\Lambda^4} \left[C_{HB}\right]^2\right] g'\,, \qquad (3.19) \\
W^I_\mu &= \left[1 + \frac{v_T^2}{\Lambda^2} C_{HW} + \frac{v_T^4}{2\Lambda^4} C^{(1)}_{W^2H^4} + \frac{3v_T^4}{2\Lambda^4} \left[C_{HW}\right]^2\right] \mathcal{W}^I_\mu\,, \\
B_\mu &= \left[1 + \frac{v_T^2}{\Lambda^2} C_{HB} + \frac{v_T^4}{2\Lambda^4} C^{(1)}_{B^2H^4} + \frac{3v_T^4}{2\Lambda^4} \left[C_{HB}\right]^2\right] \mathcal{B}_\mu\,. \qquad (3.20)
\end{aligned}$$

At this stage, there is still a $\mathcal{W}^3_{\mu\nu}\mathcal{B}^{\mu\nu}$ mixing term, as well as a separate $W^3_{\mu\nu}W^{3\mu\nu}$ term. These can both be removed by defining

$$\begin{bmatrix} \mathcal{W}^3_\mu \\ \mathcal{B}_\mu \end{bmatrix} = \begin{bmatrix} 1 + \frac{{v_T}^4}{2\Lambda^4} C^{(3)}_{W^2H^4} + \frac{3{v_T}^4}{8\Lambda^4}\left[C_{HWB}\right]^2 & -\frac{{v_T}^2}{2\Lambda^2}\begin{pmatrix} C_{HWB} + \frac{{v_T}^2}{2\Lambda^2} C^{(1)}_{WBH^4} \\ + \frac{{v_T}^2}{\Lambda^2} C_{HWB}C_{HW} \\ + \frac{{v_T}^2}{\Lambda^2} C_{HWB}C_{HB} \end{pmatrix} \\ -\frac{{v_T}^2}{2\Lambda^2}\begin{pmatrix} C_{HWB} + \frac{{v_T}^2}{2\Lambda^2} C^{(1)}_{WBH^4} \\ + \frac{{v_T}^2}{\Lambda^2} C_{HWB}C_{HW} \\ + \frac{{v_T}^2}{\Lambda^2} C_{HWB}C_{HB} \end{pmatrix} & 1 + \frac{3{v_T}^4}{8\Lambda^4}\left[C_{HWB}\right]^2 \end{bmatrix} \begin{bmatrix} \overline{\mathcal{W}}^3_\mu \\ \overline{\mathcal{B}}_\mu \end{bmatrix}. \qquad (3.21)$$

With this, we have

$$\mathcal{L}^{\text{Electroweak kin}}_{\text{SMEFT}} = -\frac{1}{2}\mathcal{W}^+_{\mu\nu}\mathcal{W}^{-\mu\nu} - \frac{1}{4}\overline{\mathcal{W}}^3_{\mu\nu}\overline{\mathcal{W}}^{3\mu\nu} - \frac{1}{4}\overline{\mathcal{B}}_{\mu\nu}\overline{\mathcal{B}}^{\mu\nu}\,, \qquad (3.22)$$

where $\mathcal{W}^\pm_{\mu\nu} \equiv \partial_\mu \mathcal{W}^\pm_\nu - \partial_\nu \mathcal{W}^\pm_\mu$, $\mathcal{W}^\pm_\mu \equiv \frac{1}{\sqrt{2}}(\mathcal{W}^1_\mu \mp i\mathcal{W}^2_\mu)$, $\overline{\mathcal{W}}^3_{\mu\nu} \equiv \partial_\mu \overline{\mathcal{W}}^3_\nu - \partial_\nu \overline{\mathcal{W}}^3_\mu$, $\overline{\mathcal{B}}_{\mu\nu} \equiv \partial_\mu \overline{\mathcal{B}}_\nu - \partial_\nu \overline{\mathcal{B}}_\mu$, and we have dropped the cubic and quartic self-coupling terms of the gauge bosons.

Note that we still have the freedom to perform the following rotation:

$$\begin{bmatrix} \overline{\mathcal{W}}^3_\mu \\ \overline{\mathcal{B}}_\mu \end{bmatrix} = \begin{bmatrix} \cos\overline{\theta}_W & \sin\overline{\theta}_W \\ -\sin\overline{\theta}_W & \cos\overline{\theta}_W \end{bmatrix} \begin{bmatrix} \mathcal{Z}_\mu \\ \mathcal{A}_\mu \end{bmatrix}, \qquad (3.23)$$

where $\mathcal{Z}_\mu$ and $\mathcal{A}_\mu$ are the physical Z-boson and photon fields. In terms of these fields, we have

$$\mathcal{L}^{\text{Electroweak kin}}_{\text{SMEFT}} = -\frac{1}{2}\mathcal{W}^+_{\mu\nu}\mathcal{W}^{-\mu\nu} - \frac{1}{4}\mathcal{Z}_{\mu\nu}\mathcal{Z}^{\mu\nu} - \frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}\,. \qquad (3.24)$$

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For completeness, we also present the results for gluons. Including the SMEFT contributions up to dimension 8, the gluon kinetic term is

$$\mathcal{L}_{\text{SMEFT}}^{\text{Gluons kin}} = -\frac{1}{4}\left[1 - \frac{2v_T^2}{\Lambda^2}\, C_{HG} - \frac{v_T^4}{\Lambda^4}\, C_{G^2H^4}^{(1)}\right] G_{\mu\nu}^A G^{A\mu\nu}\,. \tag{3.25}$$

In order to properly normalize this kinetic term, we make redefinitions similar to those in eqs. (3.19) and (3.20):

$$\bar{g}_s = \left[1 + \frac{v_T^2}{\Lambda^2}\, C_{HG} + \frac{v_T^4}{2\Lambda^4}\, C_{G^2H^4}^{(1)} + \frac{3v_T^4}{2\Lambda^4}\, [C_{HG}]^2\right] g_s\,, \tag{3.26}$$

$$G_\mu^A = \left[1 + \frac{v_T^2}{\Lambda^2}\, C_{HG} + \frac{v_T^4}{2\Lambda^4}\, C_{G^2H^4}^{(1)} + \frac{3v_T^4}{2\Lambda^4}\, [C_{HG}]^2\right] \mathcal{G}_\mu^A\,. \tag{3.27}$$

3.3.2 Mass terms

The SMEFT contributions up to dimension 8 to the mass terms of the electroweak gauge bosons after symmetry breaking are

$$\mathcal{L}_{\text{SMEFT}}^{\text{Electroweak mass}} = \frac{v_T^2}{8}\left\{\begin{array}{l} \left[1 + \frac{v_T^4}{4\Lambda^4}\left(C_{H^6}^{(1)} - C_{H^6}^{(2)}\right)\right] g^2 (W_\mu^1 W^{1\mu} + W_\mu^2 W^{2\mu}) \\ \left[\begin{array}{l} 1 + \frac{v_T^2}{2\Lambda^2}\, C_{HD} \\ + \frac{v_T^4}{4\Lambda^4}\left(C_{H^6}^{(1)} + C_{H^6}^{(2)}\right)\end{array}\right] (gW_\mu^3 - g' B_\mu)(gW^{3\mu} - g' B^\mu) \end{array}\right\}. \tag{3.28}$$

We can write W_μ^I and B_μ in terms of $\mathcal{W}_\mu^\pm$, $\mathcal{Z}_\mu$ and $\mathcal{A}_\mu$ using the transformations described in section 3.3.1. The mixing angle $\bar{\theta}_W$ of eq. (3.23) satisfies

$$\begin{aligned} \cos\bar{\theta}_W &= \frac{1}{\sqrt{\bar{g}^2+\bar{g}'^2}}\left[\begin{array}{l} \bar{g} + \frac{\bar{g}v_T^4}{8\Lambda^4}\frac{(6\bar{g}^2\bar{g}'^2 - \bar{g}^4 - 5\bar{g}'^4)}{(\bar{g}^2+\bar{g}'^2)^2}[C_{HWB}]^2 + \frac{v_T^4}{2\Lambda^4}\frac{\bar{g}\bar{g}'^2}{\bar{g}^2+\bar{g}'^2}C_{W^2H^4}^{(3)} \\ -\frac{\bar{g}'v_T^2}{2\Lambda^2}\frac{\bar{g}^2-\bar{g}'^2}{\bar{g}^2+\bar{g}'^2}\left(C_{HWB} + \frac{v_T^2}{2\Lambda^2}C_{WBH^4}^{(1)} + \frac{v_T^2}{\Lambda^2}C_{HWB}[C_{HW}+C_{HB}]\right)\end{array}\right] \\ \sin\bar{\theta}_W &= \frac{1}{\sqrt{\bar{g}^2+\bar{g}'^2}}\left[\begin{array}{l} \bar{g}' + \frac{\bar{g}'v_T^4}{8\Lambda^4}\frac{(6\bar{g}^2\bar{g}'^2 - \bar{g}'^4 - 5\bar{g}^4)}{(\bar{g}^2+\bar{g}'^2)^2}[C_{HWB}]^2 - \frac{v_T^4}{2\Lambda^4}\frac{\bar{g}^2\bar{g}'}{\bar{g}^2+\bar{g}'^2}C_{W^2H^4}^{(3)} \\ +\frac{\bar{g}v_T^2}{2\Lambda^2}\frac{\bar{g}^2-\bar{g}'^2}{\bar{g}^2+\bar{g}'^2}\left(C_{HWB} + \frac{v_T^2}{2\Lambda^2}C_{WBH^4}^{(1)} + \frac{v_T^2}{\Lambda^2}C_{HWB}[C_{HW}+C_{HB}]\right)\end{array}\right] \end{aligned} \tag{3.29}$$

up to dimension 8.

Note that, while in the SM we have $\sin\theta_W = g'/\sqrt{g^2+g'^2}$ and $\cos\theta_W = g/\sqrt{g^2+g'^2}$, these relations no longer hold in the presence of SMEFT operators. Similarly, in the SM, $e = gg'/\sqrt{g^2+g'^2}$. Including SMEFT operators, this becomes

$$\bar{e} = \frac{\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}\left[\begin{array}{l} 1 - \frac{\bar{g}\bar{g}'v_T^2 C_{HWB}}{(\bar{g}^2+\bar{g}'^2)\Lambda^2} - \frac{\bar{g}\bar{g}'v_T^4 C_{WBH^4}^{(1)}}{2(\bar{g}^2+\bar{g}'^2)\Lambda^4} + \frac{\bar{g}'^2 v_T^4 C_{W^2H^4}^{(3)}}{2(\bar{g}^2+\bar{g}'^2)\Lambda^4} \\ -\frac{\bar{g}\bar{g}'v_T^4 C_{HWB}(C_{HW}+C_{HB})}{(\bar{g}^2+\bar{g}'^2)\Lambda^4} + \frac{3\bar{g}^2\bar{g}'^2 v_T^4 [C_{HWB}]^2}{2(\bar{g}^2+\bar{g}'^2)^2\Lambda^4}\end{array}\right]. \tag{3.30}$$

The masses of the W and Z are given by

$$M_W^2 = \frac{\bar{g}^2 v_T^2}{4}\left[1+\frac{v_T^4}{4\Lambda^4}\left(C_{H^6}^{(1)}-C_{H^6}^{(2)}\right)\right], \tag{3.31}$$

$$M_Z^2 = \frac{\bar{g}_Z{}^2 v_T^2}{4}\left[1+\frac{v_T^2}{2\Lambda^2}C_{HD}+\frac{v_T^4}{4\Lambda^4}\left(C_{H^6}^{(1)}+C_{H^6}^{(2)}\right)\right], \tag{3.32}$$

where

$$\bar{g}_Z = \sqrt{\bar{g}^2+\bar{g}'^2}\left[1+\frac{\bar{g}\bar{g}' v_T^2 C_{HWB}}{(\bar{g}^2+\bar{g}'^2)\Lambda^2}+\frac{\bar{g}\bar{g}' v_T^4 C_{WBH^4}^{(1)}}{2(\bar{g}^2+\bar{g}'^2)\Lambda^4}+\frac{\bar{g}^2 v_T^4 C_{W^2H^4}^{(3)}}{2(\bar{g}^2+\bar{g}'^2)\Lambda^4}\right.$$
$$\left.+\frac{\bar{g}\bar{g}' v_T^4 C_{HWB}(C_{HW}+C_{HB})}{(\bar{g}^2+\bar{g}'^2)\Lambda^4}+\left(1-\frac{\bar{g}^2\bar{g}'^2}{(\bar{g}^2+\bar{g}'^2)^2}\right)\frac{[C_{HWB}]^2}{2\Lambda^4}\right]. \tag{3.33}$$

In the SM, the "charge" to which the Z^0 couples is $I_{3L}-Q_{em}\sin^2\theta_W$. When one adds SMEFT operators up to dimension 6, the mixing angle is changed, $\theta_W \to \bar{\theta}_W$, but the Z^0 coupling still has the same form: it couples to $I_{3L}-Q_{em}\sin^2\bar{\theta}_W$ [8]. However, when SMEFT operators up to dimension 8 are included, this no longer holds. Instead, the Z^0 couples to $I_{3L}-Q_{em}\sin^2\bar{\theta}_Z$, where

$$\sin^2\bar{\theta}_Z = \sin^2\bar{\theta}_W + \frac{v_T^4}{4\Lambda^4}[C_{HWB}]^2(\sin^2\bar{\theta}_W-\cos^2\bar{\theta}_W)\,. \tag{3.34}$$

(This was also noted in ref. [20].)

3.4 Couplings of electroweak gauge bosons to fermions

As shown in eq. (3.24), the physical electroweak gauge bosons are $\mathcal{A}_\mu$, $\mathcal{W}_\mu^\pm$ and $\mathcal{Z}_\mu$. Their effective couplings to fermions, as well as those of the gluon $\mathcal{G}_\mu^A$, take the following form:

$$\mathcal{L} = -\bar{g}_s\mathcal{G}_\mu^A j_{\mathcal{G}}^{A\mu} - \bar{e}\mathcal{A}_\mu j_{\mathcal{A}}^\mu - \frac{\bar{g}}{\sqrt{2}}\{\mathcal{W}_\mu^+ j_{\mathcal{W}}^{+\mu}+\mathcal{W}_\mu^- j_{\mathcal{W}}^{-\mu}\} - \bar{g}_Z\mathcal{Z}_\mu j_{\mathcal{Z}}^\mu\,, \tag{3.35}$$

in which the corresponding currents are

$$\begin{aligned}
j_{\mathcal{G}}^{A\mu} &= \overline{u}_{Lp}\gamma^\mu T^A u_{Lr}+\overline{d}_{Lp}\gamma^\mu T^A d_{Lr}+\overline{u}_{Rp}\gamma^\mu T^A u_{Rr}+\overline{d}_{Rp}\gamma^\mu T^A d_{Rr}\,,\\
j_{\mathcal{A}}^\mu &= -\overline{e}_{Lp}\gamma^\mu e_{Lr}+\frac{2}{3}\overline{u}_{Lp}\gamma^\mu u_{Lr}-\frac{1}{3}\overline{d}_{Lp}\gamma^\mu d_{Lr}-\overline{e}_{Rp}\gamma^\mu e_{Rr}+\frac{2}{3}\overline{u}_{Rp}\gamma^\mu u_{Rr}-\frac{1}{3}\overline{d}_{Rp}\gamma^\mu d_{Rr}\,,\\
j_{\mathcal{W}}^{+\mu} &= [W_l]_{pr}^{\rm eff}\overline{\nu}_{Lp}\gamma^\mu e_{Lr}+[W_q]_{pr}^{\rm eff}\overline{u}_{Lp}\gamma^\mu d_{Lr}+[W_R]_{pr}^{\rm eff}\overline{u}_{Rp}\gamma^\mu d_{Rr}+[W_l^{\not{L}}]_{pr}^{\rm eff}(\nu_{Lp}^T C\gamma^\mu e_{Rr})\,,\\
j_{\mathcal{W}}^{-\mu} &= [W_l]_{rp}^{\rm eff*}\overline{e}_{Lp}\gamma^\mu \nu_{Lr}+[W_q]_{rp}^{\rm eff*}\overline{d}_{Lp}\gamma^\mu u_{Lr}+[W_R]_{rp}^{\rm eff*}\overline{d}_{Rp}\gamma^\mu u_{Rr}+[W_l^{\not{L}}]_{rp}^{\rm eff*}(\overline{\nu}_{Lp}\gamma^\mu C\overline{e}_{Rr}^T)\,,\\
j_{\mathcal{Z}}^\mu &= \begin{bmatrix}[Z_{\nu_L}]_{pr}^{\rm eff}\overline{\nu}_{Lp}\gamma^\mu\nu_{Lr}+[Z_{e_L}]_{pr}^{\rm eff}\overline{e}_{Lp}\gamma^\mu e_{Lr}+[Z_{u_L}]_{pr}^{\rm eff}\overline{u}_{Lp}\gamma^\mu u_{Lr}+[Z_{d_L}]_{pr}^{\rm eff}\overline{d}_{Lp}\gamma^\mu d_{Lr}\\ +[Z_{e_R}]_{pr}^{\rm eff}\overline{e}_{Rp}\gamma^\mu e_{Rr}+[Z_{u_R}]_{pr}^{\rm eff}\overline{u}_{Rp}\gamma^\mu u_{Rr}+[Z_{d_R}]_{pr}^{\rm eff}\overline{d}_{Rp}\gamma^\mu d_{Rr}\end{bmatrix}.
\end{aligned} \tag{3.36}$$

Since $\mathrm{SU}(3)_C\times\mathrm{U}(1)_{em}$ remains unbroken, the currents involving gluons and photons are fully determined by QCD and QED. This is not the case for the $\mathcal{W}_\mu^\pm$ and $\mathcal{Z}_\mu$ gauge bosons: the fermion currents to which the $\mathcal{W}^{\pm}_{\mu}$ and $\mathcal{Z}_{\mu}$ couple are given by the following (up to dimension 8):

$$\begin{aligned}
[W_l]^{\text{eff}}_{pr} &= \delta_{pr} + \frac{v_T^2}{\Lambda^2}\, C^{(3)}_{\substack{Hl\\pr}} + \frac{v_T^4}{2\Lambda^4}\left(C^{(2)}_{\substack{l^2H^4D\\pr}} - iC^{(3)}_{\substack{l^2H^4D\\pr}} \right) ,\\
[W_q]^{\text{eff}}_{pr} &= \delta_{pr} + \frac{v_T^2}{\Lambda^2}\, C^{(3)}_{\substack{Hq\\pr}} + \frac{v_T^4}{2\Lambda^4}\left(C^{(2)}_{\substack{q^2H^4D\\pr}} - iC^{(3)}_{\substack{q^2H^4D\\pr}} \right) ,\\
[W_R]^{\text{eff}}_{pr} &= \frac{v_T^2}{2\Lambda^2}\, C^{(3)}_{\substack{Hud\\pr}} + \frac{v_T^4}{4\Lambda^4}\, C_{\substack{udH^4D\\pr}} ,\\
[W_l^{\not L}]^{\text{eff}}_{pr} &= -\frac{{v_T}^3}{2\sqrt{2}\,\Lambda^3} C_{\substack{leH^3D\\pr}} ,\\
[Z_{\nu_L}]^{\text{eff}}_{pr} &= \frac{1}{2}\delta_{pr} - \frac{v_T^2}{2\Lambda^2}\left(C^{(1)}_{\substack{Hl\\pr}} - C^{(3)}_{\substack{Hl\\pr}} \right) - \frac{v_T^4}{4\Lambda^4}\left(C^{(1)}_{\substack{l^2H^4D\\pr}} - 2C^{(2)}_{\substack{l^2H^4D\\pr}} \right) ,\\
[Z_{e_L}]^{\text{eff}}_{pr} &= \frac{1}{2} g_L^e \delta_{pr} - \frac{v_T^2}{2\Lambda^2}\left(C^{(1)}_{\substack{Hl\\pr}} + C^{(3)}_{\substack{Hl\\pr}} \right) - \frac{v_T^4}{4\Lambda^4}\left(C^{(1)}_{\substack{l^2H^4D\\pr}} + 2C^{(2)}_{\substack{l^2H^4D\\pr}} \right) ,\\
[Z_{u_L}]^{\text{eff}}_{pr} &= \frac{1}{2} g_L^u \delta_{pr} - \frac{v_T^2}{2\Lambda^2}\left(C^{(1)}_{\substack{Hq\\pr}} - C^{(3)}_{\substack{Hq\\pr}} \right) - \frac{v_T^4}{4\Lambda^4}\left(C^{(1)}_{\substack{q^2H^4D\\pr}} - 2C^{(2)}_{\substack{q^2H^4D\\pr}} \right) ,\\
[Z_{d_L}]^{\text{eff}}_{pr} &= \frac{1}{2} g_L^d \delta_{pr} - \frac{v_T^2}{2\Lambda^2}\left(C^{(1)}_{\substack{Hq\\pr}} + C^{(3)}_{\substack{Hq\\pr}} \right) - \frac{v_T^4}{4\Lambda^4}\left(C^{(1)}_{\substack{q^2H^4D\\pr}} + 2C^{(2)}_{\substack{q^2H^4D\\pr}} \right) ,\\
[Z_{e_R}]^{\text{eff}}_{pr} &= \frac{1}{2} g_R^e \delta_{pr} - \frac{v_T^2}{2\Lambda^2}\, C_{\substack{He\\pr}} - \frac{v_T^4}{4\Lambda^4}\, C^{(1)}_{\substack{e^2H^4D\\pr}} ,\\
[Z_{u_R}]^{\text{eff}}_{pr} &= \frac{1}{2} g_R^u \delta_{pr} - \frac{v^2}{2\Lambda^2}\, C_{\substack{Hu\\pr}} - \frac{v_T^4}{4\Lambda^4}\, C^{(1)}_{\substack{u^2H^4D\\pr}} ,\\
[Z_{d_R}]^{\text{eff}}_{pr} &= \frac{1}{2} g_R^d \delta_{pr} - \frac{v^2}{2\Lambda^2}\, C_{\substack{Hd\\pr}} - \frac{v_T^4}{4\Lambda^4}\, C^{(1)}_{\substack{d^2H^4D\\pr}} .
\end{aligned} \tag{3.37}$$

Here, we have defined $g_L^e \equiv -1 + 2\sin^2\overline{\theta}_Z$, $g_L^u \equiv 1 - \frac{4}{3}\sin^2\overline{\theta}_Z$, $g_L^d \equiv -1 + \frac{2}{3}\sin^2\overline{\theta}_Z$, $g_R^e \equiv 2\sin^2\overline{\theta}_Z$, $g_R^u \equiv -\frac{4}{3}\sin^2\overline{\theta}_Z$, and $g_R^d \equiv \frac{2}{3}\sin^2\overline{\theta}_Z$, where $\sin^2\overline{\theta}_Z$ is defined in eq. (3.34).

4 Matching conditions

There are four classes of LEFT operators up to dimension 6: (i) four-fermion operators, (ii) magnetic dipole moment operators, (iii) three-gluon operators, and (iv) neutrino mass terms. The matching conditions for operators that conserve both B and L involve only even-dimension SMEFT operators, and are given up to dimension 6 in JMS. For operators that violate B and/or L, the matching conditions involve either even- or odd-dimension SMEFT operators, depending on the operator; these are given to dimension 6 or dimension 5 in JMS.

In general, there are three types of contributions to the matching conditions of LEFT operators to SMEFT operators up to dimension-8: (a) *direct* contributions, (b) *indirect*

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contributions, and (c) *momentum-dependent* contributions. The classes (ii)-(iv) of LEFT operators receive only direct contributions. In the following subsections, we describe in detail the direct and indirect contributions to four-fermion operators and their origin within the SMEFT up to the dimension-8 level. We also briefly summarize the sources of the MD contributions.

4.1 Direct contributions

The dimension-6 SMEFT direct contribution is the LEFT operator itself, in which all left- and right-handed particles are replaced by the left-handed $SU(2)_L$ doublets and right-handed $SU(2)_L$ singlets to which they respectively belong. The dimension-8 contributions involve the dimension-6 SMEFT operator multiplied by a pair of Higgs fields. When the Higgs gets a vev, this generates the four-fermion LEFT operator.

The details of the computation are best illustrated with an example. Consider the LEFT operator

$$\mathcal{O}^{V,LL}_{\underset{prst}{\nu\nu}} \equiv (\overline{\nu}_{Lp}\gamma_\mu\nu_{Lr})(\overline{\nu}_{Ls}\gamma^\mu\nu_{Lt})\,. \tag{4.1}$$

It is generated by the dimension-6 SMEFT operator $Q_{\underset{prst}{ll}} \equiv (\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$. This can be seen by separating the SMEFT operator into components:

$$\frac{1}{\Lambda^2}C_{\underset{prst}{ll}}Q_{\underset{prst}{ll}} \to \frac{1}{\Lambda^2}C_{\underset{prst}{ll}}\begin{bmatrix}(\overline{\nu}_{Lp}\gamma_\mu\nu_{Lr})(\overline{\nu}_{Ls}\gamma^\mu\nu_{Lt}) + (\overline{\nu}_{Lp}\gamma_\mu\nu_{Lr})(\overline{e}_{Ls}\gamma^\mu e_{Lt}) \\ + (\overline{e}_{Lp}\gamma_\mu e_{Lr})(\overline{\nu}_{Ls}\gamma^\mu\nu_{Lt}) + (\overline{e}_{Lp}\gamma_\mu e_{Lr})(\overline{e}_{Ls}\gamma^\mu e_{Lt})\end{bmatrix}. \tag{4.2}$$

The first term is $\mathcal{O}^{V,LL}_{\underset{prst}{\nu\nu}}$.

One dimension-8 SMEFT operator that is among the matching conditions is $Q^{(1)}_{\underset{prst}{l^4H^2}} \equiv (\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)(H^\dagger H)$. Because the $SU(2)_L$ doublets l and H are involved, there are two additional dimension-8 SMEFT operators that must be included: $Q^{(2)}_{\underset{prst}{l^4H^2}} \equiv (\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu\tau^I l_t)(H^\dagger\tau^I H)$ and $Q^{(2)}_{\underset{stpr}{l^4H^2}} \equiv (\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{l}_s\gamma^\mu l_t)(H^\dagger\tau^I H)$. When the Higgs gets a vev, these three operators can also generate $\mathcal{O}^{V,LL}_{\underset{prst}{\nu\nu}}$:

$$\begin{aligned}
\frac{1}{\Lambda^4}C^{(1)}_{\underset{prst}{l^4H^2}}Q^{(1)}_{\underset{prst}{l^4H^2}} &\to \frac{v_T^2}{2\Lambda^4}C^{(1)}_{\underset{prst}{l^4H^2}}\begin{bmatrix}(\overline{\nu}_{Lp}\gamma_\mu\nu_{Lr})(\overline{\nu}_{Ls}\gamma^\mu\nu_{Lt}) + (\overline{\nu}_{Lp}\gamma_\mu\nu_{Lr})(\overline{e}_{Ls}\gamma^\mu e_{Lt}) \\ + (\overline{e}_{Lp}\gamma_\mu e_{Lr})(\overline{\nu}_{Ls}\gamma^\mu\nu_{Lt}) + (\overline{e}_{Lp}\gamma_\mu e_{Lr})(\overline{e}_{Ls}\gamma^\mu e_{Lt})\end{bmatrix}, \\
\frac{1}{\Lambda^4}C^{(2)}_{\underset{prst}{l^4H^2}}Q^{(2)}_{\underset{prst}{l^4H^2}} &\to \frac{v_T^2}{2\Lambda^4}C^{(2)}_{\underset{prst}{l^4H^2}}\begin{bmatrix}-(\overline{\nu}_{Lp}\gamma_\mu\nu_{Lr})(\overline{\nu}_{Ls}\gamma^\mu\nu_{Lt}) + (\overline{\nu}_{Lp}\gamma_\mu\nu_{Lr})(\overline{e}_{Ls}\gamma^\mu e_{Lt}) \\ - (\overline{e}_{Lp}\gamma_\mu e_{Lr})(\overline{\nu}_{Ls}\gamma^\mu\nu_{Lt}) + (\overline{e}_{Lp}\gamma_\mu e_{Lr})(\overline{e}_{Ls}\gamma^\mu e_{Lt})\end{bmatrix}, \\
\frac{1}{\Lambda^4}C^{(2)}_{\underset{stpr}{l^4H^2}}Q^{(2)}_{\underset{stpr}{l^4H^2}} &\to \frac{v_T^2}{2\Lambda^4}C^{(2)}_{\underset{stpr}{l^4H^2}}\begin{bmatrix}-(\overline{\nu}_{Lp}\gamma_\mu\nu_{Lr})(\overline{\nu}_{Ls}\gamma^\mu\nu_{Lt}) - (\overline{\nu}_{Lp}\gamma_\mu\nu_{Lr})(\overline{e}_{Ls}\gamma^\mu e_{Lt}) \\ + (\overline{e}_{Lp}\gamma_\mu e_{Lr})(\overline{\nu}_{Ls}\gamma^\mu\nu_{Lt}) + (\overline{e}_{Lp}\gamma_\mu e_{Lr})(\overline{e}_{Ls}\gamma^\mu e_{Lt})\end{bmatrix}.
\end{aligned} \tag{4.3}$$

We therefore see that the direct contribution to the matching condition of the LEFT operator $\frac{1}{\Lambda^2}\mathcal{O}^{V,LL}_{\underset{prst}{\nu\nu}}$, up to dimension 8, is

$$\frac{1}{\Lambda^2}\left[C_{\underset{prst}{ll}} + \frac{v_T^2}{2\Lambda^2}\left(C^{(1)}_{\underset{prst}{l^4H^2}} - C^{(2)}_{\underset{prst}{l^4H^2}} - C^{(2)}_{\underset{stpr}{l^4H^2}}\right)\right]. \tag{4.4}$$

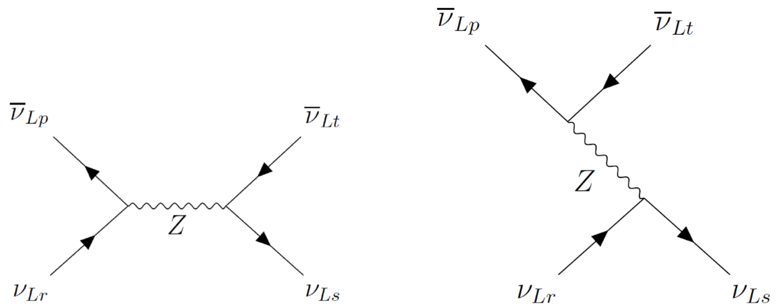

Figure 1. Z-exchange contributions to $\mathcal{O}_{\nu\nu}^{V,LL}$ with flavour indices $prst$.

The direct contributions to the matching conditions of the other LEFT four-fermion operators are calculated similarly.

4.2 Indirect contributions

A four-fermion operator can also be generated when a boson is exchanged between two fermion currents and this boson is integrated out. This produces an indirect contribution [e.g., see eq. (2.7)].

Consider once again the LEFT operator $\underset{prst}{\mathcal{O}_{\nu\nu}^{V,LL}}$ of eq. (4.1). The indirect contributions arise from the Z-exchange diagrams of figure 1, when the Z^0 is integrated out. We note that (i) there is a relative minus sign between the two diagrams, and (ii) when one Fierz transforms (see appendix C) the amplitude of the second diagram, one obtains the amplitude of the first diagram, but with an exchange of generation indices $r \leftrightarrow t$. The indirect contribution to the matching condition of this operator, up to dimension 8, is

$$-\frac{\overline{g}_Z^2}{4M_Z^2}\left([Z_{\nu_L}]^{\rm eff}_{pr}[Z_{\nu_L}]^{\rm eff}_{st}+[Z_{\nu_L}]^{\rm eff}_{pt}[Z_{\nu_L}]^{\rm eff}_{sr}\right)\,, \tag{4.5}$$

where $\overline{g}_Z$ and $[Z_{\nu_L}]^{\rm eff}_{pr}$ are defined in eqs. (3.33) and (3.37), respectively.

Another example is the LEFT operator $\underset{prst}{\mathcal{O}_{\nu e}^{V,LL}} \equiv (\overline{\nu}_{Lp}\gamma_\mu\nu_{Lr})(\overline{e}_{Ls}\gamma^\mu e_{Lt})$. Here the indirect contributions arise from the Z- and W-exchange diagrams of Fig, 2, when the heavy gauge bosons are integrated out. The indirect contribution to the matching condition of this operator, up to dimension 8, is

$$-\frac{\overline{g}_Z^2}{M_Z^2}[Z_{\nu_L}]^{\rm eff}_{pr}[Z_{e_L}]^{\rm eff}_{st}-\frac{\overline{g}^2}{2M_W^2}[W_l]^{\rm eff}_{pr}[W_l]^{\rm eff*}_{st}\,, \tag{4.6}$$

where $\overline{g}$ and $[W_l]^{\rm eff}_{pr}$ are defined in eqs. (3.19) and (3.37), respectively.

The indirect contributions from gauge-boson exchange to the matching conditions of the other LEFT four-fermion operators are calculated similarly. Most such operators can be generated via diagrams with the exchange of a Z^0. A small subset of these also involve W-exchange diagrams. And a few LEFT operators can be generated only via the exchange of a W.

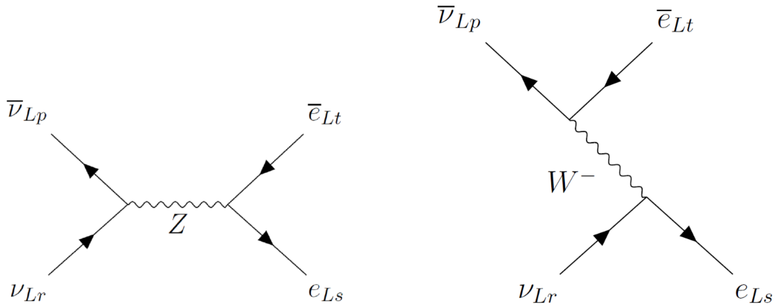

Figure 2. Z- and W-exchange contributions to $\mathcal{O}^{V,LL}_{\nu e}$ with flavour indices $prst$.

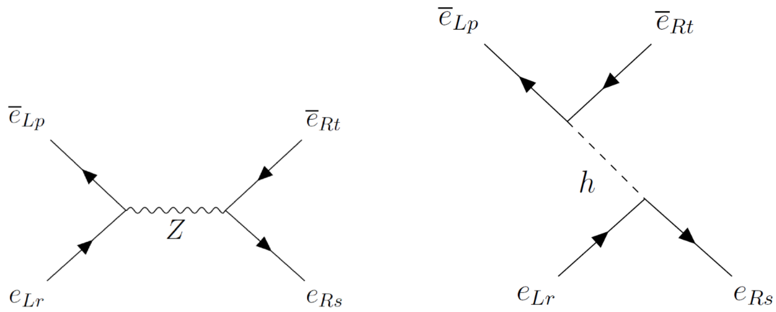

Figure 3. Z- and h-exchange contributions to $\mathcal{O}^{V,LL}_{\nu e}$ with flavour indices $prst$.

Finally, the matching conditions of certain LEFT operators receive indirect contributions from Higgs exchange. As an example, consider the operator $\mathcal{O}^{V,LR}_{\substack{ee\\prst}} \equiv (\overline{e}_{Lp}\gamma_\mu e_{Lr})(\overline{e}_{Rs}\gamma^\mu e_{Rt})$.

The indirect contributions come from the diagrams of figure 3. The Z- and h-exchange contributions are computed similarly to the previous examples. The indirect contribution to the matching condition is

$$-\frac{\overline{g_Z}^2}{M_Z{}^2}[Z_{e_L}]^{\text{eff}}_{pr}[Z_{e_R}]^{\text{eff}}_{st} - \frac{1}{2m_h{}^2}(Y_e)^{\text{eff}}_{pt}(Y_e)^{\text{eff}*}_{rs}\,. \tag{4.7}$$

The Yukawa coupling is [eq. (3.15), repeated for convenience]

$$(Y_e)^{\text{eff}}_{pr} = \frac{1+c_{H,\text{kin}}}{\sqrt{2}}\left[\frac{\sqrt{2}}{v_T}[M_e]_{pr} - \frac{v_T^2}{\Lambda^2}C_{\substack{eH\\pr}} - \frac{v_T^4}{\Lambda^4}C_{\substack{\chi eH^5\\pr}}\right].$$

The first term is $\sim m_e/v_T$ and is negligible. For this reason, JMS, which works only to dimension 6, argues that the h-exchange indirect contributions to the matching conditions are unimportant. However, when one works to dimension 8, there is a non-negligible contribution resulting from the square of the second term.

4.3 Momentum-dependent contributions

In the introduction, we noted that (i) the matching conditions can be separated into two types, momentum-independent and momentum-dependent, and (ii) in this paper, we focus

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only on the MI type. Indeed, in the above subsections, the direct and indirect contributions give rise to MI matching conditions. Still, it is a useful exercise to explore which types of SMEFT operators can produce MD matching conditions. The various sources of MD contributions are outlined below.

We first consider dimension-8 SMEFT operators. Those belonging to the $\psi^4 D^2$ class contribute directly to four-fermion operators and give rise to MD contributions that scale as p^2/Λ^4. There are also MD operators in the $\psi^2 H^4 D$ class. These contribute to four-fermion operators via Higgs exchange; the net effect scales as vp/Λ^4. However, the MI contributions scale as v^2/Λ^4. Since the momentum transfer p in low-energy processes is much smaller than v, one can safely neglect these types of MD contributions.

On the other hand, there are also MD contributions from dimension-6 SMEFT operators. Those in the $\psi^2 H^2 D$ and $\psi^2 X H$ classes contribute to four-fermion operators via the exchange of a Higgs boson or a W/Z boson, respectively. These contributions scale as $(m/v)(1/v^2)(pv/\Lambda^2) \sim (p^2/v^2)/\Lambda^2$ and $(1/v^2)(pv/\Lambda^2) \sim (p/v)/\Lambda^2$, respectively. Clearly, depending on the values of Λ and $p \sim m$, these can be comparable to the MI dimension-8 SMEFT contributions, which scale as v^2/Λ^4.

In addition, the second-order term in the expansion of the propagator can give rise to contributions of the same order, $(p^2/v^4)(v^2/\Lambda^2) \sim (p^2/v^2)/\Lambda^2$. In this case, the momentum dependence arises from the propagator, in contrast to the above contributions, where it is in the vertex. (Note that, with dimension-8 operators, this type of effect is suppressed since it scales as $(p^2/v^4)(v^2/\Lambda^4) \sim (p^2/v^2)/\Lambda^4$, which can be neglected.

4.4 Results

For all four-fermion LEFT operators, the MI matching conditions up to dimension 8 in SMEFT are determined using the techniques described above for computing the direct and indirect contributions. For the LEFT magnetic dipole moment operators, three-gluon operators and neutrino mass terms, the calculations are straightforward, as there are no indirect contributions. The matching conditions are given in the tables in appendix D.

In the literature, the matching conditions of LEFT operators to dimension-7 SMEFT operators have been calculated in ref. [32]. The results obtained there are in agreement with ours. The matching conditions of LEFT operators to dimension-8 SMEFT operators has only been performed in refs. [15, 16], where the focus was on LEFT operators that lead to lepton flavour violation. Our results agree with this analysis. Matching conditions to dimension-8 SMEFT operators have also been computed in ref. [25], but in the context of high-energy processes. Although LEFT operators were not involved, there is still some overlap, and we agree here as well. Finally, the contributions of dimension-8 SMEFT operators to the SM parameters, as described in section 3, was also examined in ref. [15], and we are in agreement.

5 Conclusions

The modern thinking is that the Standard Model is the leading part of an effective field theory, produced when the heavy new physics is integrated out. This EFT is usually

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assumed to be the SMEFT, which includes the Higgs boson. The SMEFT has been well-studied — all operators up to dimension 8 have been worked out.

When the heavy particles of the SM ($W^{\pm}$, Z^0, H, t) are also integrated out, one obtains the LEFT (low-energy EFT), applicable at scales $\ll M_W$. In order to establish how low-energy measurements are affected by the underlying NP, it is necessary to determine how the LEFT operators depend on the SMEFT operators (the matching conditions).

In ref. [8], Jenkins, Manohar and Stoffer (JMS) present a complete and non-redundant basis of LEFT operators up to dimension 6, and compute the matching to SMEFT operators up to dimension 6. However, if the low-energy observable in question is suppressed in the SM and/or is very precisely measured, this may not be sufficient. Indeed, it has been pointed out that dimension-8 SMEFT contributions may be important for electroweak precision data from LEP, lepton-flavour-violating processes, meson-antimeson mixing, and electric dipole moments.

In this paper, we extend the analysis of JMS: for all LEFT operators, we work out the complete tree-level momentum-independent matching conditions to SMEFT operators up to dimension 8. The momentum-dependent contributions will be presented elsewhere. There are direct contributions to these matching conditions for all LEFT operators, and four-fermion operators also receive indirect contributions due to the exchange of a $W^{\pm}$, Z^0 and/or H.

Should the analysis of a LEFT observable require information about dimension-8 SMEFT tree-level contributions, much of that information can be found here.

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A LEFT operators up to dimension 6

The following two tables are taken from ref. [8].

B SMEFT operators used in this paper

B.1 Even-dimensional operators

These tables list the dimension-6 [3] and dimension-8 [7] SMEFT operators that contribute to the matching conditions, separated into various categories.

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$\nu\nu$ + h.c.	
$\mathcal{O}_{\nu}$	$(\nu_{Lp}^T C \nu_{Lr})$

$(\nu\nu)X$ + h.c.	
$\mathcal{O}_{\nu\gamma}$	$(\nu_{Lp}^T C \sigma^{\mu\nu} \nu_{Lr}) F_{\mu\nu}$

$(\overline{L}R)X$ + h.c.	
$\mathcal{O}_{e\gamma}$	$\bar{e}_{Lp}\sigma^{\mu\nu} e_{Rr}\, F_{\mu\nu}$
$\mathcal{O}_{u\gamma}$	$\bar{u}_{Lp}\sigma^{\mu\nu} u_{Rr}\, F_{\mu\nu}$
$\mathcal{O}_{d\gamma}$	$\bar{d}_{Lp}\sigma^{\mu\nu} d_{Rr}\, F_{\mu\nu}$
$\mathcal{O}_{uG}$	$\bar{u}_{Lp}\sigma^{\mu\nu} T^A u_{Rr}\, G^A_{\mu\nu}$
$\mathcal{O}_{dG}$	$\bar{d}_{Lp}\sigma^{\mu\nu} T^A d_{Rr}\, G^A_{\mu\nu}$

X^3	
$\mathcal{O}_G$	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
$\mathcal{O}_{\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$

$(\overline{L}L)(\overline{L}L)$	
$\mathcal{O}_{\nu\nu}^{V,LL}$	$(\bar{\nu}_{Lp}\gamma^\mu \nu_{Lr})(\bar{\nu}_{Ls}\gamma_\mu \nu_{Lt})$
$\mathcal{O}_{ee}^{V,LL}$	$(\bar{e}_{Lp}\gamma^\mu e_{Lr})(\bar{e}_{Ls}\gamma_\mu e_{Lt})$
$\mathcal{O}_{\nu e}^{V,LL}$	$(\bar{\nu}_{Lp}\gamma^\mu \nu_{Lr})(\bar{e}_{Ls}\gamma_\mu e_{Lt})$
$\mathcal{O}_{\nu u}^{V,LL}$	$(\bar{\nu}_{Lp}\gamma^\mu \nu_{Lr})(\bar{u}_{Ls}\gamma_\mu u_{Lt})$
$\mathcal{O}_{\nu d}^{V,LL}$	$(\bar{\nu}_{Lp}\gamma^\mu \nu_{Lr})(\bar{d}_{Ls}\gamma_\mu d_{Lt})$
$\mathcal{O}_{eu}^{V,LL}$	$(\bar{e}_{Lp}\gamma^\mu e_{Lr})(\bar{u}_{Ls}\gamma_\mu u_{Lt})$
$\mathcal{O}_{ed}^{V,LL}$	$(\bar{e}_{Lp}\gamma^\mu e_{Lr})(\bar{d}_{Ls}\gamma_\mu d_{Lt})$
$\mathcal{O}_{\nu edu}^{V,LL}$	$(\bar{\nu}_{Lp}\gamma^\mu e_{Lr})(\bar{d}_{Ls}\gamma_\mu u_{Lt})$ + h.c.
$\mathcal{O}_{uu}^{V,LL}$	$(\bar{u}_{Lp}\gamma^\mu u_{Lr})(\bar{u}_{Ls}\gamma_\mu u_{Lt})$
$\mathcal{O}_{dd}^{V,LL}$	$(\bar{d}_{Lp}\gamma^\mu d_{Lr})(\bar{d}_{Ls}\gamma_\mu d_{Lt})$
$\mathcal{O}_{ud}^{V1,LL}$	$(\bar{u}_{Lp}\gamma^\mu u_{Lr})(\bar{d}_{Ls}\gamma_\mu d_{Lt})$
$\mathcal{O}_{ud}^{V8,LL}$	$(\bar{u}_{Lp}\gamma^\mu T^A u_{Lr})(\bar{d}_{Ls}\gamma_\mu T^A d_{Lt})$

$(\overline{R}R)(\overline{R}R)$	
$\mathcal{O}_{ee}^{V,RR}$	$(\bar{e}_{Rp}\gamma^\mu e_{Rr})(\bar{e}_{Rs}\gamma_\mu e_{Rt})$
$\mathcal{O}_{eu}^{V,RR}$	$(\bar{e}_{Rp}\gamma^\mu e_{Rr})(\bar{u}_{Rs}\gamma_\mu u_{Rt})$
$\mathcal{O}_{ed}^{V,RR}$	$(\bar{e}_{Rp}\gamma^\mu e_{Rr})(\bar{d}_{Rs}\gamma_\mu d_{Rt})$
$\mathcal{O}_{uu}^{V,RR}$	$(\bar{u}_{Rp}\gamma^\mu u_{Rr})(\bar{u}_{Rs}\gamma_\mu u_{Rt})$
$\mathcal{O}_{dd}^{V,RR}$	$(\bar{d}_{Rp}\gamma^\mu d_{Rr})(\bar{d}_{Rs}\gamma_\mu d_{Rt})$
$\mathcal{O}_{ud}^{V1,RR}$	$(\bar{u}_{Rp}\gamma^\mu u_{Rr})(\bar{d}_{Rs}\gamma_\mu d_{Rt})$
$\mathcal{O}_{ud}^{V8,RR}$	$(\bar{u}_{Rp}\gamma^\mu T^A u_{Rr})(\bar{d}_{Rs}\gamma_\mu T^A d_{Rt})$

$(\overline{L}L)(\overline{R}R)$	
$\mathcal{O}_{\nu e}^{V,LR}$	$(\bar{\nu}_{Lp}\gamma^\mu \nu_{Lr})(\bar{e}_{Rs}\gamma_\mu e_{Rt})$
$\mathcal{O}_{ee}^{V,LR}$	$(\bar{e}_{Lp}\gamma^\mu e_{Lr})(\bar{e}_{Rs}\gamma_\mu e_{Rt})$
$\mathcal{O}_{\nu u}^{V,LR}$	$(\bar{\nu}_{Lp}\gamma^\mu \nu_{Lr})(\bar{u}_{Rs}\gamma_\mu u_{Rt})$
$\mathcal{O}_{\nu d}^{V,LR}$	$(\bar{\nu}_{Lp}\gamma^\mu \nu_{Lr})(\bar{d}_{Rs}\gamma_\mu d_{Rt})$
$\mathcal{O}_{eu}^{V,LR}$	$(\bar{e}_{Lp}\gamma^\mu e_{Lr})(\bar{u}_{Rs}\gamma_\mu u_{Rt})$
$\mathcal{O}_{ed}^{V,LR}$	$(\bar{e}_{Lp}\gamma^\mu e_{Lr})(\bar{d}_{Rs}\gamma_\mu d_{Rt})$
$\mathcal{O}_{ue}^{V,LR}$	$(\bar{u}_{Lp}\gamma^\mu u_{Lr})(\bar{e}_{Rs}\gamma_\mu e_{Rt})$
$\mathcal{O}_{de}^{V,LR}$	$(\bar{d}_{Lp}\gamma^\mu d_{Lr})(\bar{e}_{Rs}\gamma_\mu e_{Rt})$
$\mathcal{O}_{\nu edu}^{V,LR}$	$(\bar{\nu}_{Lp}\gamma^\mu e_{Lr})(\bar{d}_{Rs}\gamma_\mu u_{Rt})$ + h.c.
$\mathcal{O}_{uu}^{V1,LR}$	$(\bar{u}_{Lp}\gamma^\mu u_{Lr})(\bar{u}_{Rs}\gamma_\mu u_{Rt})$
$\mathcal{O}_{uu}^{V8,LR}$	$(\bar{u}_{Lp}\gamma^\mu T^A u_{Lr})(\bar{u}_{Rs}\gamma_\mu T^A u_{Rt})$
$\mathcal{O}_{ud}^{V1,LR}$	$(\bar{u}_{Lp}\gamma^\mu u_{Lr})(\bar{d}_{Rs}\gamma_\mu d_{Rt})$
$\mathcal{O}_{ud}^{V8,LR}$	$(\bar{u}_{Lp}\gamma^\mu T^A u_{Lr})(\bar{d}_{Rs}\gamma_\mu T^A d_{Rt})$
$\mathcal{O}_{du}^{V1,LR}$	$(\bar{d}_{Lp}\gamma^\mu d_{Lr})(\bar{u}_{Rs}\gamma_\mu u_{Rt})$
$\mathcal{O}_{du}^{V8,LR}$	$(\bar{d}_{Lp}\gamma^\mu T^A d_{Lr})(\bar{u}_{Rs}\gamma_\mu T^A u_{Rt})$
$\mathcal{O}_{dd}^{V1,LR}$	$(\bar{d}_{Lp}\gamma^\mu d_{Lr})(\bar{d}_{Rs}\gamma_\mu d_{Rt})$
$\mathcal{O}_{dd}^{V8,LR}$	$(\bar{d}_{Lp}\gamma^\mu T^A d_{Lr})(\bar{d}_{Rs}\gamma_\mu T^A d_{Rt})$
$\mathcal{O}_{uddu}^{V1,LR}$	$(\bar{u}_{Lp}\gamma^\mu d_{Lr})(\bar{d}_{Rs}\gamma_\mu u_{Rt})$ + h.c.
$\mathcal{O}_{uddu}^{V8,LR}$	$(\bar{u}_{Lp}\gamma^\mu T^A d_{Lr})(\bar{d}_{Rs}\gamma_\mu T^A u_{Rt})$ + h.c.

$(\overline{L}R)(\overline{L}R)$ + h.c.	
$\mathcal{O}_{ee}^{S,RR}$	$(\bar{e}_{Lp} e_{Rr})(\bar{e}_{Ls} e_{Rt})$
$\mathcal{O}_{eu}^{S,RR}$	$(\bar{e}_{Lp} e_{Rr})(\bar{u}_{Ls} u_{Rt})$
$\mathcal{O}_{eu}^{T,RR}$	$(\bar{e}_{Lp}\sigma^{\mu\nu} e_{Rr})(\bar{u}_{Ls}\sigma_{\mu\nu} u_{Rt})$
$\mathcal{O}_{ed}^{S,RR}$	$(\bar{e}_{Lp} e_{Rr})(\bar{d}_{Ls} d_{Rt})$
$\mathcal{O}_{ed}^{T,RR}$	$(\bar{e}_{Lp}\sigma^{\mu\nu} e_{Rr})(\bar{d}_{Ls}\sigma_{\mu\nu} d_{Rt})$
$\mathcal{O}_{\nu edu}^{S,RR}$	$(\bar{\nu}_{Lp} e_{Rr})(\bar{d}_{Ls} u_{Rt})$
$\mathcal{O}_{\nu edu}^{T,RR}$	$(\bar{\nu}_{Lp}\sigma^{\mu\nu} e_{Rr})(\bar{d}_{Ls}\sigma_{\mu\nu} u_{Rt})$
$\mathcal{O}_{uu}^{S1,RR}$	$(\bar{u}_{Lp} u_{Rr})(\bar{u}_{Ls} u_{Rt})$
$\mathcal{O}_{uu}^{S8,RR}$	$(\bar{u}_{Lp} T^A u_{Rr})(\bar{u}_{Ls} T^A u_{Rt})$
$\mathcal{O}_{ud}^{S1,RR}$	$(\bar{u}_{Lp} u_{Rr})(\bar{d}_{Ls} d_{Rt})$
$\mathcal{O}_{ud}^{S8,RR}$	$(\bar{u}_{Lp} T^A u_{Rr})(\bar{d}_{Ls} T^A d_{Rt})$
$\mathcal{O}_{dd}^{S1,RR}$	$(\bar{d}_{Lp} d_{Rr})(\bar{d}_{Ls} d_{Rt})$
$\mathcal{O}_{dd}^{S8,RR}$	$(\bar{d}_{Lp} T^A d_{Rr})(\bar{d}_{Ls} T^A d_{Rt})$
$\mathcal{O}_{uddu}^{S1,RR}$	$(\bar{u}_{Lp} d_{Rr})(\bar{d}_{Ls} u_{Rt})$
$\mathcal{O}_{uddu}^{S8,RR}$	$(\bar{u}_{Lp} T^A d_{Rr})(\bar{d}_{Ls} T^A u_{Rt})$

$(\overline{L}R)(\overline{R}L)$ + h.c.	
$\mathcal{O}_{eu}^{S,RL}$	$(\bar{e}_{Lp} e_{Rr})(\bar{u}_{Rs} u_{Lt})$
$\mathcal{O}_{ed}^{S,RL}$	$(\bar{e}_{Lp} e_{Rr})(\bar{d}_{Rs} d_{Lt})$
$\mathcal{O}_{\nu edu}^{S,RL}$	$(\bar{\nu}_{Lp} e_{Rr})(\bar{d}_{Rs} u_{Lt})$

Table 1. The non-four-fermions LEFT operators up to dimension 6 and the dimension-6 four-fermion LEFT operators conserving B and L.

B.2 Odd-dimensional operators

There is only one dimension-5 SMEFT operator: $\epsilon^{ij}\epsilon^{kl}(l_{ip}^T C l_{kr})H_j H_l$. The basis for the dimension-7 operators used here is equivalent to those given in refs. [4, 5].

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$\Delta L = 4 + \text{h.c.}$	
$\mathcal{O}_{\nu\nu}^{S,LL}$	$(\nu_{Lp}^T C\nu_{Lr})(\nu_{Ls}^T C\nu_{Lt})$

$\Delta L = 2 + \text{h.c.}$	
$\mathcal{O}_{\nu e}^{S,LL}$	$(\nu_{Lp}^T C\nu_{Lr})(\bar{e}_{Rs} e_{Lt})$
$\mathcal{O}_{\nu e}^{T,LL}$	$(\nu_{Lp}^T C\sigma^{\mu\nu}\nu_{Lr})(\bar{e}_{Rs}\sigma_{\mu\nu} e_{Lt})$
$\mathcal{O}_{\nu e}^{S,LR}$	$(\nu_{Lp}^T C\nu_{Lr})(\bar{e}_{Ls} e_{Rt})$
$\mathcal{O}_{\nu u}^{S,LL}$	$(\nu_{Lp}^T C\nu_{Lr})(\bar{u}_{Rs} u_{Lt})$
$\mathcal{O}_{\nu u}^{T,LL}$	$(\nu_{Lp}^T C\sigma^{\mu\nu}\nu_{Lr})(\bar{u}_{Rs}\sigma_{\mu\nu} u_{Lt})$
$\mathcal{O}_{\nu u}^{S,LR}$	$(\nu_{Lp}^T C\nu_{Lr})(\bar{u}_{Ls} u_{Rt})$
$\mathcal{O}_{\nu d}^{S,LL}$	$(\nu_{Lp}^T C\nu_{Lr})(\bar{d}_{Rs} d_{Lt})$
$\mathcal{O}_{\nu d}^{T,LL}$	$(\nu_{Lp}^T C\sigma^{\mu\nu}\nu_{Lr})(\bar{d}_{Rs}\sigma_{\mu\nu} d_{Lt})$
$\mathcal{O}_{\nu d}^{S,LR}$	$(\nu_{Lp}^T C\nu_{Lr})(\bar{d}_{Ls} d_{Rt})$
$\mathcal{O}_{\nu edu}^{S,LL}$	$(\nu_{Lp}^T C e_{Lr})(\bar{d}_{Rs} u_{Lt})$
$\mathcal{O}_{\nu edu}^{T,LL}$	$(\nu_{Lp}^T C\sigma^{\mu\nu} e_{Lr})(\bar{d}_{Rs}\sigma_{\mu\nu} u_{Lt})$
$\mathcal{O}_{\nu edu}^{S,LR}$	$(\nu_{Lp}^T C e_{Lr})(\bar{d}_{Ls} u_{Rt})$
$\mathcal{O}_{\nu edu}^{V,RL}$	$(\nu_{Lp}^T C\gamma^{\mu} e_{Rr})(\bar{d}_{Ls}\gamma_\mu u_{Lt})$
$\mathcal{O}_{\nu edu}^{V,RR}$	$(\nu_{Lp}^T C\gamma^{\mu} e_{Rr})(\bar{d}_{Rs}\gamma_\mu u_{Rt})$

$\Delta B = \Delta L = 1 + \text{h.c.}$	
$\mathcal{O}_{udd}^{S,LL}$	$\epsilon_{\alpha\beta\gamma}(u_{Lp}^{\alpha T} C d_{Lr}^{\beta})(d_{Ls}^{\gamma T} C\nu_{Lt})$
$\mathcal{O}_{duu}^{S,LL}$	$\epsilon_{\alpha\beta\gamma}(d_{Lp}^{\alpha T} C u_{Lr}^{\beta})(u_{Ls}^{\gamma T} C e_{Lt})$
$\mathcal{O}_{uud}^{S,LR}$	$\epsilon_{\alpha\beta\gamma}(u_{Lp}^{\alpha T} C u_{Lr}^{\beta})(d_{Rs}^{\gamma T} C e_{Rt})$
$\mathcal{O}_{duu}^{S,LR}$	$\epsilon_{\alpha\beta\gamma}(d_{Lp}^{\alpha T} C u_{Lr}^{\beta})(u_{Rs}^{\gamma T} C e_{Rt})$
$\mathcal{O}_{uud}^{S,RL}$	$\epsilon_{\alpha\beta\gamma}(u_{Rp}^{\alpha T} C u_{Rr}^{\beta})(d_{Ls}^{\gamma T} C e_{Lt})$
$\mathcal{O}_{duu}^{S,RL}$	$\epsilon_{\alpha\beta\gamma}(d_{Rp}^{\alpha T} C u_{Rr}^{\beta})(u_{Ls}^{\gamma T} C e_{Lt})$
$\mathcal{O}_{dud}^{S,RL}$	$\epsilon_{\alpha\beta\gamma}(d_{Rp}^{\alpha T} C u_{Rr}^{\beta})(d_{Ls}^{\gamma T} C\nu_{Lt})$
$\mathcal{O}_{ddu}^{S,RL}$	$\epsilon_{\alpha\beta\gamma}(d_{Rp}^{\alpha T} C d_{Rr}^{\beta})(u_{Ls}^{\gamma T} C\nu_{Lt})$
$\mathcal{O}_{duu}^{S,RR}$	$\epsilon_{\alpha\beta\gamma}(d_{Rp}^{\alpha T} C u_{Rr}^{\beta})(u_{Rs}^{\gamma T} C e_{Rt})$

$\Delta B = -\Delta L = 1 + \text{h.c.}$	
$\mathcal{O}_{ddd}^{S,LL}$	$\epsilon_{\alpha\beta\gamma}(d_{Lp}^{\alpha T} C d_{Lr}^{\beta})(\bar{e}_{Rs} d_{Lt}^{\gamma})$
$\mathcal{O}_{udd}^{S,LR}$	$\epsilon_{\alpha\beta\gamma}(u_{Lp}^{\alpha T} C d_{Lr}^{\beta})(\bar{\nu}_{Ls} d_{Rt}^{\gamma})$
$\mathcal{O}_{ddu}^{S,LR}$	$\epsilon_{\alpha\beta\gamma}(d_{Lp}^{\alpha T} C d_{Lr}^{\beta})(\bar{\nu}_{Ls} u_{Rt}^{\gamma})$
$\mathcal{O}_{ddd}^{S,LR}$	$\epsilon_{\alpha\beta\gamma}(d_{Lp}^{\alpha T} C d_{Lr}^{\beta})(\bar{e}_{Ls} d_{Rt}^{\gamma})$
$\mathcal{O}_{ddd}^{S,RL}$	$\epsilon_{\alpha\beta\gamma}(d_{Rp}^{\alpha T} C d_{Rr}^{\beta})(\bar{e}_{Rs} d_{Lt}^{\gamma})$
$\mathcal{O}_{udd}^{S,RR}$	$\epsilon_{\alpha\beta\gamma}(u_{Rp}^{\alpha T} C d_{Rr}^{\beta})(\bar{\nu}_{Ls} d_{Rt}^{\gamma})$
$\mathcal{O}_{ddd}^{S,RR}$	$\epsilon_{\alpha\beta\gamma}(d_{Rp}^{\alpha T} C d_{Rr}^{\beta})(\bar{e}_{Ls} d_{Rt}^{\gamma})$

Table 2. The dimension-6 four-fermion LEFT operators violating B and/or L.

C Useful Fierz identities

The following Fierz identities are needed to derive the matching conditions given in this paper.

C.1 For $(\overline{L}L)(\overline{L}L)$ and $(\overline{R}R)(\overline{R}R)$ operators

In the case of a four-lepton operator, the identities take the form

$$(\overline{\nu}_{Lp}\gamma^{\mu} e_{Lt})(\overline{e}_{Ls}\gamma_{\mu}\nu_{Lr}) = (\overline{\nu}_{Lp}\gamma^{\mu}\nu_{Lr})(\overline{e}_{Ls}\gamma_{\mu} e_{Lt}). \tag{C.1}$$

In the case of a four-quark operator, color has to be taken into consideration. This is done through the identity $\delta_{\alpha\lambda}\delta_{\kappa\beta} = 2T^A_{\alpha\beta}T^A_{\kappa\lambda} + \dfrac{1}{3}\delta_{\alpha\beta}\delta_{\kappa\lambda}$. The identities are (for instance)

$$(\overline{u}_{Lp}\gamma^{\mu} d_{Lt})(\overline{d}_{Ls}\gamma_{\mu} u_{Lr}) = 2(\overline{u}_{Lp}\gamma^{\mu} T^A u_{Lr})(\overline{d}_{Ls}\gamma_{\mu} T^A d_{Lt}) + \frac{1}{3}(\overline{u}_{Lp}\gamma^{\mu} u_{Lr})(\overline{d}_{Ls}\gamma_{\mu} d_{Lt}). \tag{C.2}$$

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Classes H^n and H^nD^2	
Operator	**WC**
$(H^\dagger H)^3$	$\frac{1}{\Lambda^2} C_H$
$(H^\dagger H)^4$	$\frac{1}{\Lambda^4} C_{H^8}$
$(H^\dagger H)\Box(H^\dagger H)$	$\frac{1}{\Lambda^2} C_{H\Box}$
$(H^\dagger D_\mu H)^*(H^\dagger D^\mu H)$	$\frac{1}{\Lambda^2} C_{HD}$
$(H^\dagger H)^2(D_\mu H^\dagger D^\mu H)$	$\frac{1}{\Lambda^4} C^{(1)}_{H^6}$
$(H^\dagger H)(H^\dagger \tau^I H)(D_\mu H^\dagger \tau^I D^\mu H)$	$\frac{1}{\Lambda^4} C^{(2)}_{H^6}$

Classes X^3H^n	
Operator	**WC**
$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$\frac{1}{\Lambda^2} C_G$
$f^{ABC} \tilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$\frac{1}{\Lambda^2} C_{\tilde{G}}$
$f^{ABC}(H^\dagger H) G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$\frac{1}{\Lambda^2} C^{(1)}_{G^3H^2}$
$f^{ABC}(H^\dagger H) \tilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$\frac{1}{\Lambda^2} C^{(2)}_{G^3H^2}$

Classes ψ^2H^n	
Operator	**WC**
$(H^\dagger H)(\bar{l}_p e_r H)$	$\frac{1}{\Lambda^2} C_{\underset{pr}{eH}}$
$(H^\dagger H)(\bar{q}_p u_r H)$	$\frac{1}{\Lambda^2} C_{\underset{pr}{uH}}$
$(H^\dagger H)(\bar{q}_p d_r H)$	$\frac{1}{\Lambda^2} C_{\underset{pr}{dH}}$
$(H^\dagger H)^2(\bar{l}_p e_r H)$	$\frac{1}{\Lambda^4} C_{\underset{pr}{leH^5}}$
$(H^\dagger H)^2(\bar{q}_p u_r H)$	$\frac{1}{\Lambda^4} C_{\underset{pr}{quH^5}}$
$(H^\dagger H)^2(\bar{q}_p d_r H)$	$\frac{1}{\Lambda^4} C_{\underset{pr}{qdH^5}}$

Classes X^2H^n	
Operator	**WC**
$(H^\dagger H)(G^A_{\mu\nu} G^{A\mu\nu})$	$\frac{1}{\Lambda^2} C_{HG}$
$(H^\dagger H)(W^I_{\mu\nu} W^{I\mu\nu})$	$\frac{1}{\Lambda^2} C_{HW}$
$(H^\dagger H)(B_{\mu\nu} B^{\mu\nu})$	$\frac{1}{\Lambda^2} C_{HB}$
$(H^\dagger \tau^I H)(W^I_{\mu\nu} B^{\mu\nu})$	$\frac{1}{\Lambda^2} C_{HWB}$
$(H^\dagger H)^2 G^A_{\mu\nu} G^{A\mu\nu}$	$\frac{1}{\Lambda^4} C^{(1)}_{G^2H^4}$
$(H^\dagger H)^2 W^I_{\mu\nu} W^{I\mu\nu}$	$\frac{1}{\Lambda^4} C^{(1)}_{W^2H^4}$
$(H^\dagger \tau^I H)(H^\dagger \tau^J H) W^I_{\mu\nu} W^{J\mu\nu}$	$\frac{1}{\Lambda^4} C^{(3)}_{W^2H^4}$
$(H^\dagger H)(H^\dagger \tau^I H) W^I_{\mu\nu} B^{\mu\nu}$	$\frac{1}{\Lambda^4} C^{(1)}_{WBH^4}$
$(H^\dagger H)^2 B_{\mu\nu} B^{\mu\nu}$	$\frac{1}{\Lambda^4} C^{(1)}_{B^2H^4}$

Classes ψ^2XH^n	
Operator	**WC**
$(\bar{l}_p \sigma^{\mu\nu} e_r)\tau^I H W^I_{\mu\nu}$	$\frac{1}{\Lambda^2} C_{\underset{pr}{eW}}$
$(\bar{l}_p \sigma^{\mu\nu} e_r) H B_{\mu\nu}$	$\frac{1}{\Lambda^2} C_{\underset{pr}{eB}}$
$(\bar{q}_p \sigma^{\mu\nu} T^A u_r)\tilde{H} G^A_{\mu\nu}$	$\frac{1}{\Lambda^2} C_{\underset{pr}{uG}}$
$(\bar{q}_p \sigma^{\mu\nu} u_r)\tau^I \tilde{H} W^I_{\mu\nu}$	$\frac{1}{\Lambda^2} C_{\underset{pr}{uW}}$
$(\bar{q}_p \sigma^{\mu\nu} u_r)\tilde{H} B_{\mu\nu}$	$\frac{1}{\Lambda^2} C_{\underset{pr}{uB}}$
$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) H G^A_{\mu\nu}$	$\frac{1}{\Lambda^2} C_{\underset{pr}{dG}}$
$(\bar{q}_p \sigma^{\mu\nu} d_r)\tau^I H W^I_{\mu\nu}$	$\frac{1}{\Lambda^2} C_{\underset{pr}{dW}}$
$(\bar{q}_p \sigma^{\mu\nu} d_r) H B_{\mu\nu}$	$\frac{1}{\Lambda^2} C_{\underset{pr}{dB}}$
$(\bar{l}_p \sigma^{\mu\nu} e_r)\tau^I H (H^\dagger H) W^I_{\mu\nu}$	$\frac{1}{\Lambda^4} C^{(1)}_{\underset{pr}{leWH^3}}$
$(\bar{l}_p \sigma^{\mu\nu} e_r) H (H^\dagger \tau^I H) W^I_{\mu\nu}$	$\frac{1}{\Lambda^4} C^{(2)}_{\underset{pr}{leWH^3}}$
$(\bar{l}_p \sigma^{\mu\nu} e_r) H (H^\dagger H) B_{\mu\nu}$	$\frac{1}{\Lambda^4} C_{\underset{pr}{leBH^3}}$
$(\bar{q}_p \sigma^{\mu\nu} T^A u_r)\tilde{H} (H^\dagger H) G^A_{\mu\nu}$	$\frac{1}{\Lambda^4} C_{\underset{pr}{quGH^3}}$
$(\bar{q}_p \sigma^{\mu\nu} u_r)\tau^I \tilde{H} (H^\dagger H) W^I_{\mu\nu}$	$\frac{1}{\Lambda^4} C^{(1)}_{\underset{pr}{quWH^3}}$
$(\bar{q}_p \sigma^{\mu\nu} u_r)\tilde{H} (H^\dagger \tau^I H) W^I_{\mu\nu}$	$\frac{1}{\Lambda^4} C^{(2)}_{\underset{pr}{quWH^3}}$
$(\bar{q}_p \sigma^{\mu\nu} u_r)\tilde{H} (H^\dagger H) B_{\mu\nu}$	$\frac{1}{\Lambda^4} C_{\underset{pr}{quBH^3}}$
$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) H (H^\dagger H) G^A_{\mu\nu}$	$\frac{1}{\Lambda^4} C_{\underset{pr}{qdGH^3}}$
$(\bar{q}_p \sigma^{\mu\nu} d_r)\tau^I H (H^\dagger H) W^I_{\mu\nu}$	$\frac{1}{\Lambda^4} C^{(1)}_{\underset{pr}{qdWH^3}}$
$(\bar{q}_p \sigma^{\mu\nu} d_r) H (H^\dagger \tau^I H) W^I_{\mu\nu}$	$\frac{1}{\Lambda^4} C^{(2)}_{\underset{pr}{qdWH^3}}$
$(\bar{q}_p \sigma^{\mu\nu} d_r) H (H^\dagger H) B_{\mu\nu}$	$\frac{1}{\Lambda^4} C_{\underset{pr}{qdBH^3}}$

Classes ψ^2H^nD	
Operator	**WC**
$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{l}_p \gamma^\mu l_r)$	$\frac{1}{\Lambda^2} C^{(1)}_{\underset{pr}{Hl}}$
$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{l}_p \tau^I \gamma^\mu l_r)$	$\frac{1}{\Lambda^2} C^{(3)}_{\underset{pr}{Hl}}$
$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}_p \gamma^\mu e_r)$	$\frac{1}{\Lambda^2} C_{\underset{pr}{He}}$
$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{q}_p \gamma^\mu q_r)$	$\frac{1}{\Lambda^2} C^{(1)}_{\underset{pr}{Hq}}$
$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{q}_p \tau^I \gamma^\mu q_r)$	$\frac{1}{\Lambda^2} C^{(3)}_{\underset{pr}{Hq}}$
$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}_p \gamma^\mu u_r)$	$\frac{1}{\Lambda^2} C_{\underset{pr}{Hu}}$
$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{d}_p \gamma^\mu d_r)$	$\frac{1}{\Lambda^2} C_{\underset{pr}{Hd}}$
$i(\tilde{H}^\dagger D_\mu H)(\bar{u}_p \gamma^\mu d_r)$	$\frac{1}{\Lambda^2} C_{\underset{pr}{Hud}}$
$i(\bar{l}_p \gamma^\mu l_r)(H^\dagger \overleftrightarrow{D}_\mu H)(H^\dagger H)$	$\frac{1}{\Lambda^4} C^{(1)}_{\underset{pr}{l^2H^4D}}$
$i(\bar{l}_p \gamma^\mu \tau^I l_r)[(H^\dagger \overleftrightarrow{D}^I_\mu H)(H^\dagger H) + (H^\dagger \overleftrightarrow{D}_\mu H)(H^\dagger \tau^I H)]$	$\frac{1}{\Lambda^4} C^{(2)}_{\underset{pr}{l^2H^4D}}$
$i\epsilon^{IJK}(\bar{l}_p \gamma^\mu \tau^I l_r)(H^\dagger \overleftrightarrow{D}^J_\mu H)(H^\dagger \tau^K H)$	$\frac{1}{\Lambda^4} C^{(3)}_{\underset{pr}{l^2H^4D}}$
$i(\bar{e}_p \gamma^\mu e_r)(H^\dagger \overleftrightarrow{D}_\mu H)(H^\dagger H)$	$\frac{1}{\Lambda^4} C_{\underset{pr}{e^2H^4D}}$
$i(\bar{q}_p \gamma^\mu q_r)(H^\dagger \overleftrightarrow{D}_\mu H)(H^\dagger H)$	$\frac{1}{\Lambda^4} C^{(1)}_{\underset{pr}{q^2H^4D}}$
$i(\bar{q}_p \gamma^\mu \tau^I q_r)[(H^\dagger \overleftrightarrow{D}^I_\mu H)(H^\dagger H) + (H^\dagger \overleftrightarrow{D}_\mu H)(H^\dagger \tau^I H)]$	$\frac{1}{\Lambda^4} C^{(2)}_{\underset{pr}{q^2H^4D}}$
$i\epsilon^{IJK}(\bar{q}_p \gamma^\mu \tau^I q_r)(H^\dagger \overleftrightarrow{D}^J_\mu H)(H^\dagger \tau^K H)$	$\frac{1}{\Lambda^4} C^{(3)}_{\underset{pr}{q^2H^4D}}$
$i(\bar{u}_p \gamma^\mu u_r)(H^\dagger \overleftrightarrow{D}_\mu H)(H^\dagger H)$	$\frac{1}{\Lambda^4} C_{\underset{pr}{u^2H^4D}}$
$i(\bar{d}_p \gamma^\mu d_r)(H^\dagger \overleftrightarrow{D}_\mu H)(H^\dagger H)$	$\frac{1}{\Lambda^4} C_{\underset{pr}{d^2H^4D}}$
$i(\bar{u}_p \gamma^\mu d_r)(\tilde{H}^\dagger \overleftrightarrow{D}_\mu H)(H^\dagger H)$	$\frac{1}{\Lambda^4} C_{\underset{pr}{udH^4D}}$

Table 3. The even-dimensional non-four-fermion SMEFT operators appearing in this paper.

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Classes $(\overline{L}L)(\overline{L}L)H^n$	
Operator	**WC**
$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	$\frac{1}{\Lambda^2}C_{\substack{ll\\prst}}$
$(\overline{q}_p\gamma_\mu q_r)(\overline{q}_s\gamma^\mu q_t)$	$\frac{1}{\Lambda^2}C^{(1)}_{\substack{qq\\prst}}$
$(\overline{q}_p\gamma_\mu\tau^I q_r)(\overline{q}_s\gamma^\mu\tau^I q_t)$	$\frac{1}{\Lambda^2}C^{(3)}_{\substack{qq\\prst}}$
$(\bar{l}_p\gamma_\mu l_r)(\overline{q}_s\gamma^\mu q_t)$	$\frac{1}{\Lambda^2}C^{(1)}_{\substack{lq\\prst}}$
$(\bar{l}_p\gamma_\mu\tau^I l_r)(\overline{q}_s\gamma^\mu\tau^I q_t)$	$\frac{1}{\Lambda^2}C^{(3)}_{\substack{lq\\prst}}$
$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(1)}_{\substack{l^4H^2\\prst}}$
$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu\tau^I l_t)(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(2)}_{\substack{l^4H^2\\prst}}$
$(\overline{q}_p\gamma_\mu q_r)(\overline{q}_s\gamma^\mu q_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(1)}_{\substack{q^4H^2\\prst}}$
$(\overline{q}_p\gamma_\mu q_r)(\overline{q}_s\gamma^\mu\tau^I q_t)(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(2)}_{\substack{q^4H^2\\prst}}$
$(\overline{q}_p\gamma_\mu\tau^I q_r)(\overline{q}_s\gamma^\mu\tau^I q_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(3)}_{\substack{q^4H^2\\prst}}$
$\epsilon^{IJK}(\overline{q}_p\gamma_\mu\tau^I q_r)(\overline{q}_s\gamma^\mu\tau^J q_t)(H^\dagger\tau^K H)$	$\frac{1}{\Lambda^4}C^{(5)}_{\substack{q^4H^2\\prst}}$
$(\bar{l}_p\gamma_\mu l_r)(\overline{q}_s\gamma^\mu q_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(1)}_{\substack{l^2q^2H^2\\prst}}$
$(\bar{l}_p\gamma_\mu\tau^I l_r)(\overline{q}_s\gamma^\mu q_t)(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(2)}_{\substack{l^2q^2H^2\\prst}}$
$(\bar{l}_p\gamma_\mu\tau^I l_r)(\overline{q}_s\gamma^\mu\tau^I q_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(3)}_{\substack{l^2q^2H^2\\prst}}$
$(\bar{l}_p\gamma_\mu l_r)(\overline{q}_s\gamma^\mu\tau^I q_t)(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(4)}_{\substack{l^2q^2H^2\\prst}}$
$\epsilon^{IJK}(\bar{l}_p\gamma_\mu\tau^I l_r)(\overline{q}_s\gamma^\mu\tau^J q_t)(H^\dagger\tau^K H)$	$\frac{1}{\Lambda^4}C^{(5)}_{\substack{l^2q^2H^2\\prst}}$

Classes $(\overline{R}R)(\overline{R}R)H^n$	
Operator	**WC**
$(\overline{e}_p\gamma_\mu e_r)(\overline{e}_s\gamma^\mu e_t)$	$\frac{1}{\Lambda^2}C_{\substack{ee\\prst}}$
$(\overline{u}_p\gamma_\mu u_r)(\overline{u}_s\gamma^\mu u_t)$	$\frac{1}{\Lambda^2}C_{\substack{uu\\prst}}$
$(\overline{d}_p\gamma_\mu d_r)(\overline{d}_s\gamma^\mu d_t)$	$\frac{1}{\Lambda^2}C_{\substack{dd\\prst}}$
$(\overline{e}_p\gamma_\mu e_r)(\overline{u}_s\gamma^\mu u_t)$	$\frac{1}{\Lambda^2}C_{\substack{eu\\prst}}$
$(\overline{e}_p\gamma_\mu e_r)(\overline{d}_s\gamma^\mu d_t)$	$\frac{1}{\Lambda^2}C_{\substack{ed\\prst}}$
$(\overline{u}_p\gamma_\mu u_r)(\overline{d}_s\gamma^\mu d_t)$	$\frac{1}{\Lambda^2}C^{(1)}_{\substack{ud\\prst}}$
$(\overline{u}_p\gamma_\mu T^A u_r)(\overline{d}_s\gamma^\mu T^A d_t)$	$\frac{1}{\Lambda^2}C^{(8)}_{\substack{ud\\prst}}$
$(\overline{e}_p\gamma_\mu e_r)(\overline{e}_s\gamma^\mu e_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C_{\substack{e^4H^2\\prst}}$
$(\overline{u}_p\gamma_\mu u_r)(\overline{u}_s\gamma^\mu u_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C_{\substack{u^4H^2\\prst}}$
$(\overline{d}_p\gamma_\mu d_r)(\overline{d}_s\gamma^\mu d_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C_{\substack{d^4H^2\\prst}}$
$(\overline{e}_p\gamma_\mu e_r)(\overline{u}_s\gamma^\mu u_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C_{\substack{e^2u^2H^2\\prst}}$
$(\overline{e}_p\gamma_\mu e_r)(\overline{d}_s\gamma^\mu d_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C_{\substack{e^2d^2H^2\\prst}}$
$(\overline{u}_p\gamma_\mu u_r)(\overline{d}_s\gamma^\mu d_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(1)}_{\substack{u^2d^2H^2\\prst}}$
$(\overline{u}_p\gamma_\mu T^A u_r)(\overline{d}_s\gamma^\mu T^A d_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(2)}_{\substack{u^2d^2H^2\\prst}}$

Classes $(\overline{L}R)(\overline{L}R)H^n$	
Operator	**WC**
$(\overline{q}_p^j u_r)\epsilon_{jk}(\overline{q}_s^k d_t)$	$\frac{1}{\Lambda^2}C^{(1)}_{\substack{quqd\\prst}}$
$(\overline{q}_p^j T^A u_r)\epsilon_{jk}(\overline{q}_s^k T^A d_t)$	$\frac{1}{\Lambda^2}C^{(8)}_{\substack{quqd\\prst}}$
$(\bar{l}_p^j e_r)\epsilon_{jk}(\overline{q}_s^k u_t)$	$\frac{1}{\Lambda^2}C^{(1)}_{\substack{lequ\\prst}}$
$(\bar{l}_p^j\sigma_{\mu\nu}e_r)\epsilon_{jk}(\overline{q}_s^k\sigma^{\mu\nu}u_t)$	$\frac{1}{\Lambda^2}C^{(3)}_{\substack{lequ\\prst}}$
$(\overline{q}_p^j u_r)\epsilon_{jk}(\overline{q}_s^k d_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(1)}_{\substack{q^2udH^2\\prst}}$
$(\overline{q}_p^j u_r)(\tau^I\epsilon)_{jk}(\overline{q}_s^k d_t)(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(2)}_{\substack{q^2udH^2\\prst}}$
$(\overline{q}_p^j T^A u_r)\epsilon_{jk}(\overline{q}_s^k T^A d_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(3)}_{\substack{q^2udH^2\\prst}}$
$(\overline{q}_p^j T^A u_r)(\tau^I\epsilon)_{jk}(\overline{q}_s^k T^A d_t)(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(4)}_{\substack{q^2udH^2\\prst}}$
$(\bar{l}_p^j e_r)\epsilon_{jk}(\overline{q}_s^k u_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(1)}_{\substack{lequH^2\\prst}}$
$(\bar{l}_p^j e_r)(\tau^I\epsilon)_{jk}(\overline{q}_s^k u_t)(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(2)}_{\substack{lequH^2\\prst}}$
$(\bar{l}_p^j\sigma_{\mu\nu}e_r)\epsilon_{jk}(\overline{q}_s^k\sigma^{\mu\nu}u_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(3)}_{\substack{lequH^2\\prst}}$
$(\bar{l}_p^j\sigma_{\mu\nu}e_r)(\tau^I\epsilon)_{jk}(\overline{q}_s^k\sigma^{\mu\nu}u_t)(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(4)}_{\substack{lequH^2\\prst}}$
$(\bar{l}_p e_r H)(\bar{l}_s e_t H)$	$\frac{1}{\Lambda^4}C^{(3)}_{\substack{l^2e^2H^2\\prst}}$
$(\bar{l}_p e_r H)(\overline{q}_s d_t H)$	$\frac{1}{\Lambda^4}C^{(3)}_{\substack{leqdH^2\\prst}}$
$(\bar{l}_p\sigma_{\mu\nu}e_r H)(\overline{q}_s\sigma^{\mu\nu}d_t H)$	$\frac{1}{\Lambda^4}C^{(4)}_{\substack{leqdH^2\\prst}}$
$(\overline{q}_p u_r\tilde{H})(\overline{q}_s u_t\tilde{H})$	$\frac{1}{\Lambda^4}C^{(5)}_{\substack{q^2u^2H^2\\prst}}$
$(\overline{q}_p T^A u_r\tilde{H})(\overline{q}_s T^A u_t\tilde{H})$	$\frac{1}{\Lambda^4}C^{(6)}_{\substack{q^2u^2H^2\\prst}}$
$(\overline{q}_p d_r H)(\overline{q}_s d_t H)$	$\frac{1}{\Lambda^4}C^{(5)}_{\substack{q^2d^2H^2\\prst}}$
$(\overline{q}_p T^A d_r H)(\overline{q}_s T^A d_t H)$	$\frac{1}{\Lambda^4}C^{(6)}_{\substack{q^2d^2H^2\\prst}}$

Classes $(\not{B})\psi^4H^n$	
Operator	**WC**
$\epsilon^{\alpha\beta\gamma}\epsilon^{jk}(d_p^{\alpha T}Cu_r^\beta)(q_{js}^{\gamma T}Cl_{kt})$	$\frac{1}{\Lambda^2}C_{\substack{duq\\prst}}$
$\epsilon^{\alpha\beta\gamma}\epsilon^{jk}(q_{jp}^{\alpha T}Cq_{kr}^\beta)(u_s^{\gamma T}Ce_t)$	$\frac{1}{\Lambda^2}C_{\substack{qqu\\prst}}$
$\epsilon^{\alpha\beta\gamma}\epsilon^{jn}\epsilon^{km}(q_{jp}^{\alpha T}Cq_{kr}^\beta)(q_{ms}^{\gamma T}Cl_{nt})$	$\frac{1}{\Lambda^2}C_{\substack{qqq\\prst}}$
$\epsilon^{\alpha\beta\gamma}(d_p^{\alpha T}Cu_r^\beta)(u_s^{\gamma T}Ce_t)$	$\frac{1}{\Lambda^2}C_{\substack{duu\\prst}}$
$\epsilon^{\alpha\beta\gamma}\epsilon^{jk}(d_p^{\alpha T}Cu_r^\beta)(q_{js}^{\gamma T}Cl_{kt})(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(1)}_{\substack{lqudH^2\\prst}}$
$\epsilon^{\alpha\beta\gamma}(\epsilon\tau^I)^{jk}(d_p^{\alpha T}Cu_r^\beta)(q_{js}^{\gamma T}Cl_{kt})(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(2)}_{\substack{lqudH^2\\prst}}$
$\epsilon^{\alpha\beta\gamma}\epsilon^{jk}(q_{jp}^{\alpha T}Cq_{mr}^\beta)(u_s^{\gamma T}Ce_t)(H^{m\dagger}H_k)$	$\frac{1}{\Lambda^4}C_{\substack{eq^2uH^2\\prst}}$
$\epsilon^{\alpha\beta\gamma}\epsilon^{jk}\epsilon^{mn}(q_{mp}^{\alpha T}Cq_{jr}^\beta)(q_{ks}^{\gamma T}Cl_{nt})(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(1)}_{\substack{lq^3H^2\\prst}}$
$\epsilon^{\alpha\beta\gamma}\epsilon^{jk}(\epsilon\tau^I)^{mn}(q_{mp}^{\alpha T}Cq_{jr}^\beta)(q_{ks}^{\gamma T}Cl_{nt})(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(2)}_{\substack{lq^3H^2\\prst}}$
$\epsilon^{\alpha\beta\gamma}(\epsilon\tau^I)^{jk}\epsilon^{mn}(q_{mp}^{\alpha T}Cq_{jr}^\beta)(q_{ks}^{\gamma T}Cl_{nt})(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(3)}_{\substack{lq^3H^2\\prst}}$
$\epsilon^{\alpha\beta\gamma}(d_p^{\alpha T}Cu_r^\beta)(u_s^{\gamma T}Ce_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C_{\substack{eu^2dH^2\\prst}}$
$\epsilon^{\alpha\beta\gamma}\epsilon^{jk}\epsilon^{mn}(l_{jp}^TCq_{mr}^\alpha)(u_s^{\beta T}Cu_t^\gamma)\tilde{H}_k\tilde{H}_n$	$\frac{1}{\Lambda^4}C_{\substack{lqu^2H^2\\prst}}$
$\epsilon^{\alpha\beta\gamma}\epsilon^{jk}\epsilon^{mn}(l_{jp}^TCq_{mr}^\alpha)(d_s^{\beta T}Cd_t^\gamma)H_kH_n$	$\frac{1}{\Lambda^4}C_{\substack{lqd^2H^2\\prst}}$
$\epsilon^{\alpha\beta\gamma}\epsilon^{jk}\epsilon^{mn}(e_p^TCd_r^\alpha)(q_{js}^{\beta T}Cq_{mt}^\gamma)H_kH_n$	$\frac{1}{\Lambda^4}C_{\substack{eq^2dH^2\\prst}}$

Classes $(\overline{L}L)(\overline{R}R)H^n$	
Operator	**WC**
$(\bar{l}_p\gamma_\mu l_r)(\overline{e}_s\gamma^\mu e_t)$	$\frac{1}{\Lambda^2}C_{\substack{le\\prst}}$
$(\bar{l}_p\gamma_\mu l_r)(\overline{u}_s\gamma^\mu u_t)$	$\frac{1}{\Lambda^2}C_{\substack{lu\\prst}}$
$(\bar{l}_p\gamma_\mu l_r)(\overline{d}_s\gamma^\mu d_t)$	$\frac{1}{\Lambda^2}C_{\substack{ld\\prst}}$
$(\overline{q}_p\gamma_\mu q_r)(\overline{e}_s\gamma^\mu e_t)$	$\frac{1}{\Lambda^2}C_{\substack{qe\\prst}}$
$(\overline{q}_p\gamma_\mu q_r)(\overline{u}_s\gamma^\mu u_t)$	$\frac{1}{\Lambda^2}C^{(1)}_{\substack{qu\\prst}}$
$(\overline{q}_p\gamma_\mu T^A q_r)(\overline{u}_s\gamma^\mu T^A u_t)$	$\frac{1}{\Lambda^2}C^{(8)}_{\substack{qu\\prst}}$
$(\overline{q}_p\gamma_\mu q_r)(\overline{d}_s\gamma^\mu d_t)$	$\frac{1}{\Lambda^2}C^{(1)}_{\substack{qd\\prst}}$
$(\overline{q}_p\gamma_\mu T^A q_r)(\overline{d}_s\gamma^\mu T^A d_t)$	$\frac{1}{\Lambda^2}C^{(8)}_{\substack{qd\\prst}}$
$(\bar{l}_p\gamma_\mu l_r)(\overline{e}_s\gamma^\mu e_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(1)}_{\substack{l^2e^2H^2\\prst}}$
$(\bar{l}_p\gamma_\mu\tau^I l_r)(\overline{e}_s\gamma^\mu e_t)(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(2)}_{\substack{l^2e^2H^2\\prst}}$
$(\bar{l}_p\gamma_\mu l_r)(\overline{u}_s\gamma^\mu u_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(1)}_{\substack{l^2u^2H^2\\prst}}$
$(\bar{l}_p\gamma_\mu\tau^I l_r)(\overline{u}_s\gamma^\mu u_t)(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(2)}_{\substack{l^2u^2H^2\\prst}}$
$(\bar{l}_p\gamma_\mu l_r)(\overline{d}_s\gamma^\mu d_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(1)}_{\substack{l^2d^2H^2\\prst}}$
$(\bar{l}_p\gamma_\mu\tau^I l_r)(\overline{d}_s\gamma^\mu d_t)(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(2)}_{\substack{l^2d^2H^2\\prst}}$
$(\overline{q}_p\gamma_\mu q_r)(\overline{e}_s\gamma^\mu e_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(1)}_{\substack{q^2e^2H^2\\prst}}$
$(\overline{q}_p\gamma_\mu\tau^I q_r)(\overline{e}_s\gamma^\mu e_t)(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(2)}_{\substack{q^2e^2H^2\\prst}}$
$(\overline{q}_p\gamma_\mu q_r)(\overline{u}_s\gamma^\mu u_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(1)}_{\substack{q^2u^2H^2\\prst}}$
$(\overline{q}_p\gamma_\mu\tau^I q_r)(\overline{u}_s\gamma^\mu u_t)(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(2)}_{\substack{q^2u^2H^2\\prst}}$
$(\overline{q}_p\gamma_\mu T^A q_r)(\overline{u}_s\gamma^\mu T^A u_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(3)}_{\substack{q^2u^2H^2\\prst}}$
$(\overline{q}_p\gamma_\mu\tau^I T^A q_r)(\overline{u}_s\gamma^\mu T^A u_t)(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(4)}_{\substack{q^2u^2H^2\\prst}}$
$(\overline{q}_p\gamma_\mu q_r)(\overline{d}_s\gamma^\mu d_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(1)}_{\substack{q^2d^2H^2\\prst}}$
$(\overline{q}_p\gamma_\mu\tau^I q_r)(\overline{d}_s\gamma^\mu d_t)(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(2)}_{\substack{q^2d^2H^2\\prst}}$
$(\overline{q}_p\gamma_\mu T^A q_r)(\overline{d}_s\gamma^\mu T^A d_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(3)}_{\substack{q^2d^2H^2\\prst}}$
$(\overline{q}_p\gamma_\mu\tau^I T^A q_r)(\overline{d}_s\gamma^\mu T^A d_t)(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(4)}_{\substack{q^2d^2H^2\\prst}}$

Classes $(\overline{L}R)(\overline{R}L)H^n$	
Operator	**WC**
$(\bar{l}_p^j e_r)(\overline{d}_s q_t^j)$	$\frac{1}{\Lambda^2}C_{\substack{ledq\\prst}}$
$(\bar{l}_p^j e_r)(\overline{d}_s q_t^j)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(1)}_{\substack{leqdH^2\\prst}}$
$(\bar{l}_p e_r)\tau^I(\overline{d}_s q_t)(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(2)}_{\substack{leqdH^2\\prst}}$
$(\bar{l}_p d_r H)(\tilde{H}^\dagger\overline{u}_s l_t)$	$\frac{1}{\Lambda^4}C_{\substack{l^2udH^2\\prst}}$
$(\bar{l}_p e_r H)(\tilde{H}^\dagger\overline{u}_s q_t)$	$\frac{1}{\Lambda^4}C^{(5)}_{\substack{lequH^2\\prst}}$
$(\overline{q}_p d_r H)(\tilde{H}^\dagger\overline{u}_s q_t)$	$\frac{1}{\Lambda^4}C^{(5)}_{\substack{q^2udH^2\\prst}}$
$(\overline{q}_p T^A d_r H)(\tilde{H}^\dagger\overline{u}_s T^A q_t)$	$\frac{1}{\Lambda^4}C^{(6)}_{\substack{q^2udH^2\\prst}}$

Table 4. The even-dimensional four-fermion SMEFT operators appearing in this paper.

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Classes $\psi^2 H^n$	
Operator	**WC**
$\epsilon^{ij}\epsilon^{kl}(l_{ip}^T C l_{kr})H_j H_l$	$\frac{1}{\Lambda}C_{\substack{5\\pr}}$
$\epsilon^{ij}\epsilon^{kl}(l_{ip}^T C l_{kr})H_j H_l(H^\dagger H)$	$\frac{1}{\Lambda^3}C_{\substack{l^2H^4\\pr}}$

Class $\psi^2 H^3 D$	
Operator	**WC**
$i\epsilon^{ij}\epsilon^{kl}(l_{ip}^T C\gamma^\mu e_r)H_j H_k(D_\mu H)_l$	$\frac{1}{\Lambda^3}C_{\substack{leH^3D\\pr}}$

Class $\psi^2 H^2 X$	
Operator	**WC**
$\epsilon^{ij}\epsilon^{kl}(l_{ip}^T C\sigma_{\mu\nu} l_{kr})H_j H_l B^{\mu\nu}$	$\frac{1}{\Lambda^3}C_{\substack{l^2H^2B\\pr}}$
$\epsilon^{ij}(\epsilon\tau^I)^{kl}(l_{ip}^T C\sigma_{\mu\nu} l_{kr})H_j H_l W^{I\mu\nu}$	$\frac{1}{\Lambda^3}C_{\substack{l^2H^2W\\pr}}$

Class $\psi^4 H$	
Operator	**WC**
$\epsilon^{ij}\epsilon^{kl}(l_{ip}^T C l_{kr})(\overline{e}_s l_{jt})H_l$	$\frac{1}{\Lambda^3}C_{\substack{l^3eH\\prst}}$
$\epsilon^{ij}\epsilon^{kl}(l_{ip}^T C l_{kr})(\overline{d}_s q_{lt})H_j$	$\frac{1}{\Lambda^3}C^{(1)}_{\substack{l^2dqH\\prst}}$
$\epsilon^{ij}\epsilon^{kl}(l_{ip}^T C\sigma_{\mu\nu} l_{kr})(\overline{d}_s\sigma^{\mu\nu} q_{lt})H_j$	$\frac{1}{\Lambda^3}C^{(2)}_{\substack{l^2dqH\\prst}}$
$\epsilon^{ij}(l_{ip}^T C l_{kr})(\overline{q}_s^k u_t)H_j$	$\frac{1}{\Lambda^3}C_{\substack{l^2quH\\prst}}$
$\epsilon^{\alpha\beta\gamma}\epsilon^{ij}(q_{kp}^{\alpha T} C q_{ir}^\beta)(\overline{l}_s^k d_t^\gamma)\tilde{H}_j$	$\frac{1}{\Lambda^3}C_{\substack{q^2ldH\\prst}}$
$\epsilon^{\alpha\beta\gamma}(d_p^{\alpha T} C d_r^\beta)(\overline{l}_s d_t^\gamma)H$	$\frac{1}{\Lambda^3}C_{\substack{d^3lH\\prst}}$
$\epsilon^{\alpha\beta\gamma}(u_p^{\alpha T} C d_r^\beta)(\overline{l}_s d_t^\gamma)\tilde{H}$	$\frac{1}{\Lambda^3}C_{\substack{ud^2lH\\prst}}$
$\epsilon^{ij}(l_{ip}^T C\gamma_\mu e_r)(\overline{d}_s\gamma^\mu u_t)H_j$	$\frac{1}{\Lambda^3}C_{\substack{leduH\\prst}}$
$\epsilon^{\alpha\beta\gamma}\epsilon^{ij}(d_p^{\alpha T} C d_r^\beta)(\overline{e}_s q_{it}^\gamma)\tilde{H}_j$	$\frac{1}{\Lambda^3}C_{\substack{eqd^2H\\prst}}$

Table 5. The odd-dimensional SMEFT operators appearing in this paper.

C.2 For $(\overline{L}L)(\overline{R}R)$ operators

In the case of a four-lepton operator or a two-lepton and two-quark operator, the Fierz identities take the form

$$(\overline{\nu}_{Lp} e_{Rt})(\overline{e}_{Rs}\nu_{Lr}) = -\frac{1}{2}(\overline{\nu}_{Lp}\gamma^\mu\nu_{Lr})(\overline{e}_{Rs}\gamma_\mu e_{Rt})\,. \tag{C.3}$$

In the case of a four-quark operator, the identities take the following forms:

$$\begin{aligned}(\overline{u}_{Lp} d_{Rt})(\overline{d}_{Rs} u_{Lr}) &= -(\overline{u}_{Lp}\gamma^\mu T^A u_{Lr})(\overline{d}_{Rs}\gamma_\mu T^A d_{Rt}) \\ &\quad - \frac{1}{6}(\overline{u}_{Lp}\gamma^\mu u_{Lr})(\overline{d}_{Rs}\gamma_\mu d_{Rt})\,, \end{aligned}\tag{C.4}$$

$$\begin{aligned}(\overline{u}_{Lp} T^A d_{Rt})(\overline{d}_{Rs} T^A u_{Lr}) &= -\frac{2}{9}(\overline{u}_{Lp}\gamma^\mu u_{Lr})(\overline{d}_{Rs}\gamma_\mu d_{Rt}) \\ &\quad + \frac{1}{6}(\overline{u}_{Lp}\gamma^\mu T^A u_{Lr})(\overline{d}_{Rs}\gamma_\mu T^A d_{Rt})\,. \end{aligned}\tag{C.5}$$

C.3 For fermion-number-violating operators

The needed identities take the form

$$(\nu_{Lr}^T C\nu_{Lr})(\bar{e}_{Rs}e_{Lt}) = -\frac{1}{2}(\nu_{Lp}^T Ce_{Lt})(\bar{e}_{Rs}\nu_{Lr}) - \frac{1}{8}(\nu_{Lp}^T C\sigma_{\mu\nu}e_{Lt})(\bar{e}_{Rs}\sigma^{\mu\nu}\nu_{Lr})\,, \tag{C.6}$$

$$(\nu_{Lr}^T C\nu_{Lr})(\bar{e}_{Ls}e_{Rt}) = -\frac{1}{2}(\nu_{Lp}^T C\gamma_\mu e_{Rt})(\bar{e}_{Rs}\gamma^\mu \nu_{Rr})\,, \tag{C.7}$$

$$(\nu_{Lr}^T C\nu_{Lr})(e_{Ls}^T Ce_{Lt}) = -\frac{1}{2}(\nu_{Lp}^T Ce_{Lt})(e_{Ls}^T C\nu_{Lr}) - \frac{1}{8}(\nu_{Lp}^T C\sigma_{\mu\nu}e_{Lt})(e_{Ls}^T C\sigma^{\mu\nu}\nu_{Lr})\,. \tag{C.8}$$

D Matching conditions

D.1 $\nu\nu+$ h.c. operator

LEFT WC	**Matching**
$\Lambda C_{\substack{\nu\\pr}}$	$\frac{{v_T}^2}{2\Lambda}\left[C_{\substack{5\\pr}} + \frac{{v_T}^2}{2\Lambda^2}C_{\substack{l^2H^4\\pr}}\right]$

D.2 $(\nu\nu)X+$ h.c. and $(\overline{L}R)X+$ h.c. operators

LEFT WC (+c.c.)	**Matching (+c.c.)**
$\frac{1}{\Lambda^2}C_{\substack{\nu\gamma\\pr}}$	$\frac{{v_T}^2}{2g_Z\Lambda^3}\left[gC_{\substack{l^2H^2B\\pr}} - \frac{g'}{2}\left(C_{\substack{l^2H^2W\\pr}} - C_{\substack{l^2H^2W\\rp}}\right)\right]$
$\frac{1}{\Lambda^2}C_{\substack{e\gamma\\pr}}$	$\frac{v_T}{\sqrt{2}\,g_Z\,\Lambda^2}\left[\left(gC_{\substack{eB\\pr}} - g'C_{\substack{eW\\pr}}\right) + \frac{{v_T}^2}{2\Lambda^2}\left(gC_{\substack{leBH^3\\pr}} - g'C^{(1)}_{\substack{leWH^3\\pr}} - g'C^{(2)}_{\substack{leWH^3\\pr}}\right)\right]$
$\frac{1}{\Lambda^2}C_{\substack{u\gamma\\pr}}$	$\frac{v_T}{\sqrt{2}\,g_Z\,\Lambda^2}\left[\left(gC_{\substack{uB\\pr}} + g'C_{\substack{uW\\pr}}\right) + \frac{{v_T}^2}{2\Lambda^2}\left(gC_{\substack{quBH^3\\pr}} + g'C^{(1)}_{\substack{quWH^3\\pr}} - g'C^{(2)}_{\substack{quWH^3\\pr}}\right)\right]$
$\frac{1}{\Lambda^2}C_{\substack{d\gamma\\pr}}$	$\frac{v_T}{\sqrt{2}\,g_Z\,\Lambda^2}\left[\left(gC_{\substack{dB\\pr}} - g'C_{\substack{dW\\pr}}\right) + \frac{{v_T}^2}{2\Lambda^2}\left(gC_{\substack{qdBH^3\\pr}} - g'C^{(1)}_{\substack{qdWH^3\\pr}} - g'C^{(2)}_{\substack{qdWH^3\\pr}}\right)\right]$
$\frac{1}{\Lambda^2}C_{\substack{uG\\pr}}$	$\frac{v_T}{\sqrt{2}\,\Lambda^2}\left[C_{\substack{uG\\pr}} + \frac{{v_T}^2}{2\Lambda^2}C_{\substack{quGH^3\\pr}}\right]$
$\frac{1}{\Lambda^2}C_{\substack{dG\\pr}}$	$\frac{v_T}{\sqrt{2}\,\Lambda^2}\left[C_{\substack{dG\\pr}} + \frac{{v_T}^2}{2\Lambda^2}C_{\substack{qdGH^3\\pr}}\right]$

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The non-physical ratios g/g_Z and g'/g_Z appearing here can be expressed in terms of the corrected coupling constants $\overline{g}$ and $\overline{g}'$ and the SMEFT WC's, using the following equations:

$$\frac{g}{g_Z} = \frac{\overline{g}}{\sqrt{\overline{g}^2+\overline{g}'^2}} \left[\begin{aligned} &1+\frac{\overline{g}'^2 {v_T}^2}{(\overline{g}^2+\overline{g}'^2)\Lambda^2}(C_{HB}-C_{HW}) \\ &+\frac{\overline{g}'^2 {v_T}^4}{2(\overline{g}^2+\overline{g}'^2)\Lambda^4}(C^{(1)}_{B^2H^4}-C^{(1)}_{W^2H^4}) \\ &+\frac{{v_T}^4}{2(\overline{g}^2+\overline{g}'^2)^2\Lambda^4}\begin{pmatrix} 3\overline{g}'^4\,[C_{HB}]^2-\overline{g}'^2[4\overline{g}^2+\overline{g}'^2]\,[C_{HW}]^2 \\ +\,2\overline{g}'^2[2\overline{g}^2-\overline{g}'^2]\,C_{HW}\,C_{HB} \end{pmatrix} \end{aligned} \right], \tag{D.1}$$

$$\frac{g'}{g_Z} = \frac{\overline{g}'}{\sqrt{\overline{g}^2+\overline{g}'^2}} \left[\begin{aligned} &1+\frac{\overline{g}^2 {v_T}^2}{(\overline{g}^2+\overline{g}'^2)\Lambda^2}(C_{HW}-C_{HB}) \\ &+\frac{\overline{g}^2 {v_T}^4}{2(\overline{g}^2+\overline{g}'^2)\Lambda^4}(C^{(1)}_{W^2H^4}-C^{(1)}_{B^2H^4}) \\ &+\frac{{v_T}^4}{2(\overline{g}^2+\overline{g}'^2)^2\Lambda^4}\begin{pmatrix} 3\overline{g}^4\,[C_{HW}]^2-\overline{g}^2[4\overline{g}'^2+\overline{g}^2]\,[C_{HB}]^2 \\ +\,2\overline{g}^2[2\overline{g}'^2-\overline{g}^2]\,C_{HW}\,C_{HB} \end{pmatrix} \end{aligned} \right]. \tag{D.2}$$

D.3 X^3 operators

LEFT WC	**Matching**
$\frac{1}{\Lambda^2}\mathcal{C}_G$	$\frac{1}{\Lambda^2}\left[C_G+\frac{{v_T}^2}{2\Lambda^2}C^{(1)}_{G^3H^2}\right]$
$\frac{1}{\Lambda^2}\mathcal{C}_{\tilde{G}}$	$\frac{1}{\Lambda^2}\left[C_{\tilde{G}}+\frac{{v_T}^2}{2\Lambda^2}C^{(2)}_{G^3H^2}\right]$

D.4 $(\overline{L}L)(\overline{L}L)$ operators

LEFT WC	Matching
$\frac{1}{\Lambda^2} C^{V,LL}_{\underset{prst}{\nu\nu}}$	$\frac{1}{\Lambda^2}\left[C_{\underset{prst}{ll}} + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\underset{prst}{l^4H^2}} - C^{(2)}_{\underset{prst}{l^4H^2}} - C^{(2)}_{\underset{stpr}{l^4H^2}}\right)\right]$ $-\frac{{\overline{g_Z}}^2}{4{M_Z}^2}\left([Z_{\nu_L}]^{\rm eff}_{pr}[Z_{\nu_L}]^{\rm eff}_{st} + [Z_{\nu_L}]^{\rm eff}_{pt}[Z_{\nu_L}]^{\rm eff}_{sr}\right)$
$\frac{1}{\Lambda^2} C^{V,LL}_{\underset{prst}{ee}}$	$\frac{1}{\Lambda^2}\left[C_{\underset{prst}{ll}} + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\underset{prst}{l^4H^2}} + C^{(2)}_{\underset{prst}{l^4H^2}} + C^{(2)}_{\underset{stpr}{l^4H^2}}\right)\right]$ $-\frac{{\overline{g_Z}}^2}{4{M_Z}^2}\left([Z_{e_L}]^{\rm eff}_{pr}[Z_{e_L}]^{\rm eff}_{st} + [Z_{e_L}]^{\rm eff}_{pt}[Z_{e_L}]^{\rm eff}_{sr}\right)$
$\frac{1}{\Lambda^2} C^{V,LL}_{\underset{prst}{\nu e}}$	$\frac{1}{\Lambda^2}\left[\left(C_{\underset{prst}{ll}} + C_{\underset{stpr}{ll}}\right) + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\underset{prst}{l^4H^2}} + C^{(1)}_{\underset{stpr}{l^4H^2}} + C^{(2)}_{\underset{prst}{l^4H^2}} - C^{(2)}_{\underset{stpr}{l^4H^2}}\right)\right]$ $-\frac{{\overline{g_Z}}^2}{{M_Z}^2}[Z_{\nu_L}]^{\rm eff}_{pr}[Z_{e_L}]^{\rm eff}_{st} - \frac{\overline{g}^2}{2{M_W}^2}[W_l]^{\rm eff}_{pt}[W_l]^{\rm eff*}_{rs}$
$\frac{1}{\Lambda^2} C^{V,LL}_{\underset{prst}{uu}}$	$\frac{1}{\Lambda^2}\left[\left(C^{(1)}_{\underset{prst}{qq}} + C^{(3)}_{\underset{prst}{qq}}\right) + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\underset{prst}{q^4H^2}} - C^{(2)}_{\underset{prst}{q^4H^2}} - C^{(2)}_{\underset{stpr}{q^4H^2}} + C^{(3)}_{\underset{prst}{q^4H^2}}\right)\right]$ $-\frac{{\overline{g_Z}}^2}{2{M_Z}^2}[Z_{u_L}]^{\rm eff}_{pr}[Z_{u_L}]^{\rm eff}_{st}$
$\frac{1}{\Lambda^2} C^{V,LL}_{\underset{prst}{dd}}$	$\frac{1}{\Lambda^2}\left[\left(C^{(1)}_{\underset{prst}{qq}} + C^{(3)}_{\underset{prst}{qq}}\right) + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\underset{prst}{q^4H^2}} + C^{(2)}_{\underset{prst}{q^4H^2}} + C^{(2)}_{\underset{stpr}{q^4H^2}} + C^{(3)}_{\underset{prst}{q^4H^2}}\right)\right]$ $-\frac{{\overline{g_Z}}^2}{2{M_Z}^2}[Z_{d_L}]^{\rm eff}_{pr}[Z_{d_L}]^{\rm eff}_{st}$

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LEFT WC	Matching
$\frac{1}{\Lambda^2}C^{V1,LL}_{\substack{ud\\prst}}$	$\frac{1}{\Lambda^2}\left[\left(C^{(1)}_{\substack{qq\\prst}} + C^{(1)}_{\substack{qq\\stpr}} - C^{(3)}_{\substack{qq\\prst}} - C^{(3)}_{\substack{qq\\stpr}} + \frac{2}{3}C^{(3)}_{\substack{qq\\ptsr}} + \frac{2}{3}C^{(3)}_{\substack{qq\\srpt}}\right) + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{q^4H^2\\prst}} + C^{(1)}_{\substack{q^4H^2\\stpr}} + C^{(2)}_{\substack{q^4H^2\\prst}} - C^{(2)}_{\substack{q^4H^2\\stpr}} - C^{(3)}_{\substack{q^4H^2\\prst}} - C^{(3)}_{\substack{q^4H^2\\stpr}} + \frac{2}{3}C^{(3)}_{\substack{q^4H^2\\ptsr}} + \frac{2}{3}C^{(3)}_{\substack{q^4H^2\\srpt}} + \frac{2i}{3}C^{(5)}_{\substack{q^4H^2\\ptsr}} - \frac{2i}{3}C^{(5)}_{\substack{q^4H^2\\srpt}}\right)\right] - \frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{u_L}]^{\text{eff}}_{pr}[Z_{d_L}]^{\text{eff}}_{st} - \frac{\overline{g}^2}{6{M_W}^2}[W_q]^{\text{eff}}_{pt}[W_q]^{\text{eff}*}_{rs}$
$\frac{1}{\Lambda^2}C^{V8,LL}_{\substack{ud\\prst}}$	$\frac{4}{\Lambda^2}\left[\left(C^{(3)}_{\substack{qq\\ptsr}} + C^{(3)}_{\substack{qq\\srpt}}\right) + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(3)}_{\substack{q^4H^2\\ptsr}} + C^{(3)}_{\substack{q^4H^2\\srpt}} - iC^{(5)}_{\substack{q^4H^2\\prst}} + iC^{(5)}_{\substack{q^4H^2\\stpr}}\right)\right] - \frac{\overline{g}^2}{{M_W}^2}[W_q]^{\text{eff}}_{pt}[W_q]^{\text{eff}*}_{rs}$
$\frac{1}{\Lambda^2}C^{V,LL}_{\substack{\nu u\\prst}}$	$\frac{1}{\Lambda^2}\left[\left(C^{(1)}_{\substack{lq\\prst}} + C^{(3)}_{\substack{lq\\prst}}\right) + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{l^2q^2H^2\\prst}} - C^{(2)}_{\substack{l^2q^2H^2\\prst}} + C^{(3)}_{\substack{l^2q^2H^2\\prst}} - C^{(4)}_{\substack{l^2q^2H^2\\prst}}\right)\right] - \frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{\nu_L}]^{\text{eff}}_{pr}[Z_{u_L}]^{\text{eff}}_{st}$
$\frac{1}{\Lambda^2}C^{V,LL}_{\substack{\nu d\\prst}}$	$\frac{1}{\Lambda^2}\left[\left(C^{(1)}_{\substack{lq\\prst}} - C^{(3)}_{\substack{lq\\prst}}\right) + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{l^2q^2H^2\\prst}} - C^{(2)}_{\substack{l^2q^2H^2\\prst}} - C^{(3)}_{\substack{l^2q^2H^2\\prst}} + C^{(4)}_{\substack{l^2q^2H^2\\prst}}\right)\right] - \frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{\nu_L}]^{\text{eff}}_{pr}[Z_{d_L}]^{\text{eff}}_{st}$
$\frac{1}{\Lambda^2}C^{V,LL}_{\substack{eu\\prst}}$	$\frac{1}{\Lambda^2}\left[\left(C^{(1)}_{\substack{lq\\prst}} - C^{(3)}_{\substack{lq\\prst}}\right) + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{l^2q^2H^2\\prst}} + C^{(2)}_{\substack{l^2q^2H^2\\prst}} - C^{(3)}_{\substack{l^2q^2H^2\\prst}} - C^{(4)}_{\substack{l^2q^2H^2\\prst}}\right)\right] - \frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{e_L}]^{\text{eff}}_{pr}[Z_{u_L}]^{\text{eff}}_{st}$
$\frac{1}{\Lambda^2}C^{V,LL}_{\substack{ed\\prst}}$	$\frac{1}{\Lambda^2}\left[\left(C^{(1)}_{\substack{lq\\prst}} + C^{(3)}_{\substack{lq\\prst}}\right) + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{l^2q^2H^2\\prst}} + C^{(2)}_{\substack{l^2q^2H^2\\prst}} + C^{(3)}_{\substack{l^2q^2H^2\\prst}} + C^{(4)}_{\substack{l^2q^2H^2\\prst}}\right)\right] - \frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{e_L}]^{\text{eff}}_{pr}[Z_{d_L}]^{\text{eff}}_{st}$
$\frac{1}{\Lambda^2}C^{V,LL}_{\substack{\nu edu\\prst}} + \text{h.c.}$	$\frac{2}{\Lambda^2}\left[C^{(3)}_{\substack{lq\\prst}} + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(3)}_{\substack{l^2q^2H^2\\prst}} - iC^{(5)}_{\substack{l^2q^2H^2\\prst}}\right)\right] - \frac{\overline{g}^2}{2{M_W}^2}[W_l]^{\text{eff}}_{pr}[W_q]^{\text{eff}*}_{ts} + \text{c.c.}$

D.5 $(\overline{R}R)(\overline{R}R)$ operators

LEFT WC	Matching
$\frac{1}{\Lambda^2}C^{V,RR}_{\substack{ee\\prst}}$	$\frac{1}{\Lambda^2}\left[C_{\substack{ee\\prst}}+\frac{{v_T}^2}{2\Lambda^2}C_{\substack{e^4H^2\\prst}}\right]-\frac{\overline{g_Z}^2}{4{M_Z}^2}\left([Z_{e_R}]^{\text{eff}}_{pr}[Z_{e_R}]^{\text{eff}}_{st}+[Z_{e_R}]^{\text{eff}}_{pt}[Z_{e_R}]^{\text{eff}}_{sr}\right)$
$\frac{1}{\Lambda^2}C^{V,RR}_{\substack{eu\\prst}}$	$\frac{1}{\Lambda^2}\left[C_{\substack{eu\\prst}}+\frac{{v_T}^2}{2\Lambda^2}C_{\substack{e^2u^2H^2\\prst}}\right]-\frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{e_R}]^{\text{eff}}_{pr}[Z_{u_R}]^{\text{eff}}_{st}$
$\frac{1}{\Lambda^2}C^{V,RR}_{\substack{ed\\prst}}$	$\frac{1}{\Lambda^2}\left[C_{\substack{ed\\prst}}+\frac{{v_T}^2}{2\Lambda^2}C_{\substack{e^2d^2H^2\\prst}}\right]-\frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{e_R}]^{\text{eff}}_{pr}[Z_{d_R}]^{\text{eff}}_{st}$
$\frac{1}{\Lambda^2}C^{V,RR}_{\substack{uu\\prst}}$	$\frac{1}{\Lambda^2}\left[C_{\substack{uu\\prst}}+\frac{{v_T}^2}{2\Lambda^2}C_{\substack{u^4H^2\\prst}}\right]-\frac{\overline{g_Z}^2}{2{M_Z}^2}[Z_{u_R}]^{\text{eff}}_{pr}[Z_{u_R}]^{\text{eff}}_{st}$
$\frac{1}{\Lambda^2}C^{V,RR}_{\substack{dd\\prst}}$	$\frac{1}{\Lambda^2}\left[C_{\substack{dd\\prst}}+\frac{{v_T}^2}{2\Lambda^2}C_{\substack{d^4H^2\\prst}}\right]-\frac{\overline{g_Z}^2}{2{M_Z}^2}[Z_{d_R}]^{\text{eff}}_{pr}[Z_{d_R}]^{\text{eff}}_{st}$
$\frac{1}{\Lambda^2}C^{V1,RR}_{\substack{ud\\prst}}$	$\frac{1}{\Lambda^2}\left[C^{(1)}_{\substack{ud\\prst}}+\frac{{v_T}^2}{2\Lambda^2}C^{(1)}_{\substack{u^2d^2H^2\\prst}}\right]-\frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{u_R}]^{\text{eff}}_{pr}[Z_{d_R}]^{\text{eff}}_{st}-\frac{\overline{g}^2}{6{M_W}^2}[W_R]^{\text{eff}}_{pt}[W_R]^{\text{eff}*}_{rs}$
$\frac{1}{\Lambda^2}C^{V8,RR}_{\substack{ud\\prst}}$	$\frac{1}{\Lambda^2}\left[C^{(8)}_{\substack{ud\\prst}}+\frac{{v_T}^2}{2\Lambda^2}C^{(2)}_{\substack{u^2d^2H^2\\prst}}\right]-\frac{\overline{g}^2}{{M_W}^2}[W_R]^{\text{eff}}_{pt}[W_R]^{\text{eff}*}_{rs}$

D.6 $(\overline{L}L)(\overline{R}R)$ operators

LEFT WC	Matching
$\frac{1}{\Lambda^2}C^{V,LR}_{\substack{\nu e\\prst}}$	$\frac{1}{\Lambda^2}\left[C_{\substack{le\\prst}}+\frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{l^2e^2H^2\\prst}}-C^{(2)}_{\substack{l^2e^2H^2\\prst}}\right)\right]-\frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{\nu_L}]^{\text{eff}}_{pr}[Z_{e_R}]^{\text{eff}}_{st}$
$\frac{1}{\Lambda^2}C^{V,LR}_{\substack{ee\\prst}}$	$\frac{1}{\Lambda^2}\left[C_{\substack{le\\prst}}+\frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{l^2e^2H^2\\prst}}+C^{(2)}_{\substack{l^2e^2H^2\\prst}}\right)\right]$ $-\frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{e_L}]^{\text{eff}}_{pr}[Z_{e_R}]^{\text{eff}}_{st}-\frac{1}{2{m_h}^2}(Y_e)^{\text{eff}}_{pt}(Y_e)^{\text{eff}*}_{rs}$
$\frac{1}{\Lambda^2}C^{V,LR}_{\substack{\nu u\\prst}}$	$\frac{1}{\Lambda^2}\left[C_{\substack{lu\\prst}}+\frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{l^2u^2H^2\\prst}}-C^{(2)}_{\substack{l^2u^2H^2\\prst}}\right)\right]-\frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{\nu_L}]^{\text{eff}}_{pr}[Z_{u_R}]^{\text{eff}}_{st}$
$\frac{1}{\Lambda^2}C^{V,LR}_{\substack{eu\\prst}}$	$\frac{1}{\Lambda^2}\left[C_{\substack{lu\\prst}}+\frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{l^2u^2H^2\\prst}}+C^{(2)}_{\substack{l^2u^2H^2\\prst}}\right)\right]-\frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{e_L}]^{\text{eff}}_{pr}[Z_{u_R}]^{\text{eff}}_{st}$
$\frac{1}{\Lambda^2}C^{V,LR}_{\substack{\nu d\\prst}}$	$\frac{1}{\Lambda^2}\left[C_{\substack{ld\\prst}}+\frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{l^2d^2H^2\\prst}}-C^{(2)}_{\substack{l^2d^2H^2\\prst}}\right)\right]-\frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{\nu_L}]^{\text{eff}}_{pr}[Z_{d_R}]^{\text{eff}}_{st}$
$\frac{1}{\Lambda^2}C^{V,LR}_{\substack{ed\\prst}}$	$\frac{1}{\Lambda^2}\left[C_{\substack{ld\\prst}}+\frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{l^2d^2H^2\\prst}}+C^{(2)}_{\substack{l^2d^2H^2\\prst}}\right)\right]-\frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{e_L}]^{\text{eff}}_{pr}[Z_{d_R}]^{\text{eff}}_{st}$
$\frac{1}{\Lambda^2}C^{V,LR}_{\substack{ue\\prst}}$	$\frac{1}{\Lambda^2}\left[C_{\substack{qe\\prst}}+\frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{q^2e^2H^2\\prst}}-C^{(2)}_{\substack{q^2e^2H^2\\prst}}\right)\right]-\frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{u_L}]^{\text{eff}}_{pr}[Z_{e_R}]^{\text{eff}}_{st}$
$\frac{1}{\Lambda^2}C^{V,LR}_{\substack{de\\prst}}$	$\frac{1}{\Lambda^2}\left[C_{\substack{qe\\prst}}+\frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{q^2e^2H^2\\prst}}+C^{(2)}_{\substack{q^2e^2H^2\\prst}}\right)\right]-\frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{d_L}]^{\text{eff}}_{pr}[Z_{e_R}]^{\text{eff}}_{st}$

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LEFT WC	**Matching**
$\frac{1}{\Lambda^2}C^{V1,LR}_{\substack{uu\\prst}}$	$\frac{1}{\Lambda^2}\left[C^{(1)}_{\substack{qu\\prst}} + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{q^2u^2H^2\\prst}} - C^{(2)}_{\substack{q^2u^2H^2\\prst}}\right)\right] - \frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{u_L}]^{\text{eff}}_{pr}[Z_{u_R}]^{\text{eff}}_{st} - \frac{1}{6{m_h}^2}(Y_u)^{\text{eff}}_{pt}(Y_u)^{\text{eff}*}_{rs}$
$\frac{1}{\Lambda^2}C^{V1,LR}_{\substack{du\\prst}}$	$\frac{1}{\Lambda^2}\left[C^{(1)}_{\substack{qu\\prst}} + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{q^2u^2H^2\\prst}} + C^{(2)}_{\substack{q^2u^2H^2\\prst}}\right)\right] - \frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{d_L}]^{\text{eff}}_{pr}[Z_{u_R}]^{\text{eff}}_{st}$
$\frac{1}{\Lambda^2}C^{V8,LR}_{\substack{uu\\prst}}$	$\frac{1}{\Lambda^2}\left[C^{(8)}_{\substack{qu\\prst}} + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(3)}_{\substack{q^2u^2H^2\\prst}} - C^{(4)}_{\substack{q^2u^2H^2\\prst}}\right)\right] - \frac{1}{{m_h}^2}(Y_u)^{\text{eff}}_{pt}(Y_u)^{\text{eff}*}_{rs}$
$\frac{1}{\Lambda^2}C^{V8,LR}_{\substack{du\\prst}}$	$\frac{1}{\Lambda^2}\left[C^{(8)}_{\substack{qu\\prst}} + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(3)}_{\substack{q^2u^2H^2\\prst}} + C^{(4)}_{\substack{q^2u^2H^2\\prst}}\right)\right]$
$\frac{1}{\Lambda^2}C^{V1,LR}_{\substack{ud\\prst}}$	$\frac{1}{\Lambda^2}\left[C^{(1)}_{\substack{qd\\prst}} + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{q^2d^2H^2\\prst}} - C^{(2)}_{\substack{q^2d^2H^2\\prst}}\right)\right] - \frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{u_L}]^{\text{eff}}_{pr}[Z_{d_R}]^{\text{eff}}_{st}$
$\frac{1}{\Lambda^2}C^{V1,LR}_{\substack{dd\\prst}}$	$\frac{1}{\Lambda^2}\left[C^{(1)}_{\substack{qd\\prst}} + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{q^2d^2H^2\\prst}} + C^{(2)}_{\substack{q^2d^2H^2\\prst}}\right)\right] - \frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{d_L}]^{\text{eff}}_{pr}[Z_{d_R}]^{\text{eff}}_{st} - \frac{1}{6{m_h}^2}(Y_d)^{\text{eff}}_{pt}(Y_d)^{\text{eff}*}_{rs}$
$\frac{1}{\Lambda^2}C^{V8,LR}_{\substack{ud\\prst}}$	$\frac{1}{\Lambda^2}\left[C^{(8)}_{\substack{qd\\prst}} + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(3)}_{\substack{q^2d^2H^2\\prst}} - C^{(4)}_{\substack{q^2d^2H^2\\prst}}\right)\right]$
$\frac{1}{\Lambda^2}C^{V8,LR}_{\substack{dd\\prst}}$	$\frac{1}{\Lambda^2}\left[C^{(8)}_{\substack{qd\\prst}} + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(3)}_{\substack{q^2d^2H^2\\prst}} + C^{(4)}_{\substack{q^2d^2H^2\\prst}}\right)\right] - \frac{1}{{m_h}^2}(Y_d)^{\text{eff}}_{pt}(Y_d)^{\text{eff}*}_{rs}$
$\frac{1}{\Lambda^2}C^{V,LR}_{\substack{\nu edu\\prst}}+\text{h.c.}$	$\frac{{v_T}^2}{4\Lambda^4}\,{C_{\substack{l^2udH^2\\tprs}}}^* - \frac{\overline{g}^2}{2{M_W}^2}[W_l]^{\text{eff}}_{pr}[W_R]^{\text{eff}*}_{ts}+\text{c.c.}$
$\frac{1}{\Lambda^2}C^{V1,LR}_{\substack{uddu\\prst}}+\text{h.c.}$	$\frac{{v_T}^2}{2\Lambda^4}\left(\frac{1}{6}{C^{(5)}_{\substack{q^2udH^2\\tprs}}}^* + \frac{2}{9}{C^{(6)}_{\substack{q^2udH^2\\tprs}}}^*\right) - \frac{\overline{g}^2}{2{M_W}^2}[W_q]^{\text{eff}}_{pr}[W_R]^{\text{eff}*}_{ts} - \frac{1}{6{m_h}^2}(Y_u)^{\text{eff}}_{pt}(Y_d)^{\text{eff}*}_{rs}+\text{c.c.}$
$\frac{1}{\Lambda^2}C^{V8,LR}_{\substack{uddu\\prst}}+\text{h.c.}$	$\frac{{v_T}^2}{2\Lambda^4}\left({C^{(5)}_{\substack{q^2udH^2\\tprs}}}^* - \frac{1}{6}{C^{(6)}_{\substack{q^2udH^2\\tprs}}}^*\right) - \frac{1}{{m_h}^2}(Y_u)^{\text{eff}}_{pt}(Y_d)^{\text{eff}*}_{rs}+\text{c.c.}$

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D.7 $(\overline{L}R)(\overline{L}R)$ operators

LEFT WC (+c.c.)	**Matching (+c.c.)**
$\frac{1}{\Lambda^2}\underset{prst}{C^{S,RR}_{ee}}$	$\frac{{v_T}^2}{2\Lambda^4}\underset{prst}{C^{(3)}_{l^2e^2H^2}} + \frac{1}{2{m_h}^2}(Y_e)^{\rm eff}_{pr}(Y_e)^{\rm eff}_{st}$
$\frac{1}{\Lambda^2}\underset{prst}{C^{S,RR}_{eu}}$	$\frac{1}{\Lambda^2}\left[-\underset{prst}{C^{(1)}_{lequ}} + \frac{{v_T}^2}{2\Lambda^2}\left(-\underset{prst}{C^{(1)}_{lequH^2}} - \underset{prst}{C^{(2)}_{lequH^2}}\right)\right] + \frac{1}{{m_h}^2}(Y_e)^{\rm eff}_{pr}(Y_u)^{\rm eff}_{st}$
$\frac{1}{\Lambda^2}\underset{prst}{C^{T,RR}_{eu}}$	$\frac{1}{\Lambda^2}\left[-\underset{prst}{C^{(3)}_{lequ}} + \frac{{v_T}^2}{2\Lambda^2}\left(-\underset{prst}{C^{(3)}_{lequH^2}} - \underset{prst}{C^{(4)}_{lequH^2}}\right)\right]$
$\frac{1}{\Lambda^2}\underset{prst}{C^{S,RR}_{ed}}$	$\frac{{v_T}^2}{2\Lambda^4}\underset{prst}{C^{(3)}_{leqdH^2}} + \frac{1}{{m_h}^2}(Y_e)^{\rm eff}_{pr}(Y_d)^{\rm eff}_{st}$
$\frac{1}{\Lambda^2}\underset{prst}{C^{T,RR}_{ed}}$	$\frac{{v_T}^2}{2\Lambda^4}\underset{prst}{C^{(4)}_{leqdH^2}}$
$\frac{1}{\Lambda^2}\underset{prst}{C^{S,RR}_{\nu edu}}$	$\frac{1}{\Lambda^2}\left[\underset{prst}{C^{(1)}_{lequ}} + \frac{{v_T}^2}{2\Lambda^2}\left(\underset{prst}{C^{(1)}_{lequH^2}} - \underset{prst}{C^{(2)}_{lequH^2}}\right)\right]$
$\frac{1}{\Lambda^2}\underset{prst}{C^{T,RR}_{\nu edu}}$	$\frac{1}{\Lambda^2}\left[\underset{prst}{C^{(3)}_{lequ}} + \frac{{v_T}^2}{2\Lambda^2}\left(\underset{prst}{C^{(3)}_{lequH^2}} - \underset{prst}{C^{(4)}_{lequH^2}}\right)\right]$
$\frac{1}{\Lambda^2}\underset{prst}{C^{S1,RR}_{uu}}$	$\frac{{v_T}^2}{2\Lambda^4}\underset{prst}{C^{(5)}_{q^2u^2H^2}} + \frac{1}{2{m_h}^2}(Y_u)^{\rm eff}_{pr}(Y_u)^{\rm eff}_{st}$
$\frac{1}{\Lambda^2}\underset{prst}{C^{S8,RR}_{uu}}$	$\frac{{v_T}^2}{2\Lambda^4}\underset{prst}{C^{(6)}_{q^2u^2H^2}}$
$\frac{1}{\Lambda^2}\underset{prst}{C^{S1,RR}_{ud}}$	$\frac{1}{\Lambda^2}\left[\underset{prst}{C^{(1)}_{quqd}} + \frac{{v_T}^2}{2\Lambda^2}\left(\underset{prst}{C^{(1)}_{q^2udH^2}} - \underset{prst}{C^{(2)}_{q^2udH^2}}\right)\right] + \frac{1}{{m_h}^2}(Y_u)^{\rm eff}_{pr}(Y_d)^{\rm eff}_{st}$
$\frac{1}{\Lambda^2}\underset{prst}{C^{S8,RR}_{ud}}$	$\frac{1}{\Lambda^2}\left[\underset{prst}{C^{(8)}_{quqd}} + \frac{{v_T}^2}{2\Lambda^2}\left(\underset{prst}{C^{(3)}_{q^2udH^2}} - \underset{prst}{C^{(4)}_{q^2udH^2}}\right)\right]$
$\frac{1}{\Lambda^2}\underset{prst}{C^{S1,RR}_{dd}}$	$\frac{{v_T}^2}{2\Lambda^4}\underset{prst}{C^{(5)}_{q^2d^2H^2}} + \frac{1}{2{m_h}^2}(Y_d)^{\rm eff}_{pr}(Y_d)^{\rm eff}_{st}$
$\frac{1}{\Lambda^2}\underset{prst}{C^{S8,RR}_{dd}}$	$\frac{{v_T}^2}{2\Lambda^4}\underset{prst}{C^{(6)}_{q^2d^2H^2}}$
$\frac{1}{\Lambda^2}\underset{prst}{C^{S1,RR}_{uddu}}$	$\frac{1}{\Lambda^2}\left[-\underset{stpr}{C^{(1)}_{quqd}} + \frac{{v_T}^2}{2\Lambda^2}\left(-\underset{stpr}{C^{(1)}_{q^2udH^2}} - \underset{stpr}{C^{(2)}_{q^2udH^2}}\right)\right]$
$\frac{1}{\Lambda^2}\underset{prst}{C^{S8,RR}_{uddu}}$	$\frac{1}{\Lambda^2}\left[-\underset{prst}{C^{(8)}_{quqd}} + \frac{{v_T}^2}{2\Lambda^2}\left(-\underset{stpr}{C^{(3)}_{q^2udH^2}} - \underset{stpr}{C^{(4)}_{q^2udH^2}}\right)\right]$

D.8 $(\overline{L}R)(\overline{R}L)+$ h.c. operators

LEFT WC (+c.c.)	Matching (+c.c.)
$\frac{1}{\Lambda^2} C^{S,RL}_{\substack{eu \\ prst}}$	$\frac{{v_T}^2}{2\Lambda^4} C^{(5)}_{\substack{lequH^2 \\ prst}} + \frac{1}{{m_h}^2} (Y_e)^{\text{eff}}_{pr} (Y_u)^{\text{eff}*}_{ts}$
$\frac{1}{\Lambda^2} C^{S,RL}_{\substack{ed \\ prst}}$	$\frac{1}{\Lambda^2} \left[C_{\substack{ledq \\ prst}} + \frac{{v_T}^2}{2\Lambda^2} \left(C^{(1)}_{\substack{leqdH^2 \\ prst}} + C^{(2)}_{\substack{leqdH^2 \\ prst}} \right) \right] + \frac{1}{{m_h}^2} (Y_e)^{\text{eff}}_{pr} (Y_d)^{\text{eff}*}_{ts}$
$\frac{1}{\Lambda^2} C^{S,RL}_{\substack{\nu edu \\ prst}} + \text{h.c.}$	$\frac{1}{\Lambda^2} \left[C_{\substack{ledq \\ prst}} + \frac{{v_T}^2}{2\Lambda^2} \left(C^{(1)}_{\substack{leqdH^2 \\ prst}} - C^{(2)}_{\substack{leqdH^2 \\ prst}} \right) \right]$

D.9 $\Delta L = 4+$ h.c. operator

LEFT WC (+c.c.)	Matching (+c.c.)
$\frac{1}{\Lambda^2} C^{S,LL}_{\substack{\nu\nu \\ prst}}$	0

D.10 $\Delta L = 2+$ h.c. operators

LEFT WC (+c.c.)	Matching (+c.c.)
$\frac{1}{\Lambda^2} C^{S,LL}_{\underset{prst}{\nu e}}$	$\frac{v_T}{2\sqrt{2}\,\Lambda^3}\left[\left(C_{\underset{prst}{l^3eH}} + C_{\underset{rpst}{l^3eH}}\right) + \frac{1}{2}\left(C_{\underset{tpsr}{l^3eH}} + C_{\underset{trsp}{l^3eH}}\right)\right]$
$\frac{1}{\Lambda^2} C^{T,LL}_{\underset{prst}{\nu e}}$	$\frac{v_T}{16\sqrt{2}\,\Lambda^3}\left(C_{\underset{tpsr}{l^3eH}} - C_{\underset{trsp}{l^3eH}}\right)$
$\frac{1}{\Lambda^2} C^{S,LR}_{\underset{prst}{\nu e}}$	$\frac{\bar{g}^2}{2{M_W}^2}\left([W_l^{\not{L}}]^{\text{eff}}_{pt}[W_l]^{\text{eff}*}_{rs} + [W_l^{\not{L}}]^{\text{eff}}_{rt}[W_l]^{\text{eff}*}_{ps}\right)$
$\frac{1}{\Lambda^2} C^{S,LL}_{\underset{prst}{\nu u}}$	0
$\frac{1}{\Lambda^2} C^{T,LL}_{\underset{prst}{\nu u}}$	0
$\frac{1}{\Lambda^2} C^{S,LR}_{\underset{prst}{\nu u}}$	$\frac{v_T}{2\sqrt{2}\,\Lambda^3}\left(C_{\underset{prst}{l^2quH}} + C_{\underset{rpst}{l^2quH}}\right)$
$\frac{1}{\Lambda^2} C^{S,LL}_{\underset{prst}{\nu d}}$	$\frac{v_T}{2\sqrt{2}\,\Lambda^3}\left(C^{(1)}_{\underset{prst}{l^2dqH}} + C^{(1)}_{\underset{rpst}{l^2dqH}}\right)$
$\frac{1}{\Lambda^2} C^{T,LL}_{\underset{prst}{\nu d}}$	$\frac{v_T}{2\sqrt{2}\,\Lambda^3}\left(C^{(2)}_{\underset{prst}{l^2dqH}} - C^{(2)}_{\underset{rpst}{l^2dqH}}\right)$
$\frac{1}{\Lambda^2} C^{S,LR}_{\underset{prst}{\nu d}}$	0
$\frac{1}{\Lambda^2} C^{S,LL}_{\underset{prst}{\nu edu}}$	$-\frac{v_T}{\sqrt{2}\,\Lambda^3} C^{(1)}_{\underset{prst}{l^2qdH}}$
$\frac{1}{\Lambda^2} C^{T,LL}_{\underset{prst}{\nu edu}}$	$-\frac{v_T}{\sqrt{2}\,\Lambda^3} C^{(2)}_{\underset{prst}{l^2qdH}}$
$\frac{1}{\Lambda^2} C^{S,LR}_{\underset{prst}{\nu edu}}$	$\frac{v_T}{\sqrt{2}\,\Lambda^3} C_{\underset{prst}{l^2quH}}$
$\frac{1}{\Lambda^2} C^{V,RL}_{\underset{prst}{\nu edu}}$	$-\frac{\bar{g}^2}{2{M_W}^2}[W_l^{\not{L}}]^{\text{eff}}_{pr}[W_q]^{\text{eff}*}_{ts}$
$\frac{1}{\Lambda^2} C^{V,RR}_{\underset{prst}{\nu edu}}$	$\frac{v_T}{\sqrt{2}\,\Lambda^3} C_{\underset{prst}{leduH}}$

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D.11 $\Delta B = \Delta L = 1+$ h.c. operators

LEFT WC (+c.c.)	Matching (+c.c.)
$\frac{1}{\Lambda^2} C^{S,LL}_{\substack{udd\\prst}}$	$\frac{1}{\Lambda^2}\left[\left(C_{\substack{qqq\\rpst}} + C_{\substack{qqq\\srpt}} - C_{\substack{qqq\\rspt}}\right) + \frac{v_T^2}{2\Lambda^2}\left(C^{(1)}_{\substack{lq^3H^2\\rpst}} + C^{(1)}_{\substack{lq^3H^2\\srpt}} - C^{(1)}_{\substack{lq^3H^2\\rspt}} + C^{(2)}_{\substack{lq^3H^2\\rpst}} + C^{(2)}_{\substack{lq^3H^2\\srpt}} - C^{(2)}_{\substack{lq^3H^2\\rspt}} - C^{(3)}_{\substack{lq^3H^2\\rpst}} - C^{(3)}_{\substack{lq^3H^2\\srpt}} + C^{(3)}_{\substack{lq^3H^2\\rspt}}\right)\right]$
$\frac{1}{\Lambda^2} C^{S,LL}_{\substack{duu\\prst}}$	$\frac{1}{\Lambda^2}\left[\left(C_{\substack{qqq\\rpst}} + C_{\substack{qqq\\srpt}} - C_{\substack{qqq\\rspt}}\right) + \frac{v_T^2}{2\Lambda^2}\left(C^{(1)}_{\substack{lq^3H^2\\rpst}} + C^{(1)}_{\substack{lq^3H^2\\srpt}} - C^{(1)}_{\substack{lq^3H^2\\rspt}} - C^{(2)}_{\substack{lq^3H^2\\rpst}} - C^{(2)}_{\substack{lq^3H^2\\srpt}} + C^{(2)}_{\substack{lq^3H^2\\rspt}} + C^{(3)}_{\substack{lq^3H^2\\rpst}} + C^{(3)}_{\substack{lq^3H^2\\srpt}} - C^{(3)}_{\substack{lq^3H^2\\rspt}}\right)\right]$
$\frac{1}{\Lambda^2} C^{S,LR}_{\substack{uud\\prst}}$	$\frac{v_T^2}{2\Lambda^4} C_{\substack{eq^2dH^2\\tspr}}$
$\frac{1}{\Lambda^2} C^{S,LR}_{\substack{duu\\prst}}$	$-\frac{1}{\Lambda^2}\left[\left(C_{\substack{qqu\\prst}} + C_{\substack{qqu\\rpst}}\right) + \frac{v_T^2}{2\Lambda^2} C_{\substack{eq^2uH^2\\rpst}}\right]$
$\frac{1}{\Lambda^2} C^{S,RL}_{\substack{uud\\prst}}$	$\frac{v_T^2}{2\Lambda^4} C_{\substack{lqu^2H^2\\tspr}}$
$\frac{1}{\Lambda^2} C^{S,RL}_{\substack{duu\\prst}}$	$\frac{1}{\Lambda^2}\left[C_{\substack{duq\\prst}} + \frac{v_T^2}{2\Lambda^2}\left(C^{(1)}_{\substack{lqudH^2\\prst}} + C^{(2)}_{\substack{lqudH^2\\prst}}\right)\right]$
$\frac{1}{\Lambda^2} C^{S,RL}_{\substack{dud\\prst}}$	$\frac{1}{\Lambda^2}\left[-C_{\substack{duq\\prst}} + \frac{v_T^2}{2\Lambda^2} C^{(2)}_{\substack{lqudH^2\\prst}}\right]$
$\frac{1}{\Lambda^2} C^{S,RL}_{\substack{ddu\\prst}}$	$\frac{v_T^2}{2\Lambda^4} C_{\substack{lqd^2H^2\\tspr}}$
$\frac{1}{\Lambda^2} C^{S,RR}_{\substack{duu\\prst}}$	$\frac{1}{\Lambda^2}\left[C_{\substack{duu\\prst}} + \frac{v_T^2}{2\Lambda^2} C_{\substack{eu^2dH^2\\prst}}\right]$

D.12 $\Delta B = -\Delta L = 1+$ h.c. operators

LEFT WC (+c.c.)	Matching (+c.c.)
$\frac{1}{\Lambda^2} C^{S,LL}_{\substack{ddd\\prst}}$	0
$\frac{1}{\Lambda^2} C^{S,LR}_{\substack{udd\\prst}}$	$-\frac{v_T}{\sqrt{2}\,\Lambda^3} C_{\substack{q^2ldH\\prst}}$
$\frac{1}{\Lambda^2} C^{S,LR}_{\substack{ddu\\prst}}$	0
$\frac{1}{\Lambda^2} C^{S,LR}_{\substack{ddd\\prst}}$	$-\frac{v_T}{2\sqrt{2}\,\Lambda^3} \left(C_{\substack{q^2ldH\\prst}} - C_{\substack{q^2ldH\\rpst}} \right)$
$\frac{1}{\Lambda^2} C^{S,RL}_{\substack{ddd\\prst}}$	$-\frac{v_T}{\sqrt{2}\,\Lambda^3} C_{\substack{eqd^2H\\prst}}$
$\frac{1}{\Lambda^2} C^{S,RR}_{\substack{udd\\prst}}$	$\frac{v_T}{\sqrt{2}\,\Lambda^3} C_{\substack{ud^2lH\\prst}}$
$\frac{1}{\Lambda^2} C^{S,RR}_{\substack{ddd\\prst}}$	$\frac{v_T}{\sqrt{2}\,\Lambda^3} C_{\substack{d^3lH\\prst}}$

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