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Canonical interpretation of the $X(4140)$ state within the $^3P_0$ model

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Abstract Recently, the LHCb Collaboration has confirmed the state $X(4140)$, with a mass $M = 4146.5 \pm 4.5^{+4.6}_{-2.8}$ MeV, and a much larger width $\Gamma = 83 \pm 21^{+21}_{-14}$ MeV than the previous experimental measurements, which has confused the understanding of its nature. We will investigate the possibility of the $\chi_{c1}(3P)$ interpretation for the $X(4140)$, considering the mass spectra predicted in the quark model, and the strong decay properties within the $^3P_0$ model. We also predict the strong decay properties of the charmonium states $\chi_{c0}(3P)$ and $\chi_{c2}(3P)$. Our results show that the $X(4140)$ state with the small width given in PDG can be explained as the charmonium state $\chi_{c1}(3P)$ in the $^3P_0$ model, and high precision measurement of the width of the $X(4140)$ is crucial to understand its nature.

1 Introduction

Since the $X(3872)$ was discovered in 2003 by the Belle Collaboration [1], a lot of unexpected states (charmonium-like states or XYZ states) have been reported experimentally [2]. Most of them have strange properties, and are difficult to be interpreted as the charmonium states, which makes them more like exotic states [3–6].

In 2009, a new-threshold $X(4140)$ state was first reported in the $B^+ \to J/\psi\phi K^+$ process by the CDF Collaboration [7], with a statistical significance of the signal 3.8σ. This state was confirmed in the same process by the CMS [8] and D0 Collaborations [9,10], and also in the reanalyzed the $B^\pm \to J/\psi\phi K^\pm$ process with a larger data sample by the CDF Collaboration[11]. However, the Belle, LHCb, and Babar Collaborations have not found the signal of this state [12–14]. Since the $X(4140)$ is only seen in the $J/\psi\phi$ channel, which is OZI suppressed for the charmonium assignment, the hidden charm decay of this state disfavors the explanation of the charmonium $\chi_{cJ}(3P)$ [15]. There are a lot of theoretical interests about its properties, such as charmonium state, molecular state, tetraquark state, hybrid state, or a rescattering effect (more information can be found in the reviews [4,6]).

In 2017, the LHCb Collaboration has also confirmed this state with high statistic data [16,17][1] with a mass $4146.5 \pm 4.5^{+4.6}_{-2.8}$ MeV and a width $83 \pm 21^{+21}_{-14}$ MeV, much larger than the previous experimental measurements (see the Table 1), and the quantum numbers of this state were determined to be $J^{PC} = 1^{++}$. Thus, the $D_s^*\bar{D}_s^*$ molecular explanation, which prefers the quantum numbers $J^{PC} = 0^{++}$ or 2^{++}, is ruled out [18–25].

However, the $X(4140)$ is still the subject of much theoretical work, and there are many different suggestions about its structure [26–31]. For instance, Ref. [27] regards the $X(4140)$ as the $cs\bar{c}\bar{s}$ tetraquark ground state. The $X(4140)$ state with the assignment of the $\chi_{c1}(3P)$ state is predicted to have a small width in Ref. [28]. In Ref. [29], the partial width of the decay mode $X(4140) \to J/\psi\phi$ is predicted to be 86.9 ± 22.6 MeV, with the axial-vector tetraquark picture for the $X(4140)$. In addition, the width of the $X(4140)$ is predicted to be 80 ± 29 MeV with in the interpretation of the color triplet diquark–antidiquark state [31], and Refs. [32,33] have claimed that the structure of the $X(4140)$ may be the cusp due to the presence of the $D_s^{*+}D_s^-$ (or $D_s^{*-}D_s^+$) threshold. Recently, Ref. [34] points out that it is not possible to claim the molecular or diquark–antidiquark content of the $X(4140)$ within the QCD sum rules.

Indeed, it is natural and necessary to exhaust the possible $q\bar{q}$ description of the observed states before restoring to the more exotic assignments. While the ground states of the P-wave charmonium states, $\chi_{cJ}(1P)$, have been well established, and the first radial excitations, $\chi_{cJ}(2P)$, are predicted to have the mass around 3900 MeV [2,35–39], the $X(4140)$,

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[1] It should be stressed that LHCb have preformed twice analyses of the reaction $B^+ \to J/\psi\phi K^+$ respectively in 2012 and 2017, and the first analysis of Ref. [13] has not found the evidence of $X(4140)$.

Table 1 The experimental measurements of the $X(4140)$ (in MeV)

Exp.	Mass	Width	Sig.	Year
CDF [7]	$4143.0 \pm 2.9 \pm 1.2$	$11.7^{+8.3}_{-5.0} \pm 3.7$	3.8σ	2009
CMS [8]	$4148.0 \pm 2.4 \pm 6.3$	$28^{+15}_{-11} \pm 19$	5.0σ	2014
D0 [9]	$4159.0 \pm 4.3 \pm 6.6$	$20 \pm 13^{+3}_{-8}$	3.0σ	2014
D0 [10]	$4152.5 \pm 1.7^{+6.2}_{-5.4}$	$16.3 \pm 5.6 \pm 11.4$	4.7σ	2015
CDF [11]	$4143.4^{+2.9}_{-3.0} \pm 0.6$	$15.3^{+10.4}_{-6.1} \pm 2.5$	5.0σ	2011
LHCb [17]	$4146.5 \pm 4.5^{+4.6}_{-2.8}$	$83 \pm 21^{+21}_{-14}$	8.4σ	2017
PDG [2]	4146.8 ± 2.4	22^{+8}_{-7}		2019

with the quantum numbers of $J^{PC} = 1^{++}$, could be the second radial excitation $\chi_{c1}(3P)$, with the predicted mass of 4100–4200 MeV in the quark model [37,40]. It should be noted that the mass information alone is insufficient to classify the $X(4140)$, so its decay behaviors also need to be compared with model expectations.

In this work, taking the meson wave functions obtained from the relativistic/non-relativistic quark models, we will investigate the decay properties of the $X(4140)$ as the assignment of charmonium state in the $^3P_0$ model, and provide more information about the decay modes, since the observation of the $X(4140)$ in other channels could be useful to extract its width more precisely.

This paper is organized as follows. In Sect. 2, we will present a brief review of the $^3P_0$ decay model, and in Sect. 3, we will introduce two kinds of wave functions for the mesons. The results and discussions are shown in Sect. 4. Finally, the summary is given in Sect. 5.

2 The $^3P_0$ decay model

In this section, we will present the $^3P_0$ model, which is used to evaluate the Okubo–Zweig–Iizuka (OZI) allowed open charm decays of the $\chi_{cJ}(3P)$. The $^3P_0$ model, also known as the quark-pair creation model, was originally introduced by Micu [41] and further developed by Le Yaouanc et al. [42–44]. The $^3P_0$ model has been widely applied to study strong decays of hadrons with considerable success [45–62]. In this model, the strong decay of hadron occurs through a quark–antiquark pair created from the vacuum with the vacuum quantum number $J^{PC} = 0^{++}$, then the new quark–antiquark pair, together with the $q\bar{q}$ within the initial meson, regroups into two outgoing mesons in all possible quark rearrangement ways, as shown in Fig. 1.[2]

[2] It should be pointed out that these two diagrams of Fig. 1 are different, and they will give flavor weight factors for a specified flavor channel [47]. For instance, the process of ρ^+ (A) decay to π^+ (B) and π^0(c) can perform as left diagram by creating $\bar{d}d$ quark pair, and also can perform as right diagram by creating $\bar{u}u$ pair.

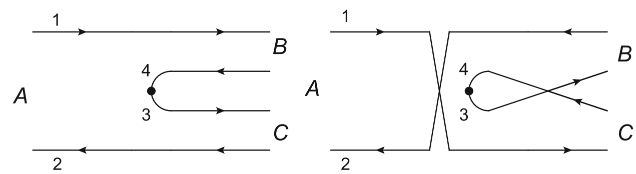

Fig. 1 The two possible diagrams contributing to $A \to BC$ in the $^3P_0$ model: (left) the quark within the meson A combines with the created antiquark to form the meson B, the antiquark within the meson A combines with the created quark to form the meson C; (right) the quark within the meson A combines with the created antiquark to form the meson C, the antiquark within the meson A combines with the created quark to form the meson B

The transition operator T of the decay $A \to BC$ in the $^3P_0$ model can be written,

$$T = -3\gamma \sum_m \langle 1m1-m|00\rangle \int d^3\boldsymbol{p}_3 d^3\boldsymbol{p}_4 \delta^3(\boldsymbol{p}_3+\boldsymbol{p}_4) \times \mathcal{Y}_1^m\left(\frac{\boldsymbol{p}_3-\boldsymbol{p}_4}{2}\right) \chi^{34}_{1,-m}\phi^{34}_0\omega^{34}_0 b^\dagger_3(\boldsymbol{p}_3) d^\dagger_4(\boldsymbol{p}_4), \quad (1)$$

where γ is a dimensionless parameter corresponding to the strength of quark–antiquark $q_3\bar{q}_4$ pair produced from the vacuum, and $\boldsymbol{p}_3$ and $\boldsymbol{p}_4$ are the momenta of the created quark q_3 and antiquark $\bar{q}_4$, respectively. $\chi^{34}_{1,-m}$, ϕ^{34}_0, and ω^{34}_0 are the spin, flavor, and color wave functions of the $q_3\bar{q}_4$, respectively. The solid harmonic polynomial $\mathcal{Y}_1^m(\boldsymbol{p}) \equiv |p|^1 Y_1^m(\theta_p, \phi_p)$ reflects the momentum–space distribution of the $q_3\bar{q}_4$.

The partial wave amplitude $\mathcal{M}^{LS}(\boldsymbol{P})$ of the decay $A \to BC$ can be given by [63,64],

$$\mathcal{M}^{LS}(\boldsymbol{P}) = \sum_{\substack{M_{J_B}, M_{J_C}, \\ M_S, M_L}} \langle LM_L SM_S|J_A M_{J_A}\rangle \langle J_B M_{J_B} J_C M_{J_C}|SM_S\rangle \times \int d\Omega\, Y^*_{LM_L} \mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}(\boldsymbol{P}), \quad (2)$$

where $\mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}(\boldsymbol{P})$ is the helicity amplitude and defined as,

$$\langle BC|T|A\rangle = \delta^3(\boldsymbol{P}_A - \boldsymbol{P}_B - \boldsymbol{P}_C)\mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}(\boldsymbol{P}). \quad (3)$$

The $|A\rangle$, $|B\rangle$, and $|C\rangle$ denote the mock meson states defined by Ref. [65].

Due to different choices of the pair-production vertex, phase space convention, employed meson space wave function, various $^3P_0$ models exist in literature. In this work, we employ the simplest vertex as introduced originally by Micu which assumes a spatially constant pair-production strength γ [41], and adopt the relativistic phase space. We will take into account the two choices of the wave functions for mesons, which will be presented in next section. Finally, the decay width $\Gamma(A \to BC)$ can be expressed in terms of the partial wave amplitude,

$$\Gamma(A \to BC) = \frac{\pi|\boldsymbol{P}|}{4M_A^2}\sum_{LS}|\mathcal{M}^{LS}(\boldsymbol{P})|^2, \quad (4)$$

where $|\boldsymbol{P}| = \frac{\sqrt{[M_A^2-(M_B+M_C)^2][M_A^2-(M_B-M_C)^2]}}{2M_A}$, and M_A, M_B, and M_C are the masses of the meson A, B, and C, respectively. The explicit expressions for $\mathcal{M}^{LS}(\boldsymbol{P})$ can be found in Refs. [53–55].

3 Wave functions

In this section, we will present two choices of wave functions for the charmonium states, charm and charmed-strange mesons, which will be used to calculate the $\chi_{cJ}(3P)$ strong decay widths.

As discussed in Ref. [37], the quenched quark models, which incorporates a coulomb term at short distances and the linear confining interaction at large distances, will not be reliable in the domain beyond the open-charm threshold. This is because the linear potential, which is expected to be dominant in this mass region, will be screened or softened by the vacuum polarization effects of dynamical fermions. We will adopt the Godfrey–Isgur model and non-relativistic quark model, modified to incorporate the screening potential to account for the screening effects. In the following, we will see that both the modified Godfrey–Isgur model and modified non-relativistic quark model could provide a nice description for the high excited charmonium states.

3.1 Non-relativistic quark model

For the wave functions of the open charm mesons in the final states, we use the non-relativistic quark model (NRQM), proposed by Lakhina and Swanson [66]. This non-relativistic quark model has been successfully used to describe the mass spectrum of charm and charmed-strange mesons [59,66], bottom mesons [62].

For the open charm mesons, the total Hamiltonian can be written as [37]

$$H = H_0 + H_{sd} + C_{q\bar{q}}, \quad (5)$$

where H_0 is the zeroth-order Hamiltonian, H_{sd} is the spin-dependent Hamiltonian, and $C_{q\bar{q}}$ is a constant. The H_0 can be compressed as

$$H_0 = \frac{\boldsymbol{p}^2}{M_r} - \frac{4}{3}\frac{\alpha_s}{r} + br, \quad (6)$$

where $\boldsymbol{p}$ is the center-of-mass momentum, r is the $q\bar{q}$ separation, $M_r = 2m_q m_{\bar{q}}/(m_q + m_{\bar{q}})$, m_q and $m_{\bar{q}}$ are the masses of quark q and anti-quark $\bar{q}$, respectively, $b = 0.14$ GeV^2 is the linear potential slope and $\alpha_s = 0.5$ is the coefficient of Coulomb potential [59,66]. The explicit expression of the H_{sd} and the corresponding parameters are given in Refs. [59,66]. We have tabulated the spectra of charm and charmed-strange mesons in Tables 2 and 3, respectively, which are same as those of Ref. [59].

For the wave functions of the charmonium states, we will use the modified non-relativistic quark model (MNRQM) by taking into account the screening effect, as discussed in Ref. [37]. When screening effect is considered, the modification can be accomplished by the transformation

$$br \to V^{scr}(r) = \frac{b(1-e^{\mu r})}{\mu}, \quad (7)$$

where $\mu = 0.0979$ GeV is the characteristic scale for color screening, and $b = 0.21$ GeV^2 [37]. The mass spectra of the charmonium states are shown in Table 4, which are same as those of Ref. [37].

3.2 Modified Godfrey–Isgur model

In addition to the non-relativistic quark model, the Godfrey–Isgur (GI) relativistic quark model [67] is one of the most successful models describing mass spectrum of mesons. Because the coupled-channel effect becomes more important for higher radial and orbital excitations, the modified relativistic quark model was proposed [68,69] and widely used to calculate mass spectrum of charm meson [68], charmed-strange meson [69], charmonium [40] and bottomonium [70]. In the relativistic quark model, the Hamiltonian of a meson system is [67]

$$\tilde{H} = \left(p^2 + m_q^2\right)^{1/2} + \left(p^2 + m_{\bar{q}}^2\right)^{1/2} \quad (8)$$

$$+\tilde{H}_{q\bar{q}}^{\rm conf} + \tilde{H}_{q\bar{q}}^{\rm so} + \tilde{H}_{q\bar{q}}^{\rm hyp}. \quad (9)$$

Table 2 The mass spectra (in MeV) of charm mesons obtained within the non-relativistic quark model (NRQM) and the modified Godfrey–Isgur Model (MGI)

$n^{2S+1}L_J$	States	PDG [2]	NRQM [59]	MGI [68]
(1^1S_0)	D	$1864.83 \pm 0.05/1869.65 \pm 0.05$	1867	1861
(1^3S_1)	D^*	$2006.85 \pm 0.05/2010.26 \pm 0.05$	2010	2020
(2^1S_0)	$D(2550)$	2564 ± 20	2555	2534
(2^3S_1)			2636	2593
(3^1S_0)			3047	2976
(3^3S_1)			3109	3015
(4^1S_0)			3464	3326
(4^3S_1)			3516	3353
$(1P)$	$D_1(2420)$	2420.8 ± 0.5	2402	2426
(1^3P_0)	$D_0^*(2400)$	2300 ± 19	2252	2365
$(1P')$	$D_1(2430)$	2427 ± 40	2417	2431
(1^3P_2)	$D_2^*(2460)$	$2460.7 \pm 0.4/2465.4 \pm 1.3$	2466	2468
$(2P)$			2886	2861
(2^3P_0)			2752	2856
$(2P')$			2929	2877
(2^3P_2)			2971	2884
$(1D)$			2693	2773
(1^3D_1)			2740	2762
$(1D')$			2789	2779
(1^3D_3)	$D_3^*(2750)$	2763.5 ± 3.4	2719	2779
$(2D)$			3145	3128
(2^3D_1)			3168	3131
$(2D')$			3215	3136
(2^3D_3)			3170	3129

where $\tilde{H}^{conf}_{q\bar{q}}$ is spin-independent potential, $\tilde{H}^{hyp}_{q\bar{q}}$ is color–hyperfine interaction, $\tilde{H}^{so}_{q\bar{q}}$ is spin–orbit interaction. The explicit expression of $\tilde{H}^{conf}_{q\bar{q}}$, $\tilde{H}^{hyp}_{q\bar{q}}$, and $\tilde{H}^{so}_{q\bar{q}}$ are given in Ref. [69]. The spin-independent potential contains a constant term, a linear confining potential, and a one-gluon exchange potential,

$$\tilde{H}^{conf}_{q\bar{q}} = c + br + \frac{\alpha_s(r)}{r} F_1 \cdot F_2. \tag{10}$$

Although the GI model has achieved great successes in describing the meson spectrum, there still exists a discrepancy between the predictions and the recent experimental observation, as discussed in Refs. [40,69]. When screening effect is considered, the modification can be accomplished by the transformation [40]

$$br \to V^{scr}(r) = \frac{b(1-e^{\mu r})}{\mu}, \tag{11}$$

where the $b = 0.2687$ GeV^2 and $\mu = 0.15$ GeV[40].

With the modified Godfrey–Isgur (MGI) model, we calculated the mass spectra of charm mesons, charmed-strange mesons, and charmonium states, as shown in Tables 2, 3, and 4, respectively, which are same as those of Refs. [40,68,69].

4 Results and discussions

The mass spectra of the charmonium states predicted by the MNRQM and MGI models are shown in Table 4. Taking into account the averaged mass of $X(4140)$, 4146.8 ± 2.4 MeV, and the quantum numbers of $I^G(J^{PC}) = 0^+(1^{++})$, we can tentatively assign the resonance $X(4140)$ as the candidate of the $\chi_{c1}(3P)$. The discrepancy between the averaged mass of $X(4140)$ and the predicted masses of $\chi_{c1}(3P)$ in both models maybe result from that the hadron loop effects (such as the $D\bar{D}$ loop), which were neglected in these two models. The hadron loop effects can give rise to mass shifts to the bare hadron states. The mass shifts induced by the hadron loop effects can present a better description of the D, D_s, charmonium states, and bottomonium states [74,75].

Table 3 The mass spectra (in MeV) of charmed-strange mesons obtained within the non-relativistic quark model (NRQM) and the modified Godfrey–Isgur Model (MGI)

$n^{2S+1}L_J$	States	PDG [2]	NRQM [59]	MGI [69]
(1^1S_0)	D_s	1968.34 ± 0.07	1969	1967
(1^3S_1)	D_s^*	2112.2 ± 0.4	2107	2115
(2^1S_0)			2640	2646
(2^3S_1)	$D_{s1}^*(2700)$	$2708.3^{+4.0}_{-3.4}$	2714	2704
(3^1S_0)			3112	3097
(3^3S_1)			3168	3136
(4^1S_0)			3511	3462
(4^3S_1)			3558	3490
$(1P)$	$D_{s1}(2536)$	2535.11 ± 0.06	2488	2531
(1^3P_0)	$D_{s0}^*(2317)$	2317.8 ± 0.5	2344	2463
$(1P')$	$D_{s1}(2460)$	2459.5 ± 0.6	2510	2532
(1^3P_2)	$D_{s2}^*(2573)$	2569.1 ± 0.8	2559	2571
$(2P)$			2958	2979
(2^3P_0)			2830	2960
$(2P')$			2995	2988
(2^3P_2)			3040	3004
$(1D)$			2788	2877
(1^3D_1)	$D_{s1}^*(2860)$	2859 ± 27	2804	2865
$(1D')$			2849	2882
(1^3D_3)	$D_{s3}^*(2860)$	2860 ± 7	2811	2883
$(2D)$			3217	3247
(2^3D_1)			3217	3244
$(2D')$			3260	3252
(2^3D_3)			3240	3251

Next, we will calculate the strong decay widths of the $X(4140)$ state as the $\chi_{c1}(3P)$ assignment. In our calculations, we take two kinds of the wave functions, by solving the Schrödinger equation in the (modified) NRQM as discussed in Sect. 3.1 (**Case A**), and in the MGI model as discussed in Sect. 3.2 (**Case B**) for the charm mesons, charmed-strange mesons, and the charmonium states. In the $^3P_0$ model, we take the same constituent quark masses as those in Eq. (5) for Case A ($m_{u/d} = 450$ MeV and $m_s = 550$ MeV), and as those in Eq. (9) for Case B ($m_{u/d} = 220$ MeV and $m_s = 419$ MeV). Another free parameter γ, the strength of quark–antiquark pair created from the vacuum, is taken to be $\gamma = 4.52 \pm 0.08$ in Case A, and $\gamma = 5.90 \pm 0.10$ for Case B, by fitting to the total widths of the well established charmonium states, $\psi(3770)$ (1^3D_1), $\psi(4040)$ (3^3S_1), $\psi(4160)$ (2^3D_1), and $\chi_{c2}(2P)$.

With the above parameters, we have calculated the partial decay widths and total decay width, as shown in Table 5 for both Case A and Case B. The total widths of $\chi_{c1}(3P)$ are 12.63 MeV for Case A, and 31.34 MeV for Case B, both of which are consistent with the average value of $\Gamma = 22^{+8}_{-7}$ MeV within errors [2]. It should be pointed out that the decay modes $D\bar{D}^*$ and $D^*\bar{D}^*$ have large decay widths, which are also consistent with the conclusions of Refs. [28,39]. We suggest to search for this state in those two channels, and to measure the width precisely, which can be shed light on its nature. We also show the dependence of the $\chi_{c1}(3P)$ decay width on the initial mass with the wave functions of Case A and Case B, respectively in Figs. 2 and 3. The decay width of the $\chi_{c1}(3P)$ state is 12.63 ± 0.45 MeV for Case A, and 31.3 ± 1.2 MeV for Case B, by taking into account the uncertainties of the $X(4140)$ mass and the strength γ.

Since the error of the LHCb measurement on the width of $X(4140)$ is quite large, we will perform a simple χ^2 study. For the results of the Case A, we have,[3]

$$\chi^2(x) = \left(\frac{x - 12.63}{0.45}\right)^2 + \left(\frac{x - 83}{30}\right)^2, \tag{12}$$

[3] For the LHCb measurement, we use 83 ± 30 MeV by square summing the errors as $\sqrt{(21^2 + 21^2)} \approx 30$ MeV.

Table 4 The mass spectra (in MeV) of charmonium states obtained within the modified non-relativistic quark model (MNRQM) and the modified Godfrey–Isgur Model (MGI), and the other predictions are also listed in this table

States	PDG [2]	MNRQM [37]	MGI [40]	[50]	[50]	[71]	[67]	[72]	[73]
$\eta_c(1^1S_0)$	2983.9 ± 0.5	2979	2981	2982	2975	2980.3	2970	2978.4	2990
$J/\psi(1^3S_1)$	3096.9 ± 0.006	3097	3096	3090	3098	3097.36	3100	3087.7	3096
$\eta_c(2^1S_0)$	3637.5 ± 1.1	3623	3642	3630	3623	3597.1	3620	3646.9	3643
$\psi(2^3S_1)$	3686.097 ± 0.025	3673	3683	3672	3676	3685.5	3680	3684.7	3703
$\eta_c(3^1S_0)$		3991	4013	4043	4064	4014.0	4060	4058.0	4054
$\psi(3^3S_1)$	4039 ± 1	4022	4035	4072	4100	4094.9	4100	4087.0	4097
$\eta_c(4^1S_0)$		4250	4260	4384	4425			4391.4	
$\psi(4^3S_1)$		4273	4274	4406	4450	4433.3		4411.4	
$h_c(1^1P_1)$	3525.38 ± 0.11	3519	3538	3516	3517	3526.9	3520	3526.9	3515
$\chi_{c0}(1^3P_0)$	3414.71 ± 0.30	3433	3464	3424	3445	3415.7	3440	3366.3	3452
$\chi_{c1}(1^3P_1)$	3510.67 ± 0.05	3510	3530	3505	3510	3508.2	3510	3517.7	3504
$\chi_{c2}(1^3P_2)$	3556.17 ± 0.07	3554	3571	3556	3550	3557.7	3550	3559.3	3532
$h_c(2^1P_1)$		3908	3933	3934	3956	3960.5	3960	3941.9	3956
$\chi_{c0}(2^3P_0)$		3842	3896	3852	3916	3843.7	3920	3842.7	3909
$\chi_{c1}(2^3P_1)$		3901	3929	3925	3953	3939.7	3950	3935.0	3947
$\chi_{c2}(2^3P_2)$	3927.2 ± 2.6	3937	3952	3972	3979	3993.7	3980	3973.1	3969
$h_c(3^1P_1)$		4184	4200	4279	4318			4309.7	4278
$\chi_{c0}(3^3P_0)$		4131	4177	4202	4292			4207.6	4242
$\chi_{c1}(3^3P_1)$		4178	4197	4271	4317			4298.7	4272
$\chi_{c2}(3^3P_2)$		4208	4213	4317	4337			4352.4	
$\psi(1^1D_2)$		3796	3848	3799	3837	3823.6	3840	3815.1	3812
$\psi(1^3D_1)$	3773.13 ± 0.35	3787	3830	3785	3819	3803.8	3820	3808.8	3796
$\psi_2(1^3D_2)$		3798	3848	3800	3838	3823.8	3840	3820.1	3810
$\psi_3(1^3D_3)$		3799	3859	3806	3849	3831.1		3812.6	
$\psi(2^1D_2)$		4099	4137	4158	4208	4190.7	4210	4164.9	4166
$\psi(2^3D_1)$		4089	4125	4142	4194	4164.2	4190	4154.4	4153
$\psi_2(2^3D_2)$		4100	4137	4158	4208	4189.1	4210	4168.7	4160
$\psi_3(2^3D_3)$		4103	4144	4167	4217	4202.3		4166.1	

which is minimized for $x = 12.65$ with $\chi_0^2 = 5.50$, and the corresponding probability $p(\chi^2 > \chi_0^2) = 0.019 < 0.05$. Then we can conclude that the theoretical width 12.63 ± 0.45 MeV of Case A is smaller than the LHCb result $83 \pm 21^{+21}_{-14}$ MeV at the 95% CL. On the other hand, for the results of the Case B, we have,

$$\chi^2(x) = \left(\frac{x - 31.3}{1.2}\right)^2 + \left(\frac{x - 83}{30}\right)^2, \quad (13)$$

which is minimized for $x = 31.4$ with $\chi_0^2 = 2.97$, and the corresponding probability $p(\chi^2 > \chi_0^2) = 0.085 > 0.05$. It implies that the value 31.3 ± 1.2 MeV of Case B is not significant smaller than the LHCb measurement from a statistical point of view.

In addition, it is also easy to find that there are large discrepancies between the Case A and Case B, since the corresponding χ_0^2 reads 212.2. Generally speaking, the different space wave functions would lead to different decay widths. Especially, if the overlap is near to the nodes of space wave functions, the decay width would strongly depend on the details of wave functions, and the small wave function difference could generate a large discrepancy of the decay width. The difference between the predictions in case A and case B provides a chance to distinguish two models. Thus, if the small width of the $X(4140)$ is confirmed in future high-precision measurements, the $X(4140)$ could be explained as the charmonium state $\chi_{c1}(3P)$. Indeed, the $B^+ \to J/\psi\phi K^+$ decay was investigated in Ref. [76], where the $X(4140)$, with the small width $\Gamma = 19$ MeV, and the molecular state $X(4160)$ were taken into account, and it was found that the low $J/\psi\phi$ invariant mass distributions were better described compared with the analysis in Refs. [16,17] where only the $X(4140)$ resonance was considered. Thus, the high-precision

Table 5 Decay widths of the $\chi_{c0}(3P)$, $\chi_{c1}(3P)$ and $\chi_{c2}(3P)$ states (in MeV). The mass of the $\chi_{c1}(3P)$ is taken to be the one of the $X(4140)$, and the masses of the $\chi_{c0}(3P)$ and $\chi_{c2}(3P)$ are taken from the Table 4, respectively for Case A and Case B

State	Channel	Mode	Γ (Case A)	Γ (Case B)
$\chi_{c0}(3P)$	$0^+ \to 0^-0^-$	$D\bar{D}$	10.58	0.22
		$D_s^+D_s^-$	0.37	1.87
	$0^+ \to 1^-1^-$	$D^*\bar{D}^*$	16.28	35.95
Total width			27.23	38.03
$\chi_{c1}(3P)$	$1^+ \to 0^-1^-$	$D\bar{D}^*$	4.54	14.48
		$D_s\bar{D}_s^*$	1.23	0.70
	$1^+ \to 1^-1^-$	$D^*\bar{D}^*$	6.86	16.17
Total width			12.63	31.34
$\chi_{c2}(3P)$	$2^+ \to 0^-0^-$	$D\bar{D}$	7.71	8.79
		$D_s^+D_s^-$	0.63	0.10
	$2^+ \to 0^-1^-$	$D\bar{D}^*$	20.04	11.34
		$D_s\bar{D}_s^*$	0.17	0.13
	$2^+ \to 1^-1^-$	$D^*\bar{D}^*$	11.33	26.87
Total width			39.89	47.23

measurement about the $X(4140)$ width is necessary to shed light on its possible nature.

Studying the strong decay properties of the $\chi_{c0}(3P)$ and $\chi_{c2}(3P)$ states is also useful to search for those states, and understand the family of the charmonium states. The decay widths of the $\chi_{c0}(3P)$ and $\chi_{c2}(3P)$ are tabulated in Table 5, and the initial mass dependences of the total widths are also shown in Figs. 2 and 3, respectively corresponding to the results of Case A and Case B. The total decay width of $\chi_{c0}(3P)$ is about 25 ± 3 MeV for Case A with the predicted mass 4131 ± 30 MeV, and about 35 ± 5 MeV for Case B with the predicted mass 4177 ± 30 MeV. For the $\chi_{c2}(3P)$, the total decay width is predicted to be about 35 ± 5 MeV for Case A with the predicted mass 4208 ± 30 MeV, and about 43 ± 5 MeV for Case B with the predicted mass 4213 ± 30 MeV. In the energies region of $4100 \sim 4250$ [2], there is one state $X(4160)$, with $M = 4156^{+29}_{-25}$ MeV and $I^G(J^{PC} = ?^?(?^{??})$, but with $\Gamma = 139^{+110}_{-60}$ MeV, which is much larger than the predicted total widths of the $\chi_{cJ}(3P)$. Indeed, among the different interpretations of the $X(4160)$, the $D_s^*\bar{D}_s^*$ molecular nature has been widely studied in Refs. [76–79].

Finally, we would like to discuss about the uncertainties of the charmonium spectrum. We have extracted the wave functions from the MGI and MNRQM, and have not taken into account the error of the parameters. Since the errors of the established charmonium state are very small, we could expect that the errors of the predictions of these two models are also very small. The more important is that, the information of its quantum numbers $J^{PC} = 1^{++}$ and the mass 4146.8 ± 2.4

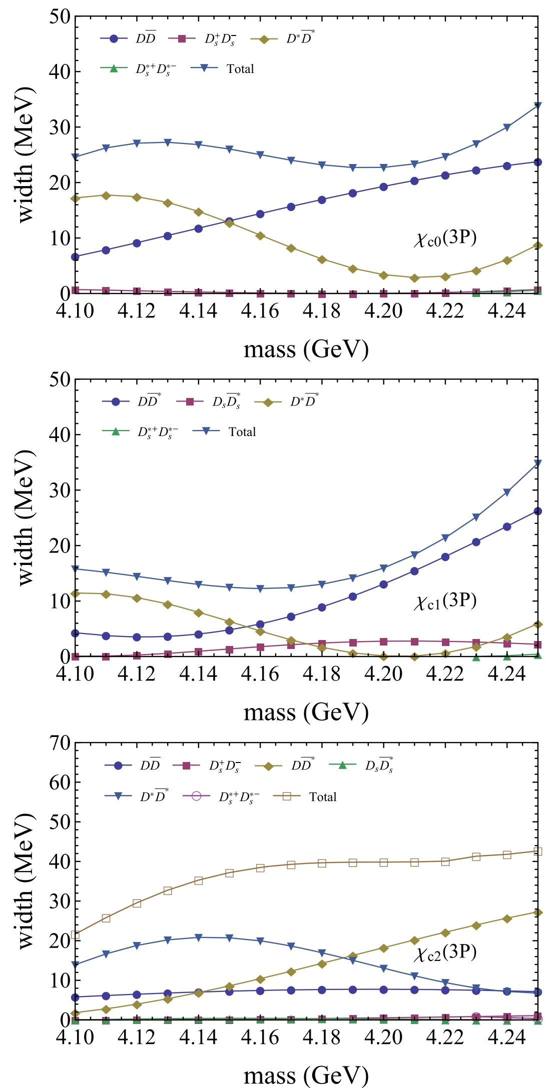

Fig. 2 The dependences of the widths of $\chi_{c0}(3P)$, $\chi_{c1}(3P)$ and $\chi_{c2}(3P)$ on the initial state mass with the wave functions of Case A

are enough for one to obtain its possible assignment, and then we could calculate the decay width with this assignment.

5 Summary

We have investigated the strong decay properties of the $X(4140)$ with the assignment of the $\chi_{c1}(3P)$ states in the $^3P_0$ model, where the modified non-relativistic quark model (Case A) and the modified Godfrey–Isgur relativistic quark model (Case B), both taking into account the screening effect, are used to extract the wave functions for the mesons. The only free parameter γ, the strength of the quark–antiquark pair created from the vacuum, is taken by fitting to the widths of the four well established charmonium states, $\psi(3770)$ (1^3D_1), $\psi(4040)$ (3^3S_1), $\psi(4160)$ (2^3D_1), and $\chi_{c2}(2P)$.

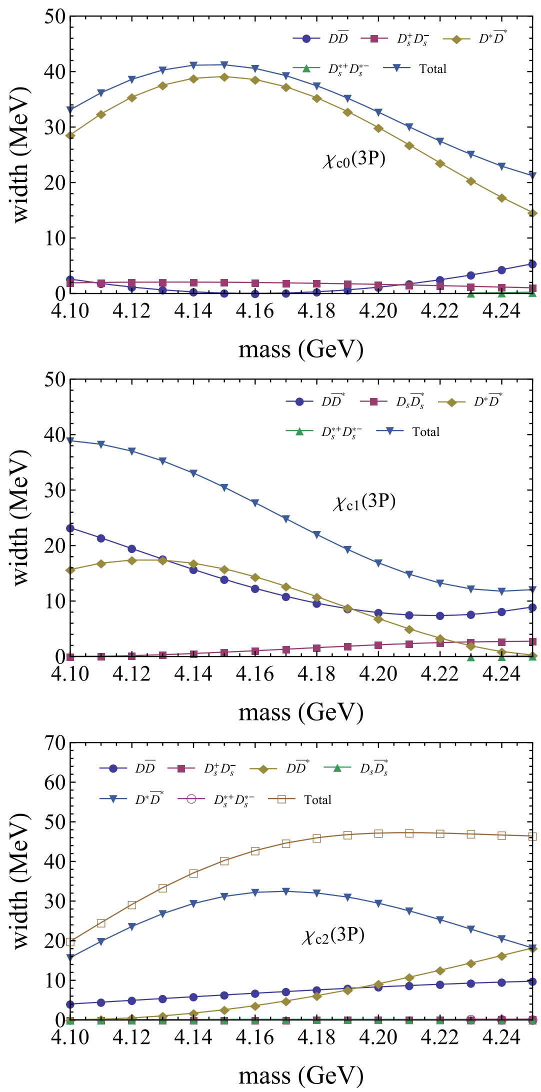

Fig. 3 The dependences of the widths of $\chi_{c0}(3P)$, $\chi_{c1}(3P)$ and $\chi_{c2}(3P)$ on the initial state mass with the wave function of Case B

The total decay width of the $\chi_{c1}(3P)$ is predicted to be 12.63 $\pm$ 0.45 MeV for Case A, and 31.3 $\pm$ 1.2 MeV for Case B, both of which support a narrow width for the $X(4140)$ resonance. Thus, we conclude that, the $X(4140)$, with a small width, could be explained as the charmonium state $\chi_{c1}(3P)$, and the high-precision measurement about the $X(4140)$ could shed light on its nature.

We have also performed a simple χ^2 study, which shows that the value 12.63 $\pm$ 0.45 MeV of Case A is smaller than the LHCb measurement at the 95% CL, and the one 31.3 $\pm$ 1.2 MeV of Case B is not significant smaller than the LHCb measurement from a statistical point of view.

We also show the strong decay properties of $\chi_{c0}(3P)$ and $\chi_{c2}(3P)$, and the total widths of the $\chi_{c0}(3P)$ and $\chi_{c2}(3P)$ are predicted be about 20 $\sim$ 40 MeV and 30–50 MeV, respectively. By comparing with the width of the $X(4160)$, we find it is difficult to interpretation the $X(4160)$ as the charmonium states $\chi_{cJ}(3P)$.

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