

A new quantum - relativistic model of tachyons

Luigi Maxmilian Caligiuri

Foundation of Physics Research Center (FoPRC), Cosenza, IT

luigimaxmilian.caligiuri@foprc.org; max.caligiuri@gmail.com

Abstract. The possible existence of tachyons, namely hypothetical particles always moving faster than light in vacuum, is still controversial due to the common belief they would violate the principle of macroscopic causality, despite their existence is not forbidden, in principle, by the Einstein's Special Theory of Relativity. Indeed, the main difficulties with tachyons are rather related to other two questions: their expected imaginary rest mass and the lack of a physical mechanism able to explain their existence especially from the standpoint of quantum theory. In this paper we discuss a novel physical mechanism, based on QED coherence in matter and on a reformulation of Special Theory of Relativity in the space-like region, able to explain a possible origin of tachyons as well as their real-value rest mass, ultimately related to the coherent dynamics of quantum vacuum.

1. Introduction

In Physics the concept of faster-than-light (FTL) motion refers to a phenomenon or a process in which the propagation velocity of some field or particle is, apparently or actually, greater than the value c of velocity of light in the vacuum.

This "definition" leads at least to three wide categories of physical situations in which this kind of behavior has been theoretically and/or experimentally conjectured and/or observed: 1) propagation of electromagnetic waves in specific media or under special boundary conditions (including, for example, anomalous dispersion, evanescent and localized wave equation solutions, and quantum and/or classical tunneling through barriers); 2) observations of astrophysical sources showing an apparent or real (this is yet matter of discussion among scientists) FTL velocity, like quasars and 3) classical or quantum theory of tachyons, supposing the existence of particles (charged or not) and relative antiparticles (or anti-tachyons) always moving at a velocity greater than of the speed of light in vacuum.

The FTL motion arising from quantum tunneling of evanescent photons has already been studied in our previous works [1] and its connection with the model here discussed will be further considered in a subsequent paper. In the so-called "Tachyons Hypthotesys" [2,3] the invariant light velocity c , assumed by Special Theory of Relativity (STR), would act as a lower velocity limit. The "classical" theory of tachyons (CTC) has been broadly developed in a series of papers [4,5,6,7] and, syntetically, it can be summarized in the three following assumptions: 1) Tachyons adhere to STR; 2) the Minkowskian geometry is still valid for them and 3) the observables like energy and momentum, being conserved and then measurable in the interactions, are physically meaningful and real.



This implies the energy momentum four-vector p_μ of a tachyon still satisfies the invariant relation:

$$p_\mu p^\mu = m_0^2 c^4 \quad (1)$$

also in the space-like region, i.e. when $p_\mu p^\mu < 0$ or $v > c$. It is then postulated that tachyons are particles always travelling with velocity greater than c , having real energy and momentum and, consequently, imaginary rest mass $m_0 = i \cdot \mu$ ($\mu \in R$).

This makes the invariant equation (1) to become

$$p_\mu p^\mu = -\mu^2 c^4 \quad (2)$$

namely it is always space-like. As shown by Recami [4,5] the energy and momentum of a tachyon moving, for example, along the x-axis with velocity $v = v_x = \beta c$, with $\beta^2 = (v_x/c)^2 > 1$, can be written as

$$p_x = \pm \frac{\mu \beta c}{\sqrt{\beta^2 - 1}}; \quad E = \pm \frac{\mu c^2}{\sqrt{\beta^2 - 1}} \quad (3)$$

Furthermore, according to the “switching procedure” [6], negative-energy objects travelling forward in time cannot exist so we must consider, in equation (3), $E > 0$. The main features of “classical” tachyons has been extensively studied in literature [4,7] to which the reader can refer for a more detailed treatment.

We have already shown [1] that, even in the context of CTC, the possible occurrence of FTL motion wouldn't necessarily violates any fundamental physical principle, as, first of all, the principle of macroscopic causality.

On the other hand it is clear the CTCs so far proposed are affected by a serious difficulty, namely the arising of a purely imaginary value of tachyon rest mass that appears, in those formulations, as a logic and mathematical need but whose physical meaning is undetermined when referred to a measurable quantity as the rest mass should be. Furthermore, if we assume tachyons to be elementary particles, we should be able to describe their origin within a suitable quantum framework.

In this paper we propose and discuss a novel relativistic quantum model able to suggest a possible origin of tachyons without requiring an imaginary value for their rest mass, so overcoming the main difficulties of the “classical” approaches. Our model is based on the integration between the Coherent Quantum Field Theory (CQFT), already proposed by this author in some previous works [1,8], and a reformulation of the STR in the space-like region [9].

2. Faster-than-light motion and causality principle

As it is known, one of the most frequent criticism against the hypothesis of superluminal phenomena concerns the presumed violation of causality principle they would determine, since the commonly accepted formulation of STR strictly forbids the superluminal propagation of a signal carrying energy or information.

We must now recall the principle of causality can be enunciated in microscopic or macroscopic terms. The formulation in macroscopic terms is generally expressed through the following two statements:

- a) cause always precedes its effects (also named “primitive” principle of causality);
- b) no signal can propagate faster than the velocity of light in vacuum (“relativistic” or Einstein’s principle of causality).

The microscopic formulation of causality principle, on the other hand, rules quantum causality and claims that any commutator of quantum field operators between two space-like separated points is zero.

One of the most common mistakes, characterizing the ingenuous and acritical interpretations of Einstein’s STR is to believe the two formulations a) and b) are equivalent and this give rise to the misconception that superluminal signals cannot carry energy or information or cannot exist at all.

As already discussed [10], the question of macroscopic causality is fundamentally related to the concept of simultaneity on which STR is based but that nevertheless results purely conventional. This conventionality is related to the arbitrariness of the choice of the synchronization procedure of distant clocks and doesn't depend upon the state of motion of the observers. In the Einstein's procedure this arbitrariness is overcome by assuming the postulate of invariance of light velocity in all the inertial frames. Nevertheless, as known to Einstein itself [11], this invariant velocity, equal to c in the vacuum, cannot be measured without previously adopting a convenient procedure of synchronization of distant clocks [12], that isn't necessarily related to true properties of physical states, being instead only conventional.

In order to explain this concept Einstein wrote, in 1916, by considering the middle point of a segment AB whose extremes have been "simultaneously" stroked by two lightning: "That light requires the same time to traverse the path $A \rightarrow M$ as for the path $B \rightarrow M$ is in reality neither a supposition nor a hypothesis about the physical nature of light, but a stipulation which I can make of my own free will in order to arrive at a definition of simultaneity" [11].

Poincaré already stated this concept in 1899 by writing: "The simultaneity of two events should be fixed in such a way that the natural laws become as simple as possible. In other words all these rules, all these definitions are only the result of an implicit convention" [12]. This synchronization is then substantially conventional and is not necessarily related to true properties of physical reality [11,13].

The definition of simultaneity then strictly depends on the synchronization procedure of distant clocks and on the measurement of the so-called "one-way" velocity of light.

The choice of a particular clock synchronization method among all the possible ones determines the form of the space-time transformations between two inertial frames S_0 and S' (for simplicity supposed in motion with relative velocity $v = v_x$) that includes, in particular, space-time transformation defining the Lorentz's group [1].

2.1. Causality Principle and Zeeman theorem

The principle of macroscopic causality can be enunciated in the form (which we'll use in the following discussion) of the Zeeman theorem [9].

It relates macroscopic causality with the Lorentz group of transformations stating that causality implies Lorentz group and vice versa. More specifically, if $x^\mu (x^0 = t, x^1, x^2, x^3)$ is a space-time point of a Minkowski manifold M , such as

$$-\infty < x^\mu < \infty \quad \mu = 0,1,2,3 \quad (4)$$

we can define an ordering relation " $<$ " between two events x^μ and y^μ that is also transitive

$$x < y \quad (5)$$

by the two conditions:

$$x^0 < y^0 \quad (6)$$

$$(y - x)^2 > 0 \quad (7)$$

with the metric $g_{\mu\nu} = \text{diag}(+, -, -, -)$.

The relations (6) and (7) mean that two events are macroscopically causally related if the space-time interval between them is time-like and if x occurs before y . Now we consider the class of applications $F : M \rightarrow M$ such as, $\forall x, y \in M$

$$x < y \Leftrightarrow F(x) < F(y) \quad (8)$$

This class of applications then preserves the temporal orders of two events and then, by means of the above interpretation, their causal relationship in M . According to the Zeeman's theorem, the application F is an automorphism and the set it forms constitutes a group generated by the

orthochrone Lorentz group (including parity inversion but not temporal inversion) with respect space and time translations and dilatations, the transformations F being linear and continuous as well.

3. Formulation of special theory of relativity in the space-like region

It has been shown [9] it is possible, basing only on considerations of groups theory, on the postulate of invariance of c and on three general axioms, to formulate a Special Theory of Relativity in the space-like region of space-time characterized by a “tachyonic metric”.

These axioms are fully justified by studying the symmetry properties of a purely mathematical group, namely the group $SO(4;C)$ of rotation in the complex plane, from which we can deduce three and only three “physical groups”, characterized by the value of relative velocity v : the usual Lorentz’s group or “bradyonic” or “subluminal” group with $v < c$, the “luminal” group with $v = c$ and the “tachyonic” or “superluminal group” with $v > c$.

More specifically, the groups $SO(3,1;C)$ and $SO(1,3;C)$ of 4×4 complex matrices Λ and $\tilde{\Lambda}$, respectively characterized by a complex metric of signature $(+---)$ and $(-+++)$, are isomorph to $SO(4;C)$ and isomorph to each other. Consequently, there will be two complete and orthochrone Lorentz groups, with the same Lie algebra and isomorph, namely L_+^T and \tilde{L}_+^T , respectively conserving the real metric $(+---)$ and $(-+++)$, \tilde{L}_+^T being the group of 4×4 real matrices of the superluminal Lorentz transformations $\tilde{\Lambda}$ that is isomorph to the group L_+^T of the 4×4 real matrices of the subluminal (usual) Lorentz transformations Λ . The groups L_+^T and \tilde{L}_+^T describe very different physical properties but their isomorphism reveals their common mathematical origin.

Moreover, the existence of a tachyonic group allows us to define the concepts of “tachyonic reference frame” (TRF), “tachyonic matter” and proper time for tachyons as well as the usual (bradyonic) Lorentz group defines the concepts of “ordinary reference frame” (ORF), “bradyonic matter” and proper time for bradyons.

This approach leads to a theory of tachyons, deeply alternative to those so far considered [1,14], based on a sort of “super-covariance” principle, having as a consequence, in particular, a real “rest” mass for tachyons.

The generalization of STR to space-like regions is based on the three following axioms [9].

Axiom 1:

There exist a “tachyonic matter” and, consequently it makes sense to define a tachyonic (or superluminal) inertial frame (associated to a TRF), characterized by superluminal space and time coordinates, different from the subluminal inertial frame (associated to an ORF).

Axiom 2:

The relative velocity of two TRFs is always greater than c while the relative velocity of two ORFs is always lower than c .

Axiom 3:

In a TRF the world lines of every particle are always space-like and a succession of superluminal events placed on a space-like world line are related by the following metric:

$$dS^2 = G_{\mu\nu} dX^\mu dX^\nu \quad \mu, \nu = 1, 2, 3, 4 \quad (9)$$

that is a real function of the coordinates X^μ defined in a TRF (see axiom 1) and such as $dS^2 > 0$.

The metric (9) has the signature $(-, +, +, +)$ and defines a superluminal manifold $\tilde{E}_4(dS^2)$ as a pseudo-Euclidean space whose metric tensor is given by:

$$[G_{\mu\nu}] = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (10)$$

in the same way as the (usual) metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \quad \mu, \nu = 1, 2, 3, 4 \quad (11)$$

where

$$[g_{\mu\nu}] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad (12)$$

with the signature $(+, -, -, -)$ defines the subluminal manifold $E_4(ds^2)$ and an ORF.

We note that

$$ds^2 = c^2 dt^2 - \sum (dx^i)^2 = dt^2 \left(c^2 - \frac{\sum (dx^i)^2}{dt^2} \right) = c^2 dt^2 (1 - \beta^2), \quad \beta = \frac{v}{c} \quad (13)$$

while

$$dS^2 = -c^2 dT^2 + \sum (dX^i)^2 = dT^2 \left(\frac{\sum (dX^i)^2}{dT^2} - c^2 \right) = c^2 dT^2 (\beta^2 - 1), \quad \beta = \frac{v}{c} \quad (14)$$

so, we have

$$ds^2 > 0 \quad dS^2 < 0, \quad \beta < 1 \quad v < c \quad (15)$$

and

$$ds^2 < 0 \quad dS^2 > 0, \quad \beta > 1 \quad v > c \quad (16)$$

As known [9] the metric (11) can describe only a manifold in which all the velocities are always lower than c , so we can say the only way to represent a manifold in which all the velocities are always higher than c is to consider the metric given by equation (9).

It is important to note the choice of the metric (9) or the metric (11) is conventional for each of the two manifolds but once made for one is automatically determined for the other. If we make the assumption above, ds will be real for two subluminal events measured in the ORF (characterized by a relative velocity $v < c$) and purely imaginary in the subluminal “elsewhere”, while dS will be real for two superluminal events measured in TRF (characterized by a relative velocity $v > c$) and purely imaginary in the superluminal “elsewhere”. This ensures the two space-time intervals to be considered as measurable physical quantities within their respective “light-cones”.

The metric (9) also implies that, in a TRF, not only the world-lines will be space-like but they are space-like axis as well in the usual sense of STR. In fact, if we assume

$$X_4 = icT \quad (17)$$

we also have

$$dS^2 = dX_4^2 + \sum_{i=1}^3 (dX^i)^2 > 0 \quad (18)$$

so dS^2 remains positive, and it can be considered as a true space axis, while if we consider the symmetrical subluminal case of space-like separation

$$ds^2 = c dt^2 - \sum_{i=1}^3 (dx^i)^2 > 0 \quad (19)$$

and assume

$$x_4 = ict \quad (20)$$

we obtain

$$ds^2 = dx_4^2 + \sum_{i=1}^3 (dx^i)^2 < 0 \quad (21)$$

namely ds^2 becomes negative, and it can be considered as a true time axis in the sense of STR.

We can express the linear transformations between two tachyonic inertial frames R and R' of \tilde{E}_4 in the form ($\mu, \nu = 0, 1, 2, 3$)

$$X'_\mu = \Lambda_{\mu\nu} X_\nu \quad (22)$$

with the invariant quantity

$$X^2 = G_{\mu\nu} X_\mu X_\nu \quad (23)$$

where X'_μ and X_ν respectively indicate the coordinate in R' and R and the metric coefficients $G_{\mu\nu}$ are given by ($i, j = 1, 2, 3$)

$$G_{00} = -1, \quad G_{ij} = \delta_{ij} \quad (24)$$

where δ_{ij} is the Kronecker delta and the invariant (23) is ($X_0 = cT$)

$$X^2 = -X_0^2 + \sum_{i=1}^3 X_i^2 \quad (25)$$

It can be easily shown the transformation (22) form the group of 4×4 matrices that leave X^2 unchanged, the identity being represented by the transformation $X'_\mu = X_\mu$ and the metric tensor by the matrix $G = G^T$ given by equation (10). The invariance condition can be then written in matrix form as:

$$G_{\mu\nu} \Lambda_{\mu\beta} \Lambda_{\nu\rho} X_\beta X_\rho = G_{\beta\rho} X_\beta X_\rho \quad (26)$$

if

$$G_{\mu\nu} \Lambda_{\mu\beta} \Lambda_{\beta\rho} = G_{\beta\rho} \quad (27)$$

namely, in compact form

$$\Lambda^T G \Lambda = G \quad (28)$$

The equation (28) and the symmetry of the matrix G leave only six independent parameters in the transformation matrix Λ . It represents a rotation in \tilde{E}_4 and its elements respectively correspond to the three components of the relative velocity v between R' and R and to the three Euler angles defining the relative orientation between the systems.

The invariance property can be written as

$$X_4^2 + \sum_{i=1}^3 X_i^2 = (X'_4)^2 + \sum_{i=1}^3 (X'_i)^2 \quad (29)$$

where

$$X_4 = iX_0 = icT; \quad X'_4 = iX'_0 = icT' \quad (30)$$

We can now write equation (22) in the form

$$X'_\mu = \alpha_{\mu\nu} X_\nu \quad (31)$$

where the coefficients $\alpha_{\mu\nu}$ are the elements of an orthogonal matrix, namely

$$\alpha_{\mu\nu} \alpha_{\lambda\nu} = \delta_{\mu\lambda} \quad (32)$$

These coefficients can be obtained through the same procedure used in the case of usual Lorentz transformations and $SO(4;C)$, namely by considering the two-dimensional rotations with one imaginary coordinate between different complex planes. This means, in this context, we can consider the transformations for which, for example, the spatial axis X_3' coincides with X_3 and moves with respect to it with constant velocity $v = \beta c > c$, while the other ones are parallel to each other $X_2' = X_2$ and $X_1' = X_1$. Equation (29) then reduces to

$$X_3^2 + X_4^2 = (X_3')^2 + (X_4')^2 \quad (33)$$

and the transformation matrix becomes

$$[\alpha_{\mu\nu}] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha_{33} & \alpha_{34} \\ 0 & 0 & \alpha_{43} & \alpha_{44} \end{pmatrix} \quad (34)$$

while the orthogonality condition is written as

$$\begin{cases} \alpha_{33}^2 + \alpha_{34}^2 = 1 \\ \alpha_{43}^2 + \alpha_{44}^2 = 1 \\ \alpha_{33}\alpha_{43} + \alpha_{34}\alpha_{44} = 0 \end{cases} \quad (35)$$

In the reference frame OX' the X_1' axis is defined by

$$X_4' = 0 \quad (36)$$

a condition that agrees with the axiom 1 and means that every point of X' move with uniform velocity $v = \beta c$ with respect to OX .

At time T we then have $X_3 = vT$ and, using equation (30)

$$X_3 = -i\beta X_4 \quad (37)$$

that, substituted in equation (31), gives

$$X_4' = \alpha_{43}X_3 + \alpha_{44}X_4 = X_4(\alpha_{44} - i\beta\alpha_{43}) \quad (38)$$

By virtue of equation (35) we have

$$\alpha_{44} = i\beta\alpha_{43} \quad (39)$$

and, from the second equation of equation (35), we obtain

$$(\beta^2 - 1)\alpha_{43}^2 = -1 \quad (40)$$

The coefficients of equation (34) are then given by

$$\alpha_{43} = \frac{\pm i}{\sqrt{\beta^2 - 1}}; \quad \alpha_{44} = \frac{\beta}{\sqrt{\beta^2 - 1}} \quad (41)$$

the sign of the first expression in equation (41) can be determined by imposing the transformation Λ to be proper, namely it doesn't invert the direction of space and time. This condition is equivalent to $\det \Lambda = +1$ that gives the sign + in equation (41). From equations (35) we deduce

$$\alpha_{33} = -\alpha_{34} \frac{\alpha_{44}}{\alpha_{43}} = -i\beta\alpha_{34} \quad (42)$$

that, used in the first of them, gives

$$\alpha_{34}^2 (\beta^2 - 1) = -1 \quad (43)$$

and we finally obtain

$$\alpha_{34} = \frac{i}{\sqrt{\beta^2 - 1}}; \quad \alpha_{33} = \frac{\beta}{\sqrt{\beta^2 - 1}} \quad (44)$$

where, as above, the sign in the first of equation (44) has been chosen in order to obtain a proper transformation. We are now in position to write the explicit form of the transformation matrix

$$(\alpha_{\mu\nu}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{\beta}{\sqrt{\beta^2 - 1}} & \frac{i}{\sqrt{\beta^2 - 1}} \\ 0 & 0 & \frac{-i}{\sqrt{\beta^2 - 1}} & \frac{\beta}{\sqrt{\beta^2 - 1}} \end{pmatrix} \quad (45)$$

The space and time transformations between two TRFs are then given by

$$\begin{cases} X_3' = \frac{\beta X_3 - X_0}{\sqrt{\beta^2 - 1}} & X_0' = \frac{\beta X_0 - X_3}{\sqrt{\beta^2 - 1}} \\ X_1' = X_1 & X_2' = X_2 \end{cases} \quad (46)$$

and, their inverse, by $(\alpha_{\mu\nu})^T$

$$\begin{cases} X_3 = \frac{\beta X_3' + X_0'}{\sqrt{\beta^2 - 1}} & X_0 = \frac{\beta X_0' + X_3'}{\sqrt{\beta^2 - 1}} \\ X_1 = X_1' & X_2 = X_2' \end{cases} \quad (47)$$

It can be easily proven [9] the matrices $(\alpha_{\mu\nu})$ form an homogeneous and proper group of orthochrones transformations (since $\alpha_{44} > 0$), preserving the metric dS^2 , whose unitary element is

$$X'_\mu = X_\mu \quad (48)$$

corresponding to the limit $\beta \rightarrow \infty$ ($\beta \gg 1$ or $v \gg c$). The analysis performed by means of group theory shows this “tachyonic” or superluminal group is symmetric with respect to the subluminal usual Lorentz group.

It is interesting to observe we can assume, using equation (41) and equation (44)

$$\alpha_{33} = \cos \Theta; \quad \alpha_{34} = -\sin \Theta; \quad \alpha_{43} = \sin \Theta; \quad \alpha_{44} = \cos \Theta \quad (49)$$

where

$$\sin \Theta = \frac{-i}{\sqrt{\beta^2 - 1}}; \quad \cos \Theta = \frac{\beta}{\sqrt{\beta^2 - 1}}; \quad \cot \Theta = i\beta \quad (50)$$

since equation (50) satisfy the conditions (40), (42) and (43) where $0 < \Theta < \pi/4$ respectively for $\beta \rightarrow \infty$ and $\beta = 1$.

The transformation matrix can be then written as

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos \Theta & -\sin \Theta \\ 0 & 0 & \sin \Theta & \cos \Theta \end{pmatrix} \quad (51)$$

which contains the 2×2 sub-matrix

$$\begin{pmatrix} \cos \Theta & -\sin \Theta \\ \sin \Theta & \cos \Theta \end{pmatrix} \quad (52)$$

representing an operator belonging to the group $SO(2;C)$ of rotations in the complex plane whose inverse is given by

$$\begin{pmatrix} \cos \Theta & \sin \Theta \\ -\sin \Theta & \cos \Theta \end{pmatrix} \quad (53)$$

The group $SO(4;C)$ and its two sub-groups $SO(2;C)$ are purely mathematical groups and can be represented, in a completely general way, by the unitary hermitian operator of $SO(2;C)$

$$\begin{pmatrix} \cos \vartheta & \mp \sin \vartheta \\ \pm \sin \vartheta & \cos \vartheta \end{pmatrix} \quad (54)$$

It is then clear how, by starting from the mathematical group $SO(4;C)$, we can obtain three transformation groups having a physical meaning and containing real elements only: the ‘‘usual’’ Lorentz subluminal group, the tachyonic or superluminal group and the luminal group, corresponding to the ‘‘liming angle’’ $\vartheta \rightarrow \pi/4$, namely the transformations between referential frames in which the relative velocities are always equal to c .

So, we can consider $SO(4;C)$ as a group describing a unique physical reality of which the three groups above (respectively associated to bradyons, tachyons and photons) are just different manifestations, according to the reference frame used for their description.

Mathematically, this can be easily view since we can obtain, from the general form equation (54), the Lorentz subluminal transformation by making in equation (51) the position

$$\vartheta = \phi; \quad \operatorname{tg} \phi = i \frac{v}{c} = i\beta \quad (55)$$

where ϕ represents the angle between the time-like world lines, namely the time axes of the two inertial frames related by the coordinate transformation. This angle is a function of the real parameter v (that has the dimensions of a velocity), being $0 \leq \phi < \pi/4$ (respectively for $\beta = 0$ and $\beta \rightarrow 1$). In this case, as usual, we’ll have $ds^2 < 0$ if $x_4 = ict$ and $ds^2 > 0$ if we use the reduced Galilean coordinates for which $x_4 = ct$. The relative velocities will be then always lower than c and the reference system coincides with the usual Lorentz one (ORF).

On the other hand, we can get the coordinates transformation of the tachyonic group by starting from the $SO(4;C)$ group and assuming in equation (51)

$$\vartheta = \Theta; \quad \cot \Theta = i \frac{v}{c} = i\beta \quad (56)$$

where Θ now represents the angle between the space-like world lines, namely the space axes of the two inertial frames related by the coordinates transformation, with $0 < \phi \leq \pi/4$ (respectively for $\beta \rightarrow \infty$ and $\beta = 1$). In this case we always have

$$dS^2 > 0 \quad (57)$$

both if using the coordinate $X_4 = icT$ or the coordinate $X_0 = cT$.

This latest result further justifies the reasonability of the three axioms previously discussed about the existence of the TRF and tachyonic matter.

The group analysis of the tachyonic (with coordinates X_μ) and bradyonic (with coordinates x_μ) Lorentz groups then shows the existence, for both of them, of the unit element ($\Lambda = I$), that is respectively

$$X'_\mu = X_\mu \quad (58)$$

for $\beta \rightarrow \infty$ ($\beta \gg 1$ or $v \gg c$) and

$$x'_\mu = x_\mu \quad (59)$$

for $\beta \rightarrow 0$ ($\beta \ll 1$ or $v \ll c$).

It is interesting to note the limiting value $\beta = 0$ no longer corresponds to a “relative velocity” as it is rather defines, within the subluminal manifold of the universe, a “bradyonic reference” as, in the same way the limiting value $\beta \rightarrow \infty$ will define a “tachyonic reference”. The existence of the unitary element both in the subluminal and superluminal groups L_+^4 and \tilde{L}_+^4 is more than just a mathematical fact but it has a deep physical meaning: the possibility to respectively define a “bradyonic body”, namely an aggregate of (bradyonic) elementary particles characterized by a mutual kinematical relationship such as $v \ll c$ and a “tachyonic body”, namely an “aggregate” of (tachyonic) elementary particles whose mutual kinematical relationship is such as $v \gg c$.

It is easy to show the superluminal Lorentz group \tilde{L}_+^4 satisfies the Zeeman theorem on macrocausality, since this group is orthochrone. In fact, for any two events X and Y of \tilde{E}_4 , the condition $X < Y$ is equivalent to

$$X^0 < Y^0 \quad (60)$$

$$(Y - X)^2 > 0 \quad (61)$$

if we assume the metric (9). Then if X happens before Y as given by equation (60), the equation (61) means the interval between X and Y is space-like so preserving the macrocausality relation in \tilde{E}_4 . It is interesting to note this is not the case with the CTC in which tachyons are able to reverse the temporal order between two events.

3.1. Relativistic dynamics in the tachyonic manifold

We'll refer the “usual” pseudo-Euclidean manifold E_4 to a general curvilinear coordinates system y^α ($\alpha, \beta = 1, 2, 3, 4$) expressing the metric

$$ds^2 = g_{\alpha\beta} dy^\alpha dy^\beta \quad (62)$$

and consider the unitary four-velocity vector of a material point M , moving in E_4 ,

$$u^\alpha = \frac{dy^\alpha}{ds} \quad (63)$$

with $u_\mu u^\mu = 1$. It is immediate to calculate the components of u in the reduced galileian coordinates ($y^1 = x^1, y^2 = x^2, y^3 = x^3, y^4 = x^4 = ct$)

$$u^k = \frac{dx^k}{ds} = \frac{dx^k}{dt} \frac{dt}{ds} = v^k \frac{dt}{ds} \quad (64)$$

$$u^4 = \frac{dx^4}{ds} = c \frac{dt}{ds} \quad (65)$$

where $\vec{v} = (v^1, v^2, v^3)$. If $v = \beta c$ is the velocity of M we have, using equation (62)

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 = c^2 dt^2 (1 - \beta^2) \quad (66)$$

and then

$$\frac{dt}{ds} = \frac{1}{c\sqrt{1 - \beta^2}} \quad (67)$$

By inserting equation (67) in equation (64) and in equation (65) we can write

$$u^k = \frac{v^k}{c\sqrt{1 - \beta^2}}; \quad u^4 = \frac{1}{\sqrt{1 - \beta^2}} \quad (68)$$

We can follow a similar reasoning in the case of tachyonic manifold \tilde{E}_4 equipped with the metric (9), and consider a point M of curvilinear coordinates (y^ν) moving with velocity $|v| = \beta c > c$.

In this case

$$dS^2 = G_{\alpha\beta} dy^\alpha dy^\beta \quad (69)$$

with $G_{\alpha\beta} = G_{\alpha\beta}(y^\nu)$ and $\alpha, \beta, \nu = 1, 2, 3, 4$.

The unitary velocity four-vector is then defined as

$$u^\alpha = \frac{dy^\alpha}{dS} \quad (70)$$

and we calculate its components with respect to a system of real coordinate $(X^i, X^4 = cT; \quad i = 1, 2, 3)$ of a tachyonic reference with the metric (69) that gives

$$\frac{dT}{dS} = c \frac{1}{\sqrt{\beta^2 - 1}} \quad (71)$$

and then, if $v^i = dX^i/dT$ are the components of three-velocity ($i = 1, 2, 3$)

$$u^i = \frac{v^i}{c\sqrt{\beta^2 - 1}}; \quad u^4 = \frac{1}{\sqrt{\beta^2 - 1}} \quad (72)$$

This result makes it natural to generalize the Einstein covariance principle [15], referred to the equations of physics, to tachyonic manifold, so establishing a “super-covariance” principle. If we make such an assumption, we can characterize the dynamical features of a tachyon by means of a **real** parameter μ having the dimensions of a mass so that the fundamental equation of tachyon dynamics is written as ($\alpha = 1, 2, 3, 4$)

$$\mu c^2 \frac{\nabla u^\alpha}{dS} = F^\alpha \quad (73)$$

where ∇u^α designates the absolute differential of u^α , namely

$$\nabla u^\alpha = du^\alpha + \omega_h^\alpha u^h \quad (74)$$

with

$$\omega_h^\alpha = \Gamma_l^{\alpha h} dy^l \quad (75)$$

$\Gamma_l^{\alpha h}$ being the Christoffel symbol

$$\Gamma_l^{\alpha h} = \frac{1}{2} g^{\alpha s} \left(\frac{\partial g_{sh}}{\partial x^l} + \frac{\partial g_{ls}}{\partial x^h} + \frac{\partial g_{lh}}{\partial x^s} \right) \quad (76)$$

The parameter c is the light velocity, whose introduction at this stage is required by dimensional and homogeneity reasons, F^α is the four-force and $\nabla u^\alpha/dS$ is the four-acceleration with respect to S .

If we now consider a tachyonic reduced Galilean coordinates system we have $\nabla u^\alpha = du^\alpha$ and equation (73) becomes

$$\mu c^2 \frac{du^\alpha}{dS} = F^\alpha \quad (77)$$

Within this dynamic, the principle of inertia is also valid and it states an isolated tachyon is described by a world-line being a geodetic of \tilde{E}_4 with $dS^2 > 0$, defined by

$$\frac{du^i}{dS} = 0; \quad \frac{du^4}{dS} = 0 \quad (78)$$

A standard calculation [16] shows the four-vector energy-impulse of a tachyonic particle can be written as ($\alpha = 1, 2, 3, 4$)

$$p^\alpha = \mu c u^\alpha \quad (79)$$

that, using equation (69), gives

$$\tilde{p}^i = \frac{\mu v^i}{\sqrt{\beta^2 - 1}}; \quad \tilde{p}^4 = \frac{\mu c}{\sqrt{\beta^2 - 1}} = \frac{\tilde{E}}{c} \quad (80)$$

The tachyonic metric implies the norm of \tilde{p} to be

$$\tilde{p}^2 = \tilde{p}_\alpha \tilde{p}^\alpha = \sum_i (\tilde{p}^i)^2 - \tilde{p}_4^2 \quad (81)$$

that, using equation (80), gives

$$\tilde{p}^2 = \mu^2 c^2 \quad (82)$$

In the case of a tachyonic “boost”, this gives

$$\tilde{p} = \frac{\mu \beta c}{\sqrt{\beta^2 - 1}}; \quad \tilde{E} = \frac{\mu c^2}{\sqrt{\beta^2 - 1}} \quad (83)$$

and the equation (81) can be written as

$$\tilde{p}^2 c^2 - \tilde{E}^2 = \mu^2 c^4 \quad (84)$$

that can be viewed as the “translation” of the metric property of \tilde{E}_4 in the energy-momentum space.

It is extremely important to note, at this stage, the equation (84) is the same of that given by the CTCs so far proposed, namely the equation (3).

In fact, using the “ordinary” metric (11) in equation (2) and considering equation (3) we obtain

$$p_\mu p^\mu = -(\vec{p})^2 + \frac{E^2}{c^2} = -\mu^2 c^2 \quad (85)$$

that is identical to equation (84).

Nevertheless, equation (85) and equation (84) are based upon completely different physical hypotheses. In particular, the first one is obtained by considering the subluminal Lorentz group of transformations L_+^T , referred to ORF and the metric (11), and assuming, without any physical motivation, the proper mass of a free tachyon as purely imaginary $m_0 = i\mu$.

On the other hand, equation (84) is directly deduced from the general equation of dynamics of the tachyonic manifold deduced through the analysis of the mathematical superluminal Lorentz group \tilde{L}_+^T , the mass of a free tachyon being real.

Furthermore, according to CTC, the tachyon rest mass cannot be directly measured being an imaginary quantity, while the tachyonic equation of dynamics (73) can be used to establish a relationship between physical observable quantities measured in an ORF.

Let's consider a tachyon moving with velocity $\bar{v} > c$ ($\bar{\beta} > 1$), the superscript \bar{a} indicating the tachyon's property a measured with respect to an ORF. As already discussed, such measured velocity shouldn't be interpreted as a relative velocity but simply as a quantity measured in the ORF having the dimension of a velocity. Such a tachyonic object moves between the points $A(\bar{x}^i, \bar{t})$ and $B(\bar{x}^i + d\bar{x}^i, \bar{t} + d\bar{t})$ in the ORF ($i = 1, 2, 3$).

With respect this reference frame, the world line described of such tachyon will be space-like as defined by the measured metric (with signature $-+++$), namely

$$d\bar{s}^2 = -c^2 d\bar{t}^2 + \sum_{i=1}^3 (d\bar{x}^i)^2 \quad (86)$$

and its motion satisfies the equation

$$\bar{\mu} c^2 \frac{d\bar{u}^\alpha}{d\bar{s}} = \bar{F}^\alpha \quad (87)$$

from which it follows

$$\bar{p}^i = \frac{\bar{\mu}\bar{v}^i}{\sqrt{\bar{\beta}^2 - 1}}; \quad \bar{p}^4 = \frac{\bar{\mu}c}{\sqrt{\bar{\beta}^2 - 1}} = \frac{\bar{E}}{c} \quad (88)$$

As above, for a boost, equation (88) simplify as

$$\bar{p} = \frac{\bar{\mu}\bar{\beta}c}{\sqrt{\bar{\beta}^2 - 1}}; \quad \bar{p}_4 = \frac{\bar{E}}{c} = \frac{\bar{\mu}c}{\sqrt{\bar{\beta}^2 - 1}} \quad (89)$$

It is important to remind the quantities \bar{p} , \bar{E} , $\bar{\beta}$ and $\bar{\mu}$ represent physical properties of a tachyon being “measured” in the ORF.

For $\bar{v} \gg c$ ($\bar{\beta} \gg 1$) we have, by equation (89), \bar{p} is finite and

$$\bar{p} \rightarrow \bar{\mu}c; \quad \bar{\mu} \sim \frac{\bar{p}}{c} \quad (90)$$

Equation (90) then defines a physical (real) rest mass that is, in principle, experimentally accessible through a momentum measurement. In a symmetrical way, a bradyon will be observed, in a tachyonic reference TRF, as characterized by a time-line world-line, according to the metric (where now the superscript \bar{a} indicates the bradyon’s property a measured with respect to TRF)

$$d\bar{S}^2 = c^2 d\bar{T}^2 - \sum_{i=1}^3 (d\bar{X}^i)^2 \quad (91)$$

and a velocity $\bar{v} < c$ ($\bar{\beta} < 1$) so that, in this case

$$\bar{p} = \frac{\bar{m}\bar{\beta}c}{\sqrt{1 - \bar{\beta}^2}}; \quad \bar{p}_4 = \frac{\bar{E}}{c} = \frac{\bar{m}c}{\sqrt{1 - \bar{\beta}^2}} \quad (92)$$

where \bar{m} is the proper bradyonic mass “measured” in the TRF and we have, if $v \ll c$ ($\bar{\beta} \ll 1$)

$$\bar{p}_4 = \frac{\bar{E}}{c} \rightarrow \bar{m}c \quad (93)$$

Equation (93) then defines a physical (real) rest mass that is, in principle, experimentally accessible through an energy measurement.

The above discussion shows another meaningful result: both the tachyonic and bradyonic rest masses are scalar invariants and then have physical meaning so we can write

$$\mu = \bar{\mu}; \quad m = \bar{m} \quad (94)$$

3.2. Quantum theory of tachyons: an introduction

A quantum theory of tachyons can be formulated, according to the usual procedure, by substituting the dynamical tachyon variables with the corresponding linear operators acting on a proper tachyonic quantum wave-function $\tilde{\Psi}(X_1, X_2, X_3, X_4 = icT)$.

We then start from the equation

$$\tilde{E}^2 - \tilde{p}^2 c^2 = -\mu^2 c^4 \quad (95)$$

and consider the operator equivalence

$$\tilde{E} \equiv \tilde{H} \rightarrow i\hbar \frac{\partial}{\partial T}; \quad \tilde{p} \rightarrow -i\hbar \nabla \quad (96)$$

where $\nabla = (\partial_{X_1}, \partial_{X_2}, \partial_{X_3})$. Using equation (96) in equation (95) we obtain

$$\left[\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial T^2} + \xi^2 \right] \tilde{\Psi}(X_\mu) = 0 \quad (97)$$

where $\xi^2 = \mu^2 c^2 / \hbar^2$.

Equation (97) represents the Klein-Gordon equation for a spinless tachyon, namely a tachyonic boson, that when written in explicit form

$$\left[-\frac{1}{c^2} \frac{\partial^2}{\partial T^2} + \frac{\partial^2}{\partial X_1^2} + \frac{\partial^2}{\partial X_2^2} + \frac{\partial^2}{\partial X_3^2} \right] \tilde{\Psi}(X_\mu) = -\xi^2 \tilde{\Psi}(X_\mu) \quad (98)$$

it manifests, in the left-side member, the tachyonic signature $(-+++)$ of \tilde{E}_4 and its invariance under the transformation $\Lambda_{\mu\nu}$ given by equation (45).

If we compare this latter equation with the usual Klein-Gordon equation for a bradyonic quantum wave-function $\Psi(x_\mu)$, namely

$$\left[\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x_1^2} - \frac{\partial^2}{\partial x_2^2} - \frac{\partial^2}{\partial x_3^2} \right] \Psi(x_\mu) = -\xi^2 \Psi(x_\mu) \quad (99)$$

characterized by the bradyonic signature $(+---)$ of E_4 , we can write a “generalized” Klein-Gordon equation in the form

$$\square \Psi(x_\mu) = K \Psi(x_\mu) \quad (100)$$

where $\square = -(1/c^2) \partial_t^2 + \partial_{x_1}^2 + \partial_{x_2}^2 + \partial_{x_3}^2$ and the x_μ can now represent both the tachyonic or bradyonic coordinates and K is a real parameter. We distinguish three cases according to the sign of K .

If $K = \xi^2 = m^2 c^2 / \hbar^2 > 0$, equation (100) reduces to the usual **bradyonic** Klein-Gordon equation

$$\square \Psi(x_\mu) = \xi^2 \Psi(x_\mu) \quad (101)$$

for $K = -\xi^2 = -\mu^2 c^2 / \hbar^2 < 0$, it represents the **tachyonic** Klein-Gordon equation (with $x_\mu = X_\mu$)

$$\square \tilde{\Psi}(X_\mu) = -\xi^2 \tilde{\Psi}(X_\mu) \quad (102)$$

while, for $K = 0$, it becomes

$$\square \Psi(x_\mu) = 0 \quad (103)$$

which describes a spinless particle with zero rest mass, namely a photon.

It can be shown the general Klein-Gordon equation (100) can be written as a tensorial equation in generic curvilinear coordinates y^α as

$$g^{\mu\nu} \left[\frac{\partial^2}{\partial y^\alpha \partial y^\nu} - \Gamma_{\mu \nu}^{\alpha} \frac{\partial}{\partial y^\alpha} \right] \Psi(y^\alpha) = -\xi^2 \Psi(y^\alpha) \quad (104)$$

where $\mu, \nu, \alpha = 1, 2, 3, 4$. According if the metric tensor $(g^{\mu\nu}) = (g_{\mu\nu})$ is given by equation (9) or equation (11) the equation (104) respectively becomes the tachyonic equation (102) or the bradyonic equation (101).

The Klein-Gordon equation (103) describing the photon is instead obtained, regardless of the form of $g^{\mu\nu}$, when $\xi = 0$ ($\mu = 0$).

We then deduce it exists, also in this case as for the equation of dynamics (73), a supercovariance principle according to which the same equation (104) can represent bradyons or tachyons depending on the form of metric tensor $g^{\mu\nu}$ and the coordinates system.

The very fundamental result is that the identification of a particle as a tachyon or bradyon can be formally interpreted “just” as a change of space-time metric. We’ll show in this work the change of such metric is not a purely mathematical choice but a specific consequence of a physical process related to quantum vacuum dynamics.

Due to the invariance of tachyon rest mass $\mu = \bar{\mu}$, for $\beta \gg 1$ we can write a Klein – Gordon quantum equation for a tachyonic particle in a coordinate system \bar{x}_μ measured in a ORF that is

$$\square\bar{\Psi}(\bar{x}_\mu) = \frac{\bar{\mu}^2 c^2}{\hbar^2} \bar{\Psi}(\bar{x}_\mu) \quad (105)$$

It is immediate to show the tachyonic Klein-Gordon equation (98) admits the plane-wave solution

$$\tilde{\Psi}(X_i, T) = \exp\left[\frac{i}{\hbar}(\tilde{\vec{p}} \cdot \tilde{\vec{r}} - \tilde{E}T)\right] = \exp\left[\frac{i}{\hbar}\left(\sum_{i=1}^3 \tilde{p}_i \tilde{X}_i - \tilde{E}T\right)\right] \quad (106)$$

and to calculate the frequency and wavelength associated to a tachyon. To this aim we write (106) in the form

$$\tilde{\Psi}(X_i, T) = \exp\left[2\pi i\left(\sum_{i=1}^3 \frac{\tilde{p}_i}{h} \tilde{X}_i - \frac{\tilde{E}}{h} T\right)\right] = \exp\left[i\left(\sum_{i=1}^3 \tilde{k}_i \tilde{X}_i - \tilde{\omega}T\right)\right] \quad (107)$$

where $\tilde{k} = (\tilde{k}_x, \tilde{k}_y, \tilde{k}_z)$ and $\tilde{\omega}$ respectively indicate the wave-number three-vector and the angular pulsation of a generic plane wave measured in TRF. We can then write the tachyonic Einstein - De Broglie relations

$$\begin{cases} \tilde{E} = \hbar\tilde{\omega} = h\tilde{\nu} \\ \tilde{\vec{p}} = \hbar\tilde{\vec{k}} \end{cases} \quad (108)$$

where ν is the plane-wave frequency measured in the TRF.

Then, for a tachyon moving with respect a TRF with velocity $v > c$, the associated quantum plane-wave is characterized by a frequency $\tilde{\nu}$ and wave-length $\tilde{\lambda}$ given by

$$\tilde{\nu} = \frac{\tilde{E}}{h} = \frac{\mu c^2}{h\sqrt{\beta^2 - 1}}; \quad \tilde{\lambda} = \frac{2\pi}{|\tilde{\vec{k}}|} = \frac{h}{|\tilde{\vec{p}}|} = \frac{h\sqrt{\beta^2 - 1}}{\mu|\tilde{\vec{v}}|} \quad (109)$$

where we have used equation (80). The physical interpretation of such relations is also preserved within the tachyonic wave-like formulation of quantum mechanics as we can see by calculating the phase and group velocity of a tachyonic plane wave.

The tachyonic phase-velocity is given by

$$\tilde{v}_\varphi = \tilde{\lambda}\tilde{\nu} \quad (110)$$

that, using equation (109), becomes

$$\tilde{v}_\varphi = \frac{h\sqrt{\beta^2 - 1}}{\mu|\tilde{\vec{v}}|} \frac{\mu c^2}{h\sqrt{\beta^2 - 1}} = \frac{c^2}{|\tilde{\vec{v}}|} \quad (111)$$

which formally coincides with the expression of bradyonic phase velocity. The tachyonic group velocity can be defined as

$$|\tilde{v}_g| = \frac{d\tilde{\omega}}{d|\tilde{\vec{k}}|} \quad (112)$$

that can be easily calculated by noting that, by equation (108) and equation (109)

$$d\tilde{\omega} = -\frac{\mu c^2}{\hbar} \beta (\beta^2 - 1)^{-\frac{3}{2}} d\beta \quad (113)$$

$$d|\tilde{\vec{k}}| = -\frac{\mu c}{\hbar} \frac{d\beta}{(\beta^2 - 1)^{\frac{3}{2}}} \quad (114)$$

and then

$$|\tilde{v}_g| = \beta c = |\tilde{\vec{v}}| \quad (115)$$

Since for a tachyon $|\tilde{\vec{v}}| > c$, we have

$$\tilde{v}_\varphi < c; \quad \tilde{v}_g > c \quad (116)$$

so the quantum wave associated to a tachyon moves, with respect to TRF, with a group velocity larger than c as we expect within the picture of tachyonic wave quantum mechanics.

4. Coherent dynamics of quantum vacuum and possible condensed matter origin of tachyons

We recently proposed a model, named CMH (Caligiuri Mass-Matter Hypothesis), based on the coherent dynamics of the physical vacuum of quantum field theory (CQFT), according to which inertia and matter (even in form of elementary particles) originate by such dynamics.

It has been shown [8,17] that, under suitable boundary conditions, the quantum vacuum fluctuations are able to couple so strong with a matter systems, through its proper resonances, to induce the system to “runaway”, through a “Superradiant Phase Transition” (SPT), from the a Perturbative Ground State, characterized by the quantum zero point oscillations of e.m. field and matter, towards a more stable (true) ground state, named the Coherent Ground State (CGS), in which both the e.m. field and matter system oscillate in phase with each other at a common frequency ω_{coh} .

The resulting coherent state is characterized by a collective common behavior of the quantized e.m. and matter fields appearing as a macroscopic quantum object in which atoms and molecules lose its individuality to become part of a whole electromagnetic field + matter entangled system, similar, in many regards, to that characterizing a Bose-Einstein Condensate (BEC). On the other hand, macroscopic quantum phenomena are well known to be a reality, as experimentally demonstrated, for example, by the occurrence of superfluidity and superconductivity.

Two of the most remarkable consequences of the coherent phase transition are: a) the release of a quantity of energy from the material systems to its surround environment during the SPT, the so called “energy gap”, associated to the transition (condensation) from PGS to CGS; b) the formation, inside the macroscopic quantum coherent e.m. + matter system, of the so-called “Coherence Domains” (CDs), namely the smallest spatial regions in which the coherent evolution of the e.m. + matter field takes place. The coherent dynamics in fact determines an extended oscillating polarization field able to correlate a high number of elementary matter electric dipoles so generating stable and ordered structures in macroscopic spatial regions as the CDs.

In particular, these could be the result of the coupling amplification between Zero-Point matter and gauge fields due to the coherent interaction. As a result of this process, manifesting itself as a spontaneous SPT due to quantum fluctuations of universal space viewed like a Bose gas described by a “classical” density, the matter – wave field acquires, within a single or few coherent domains, a non-negligible and stable amplitude sustained by the coherent interaction with a radiation field so leading to a “condensation” of both matter and radiation fields out from quantum vacuum.

As known, in QFT the ground state of the free e.m. field is considered as an infinite set of one-dimensional harmonic oscillators, $A_{j,\vec{k}r}$ (where $j = 1,2,3$ indicates the spatial component of vector potential, \vec{k} is the wave vector and $r = 1,2$ is the transverse polarization) performing their uncorrelated zero-point-oscillations. Its “excited” states, characterized by the occupation numbers $n_{\vec{k}r}$ of the mode oscillators, are interpreted as the number of quanta of that field (photons) corresponding to wave-number \vec{k} , energy $E_{\vec{k}r} = |\vec{k}| = \omega_{\vec{k}}$ and transverse polarization r .

Nevertheless, this model contains a deep limitation when applied to a more realistic case where the e.m. field interacts with its sources: it gives rise to strongly non-linear QFT whose solution cannot be determined unless a perturbative approach is adopted. This implies to consider the e.m. field + matter system only weakly interacting, and calculate the perturbation introduced in the decoupled system by means of asymptotic expansion in interaction term, characterized by a coupling parameter.

This theoretical approach ignores any information about phase coherence, preventing it to play any important role in the quantum evolution of the system. If we consider the quantum phase operator Φ

$$\hat{a} = e^{i\Phi} N^{\frac{1}{2}} \quad (117)$$

we have

$$[N, \Phi] = i \quad (118)$$

that gives, according to Heisenberg principle

$$\Delta N \cdot \Delta \Phi \geq \frac{1}{2} \quad (119)$$

Inequality (119) implies a quantum state characterized by a well-defined occupation number, as the eigenstates of the free e.m. Hamiltonian, has a completely undetermined phase. Equation (117) allows us to define a coherent quantum state $|\alpha\rangle$ as the normalized ($\langle\alpha|\alpha\rangle = 1$) eigenvector (of both matter and radiation fields) of the annihilation operator $a_n|\alpha_n\rangle = \alpha_n|\alpha_n\rangle$ as

$$|\alpha\rangle = e^{-\frac{\alpha^* \alpha}{2}} \sum_n \frac{\alpha^n}{\sqrt{n!}} |n\rangle \quad (120)$$

A coherent state is then constituted by an infinite superposition of eigenvectors of operator $N = a^\dagger a$, whose number of phased quanta is Poisson distributed around the mean value $\langle N \rangle = |\alpha|^2$.

The general evolution of a quantum matter system can be achieved [1,8,17] by considering a matter wave field described by the wavefunction

$$\Psi(\vec{x}, \alpha, t) = \sum_n a_n(\alpha) \varphi_n(\vec{x}, t) \quad (121)$$

in which \vec{x} represents the spatial coordinates, α a set of variables describing to the “internal” status of an elementary constituent, $\{\varphi_n(\vec{x}, t)\}$ a complete set of orthonormal base functions that diagonalizes the system Hamiltonian H (including the eventual short – range interactions)

$$H \varphi_n(\vec{x}, t) = E_n \varphi_n(\vec{x}, t) \quad (122)$$

and a_n is the annihilation operator of single-particle level in the state $|n\rangle$ satisfying the equal-time commutation relation $[a_n(t), a_m^\dagger(t)] = \delta_{nm}$. The matter field interacts with a quantized electromagnetic radiation field

$$\vec{A}(\vec{x}, t) = \sum_{\vec{k}, r} \frac{1}{\sqrt{2\omega_k V}} \left[\vec{\varepsilon}_{\vec{k}, r} a_{\vec{k}, r}(t) e^{-i(\omega t - \vec{k} \cdot \vec{x})} + h.c. \right] \quad (123)$$

in which V is the quantizing volume, $\vec{\varepsilon}_{\vec{k}, r}$ the polarization vectors and $a_{\vec{k}, r}(t)$ the field amplitudes, obeying the equal-time commutation relation $[a_{\vec{k}, r}(t), a_{\vec{k}', r'}^\dagger(t)] = \delta_{(\vec{k}, k)} \delta_{(r', r)}$.

By applying [8,17,18] the stationary action principle to the full Lagrangian of the interacting system

$$\bar{L}_{full} = \bar{L}_m + \bar{L}_{em} \quad (124)$$

where \bar{L}_m is the “matter Lagrangian” [8,17,18] and \bar{L}_{em} is the “electromagnetic Lagrangian” given by

$$\bar{L}_{em} = \sum_{\vec{k}, r} \left[\frac{i}{2} \left(\bar{a}_{\vec{k}, r}^\dagger \dot{\bar{a}}_{\vec{k}, r} - \dot{\bar{a}}_{\vec{k}, r}^\dagger \bar{a}_{\vec{k}, r} \right) + \frac{1}{2\omega_k} \dot{\bar{a}}_{\vec{k}, r}^\dagger \dot{\bar{a}}_{\vec{k}, r} \right] \quad (125)$$

allows us to describe the matter – e.m. field interaction through a couple of classical Euler – Lagrange equations for the fields $\bar{\Psi}(\vec{x}, \delta, t) \equiv N^{-\frac{1}{2}} \Psi(\vec{x}, \delta, t)$ and $\bar{a}_{\vec{k}, r}(t) \equiv N^{-\frac{1}{2}} a_{\vec{k}, r}(t)$ that we can write as

$$\bar{\Psi}(\vec{x}, \delta, t) = \varphi(\vec{x}, \delta, t) + \frac{1}{\sqrt{N}} Q(\vec{x}, \delta, t) \quad (126)$$

$$\bar{a}_{\vec{k}r}(t) = \alpha_{\vec{k}r}(t) + \frac{1}{\sqrt{N}} q_{\vec{k}r}(\vec{x}, \delta, t) \quad (127)$$

where the functions $Q(\vec{x}, \delta, t)$ and $q(\vec{x}, \delta, t)$ respectively represent the quantum fluctuations of the matter and e.m. field around the their “classical” paths.

The above-mentioned Euler - Lagrange equations for the “classical” amplitudes can be then obtained by the variational principle

$$\delta \int_{t_i}^{t_f} (\bar{L}_m + \bar{L}_{em}) dt = 0 \quad (128)$$

respectively by varying with respect to φ^* and $\alpha_{\vec{k}r}^*$, namely

$$i \frac{\partial}{\partial t} \varphi(\vec{x}, t) = H_{0m} \varphi(\vec{x}, t) + e \sqrt{\frac{N}{V}} \sum_{\vec{k}r} \frac{1}{\sqrt{2\omega_{\vec{k}}}} \left[\alpha_{\vec{k}r}(t) e^{-i(\omega_{\vec{k}}t - \vec{k} \cdot \vec{x})} \vec{\varepsilon}_{\vec{k}r} + c.c. \right] \cdot \vec{J} \varphi(\vec{x}, t) \quad (129)$$

and

$$i \dot{\alpha}_{\vec{k}r}(t) - \frac{1}{2\omega_{\vec{k}}} \ddot{\alpha}_{\vec{k}r}(t) = e \sqrt{\frac{N}{V}} \frac{e^{-i\omega_{\vec{k}}t}}{\sqrt{2\omega_{\vec{k}}}} \vec{\varepsilon}_{\vec{k}r} \cdot \int e^{-i\vec{k} \cdot \vec{x}} \varphi^*(\vec{x}, t) \vec{J} \varphi(\vec{x}, t) d^3x \quad (130)$$

where \vec{J} is the electromagnetic current density operator connecting the transitions between a couples of states.

The interaction between matter and e.m. field automatically “selects”, among the ZPF modes, that of frequency $\omega_{\vec{k}}$ resonating with one of the possible oscillation modes of matter field $\omega_{\vec{k}} = \omega_0 = |E_1 - E_0|$ [8,17].

If $\varphi_i(\vec{x}, \delta, t)$ describes a two-levels ($i = 1, 2$), respectively associated to energies E_1 and E_0 , quantum matter system we obtain, assuming for simplicity $J_i = J \delta_{i1} \sigma_1$ and neglecting the kinetic terms, the so-called “coherent equations”

$$i \frac{\partial}{\partial t} \varphi_1 = E_1 \varphi_1 + eJ \sqrt{\frac{N}{V}} \sum_{\vec{k}r} \frac{1}{\sqrt{2\omega_{\vec{k}}}} \left[\alpha_{\vec{k}r} e^{-i(\omega_{\vec{k}}t - \vec{k} \cdot \vec{x})} \vec{\varepsilon}_{\vec{k}r,1} + c.c. \right] \varphi_0 \quad (131)$$

$$i \frac{\partial}{\partial t} \varphi_0 = E_2 \varphi_0 + eJ \sqrt{\frac{N}{V}} \sum_{\vec{k}r} \frac{1}{\sqrt{2\omega_{\vec{k}}}} \left[\alpha_{\vec{k}r} e^{-i(\omega_{\vec{k}}t - \vec{k} \cdot \vec{x})} \vec{\varepsilon}_{\vec{k}r,1} + c.c. \right] \varphi_1 \quad (132)$$

$$i \dot{\alpha}_{\vec{k}r} - \frac{1}{2\omega_{\vec{k}}} \ddot{\alpha}_{\vec{k}r} = eJ \sqrt{\frac{N}{V}} \frac{e^{-i\omega_{\vec{k}}t}}{\sqrt{2\omega_{\vec{k}}}} \vec{\varepsilon}_{\vec{k}r,1} \int e^{-i\vec{k} \cdot \vec{x}} [\varphi_1^* \varphi_0 + \varphi_1 \varphi_0^*] d^3x \quad (133)$$

If we now assume the “interaction representation” $\varphi_i(\vec{x}, t) = e^{-iE_i t} \beta_i(\vec{x}, t)$ and “rotating-wave” approximation we can neglect the rapid time-oscillating exponential terms and retain only the modes such that $|\vec{k}| = \omega_0$ so the (131)-(133) become

$$i \frac{\partial}{\partial t} \beta_1 = eJ \sqrt{\frac{N}{V}} \frac{1}{\sqrt{2\omega_0}} \sum_{|\vec{k}|=\omega_0, r} e^{i\vec{k} \cdot \vec{x}} \alpha_{\vec{k}r} \vec{\varepsilon}_{\vec{k}r,1} \beta_0 \quad (134)$$

$$i \frac{\partial}{\partial t} \beta_0 = eJ \sqrt{\frac{N}{V}} \frac{1}{\sqrt{2\omega_0}} \sum_{|\vec{k}|=\omega_0, r} e^{-i\vec{k} \cdot \vec{x}} \alpha_{\vec{k}r}^* \vec{\varepsilon}_{\vec{k}r,1} \beta_1 \quad (135)$$

$$i\dot{\alpha}_{\vec{k}r} - \frac{1}{2\omega_0}\ddot{\alpha}_{\vec{k}r} = eJ\sqrt{\frac{N}{V}}\frac{e^{-i\omega_{\vec{k}}t}}{\sqrt{2\omega_0}}\varepsilon_{\vec{k}r,1}\int e^{-i\vec{k}\cdot\vec{x}}\beta_0^*\beta_1d^3x \quad (136)$$

The condition $|\vec{k}| = \omega_0$ limits the dynamic evolution described by the CEs in a space-structure, composed by an array of the so-called coherence domains (CD), whose spatial extension is of order of $L_{CD} \simeq \lambda = 2\pi/\omega_0$, namely the wavelength of the resonating selected e.m. modes in which the matter-wave field and e.m. amplitudes vary very slowly with space.

This allows us to write, inside a CD

$$\beta_i(\vec{x}, t) \simeq \frac{1}{\sqrt{L_{CD}^3}}\beta_i(t) \quad (137)$$

i.e. we neglect the spatial dependence of β inside the CD.

The CEs will then become

$$i\frac{\partial}{\partial t}\beta_1(t) = eJ\sqrt{\frac{N}{V}}\frac{1}{\sqrt{2\omega_0}}\sum_r\int d\Omega_{\vec{k}}\alpha_{\vec{k}r}\varepsilon_{\vec{k}r,1}\beta_0(t) \quad (138)$$

$$i\frac{\partial}{\partial t}\beta_0(t) = eJ\sqrt{\frac{N}{V}}\frac{1}{\sqrt{2\omega_0}}\sum_r\int d\Omega_{\vec{k}}\alpha_{\vec{k}r}^*\varepsilon_{\vec{k}r,1}\beta_1(t) \quad (139)$$

$$i\dot{\alpha}_{\vec{k}r} - \frac{1}{2\omega_0}\ddot{\alpha}_{\vec{k}r} = eJ\sqrt{\frac{N}{V}}\frac{e^{-i\omega_{\vec{k}}t}}{\sqrt{2\omega_0}}\varepsilon_{\vec{k}r,1}\beta_0^*(t)\beta_1(t) \quad (140)$$

where we have considered the number of quantized modes of e.m. field inside the volume L_{CD}^3 , such that $|\vec{k}| = \omega_0$, is given by $n = 4\pi$.

Introducing the adimensional time $\tau = \omega_0 t$ the (138)-(140) can be rewritten as

$$\dot{\beta}_1 = -igA\beta_0 \quad (141)$$

$$\dot{\beta}_2 = -igA^*\beta_1 \quad (142)$$

$$\frac{i}{2}\ddot{A} + \dot{A} = -ig\beta_1\beta_0^* \quad (143)$$

where

$$A(\tau) \equiv \sum_r\left(\frac{3}{4\pi}\right)^{\frac{1}{2}}\int d\Omega_{\vec{k}}\varepsilon_{\vec{k}r,1}\alpha_{\vec{k}r}(\tau) \quad (144)$$

$$g \equiv \frac{eJ}{\sqrt{2\omega_0^3}}\left(\frac{4\pi}{3}\right)^{\frac{1}{2}}\left(\frac{N}{V}\right)^{\frac{1}{2}} \quad (145)$$

the factor g representing a ‘‘coupling’’ constant between matter-wave and e.m. field in the coherent interaction.

The short-time behavior of the system can be studied [1,8,17] by differentiating the (143) and substituting it in the r.h.s. of equation (142), so obtaining

$$\frac{i}{2}\ddot{A}(\tau) + \dot{A}(\tau) + g^2\Delta(0)A(\tau) = 0 \quad (146)$$

where $\Delta(0) \equiv \beta_1^*(0)\beta_1(0) - \beta_2^*(0)\beta_2(0)$ and the algebraic associated equation ($A(\tau) \sim e^{ia\tau}$) is

$$a^3/2 - a^2 + \Delta(0)g^2 = 0 \quad (147)$$

Equation (147) has exactly three solutions (real or complex). The **spontaneous** ‘‘decay’’ towards the

coherent state will occur when the value of g is such that equation (147) has only one real solution, the other two complex-conjugate, ones just describing the exponential increase of $A(\tau)$ able to overcome its nearly zero initial value and create the coherent tuning field.

The real solutions of equation (147) correspond to ground state un-coherent oscillations while the complex conjugate ones, obtained when

$$g^2\Delta(0) > 0 \vee g^2\Delta(0) < -\frac{16}{27} \quad (148)$$

describe to the “condensation” of a strong coherent (classical) e.m. field $A(\tau)$ from the PGS, exponentially driving the systems away from this state towards a new state, the coherent ground state (CGS), where $A \sim O(g)$. The most interesting case happens when $\Delta(0) = -1$, namely when all the “ether” oscillators are initially in their ground state, then the transition occurs when

$$g^2 > g_c^2 = \frac{16}{27} \quad (149)$$

showing the presence of a threshold associated to a critical value of coupling constant g , in turn proportional, through factors related to the features of the specific matter-e.m. field interacting system, to the “density” N/V .

The CGS is then characterized by the stationary solutions of equations (141)-(143) ($i = 1, 2$)

$$\beta_i(\tau) = b_i e^{i\theta_i(\tau)} \quad (150)$$

$$A(\tau) = a e^{i\chi(\tau)} \quad (151)$$

where $a, b_i \in R^+$. Posing $b_1 = \cos \xi$ and $b_2 = \sin \xi$ with $0 \leq \xi \leq \pi/2$, it can be shown [8,17] equations (150)-(151) admit the solution

$$\begin{aligned} a^2(1 - \dot{\chi}) + \sin^2 \xi &= Q(0) \\ \dot{\theta}_1 &= g a \tan \xi \\ \dot{\theta}_2 &= g a \cot \xi \\ \dot{\chi} &= 1 \pm \sqrt{1 - g \frac{\sin 2\xi}{a}} \\ \dot{\xi} &= \dot{\theta}_1 - \dot{\theta}_2 \end{aligned} \quad (152)$$

This allows us to writing the energy of the coherent state as

$$H = Q(0) + a^2 \frac{\dot{\chi}^2}{2} - g a \sin 2\xi \quad (153)$$

showing the CGS is energetically favoured with respect the PGS and then it represents the true and stable ground state of interacting matter-e.m. field.

The coherent dynamics thus generates an overall energy gap $\Delta E = E_{CGS} - E_{PGS}$ given by

$$\Delta E = N\omega_0 \left(a^2 \frac{\dot{\chi}^2}{2} - g a \sin 2\xi \right) < 0 \quad (154)$$

ensuring the stability of CGS with respect the quantum fluctuations characterizing the PGS.

One of the most important consequences of this dynamics concerns the frequency ω_{coh} of coherent e.m. field coupled with matter inside a CD that can be written as [8,17]

$$\omega_{coh} = \omega_0 |1 - \dot{\chi}| = \omega_0 \sqrt{1 - g \frac{\sin 2\xi}{a}} \quad (155)$$

In fact, from equation (155) we see

$$\omega_{coh} < \omega_0 = \frac{2\pi}{\lambda_0} \quad (156)$$

where λ_0 is the wavelength of the “free” e.m. field interacting with the matter system.

This is a purely quantum effect and results in a frequency rescaling of the coherent e.m. field arising from quantum vacuum with respect the free e.m. field, so that we can interpret the latter as composed by photons of angular frequency ω_0 , while the coherent e.m. (“tuned” with matter field) composed by photons characterized by a “reduced” frequency ω_{coh} .

For a “free” photon of energy E , the Einstein equation is, as known

$$E^2 - p^2c^2 = \hbar^2\omega^2 - (\hbar|\vec{k}|)^2c^2 = \hbar^2\omega^2 - \hbar^2\left(\frac{2\pi}{\lambda}\right)^2 = 0 \quad (157)$$

and can be interpreted as the results of a “luxonic” metric associated to the photon Klein - Gordon equation (103), namely

$$ds^2 = c^2dt^2 - \sum_{i=1}^3 dx_i^2 = \sum_{i=1}^3 dx_i^2 - c^2dt^2 = 0 \quad (158)$$

On the other hand, for a photon belonging to the coherent field we have, by equation (156)

$$E^2 - p^2c^2 = \hbar^2\omega_{coh}^2 - \hbar^2\left(\frac{2\pi}{\lambda_0}\right)^2 = \hbar^2\omega_{coh}^2 - \hbar^2\omega_0^2 = -M^2 < 0 \quad (159)$$

that is identical to equation (95) if we assume

$$M^2 = \mu^2c^4 \quad (160)$$

that is

$$M = \mu c^2 \quad (161)$$

We can interpret this result by saying the coherent photon is a particle obeying the Klein-Gordon equation (102) whose rest energy is given by equation (161) namely, in other words, that the coherent photon is a spinless tachyon with real rest mass μ . We can then associate to this tachyon the energy \tilde{E} and momentum \tilde{p} given by equation (80).

If we trust in the mass-energy equivalence given by STR, we must conclude the energy rescaling due to the quantum coherent interaction determining $\omega_0 \rightarrow \omega_{coh}$ makes the free photon to acquire a real mass μ and, in doing this, to break the “light barrier” moving on a space-like word-line (a true space-like axis) associated to the metric (9) (that gives the tachyonic Klein-Gordon equation describing the coherent photon).

Equation (159) allows us also to relate the tachyon mass μ to the quantum coherent system from which it originates. In fact, by recalling equation (155), we have from equation (159) and assuming $\hbar = c = 1$

$$\omega_0^2 \left(1 - g \frac{\sin 2\xi}{a}\right) - \omega_0^2 = -\omega_0^2 g \frac{\sin 2\xi}{a} = -\mu^2 \quad (162)$$

and finally

$$\mu = \omega_0 \sqrt{g \frac{\sin 2\xi}{a}} = \left(\frac{\sin 2\xi}{a}\right)^{\frac{1}{2}} \left(\frac{2\pi}{3} \frac{N}{V} eJ\omega_0\right)^{\frac{1}{4}} \quad (163)$$

According to equation (163) the rest mass of a tachyon is proportional to the density of the matter system undergoing the superradiant phase transition as well as to ω_0 , so that the closest are the two energy levels involved in the transition, the smaller is the tachyon rest mass.

On the other hand we know the QED coherent dynamics selects, among the many possible couples of transition $|E_2 - E_1| = \omega_0$ allowed to the matter system, only the one with $\omega = \omega_0$ for which the transition amplitude is the highest, neglecting all the others [8,17,19].

Then, for a given quantum system experiencing such coherent behavior, a tachyonic mass spectrum $\{\mu_i\}$ can be associated to the set of transition frequency $\{\omega_{0,i}\}$. It must be stressed the frequency rescaling of the coherent e.m. field coupled to matter, given by (155), isn't just a mathematical "artifact" but the result of a quantum process able to create, inside the CD, a field of bosonic tachyons, all characterized by the same value of rest mass μ .

It is interesting to obtain the wavelength of tachyons belonging to the coherent field generated inside the CD as a function of ω_0 . To this aim we remember that, for the tachyon field, $\tilde{E} = \omega_{coh}$, then we have, using the left one of equations (109)

$$\frac{\mu}{\sqrt{\beta^2 - 1}} = \omega_{coh} \Rightarrow \beta^2 = \frac{\mu^2 + \omega_{coh}^2}{\omega_{coh}^2} \quad (164)$$

that, substituted in the right one of equations (109) gives, after few algebra

$$\tilde{\lambda} = \lambda_{tc} = \left(\frac{1}{\mu^2 + \omega_{coh}^2} \right)^{\frac{1}{2}} \quad (165)$$

If we now use equations (163), (155) and (145) in equation (165) we finally have, after some tedious but simple mathematical passages

$$\lambda_{tc} = \left\{ \omega_0^2 + \omega_0^2 \left(\frac{2\pi N}{3V} eJ \right)^{\frac{1}{2}} \frac{\sin 2\xi}{a} \left[1 - (eJ)^{\frac{1}{2}} \right] \right\}^{-\frac{1}{2}} \quad (166)$$

which can also be expressed as a function of $L_{CD} \simeq 2\pi/\omega_0$, namely

$$\lambda_{tc} = \left\{ \left(\frac{2\pi}{L_{CD}} \right)^2 + \left(\frac{2\pi}{L_{CD}} \right)^{\frac{1}{2}} \left(\frac{2\pi N}{3V} eJ \right)^{\frac{1}{2}} \frac{\sin 2\xi}{a} \left[1 - (eJ)^{\frac{1}{2}} \right] \right\}^{-\frac{1}{2}} \quad (167)$$

We can easily study the behavior of λ_{tc} as a function of ω_0 (or, that's the same, as a function of L_{CD}); we then write λ_{tc} for simplicity as

$$\lambda_{tc}(\omega_0) \sim \left[\omega_0^2 + C(\omega_0)^{\frac{1}{2}} \right]^{-\frac{1}{2}} \quad (168)$$

since we have, from equations (155) and (145)

$$\omega_{coh}^2 \sim \omega_0^2 - C_1\sqrt{\omega_0}, \quad \omega_0^2 > C_1\sqrt{\omega_0} \quad (169)$$

in which the coefficient $C_1 > 0$ doesn't depend on ω_0 and, from (163)

$$\mu^2 \sim C_2\sqrt{\omega_0} \quad (170)$$

where, as above, $C_2 > 0$ doesn't depend on ω_0 . So by inserting equations (169) and (170) in equation (165) we have

$$\lambda_{tc}(\omega_0) \sim \left(\omega_0^2 + C\sqrt{\omega_0} \right)^{-\frac{1}{2}} \quad (171)$$

where $C = C_2 - C_1$ and $\omega_0^2 + C\sqrt{\omega_0} > 0$. For low values of ω_0 (low energies)

$$\lambda_{tc}(\omega_0) \sim (C\sqrt{\omega_0})^{-\frac{1}{2}} \quad (172)$$

and the tachyon wavelength strongly depends on the physical features of the matter + e.m. system undergoing SPT; on the other hand, when the value of ω_0 is high, we obtain

$$\lambda_{tc}(\omega_0) \sim \frac{1}{\omega_0} \quad (173)$$

so that the tachyon wavelength substantially coincides with the dimension of the coherent domain L_{CD} in agreement with the quantum picture of a tachyonic field generated inside the CD.

We have also shown [8] universal empty space, “filled” with Zero – Point fields of every type of matter and radiation, could be interpreted as the ground state of a Bose condensate gas that must be treated perturbatively since its long-range correlation is very different, in nature, from the coherent state extending on large scales originated by a totally different type of interaction between matter wave and radiating fields, based on the strongly enhanced between them occurring when the density of elementary quanta N/V is sufficiently high.

In the PGS, in fact, any matter-wave or radiating field can perform only Zero-Point incoherent oscillations, and no matter nor radiation can exist in stable form. In this condition, the simple presence of interaction is not able to change the perturbative nature of such state. This is just the case of the typical quantum process of temporarily creation of virtual pairs of particles out of “nothing”. Then, in PGS, since $|\varphi| \sim O(N^{-1/2})$, N being the number of matter-wave quanta belonging to a state $|\Sigma\rangle$ contained in a given volume V , all the particles remains in a “virtual” state characterized by incoherent fluctuations around a zero mean value. In this state, also the mass of the related quantum fields assumes a “zero-point” meaning.

But, as we have already shown [8], when, for given matter-wave and radiation fields, the condition $g^2 \geq g_c^2$ is locally satisfied, the coherent transition from $|PGS\rangle$, in which a particle is “virtual”, to $|CGS\rangle$, where it acquires a stable existence as matter, can take place and mass emerges (“condenses”) from QV as rest mass of that specific particle, being described by the matter-wave field whose amplitude is now no more negligible.

In this case we can picture the two-levels involved in the SPT as respectively represented by the PGS state of the particle, namely $\varphi_0 \equiv |\varphi\rangle_{PGS}$, and its coherent stable state, that is $\varphi_1 \equiv |\varphi\rangle_{CGS}$, resulting from SPT and representing the true ground state of the particle itself in which it acquires its rest mass. Consequently [8], the “materialization” of a virtual particle, due to the spontaneous or induced density fluctuation of universal QV, can be described by CE equations for both bosons and fermions when solved for matter-wave fields instead of radiation field. We then obtain, if the radiation field can be described by a Lagrangian having the form of equation (125), from equation (143) and for the initial condition $\beta_0 \sim 1$, the equation

$$\frac{i}{2}\ddot{\beta}_1 + \ddot{\beta}_1 + ig\beta_1 = 0 \quad (174)$$

representing the coherent evolution of the amplitude of the matter field that, like the e.m. one, undergoes the coherent exponential runaway. We can so identify [8] the amplitude transition $\beta_0 \rightarrow \beta_1$ with the matter-wave field amplitude oscillation describing the particle, namely $\varphi_0 \equiv |\varphi\rangle_{PGS} \rightarrow \varphi_1 \equiv |\varphi\rangle_{CGS}$, whose energy is $E_1 - E_0 = \omega_0$. This dynamics then leads to the creation of one (in this case substantially coinciding with the physical size of the particle) or more CDs whose size is of order of $\lambda_{CD} \simeq 2\pi/\omega_0$ and also to the arising of a correspondent inertial mass.

If the process is driven by the e.m field as previously discussed, the coherent dynamics leads to the creation of tachyons, due to the frequency rescaling of coherent photons namely

$$\omega_0 \rightarrow \omega_{coh} = |1 - \chi| \omega_0 = [E(\varphi_1) - E(\varphi_2)] \sqrt{1 - g \frac{\sin 2\xi}{a}} \quad (175)$$

whose rest mass is given by equation (163). According to this model, the “materialization” of a particle out of QV would be accompanied by the creation of a tachyon field of spin zero and real rest mass μ .

More generally, we could say the coherent e.m. field itself, composed of superradiant photons, “filling” each CD, is actually a tachyon field characterized by the above described features.

According to our model, the creation of a tachyon field would be a process always and spontaneously occurring in condensed matter when the conditions for the SPT from PGS to CGS (density of elementary matter quanta higher than a threshold value) are satisfied (almost always in the case of stable condensed matter) but also in the case of the “materialization” of virtual particles from QV when the local density of quantum vacuum virtual quanta is sufficiently high.

This moreover suggests it is possible to induce the creation of tachyons by favoring the occurrence of the conditions for the STP from QV to arise.

The quantum - field structure of the tachyons ensemble so generated inside CD is further revealed by the intervention of tachyonic Klein – Gordon equation (102) in our treatment. Such equation is, in fact, a truly quantum field equation in which the wave-function $\tilde{\Psi}$ plays, as usual, the role of an Hermitian operator of the tachyonic field (of spin zero and mass μ) that can be decomposed as

$$\tilde{\Psi} = \tilde{\Psi}_1 + i\tilde{\Psi}_2 \quad (176)$$

in which the real and complex components of the field, associated to its two degrees of freedom, represent the electric charge of a charged tachyon particle. This allows us to develop, as it will be discussed in a forthcoming paper, a quantum field theory of charged tachyons in which we can respectively define the creator and annihilation operators $\tilde{\Psi}^\dagger$ and $\tilde{\Psi}$, and also to consider a tachyons field as a true tachyon-antitachyon field.

5. Summary and further developments

In the present work we have proposed a novel model able to explain the possible origin of tachyons and their existence as particles with real rest mass. Firstly, we have considered the isomorphism of the $SO(3,1;C)$ and $SO(1,3;C)$ groups from which it is possible to deduce two complete orthochrones isomorphic Lorentz groups named L_+^T and \tilde{L}_+^T which respectively correspond to subluminal and superluminal set of coordinates transformations and are characterized by real metric of signature $(+ - - -)$ and $(- + + +)$. The properties of \tilde{L}_+^T allow us to define a “tachyonic referential frame” (TRF) and a “tachyonic matter” as well as to develop a Special Theory of Relativity in the space-like variety in which every couple of events are separated by space-like space-time interval. Furthermore, on this basis, a tachyonic classical dynamics and a quantum theory can be developed in which the tachyon rest mass always assumes real values.

In particular, within this framework, a Klein – Gordon like equation for a tachyon has been discussed showing it formally satisfies the same Einstein - De Broglie relations as in the case of subluminal particles.

On the other hand, we have already proved, within the picture of QED coherence in condensed matter, the time evolution of any quantized system composed by matter interacting with an e.m. field (or even with a radiation field described by a similar Lagrangian function), under suitable conditions (almost always verified just in condensed matter), spontaneously determines the emergence of a coherent e.m. field oscillating in phase with all the matter constituents of the system. These coherent oscillations of matter and e.m. field are confined within defined spatial regions, called “coherent

domains”, in which the coherent e.m. field photons are all characterized by a “rescaled” frequency lower than the frequency associated to a free e.m. field of equal wavelength.

We have shown this frequency rescaling, due to a pure quantum process of mutual amplification of ZPF oscillations of matter and e.m. field quanta, can be viewed as the creation of tachyons whose energy correspond to this rescaled frequency.

More precisely, according to this picture, the e.m. field inside the CD would not be other than just a tachyon quantum field of spin and electric charge equal to zero.

The proposed model doesn't suffer from some of the most critical issues of the classical theories about tachyons so far proposed. In particular, on one side the space-like metric of \tilde{L}_+ assigns to tachyons a real (and then measurable) rest mass and on the other the study of the QED coherence in condensed matter gives us a physical mechanism, well-founded on QFT basis, able to explain the possible origin of tachyons (also predicting the dependence of their mass and wavelength on the features of the physical system from which they originate) not only inside condensed matter but also directly from quantum vacuum, as a consequence of a local increase of ZPF quanta density giving rise, under suitable conditions, to a coherent superradiant phase transition.

The present work concerned the case of electrically uncharged and spinless tachyons but its generalization to charged fermions will be presented in forthcoming publications. We think this model could be also applied not only to the field of elementary particles, cosmology and energy production but also used as the basis for a whole reformulation of relativistic quantum field theory.

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