NEUTRON STARS WITH REALISTIC EQUATIONS OF STATE IN SCALAR-TENSOR THEORIES OF GRAVITY

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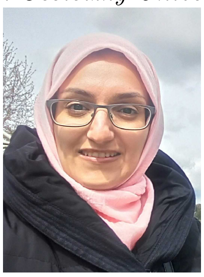

In this study, we address static spherically symmetric realistic neutron stars as well as slowly rotating ones in Scalar-Tensor Theories (STT) of gravity. We consider several realistic Equations Of State (EOS) with the values of the scalar field coupling constants constrained by current observations. We already know that in addition to the solutions of General Relativity (GR) with a trivial scalar field, STT can allow for solutions with a nontrivial scalar field, which are energetically more favorable. Our numerical results show the existence of neutron star models with a nontrivial scalar field also in the case of alternative scalar field coupling functions. We further compute the moments of inertia of neutron stars in STT which are slowly rotating. We present various results to illustrate the deviation of our models from GR.

1 Introduction

The properties of neutron stars as obtained in GR with realistic EOS have been tested successfully by observations in diverse astrophysical scenarios. However, there are various reasons to expect that GR will be superseded by a generalized theory of gravity.[1] STT are one of the most natural extensions of GR. In these theories a scalar field is included as an additional mediator of the gravitational interaction besides the metric tensor mediator of GR. Some of the alternative theories of GR like STT are indistinguishable from GR in the weak field regime but can differ significantly for strong fields. Neutron stars are good examples of the strong field regime because of their high densities and compactness. Therefore they provide an ideal laboratory for testing gravity.

The study of neutron stars in STT revealed an interesting phenomenon called spontaneous scalarization.[2,3] When a scalar field is nonminimally coupled to gravity, a nontrivial configuration of the scalar field can appear, because the scalar field nonlinearities can intensify the attractive nature of the scalar field interactions. Then it can become energetically favorable to generate more scalar-field energy without any external source. This clearly leads to neutron star models which can possess significantly different properties than their GR counterparts as shown in the static and slowly rotating case by Damour and Esposito-Farese [2,3] and for rapidly rotating neutron stars by Doneva *et al.*[4] Here we address the effect of alternative scalar field coupling functions in STT on static spherically symmetric and slowly rotating neutron stars for several realistic EOS.

2 Basic Equations and Numerical Method

We consider the gravitational action in the Einstein frame

$$S = \frac{1}{16\pi G_*}\int d^4x\sqrt{-g}\left[R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - 4V(\varphi)\right] + S_m[\Psi_m; \mathcal{A}^2(\varphi)g_{\mu\nu}], \quad (1)$$

where G_* is the gravitational constant, and the Ricci scalar curvature R is defined with respect to the metric in Einstein frame $g_{\mu\nu}$. The matter action involves the non-minimal coupling function $\mathcal{A}(\varphi)$ to the scalar field φ. We here assume a vanishing scalar potential $V(\varphi) = 0$, and set $G_* = 1$ and $c = 1$. The Einstein-matter equations then read

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi T_{\mu\nu} + 2\partial_\mu\varphi\partial_\nu\varphi - g_{\mu\nu}g^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi, \quad (2)$$

$$\nabla^\mu\nabla_\mu\varphi = -4\pi k(\varphi)T, \quad (3)$$

where $k(\varphi) = \frac{d\ln(\mathcal{A}(\varphi))}{d\varphi}$. The energy momentum tensor is given by $T_{\mu\nu} = (\varepsilon + p)u_\mu u_\nu + pg_{\mu\nu}$, where ε, p and u_μ are the energy density, the pressure and the four-velocity in the Einstein frame, which are related to the corresponding quantities in the Jordan frame by $\varepsilon = \mathcal{A}^4(\varphi)\tilde{\varepsilon}$, $p = \mathcal{A}^4(\varphi)\tilde{p}$, and $u_\mu = \mathcal{A}^{-1}(\varphi)\tilde{u}_\mu$. From the Bianchi identities the conservation law for the Einstein frame energy-momentum tensor $\nabla_\mu T^\mu{}_\nu = k(\varphi)T\partial_\nu\varphi$ is obtained.

In our study we have considered two STT with the following coupling functions

1. $\mathcal{A}(\varphi) = \exp(\frac{1}{2}\beta_1\varphi^2)$,
2. $\mathcal{A}(\varphi) = \frac{1}{\cosh(\beta_2\varphi)}$,

where the first coupling function has been investigated before, [2,3,4] whereas the second coupling function is new. There are observational constraints on the possible values of β, that should be taken into account. [5] Considering $\frac{d^2\ln(\mathcal{A}(\varphi))}{d\varphi^2}|_{\varphi=0} > -4.8$, we have employed β_1 = -4.5 and -4.8 for the exponential coupling function, while we have used $\beta_2 = \sqrt{4.8}$ and $\sqrt{4.5}$ for the new coupling function.

The metric describing a slowly rotating neutron star can be chosen as

$$ds^2 = -e^{f(r)}dt^2 + \frac{1}{n(r)}dr^2 + r^2d\theta^2 + r^2\sin^2\theta(d\phi + \omega(r)dt)^2, \quad (4)$$

where the metric function $\omega(r)$ vanishes in the static case. With the metric ansatz Eq. (4) and the scalar field $\varphi = \varphi(r)$, the pressure $p = p(r)$, and the energy density $\varepsilon = \varepsilon(r)$, the Einstein field equations for static neutron stars reduce to the following system of ordinary differential equations to be solved numerically

$$\frac{dn}{dr} = -\frac{1}{r}\left[8\pi r^2\mathcal{A}^4(\varphi)\tilde{\varepsilon} + nr^2\left(\frac{d\varphi}{dr}\right)^2 + n - 1\right], \quad (5)$$

$$\frac{df}{dr} = \frac{1}{nr}\left[8\pi r^2\mathcal{A}^4(\varphi)\tilde{p} + nr^2\left(\frac{d\varphi}{dr}\right)^2 - n + 1\right], \quad (6)$$

$$\frac{d^2\varphi}{dr^2} = \frac{4\pi r\mathcal{A}^4(\varphi)}{nr}\left[r\left(\frac{d\varphi}{dr}\right)(\tilde{\varepsilon} - \tilde{p}) + k(\varphi)(\tilde{\varepsilon} - 3\tilde{p})\right] - \left(\frac{d\varphi}{dr}\right)\frac{(n+1)}{nr}, \quad (7)$$

$$\frac{d\tilde{p}}{dr} = -(\tilde{\varepsilon} + \tilde{p})\left[\frac{4\pi r\mathcal{A}^4(\varphi)\tilde{p}}{n} + \frac{r}{2}\left(\frac{d\varphi}{dr}\right)^2 + k(\varphi)\left(\frac{d\varphi}{dr}\right) - \frac{n-1}{2nr}\right]. \quad (8)$$

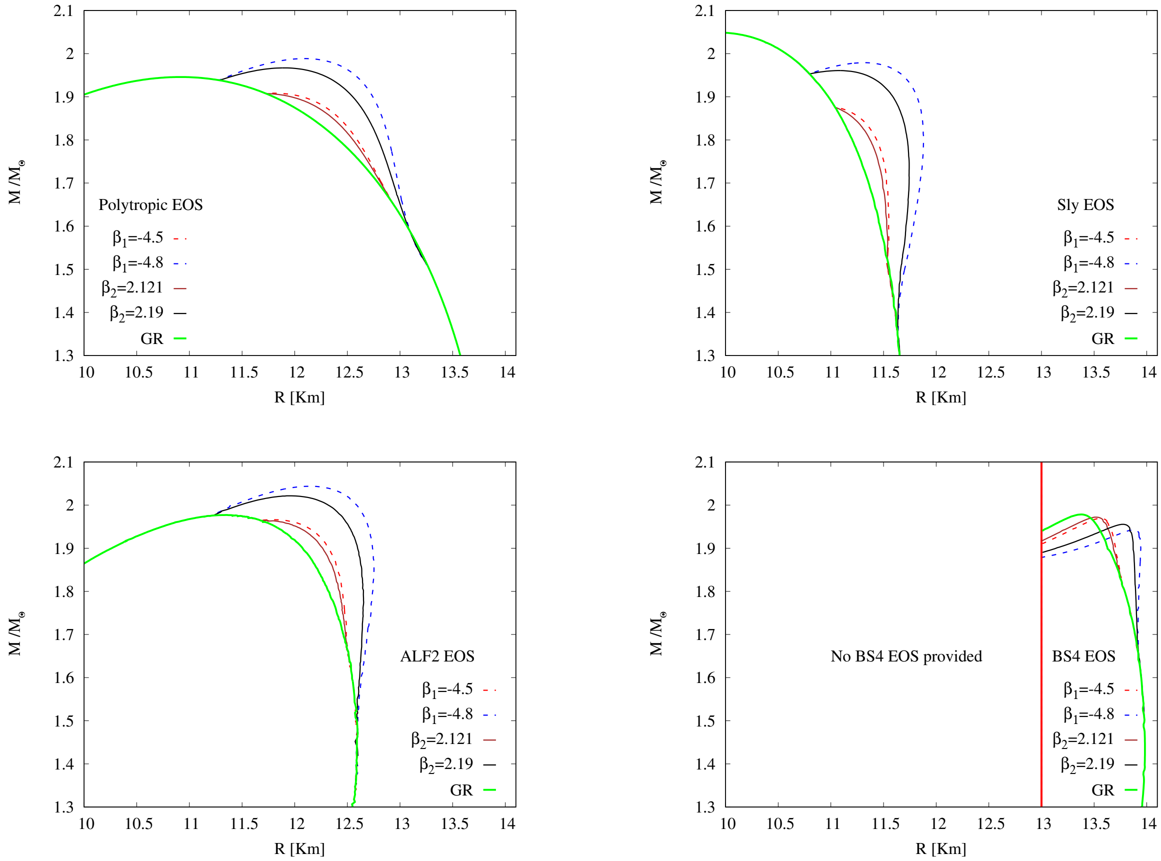

Figure 1 – Mass M vs radius R for different EOS

We have first solved these equations to obtain the spherically symmetric neutron stars (static case). Then we have considered the first order perturbation equation for the slowly rotating case

$$\frac{d^2\omega}{dr^2} = \frac{4\pi r \mathcal{A}^4(\varphi)}{n}(\tilde{\varepsilon}+\tilde{p})\left[\left(\frac{d\omega}{dr}\right)+\frac{4\omega}{r}\right]+\left(\frac{d\omega}{dr}\right)\left[r\left(\frac{d\varphi}{dr}\right)^2-\frac{4}{r}\right]. \tag{9}$$

In solving these ODEs numerically, we have used the COLSYS (COLlocation for SYStems) code. For the static solutions we have chosen the boundary conditions $n(\infty)=1$, $f(\infty)=0$, $\tilde{p}(0)=\tilde{p}_c$, $\varphi(\infty)=0$, $\left(\frac{d\varphi}{dr}\right)(0)=0$, and we have assumed slow rigid rotation with $\omega(0)=\omega_c$ and $\omega(\infty)=0$.

3 Equations of State

For the static and slowly rotating neutron stars in STT we have studied a polytropic EOS and several realistic ones. In a polytropic EOS the pressure $\tilde{p}$ and the baryonic mass density $\tilde{\rho}$ are related according to $\tilde{\varepsilon}=K\frac{\tilde{\rho}^{\Gamma}}{\Gamma-1}+\tilde{\rho}$, $\tilde{p}=K\tilde{\rho}^{\Gamma}$, where we have chosen for the polytropic constant $K=1186.0$, and for the adiabatic index $\Gamma=1+\frac{1}{N}$ the polytropic index $N=0.7463$.

We have also used various realistic EOS. But due to lack of space we here present only some examples of our results, which have been obtained using SLy [6] for nuclear matter, ALF2 [7] for quark matter, and BS4 [8] for hybrid quark-nuclear matter.

4 Results

Let us now discuss some of our results obtained in the study of static and slowly rotating neutron stars in STT with the two coupling functions $\mathcal{A}(\varphi)=\exp(\frac{1}{2}\beta_1\varphi^2)$ and $\mathcal{A}(\varphi)=1/\cosh(\beta_2\varphi)$, using realistic EOS. Fig. 1 illustrates the effects of the spontaneous scalarization on the mass–radius relationship of static neutron stars. Here the mass $M/M_{\odot}$ is shown versus the radius

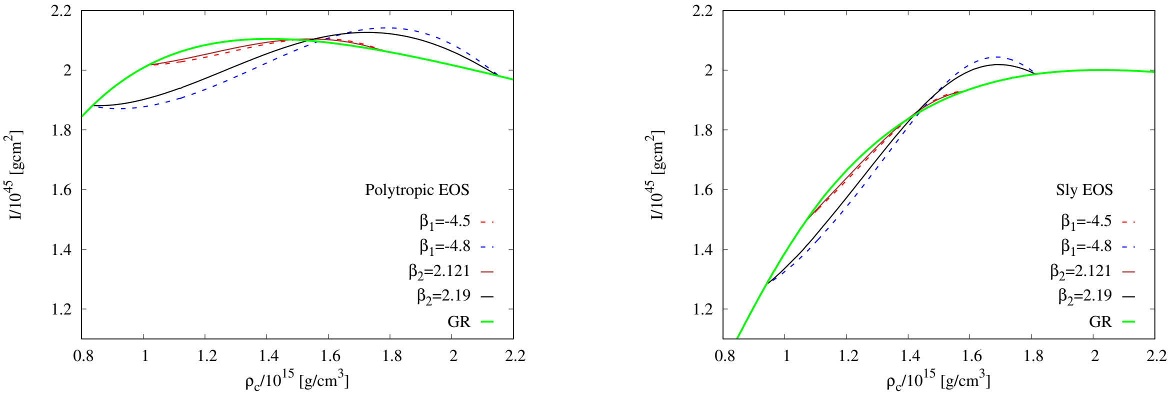

Figure 2 – Moment of inertia I vs central density ρ_c for two EOS

R for a polytropic EOS and for three realistic ones. For comparison the GR results are also exhibited. Also, both coupling functions are used, where the respective values of the coupling constants are chosen to obtain the same quadratic expansion coefficients of $\mathcal{A}(\varphi)$. Then the scalarization arises for both coupling functions at the same GR configurations, independent of the EOS. Clearly, since the exponential coupling function is smaller for larger φ than the new coupling function, the effects concerning the scalarization are larger for the exponential coupling. Thus the results for the new coupling function are closer to the GR results. We observe that for larger values of $|\beta_i|$ the maximum mass is reached at a larger radius. For smaller values of $|\beta_i|$ the maximum mass is reached only close to the upper bifurcation point for the polytropic EOS and ALF2. For Sly the maximum mass is smaller than in GR. Fig. 2 shows the moment of inertia I versus the central density ρ_c for two EOS, the polytropic EOS and the Sly EOS, to present also an example for a realistic EOS (see also Staykov *et al.*[9]).

In further studies we might consider rapidly rotating neutron stars with realistic EOS in STT with new coupling functions. It would also be interesting to study a massive scalar field for new coupling functions. Indeed, Yazadjiev *et al.*[5,10] have studied a massive scalar field for slowly and rapidly rotating neutron stars and shown that the inclusion of a mass term for the scalar field leads to very interesting new results.

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References

1. E. Berti *et al*, *Class. Quant. Grav.* **32**, 243001 (2015).
2. T. Damour and G. Esposito-Farese, *Phys. Rev. Lett.* **70**, 2220 (1993).
3. T. Damour and G. Esposito-Farese, *Phys. Rev.* D **54**, 1474 (1996).
4. D.D. Doneva *et al*,*Phys. Rev.* D **88**, 084060 (2013).
5. S.S. Yazadjiev, D.D. Doneva and D. Popchev, *Phys. Rev.* D **93**, 084010 (2016).
6. F. Douchin and P. Haensel, *A&A* **380**, 151 (2001).
7. M. Alford, M. Braby, M. Paris, and S. Reddy, *ApJ* **629**, 969 (2005).
8. L. Bonanno and A. Sedrakian, *A&A* **539**, A16 (2012).
9. K. V. Staykov, D. D. Doneva, and S. S. Yazadjiev, *Phys. Rev.* D **93**, 084010 (2016).
10. D.D. Doneva, S.S. Yazadjiev, *JCAP* **1611**, 11, 019 (2016).