

B_c meson spectrum via Dyson-Schwinger equation and Bethe-Salpeter equation approach

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We predict the masses of the low-lying B_c mesons with $J^P = 0^-, 1^-, 0^+, 1^+, 2^+$, using a flavor-dependent interaction pattern which gives an unified successful description of the light, heavy-light, and heavy mesons and is also applicable to the radial excited heavy mesons. The errors are controlled carefully. With the errors from the rainbow ladder (RL) approximation subdued, our predictions are consistent with the lattice QCD (IQCD) and quark model results, which support strongly that the flavor-dependent interaction pattern is reasonable. Our predictions provide significant guides to the experiment search of the B_c mesons.

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I. INTRODUCTION

The B_c mesons are the only heavy mesons containing two different flavor valence quarks, one charm quark and one bottom quark. The quark flavor difference forbids their annihilation into gluons or photons, so that the ground state pseudoscalar $B_c(1S)$ can only decay weakly, which makes it particularly interesting for the study of weak interaction. The excited B_c states lying below the BD production threshold decay into the ground state via hadronic or radiative transitions. As a result, the B_c mesons are supposed to be much more stable than their counterparts in the $c\bar{c}$ and $b\bar{b}$ systems, and spin splittings can be studied in detail. However, in the experimental aspect, the B_c mesons are much less explored than the charmonium and bottomonium due to the small production rate, as the dominant production mechanism requires the production of both $c\bar{c}$ and $b\bar{b}$ pairs. The ground state pseudoscalar

$B_c(1S)$ meson was first observed in 1998 by the CDF collaboration at the Tevatron collider [1]. The vector meson, $B_c^*(1S)$, decays into $B_c(1S)$ via radiative transition, $B_c^*(1S) \rightarrow B_c(1S)\gamma$. It's challenging to detect the emitted photon due to the low energy. The vector meson mass, $M_{B_c^*(1S)}$, is still not determined. Until recently two excited B_c states were reported by the CMS collaboration [2] and the LHCb collaboration [3], pushing forward the study of the B_c mesons. More B_c mesons are expected to be discovered [4]. In the theoretical aspect, the predictions of the B_c mesons have existed since 40 years ago [5], and various model calculations occurred in these years [6–14]. However, these predictions vary widely. For example, the 1S-spin splitting, i.e., $M_{B_c^*(1S)} - M_{B_c(1S)}$, spreads over a range of 40–90 MeV [6–14]. The predictions of the excited states are even more scattered. First principle calculations are of course essential to study the B_c mesons with containable precision. However, there are only a few first principle lattice QCD (IQCD) studies of the B_c mesons [15–19]. Until recently a IQCD calculation predicts the B_c meson masses with $J^P = 0^-, 1^-, 0^+, 1^+$ with high precision [19], where J is the angular momentum and P is the P -parity (C in the later text is the C -parity).

The Dyson-Schwinger equation and Bethe-Salpeter equation (DSBSE) approach, a nonperturbative quantum chromodynamic (QCD) approach solving QCD via the

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basic degree of freedom, i.e., the quarks and the gluons, is complementary to IQCD. In this approach, a meson corresponds to a pole in the quark-antiquark scattering kernel [20], and the amplitude is solved by the Bethe-Salpeter equation (BSE) [21]. The propagators of quarks and gluons in the scattering kernel are then solved by the Dyson-Schwinger equations (DSE) [22,23]. However, the DSEs are an infinite tower of coupled equations and no exact solution has been gained. A practical way to solve the problem of the hadron spectrum is building a quark-gluon-vertex, constructing a scattering kernel, the gluon propagator being modeled by an effective interaction, then the BSE of the hadron amplitude and DSE of the quark propagator could be solved consistently. While the forms of the quark-gluon-vertex and the scattering kernel have been investigated [24], the most widely used and technically simple one is the rainbow ladder (RL) approximation. This scheme has gained many phenomenological successes in the hadron spectrum, the form factors, the distribution functions, etc. [25–27]. The RL approximation is perfect for the light pseudoscalar mesons [28,29] and fairly good for the light vector mesons [30]. A possible way to solve the light meson with orbit angular momentum $L \geq 1$ is given in Ref. [24]. The mass spectra of the charmonium and bottomonium with angular momentum $J < 3$ have been investigated in the RL approximation, with relative errors of a few percent [31–33]. However, the original RL approximation fails to describe the heavy-light mesons due to the lack of the flavor asymmetry. In Ref. [34] we build an interaction model, which takes into account the quark flavor dependence of the effective quark-antiquark interaction properly and preserves the axial-vector Ward Takahashi identity excellently. Our model not only gives a successful unified description of the light, heavy-light, and heavy pseudoscalar and vector mesons, but also reduces the deviation between the theoretical and experimental value of the $B_c(1S)$ meson mass from 110 MeV [35] to about 15 MeV. In Ref. [36] our model is applied to the radial excited $B_c(2S)$ and $B_c^*(2S)$ mesons. The mass spectrum is reasonable and we predict the decay constants of the radial excited B_c mesons for the first time.

In this article, we extend our study to B_c mesons with $J^P = 0^-, 1^-, 0^+, 1^+$ and 2^+ . In Sec. II we give a complete introduction of our model and the formulas. The results are analyzed in Sec. III. We summarize our investigations in Sec. IV.

II. OUR MODEL

In this Poincaré covariant framework the quark propagator is solved by the Gap equation [20,22,23]

$$S_f^{-1}(k) = Z_2(i\gamma \cdot k + Z_m m_f) + \frac{4}{3}\bar{g}^2 Z_1 \int_{dq}^\Lambda D_{\mu\nu}(l)\gamma_\mu S_f(q)\Gamma_\nu^f(k, q), \quad (1)$$

where S_f is the quark propagator. Z_1, Z_2, Z_m are the renormalization constants of the quark-gluon-vertex, the quark field, and the quark mass, respectively. $D_{\mu\nu}$ is the gluon propagator, Γ_ν^f the quark-gluon-vertex. m_f is the current quark mass, \bar{g} the coupling constant. $f = \{u, d, s, c, b, t\}$ represents the quark flavor. $l = k - q$, with l the momentum of the gluon, k and q the momenta of outer and inner quark. $\int_{dq}^\Lambda = \int^\Lambda d^4q/(2\pi)^4$ stands for a Poincaré invariant regularized integration, with Λ the regularization scale. The Bethe-Salpeter amplitude (BSA) of the meson is solved by the BSE [20,21],

$$[\Gamma^{fg}(k; P)]_\beta^\alpha = \int_{dq}^\Lambda [K^{fg}(k, q; P)]_{\sigma\beta}^{\alpha\delta} [\chi^{fg}(q; P)]_\delta^\sigma, \quad (2)$$

where Γ^{fg} is the BSA, K^{fg} the quark-antiquark scattering kernel. k and P are the relative and the total momentum of the meson, with $P^2 = -M^2$ and M the meson mass. $\chi^{fg}(q; P) = S_f(q_+)\Gamma^{fg}(q; P)S_g(q_-)$, where χ^{fg} is the wave function, and $q_+ = q + \iota P/2$, $q_- = q - (1 - \iota)P/2$. ι is the partitioning parameter describing the momentum partition between the quark and antiquark and does not affect the physical observables. α, β, σ , and δ are the Dirac indexes.

The RL approximation is making the following replacement in Eqs. (1) and (2) [34],

$$\bar{g}^2 Z_1 D_{\mu\nu}(l)\Gamma_\nu^f(k, q) \rightarrow [Z_2]^2 \tilde{D}_{\mu\nu}^f(l)\gamma_\nu, \quad (3)$$

$$[K^{fg}(k, q; P)]_{\sigma\beta}^{\alpha\delta} \rightarrow -\frac{4}{3}[Z_2]^2 \tilde{D}_{\mu\nu}^{fg}(l)[\gamma_\mu]_\sigma^\alpha [\gamma_\nu]_\beta^\delta, \quad (4)$$

where $\tilde{D}_{\mu\nu}^{fg}(l) = (\delta_{\mu\nu} - \frac{l_\mu l_\nu}{l^2})\mathcal{G}^{fg}(l^2)$ and $\tilde{D}_{\mu\nu}^f(l) = \tilde{D}_{\mu\nu}^{ff}(l)$ are the effective quark-antiquark interactions. The dressed function $\mathcal{G}^{fg}(s)$ is composed of a flavor-dependent infrared part and a flavor-independent ultraviolet part [34],

$$\mathcal{G}^{fg}(s) = \mathcal{G}_{\text{IR}}^{fg}(s) + \mathcal{G}_{\text{UV}}(s), \quad (5)$$

$$\mathcal{G}_{\text{IR}}^{fg}(s) = 8\pi^2 \frac{D_f D_g}{\omega_f^2 \omega_g^2} e^{-s/(\omega_f \omega_g)}, \quad (6)$$

$$\mathcal{G}_{\text{UV}}(s) = \frac{8\pi^2 \gamma_m \mathcal{F}(s)}{\ln[\tau + (1 + s/\Lambda_{\text{QCD}}^2)^2]}, \quad (7)$$

where $s = l^2$. $\mathcal{G}_{\text{IR}}^{fg}(s)$ is the infrared interaction responsible for dynamical chiral symmetry breaking (DCSB), with $(D_f^2 \omega_f)^{1/3}$ expressing the interaction strength and ω_f the interaction width in the momentum space. Although the interaction form in Ref. [37] is used herein, the Maris-Tandy form is equally successful [28–30]. f and g label the quark flavors. The equality in f and g means that the quark and antiquark contribute equally to the interaction strength and width. $\mathcal{G}_{\text{UV}}(s)$ keeps the one-loop perturbative QCD limit in the ultraviolet. As we are dealing with 5 active

quarks, $\mathcal{G}_{UV}(s)$ is independent of the quark flavors. $\mathcal{F}(s) = [1 - \exp(-s^2/[4m_t^4])]/s$, $\gamma_m = 12/(33 - 2N_f)$, with $m_t = 1.0$ GeV, $\tau = e^{10} - 1$, $N_f = 5$, and $\Lambda_{\text{QCD}} = 0.21$ GeV. The values of m_t and τ are chosen so that $\mathcal{G}_{UV}(s)$ is well suppressed in the infrared, and the dressed function $\mathcal{G}_{\text{IR}}^{fg}(s)$ is qualitatively right in the limit $m_f \rightarrow \infty$ or $m_g \rightarrow \infty$.

The parameters D_f and ω_f that express the flavor-dependent quark-antiquark interaction are fitted by the physical observables. See Ref. [34] for the details. With the parameters well fitted, the axial-vector Ward-Takahashi identity (av-WTI) is perfectly satisfied [34], which guarantees the ground state pseudoscalar mesons as Goldstone bosons of DCSB. Our model gives a successful and unified description of the light, heavy, and heavy-light ground pseudoscalar and vector mesons [34].

III. RESULTS

Three sets of parameters of the charm and bottom system corresponding to the varying of the interaction width are listed in Table I. The current quark masses, defined in Ref. [36], are $\check{m}_c = 1.31$ GeV and $\check{m}_b = 4.27$ GeV.

The masses of the charmonium are listed in Table II. $M_{c\bar{c}}^{\text{RL}}$ is our RL approximation result. $M_{c\bar{c}}^{\text{expt.}}$ is the experiment value [38]. $\Delta M_{c\bar{c}}^{\text{RL}}$ is the deviation of the RL results from the experiment value,

$$\Delta M_{c\bar{c}}^{\text{RL}} = M_{c\bar{c}}^{\text{RL}} - M_{c\bar{c}}^{\text{expt.}}. \quad (8)$$

TABLE I. Three sets of the parameters ω_f and D_f (in GeV) of the charm and bottom system [34].

Flavor	Para-1		Para-2		Para-3	
	ω_f	D_f^2	ω_f	D_f^2	w_f	D_f^2
c	0.690	0.645	0.730	0.599	0.760	0.570
b	0.722	0.258	0.766	0.241	0.792	0.231

TABLE II. Masses (in MeV) of the charmonium with $J^{PC} = 0^{-+}, 1^{-+}, 0^{++}, 1^{+-}, 1^{++}, 2^{++}$, the normal states in the quark model. $M_{c\bar{c}}^{\text{RL}}$ is our RL approximation result. $M_{c\bar{c}}^{\text{expt.}}$ is the experiment value [38]. $\Delta M_{c\bar{c}}^{\text{RL}} = M_{c\bar{c}}^{\text{RL}} - M_{c\bar{c}}^{\text{expt.}}$ is the deviation of our results from the experiment value. Three sets of parameters in Table I are used in our calculation.

J^{PC}		0 ⁻⁺	1 ⁻⁺	0 ⁺⁺	1 ⁺⁻	1 ⁺⁺	2 ⁺⁺
$M_{c\bar{c}}^{\text{RL}}$	Para-1	2984	3134	3327	3400	3417	3497
	Para-2	2984	3132	3331	3416	3426	3511
	Para-3	2984	3130	3332	3426	3431	3518
$\Delta M_{c\bar{c}}^{\text{RL}}$	Para-1	0	37	-88	-125	-94	-59
	Para-2	0	35	-84	-109	-85	-45
	Para-3	0	33	-83	-99	-80	-38
$M_{c\bar{c}}^{\text{expt.}}$		2984	3097	3415	3525	3511	3556

The mass of the pseudoscalar ($J^{PC} = 0^{-+}$) meson is used to fit the parameters, so there is no deviation for it. For all the other mesons, the deviations are less than 3.5%. Three sets of parameters are used in our calculation. The uncertainties due to the varying of the parameters are very small. The absolute uncertainty is 4 MeV for the vector ($J^{PC} = 1^{-+}$) meson, and 5 MeV for the scalar meson ($J^{PC} = 0^{++}$). For other mesons ($J^{PC} = 1^{+-}, 1^{++}, 2^{++}$) the absolute uncertainties are less than 26 MeV, with the relative uncertainties less than 0.8%.

The masses of the bottomonium are listed in Table III. $M_{b\bar{b}}^{\text{RL}}$ and $M_{b\bar{b}}^{\text{expt.}}$ are our RL approximation result and the experiment value [38], respectively. $\Delta M_{b\bar{b}}^{\text{RL}}$ is the deviation, defined by

$$\Delta M_{b\bar{b}}^{\text{RL}} = M_{b\bar{b}}^{\text{RL}} - M_{b\bar{b}}^{\text{expt.}}. \quad (9)$$

There is no mass deviation for the pseudoscalar meson as it is used to fit the parameters. For all the other mesons, the deviations are less than 1.1%. The uncertainty due to the varying of the parameters is very small for the vector meson (less than 1 MeV). For other mesons ($J^{PC} = 0^{++}, 1^{+-}, 1^{++}, 2^{++}$) the absolute uncertainties are less than 17 MeV, with the relative uncertainties less than 0.2%.

The results in Tables II and III have two-sided meanings. On one hand, the RL approximation is reasonable for the charmonium and bottomonium system. The mass deviations are less than 3.5% for the charmonium and less than 1.1% for the bottomonium. However, the absolute errors do not decrease for the P-wave states, indicating that in the RL approximation some interactions are still lacking even for the heavy system. On the other hand, the results are stable when the parameters change. The uncertainties due to the varying of the parameters are small, less than 0.8% for the charmonium and less than 0.2% for the bottomonium, so the errors [defined by Eqs. (8) and (9)] of the RL approximation could be estimated quantitatively. The interaction, Eqs. (5)–(7), expresses the flavor dependence properly, so that the errors are of the same order for both the open flavor mesons and the $q\bar{q}$ mesons [34]. As the masses of the B_c mesons are approximately

TABLE III. Masses (in MeV) of the bottomonium. The meanings of the quantities are the same as in Table II.

J^{PC}		0 ⁻⁺	1 ⁻⁺	0 ⁺⁺	1 ⁺⁻	1 ⁺⁺	2 ⁺⁺
$M_{b\bar{b}}^{\text{RL}}$	Para-1	9399	9453	9754	9793	9788	9820
	Para-2	9399	9453	9762	9805	9799	9833
	Para-3	9399	9453	9765	9810	9804	9835
$\Delta M_{b\bar{b}}^{\text{RL}}$	Para-1	0	-7	-105	-106	-106	-92
	Para-2	0	-7	-97	-94	-94	-79
	Para-3	0	-7	-94	-89	-89	-77
$M_{b\bar{b}}^{\text{expt.}}$		9399	9460	9859	9899	9893	9912

$M_{c\bar{b},J^P} \approx (M_{c\bar{c},J^P} + M_{b\bar{b},J^P})/2$, the mass errors of the B_c mesons are assumed to be:

$$\Delta M_{c\bar{b},J^P}^{\text{RL}} = \frac{1}{N_{\text{normal}}} \sum_C (\Delta M_{c\bar{c},J^P}^{\text{RL}} + \Delta M_{b\bar{b},J^P}^{\text{RL}})/2, \quad (10)$$

with J^{PC} the normal states. The sum is over the normal states, and N_{normal} is the number of the normal states. For example, if $J^P = 0^-$, then the corresponding normal $c\bar{c}$ or $b\bar{b}$ state is $J^{PC} = 0^{++}$, and $N_{\text{normal}} = 1$. $\Delta M_{c\bar{b},0^-}^{\text{RL}} = (\Delta M_{c\bar{c},0^{++}}^{\text{RL}} + \Delta M_{b\bar{b},0^{++}}^{\text{RL}})/2$. It is the same case for the $J^P = 1^-, 0^+, 2^+$ mesons. If $J^P = 1^+$, then both $J^{PC} = 1^{++}$ and 1^{+-} are the normal states and $N_{\text{normal}} = 2$. The $J^P = 1^+$ B_c mesons are the mixing states of the 1P_1 state and 3P_1 state [14]. We do not bother to investigate the mixing. The mass errors of the $J^P = 1^+$ mesons are estimated to be $\Delta M_{c\bar{b},1^+}^{\text{RL}} = (\Delta M_{c\bar{c},1^{++}}^{\text{RL}} + \Delta M_{c\bar{c},1^{+-}}^{\text{RL}} + \Delta M_{b\bar{b},1^{++}}^{\text{RL}} + \Delta M_{b\bar{b},1^{+-}}^{\text{RL}})/4$, i.e., averaging the C -parity. In Tables II and III, the difference of $\Delta M_{c\bar{c}}^{\text{RL}}$ with $J^{PC} = 1^{++}$ and $J^{PC} = 1^{+-}$ is less than 31 MeV, and there is no difference for $\Delta M_{b\bar{b}}^{\text{RL}}$ for these two states, so taking the C -parity average leads no more than 8 MeV error for $\Delta M_{c\bar{b},1^+}^{\text{RL}}$.

The relation Eq. (10) is also supported by the masses of the radial excited states [36]. The center values of the excited state masses are cited in Table IV. $(\Delta M_{\eta_c(2S)}^{\text{RL}} + \Delta M_{\eta_b(2S)}^{\text{RL}})/2 = -58$ MeV, while $\Delta M_{B_c^+(2S)}^{\text{RL}} = -59$ MeV. The direct calculation of the B_c meson masses in the RL approximation and the mass errors due to the RL approximation are listed in Table V. The modified mass is defined by

$$\bar{M}_{c\bar{b}}^{\text{RL}} = M_{c\bar{b}}^{\text{RL}} - \Delta M_{c\bar{b}}^{\text{RL}}. \quad (11)$$

With the errors from the RL approximation subdued, $\bar{M}_{c\bar{b}}^{\text{RL}}$ is our prediction for the B_c mesons. There are two kinds of other errors for all the B_c mesons, which are estimated as following. $M_{c\bar{c},0^{++}}^{\text{RL}}$ and $M_{b\bar{b},0^{++}}^{\text{RL}}$ are used to fit the parameters, so the error of $M_{c\bar{b},0^-}^{\text{RL}}$ is totally due to the interaction pattern, Eqs. (5)–(7). The error due to this interaction pattern, the first error, is about 15 MeV. The results vary by a few MeVs as the parameters change, which is the second error. These errors due to the varying of the parameters are

TABLE IV. The masses (in MeV) of the first radial excited states of the charm-bottom system with $J^P = 0^-$ (cited from Ref. [36]). The experiment data for $M_{\eta_c(2S)}$ and $M_{\eta_b(2S)}$ are taken from Ref. [38], and that for $M_{B_c^+(2S)}$ is taken from Ref. [3].

State	$\eta_c(2S)$	$B_c^+(2S)$	$\eta_b(2S)$
RL	3606	6813	9915
Experiment	3638	6872	9999
ΔM^{RL}	-32	-59	-84

TABLE V. Masses of the B_c Mesons (in MeV). $M_{c\bar{b}}^{\text{RL}}$ is the direct RL result. $\Delta M_{c\bar{b}}^{\text{RL}}$ is the error of the RL approximation defined by Eq. (10). $\bar{M}_{c\bar{b}}^{\text{RL}}$ is the modified mass, defined by Eq. (11).

J^P		0^-	1^-	0^+	1_1^+	1_2^+	2^+
$M_{c\bar{b}}^{\text{RL}}$	Para-1	6293	6360	6608	6642	6677	6721
	Para-2	6290	6357	6612	6649	6686	6731
	Para-3	6287	6354	6612	6651	6688	6733
$\Delta M_{c\bar{b}}^{\text{RL}}$	Para-1	0	15	-97	-108	-108	-75
	Para-2	0	14	-91	-96	-96	-62
	Para-3	0	13	-89	-89	-89	-57
$\bar{M}_{c\bar{b}}^{\text{RL}}$	Para-1	6293	6345	6705	6750	6785	6796
	Para-2	6290	6343	6703	6745	6782	6793
	Para-3	6287	6341	6701	6740	6777	6790

much smaller than those of the charmonium and the bottomonium. The uncertainties of the parameters cancel by using Eq. (10), i.e., inferring the errors of the B_c mesons as the intermediate of the charmonium and the bottomonium. For the $J^P = 1^+$ B_c mesons, there is a third error due to the C -parity average in Eq. (10), which is about 8 MeV.

Our predictions of the B_c mesons and the estimated errors are listed in the second column of Table VI. Hitherto the only experiment data for the B_c spectrum is the pseudoscalar meson mass, as the production rate of the B_c mesons is much lower than those of the charmonium and the bottomonium. Our results are consistent with the recent IQCD predictions (with $J^P = 0^-, 1^-, 0^+, 1^+$), which are listed in the forth column. Our results are also consistent with the quark model predictions. The quark model predictions from Ref. [14] are listed in the third column, where the masses with $J^P = 0^-, 1^-$ are the input values and others are the outputs. The mass splitting of the $1S$ state of our result is $M_{B_c(1^-)} - M_{B_c(0^-)} = 53$ MeV, consistent with the IQCD result 55 MeV. The mass splittings of the $1P$ states of our results are $(M_{B_c(1_1^+)} + M_{B_c(1_2^+)})/2 - M_{B_c(0^+)} = 60$ MeV and $M_{B_c(2^+)} - M_{B_c(0^+)} = 90$ MeV, while the

TABLE VI. Masses of the B_c Mesons (in MeV). $\bar{M}_{c\bar{b}}^{\text{RL}}$ is our prediction. The first error is due to the interaction pattern Eqs. (5)–(7). The second error is due to the varying of the parameters. For $J^P = 1^+$ mesons, the third error is due to the C -parity average in Eq. (10). $M_{c\bar{b}}^{\text{QM}}$ is the quark model result [14], and the underlined ones are the input values. $M_{c\bar{b}}^{\text{LQCD}}$ is the IQCD prediction [19]. $M_{c\bar{b}}^{\text{expt.}}$ is the experiment value [38].

J^P	$\bar{M}_{c\bar{b}}^{\text{RL}}$	$M_{c\bar{b}}^{\text{QM}}$	$M_{c\bar{b}}^{\text{LQCD}}$	$M_{c\bar{b}}^{\text{expt.}}$
0^-	6290(15)(3)	<u>6271</u>	6276(7)	6275(1)
1^-	6343(15)(2)	<u>6326</u>	6331(7)	...
0^+	6703(15)(2)	6714	6712(19)	...
1_1^+	6745(15)(5)(8)	6757	6736(18)	...
1_2^+	6781(15)(4)(8)	6776
2^+	6793(15)(3)	6787

quark model results are 53 MeV and 73 MeV, respectively. Our results of the B_c meson masses also have two-sided meanings. On one hand, we predict the masses of the B_c mesons (with $J^P = 0^-, 1^-, 0^+, 1^+, 2^+$), providing a significant guide to the experimental search for the B_c mesons. On the other hand, the consistency of our results with other predictions supports that the flavor-dependent interaction pattern, Eqs. (5)–(7), is reasonable. This pattern leads an error about 15 MeV for the B_c mesons.

IV. SUMMARY

In this paper, we predict the masses of the B_c mesons with $J^P = 0^-, 1^-, 0^+, 1^+, 2^+$ using a flavor-dependent interaction pattern via the Dyson-Schwinger equation and the Bethe-Salpeter equation approach. This interaction pattern, composed of a flavor-dependent infrared part and a flavor-independent ultraviolet part, gives an unified successful description of the pseudoscalar and vector light, heavy-light, and heavy mesons. This interaction pattern could also be applied to the radial excited heavy mesons. Herein we control the errors carefully. Besides the error from the RL approximation, two other kinds of errors are considered. One is the error from the interaction pattern; the other is the error from the varying of the parameters. For the

$J^P = 1^+$ B_c mesons, a third error due to the C -parity average is also considered. With the errors from the RL approximation subdued, our predictions are consistent with the IQCD and quark model results. Our results have two-sided meanings. On one hand, we predict the B_c meson masses with errors well controlled, providing a significant guide to the experiment search. On the other hand, the excellent consistency of our predictions with the results from other approaches also supports strongly that the flavor-dependent interaction pattern, Eqs. (5)–(7), is reasonable.

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