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Abstract

Beam instability issues are studied in a quasi-isochronous muon collider. The problems of beam breakup, strong head-tail, and longitudinal microwave instabilities are discussed. We find that in the present design, the muon beam may be susceptible to a microwave instability.

I. INTRODUCTION

To achieve the high luminosity required in a 2 TeV muon-muon collider and to overcome partially the hour-glass effect, the muon bunch has to be extremely short, within 3 mm (rms), or, equivalently, 10 ps. Therefore, the time slip per turn of a muon particle must be much less than 10 ps for the muons to remain bunched. This implies for the proposed 2-TeV muon-muon collider [1] — a ring with approximately 1 km radius and a full momentum spread of $\pm 0.5\%$ — that the phase-slip factor η must have an absolute value much less than 1×10^{-4} . For this reason, an isochronous collider ring is preferred. In this paper, first the problem of beam breakup in a perfectly isochronous ring is addressed in Sec. II. Then, in Sec. III, unpreventable deviations of a ring from isochronicity and the strong head-tail instability are discussed. Finally, in Sec. IV, it is shown that in the present design, the muon bunches may have surpassed the lower limit for excitation of longitudinal microwave instabilities.

II. BEAM BREAKUP

We first assume that the collider ring is isochronous. If so, the relative longitudinal positions of the bunch particles will not change. The tail of the bunch is continuously deflected by the wakefields emanating from the head increasing transverse emittances and eventually causing beam breakup. Intuitively this effect can be studied using a two-particle model [2]. Here, the N -particle bunch is modeled in Fig. 1 as two macro particles each with charge $eN/2$ separated by a distance $\hat{z} = 3$ mm apart. The transverse motion of the front or head particle, y_1 , is just free betatron oscillation,

$$y_1''(s) + k_\beta^2 y_1(s) = 0, \quad (2.1)$$

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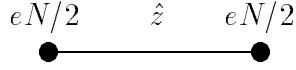


Figure 1: The two-particle model.

where $k_\beta = \nu_\beta/R$ is the betatron wave number, ν_β the betatron tune, R the ring radius, and s is the path length measured along the ideal closed orbit. The rear or tail particle sees the transverse wake $W_1(\hat{z})$ of the front particle and satisfies a driven equation,

$$y_2''(s) + k_\beta^2 y_2(s) = -e \left(\frac{Ne}{2} \right) \left(\frac{W_1(\hat{z})}{2\pi R} \right) \frac{1}{E} y_1(s), \quad (2.2)$$

where E is the energy of particles. The solution is

$$y_1(s) = \hat{y} \cos k_\beta s, \quad (2.3)$$

$$y_2(s) = \hat{y} \left[\cos k_\beta s + \Upsilon \frac{s}{L} \sin k_\beta s \right], \quad (2.4)$$

where \hat{y} is the initial betatron amplitude of the bunch, and

$$\Upsilon = -\frac{e^2 N W_1(\hat{z}) L}{8\pi R E k_\beta} \quad (2.5)$$

denotes the growth in amplitude of the tail after traveling a length L .

A. Resistive Wall

The dipole wake due to the resistivity of the vacuum chamber wall is

$$W_1(\hat{z}) = -\frac{4c}{b^3} \sqrt{\frac{Z_0 \rho}{4\pi}} \frac{r}{|\hat{z}|^{1/2}}, \quad (2.6)$$

where c is the velocity of light, b the beam pipe radius, $Z_0 \approx 377$ Ohms is the free-space impedance, and ρ the resistivity of the wall. Equation (2.6) is correct in the region

$$\frac{b}{\chi} \gg |\hat{z}| \gg b\chi^{1/3}, \quad \chi = \frac{\rho}{bZ_0}. \quad (2.7)$$

We can now compute the time $\tau = L/c$ for the amplitude of the tail particle to double by letting $\Upsilon = 1$. For a muon bunch containing $N = 2 \times 10^{12}$ particles in a stainless steel beam pipe with $\rho = 0.74 \mu\Omega\text{-m}$, the results are listed in

Table I: Tail amplitude doubling times for a stainless-steel beam pipe

b (cm)	χ	b/χ (m)	$\chi^{1/3}b$ (m)	τ (ms)	No. of Turns
1.0	1.96×10^{-7}	5.09×10^4	5.81×10^{-5}	0.12	6.1
2.5	7.85×10^{-8}	3.18×10^5	1.07×10^{-4}	1.90	95
5.0	3.93×10^{-8}	1.27×10^6	1.70×10^{-4}	15.2	760

Table II: Tail amplitude doubling times for an aluminum beam pipe

b (cm)	χ	b/χ (m)	$\chi^{1/3}b$ (m)	τ (ms)	No. of Turns
1.0	7.03×10^{-9}	1.42×10^6	1.91×10^{-5}	0.64	32
2.5	2.81×10^{-9}	8.89×10^6	3.52×10^{-5}	10.0	502
5.0	1.41×10^{-9}	3.55×10^7	5.60×10^{-5}	80.4	4018

Table I. For an aluminum beam pipe with $\rho = 0.0265 \mu\Omega\text{-m}$, the results are listed in Table II.

We see that Eq. (2.7) is satisfied in all the cases so that the wake potential described by Eq. (2.6) is valid. As is expected, the bunch is much more stable inside an aluminum beam pipe than in a stainless-steel beam pipe. Due to the short life-time of the muons, the bunch is designed to stay in the collider for only ~ 1000 turns. Even over this short length of time, as is clear from the table, the increase in the tail amplitude is serious for stainless-steel beam pipe, and is only marginal for aluminum beam pipe (and only then if the pipe radius is less than 2.5 cm). Note that the increase in tail amplitude is inversely proportional to the beam-pipe radius to the third power (b^3).

B. Broad-Band Impedance

For a ring with longitudinal broad-band impedance of peak value Z_{\parallel}/n , a figure of merit Q , and centered at angular frequency ω_r , the corresponding transverse broad-band impedance has a peak value given roughly by

$$Z_{\perp} = \frac{2R}{b^2} \frac{Z_{\parallel}}{n}. \quad (2.8)$$

Thus, the dipole wake at a short distance \hat{z} is

$$W_1(\hat{z}) = -\frac{2\hat{z}R\omega_r^2}{Qb^2c} \frac{Z_{\parallel}}{n}. \quad (2.9)$$

If the broad band is centered at the cutoff frequency, or $\omega_r \approx c/b$, the doubling

time τ for the tail amplitude can be obtained from the relation $\Upsilon = 1$:

$$\tau \approx \frac{2b^4 \nu_\beta (E/e)}{I_b R^2 \hat{z}} \frac{Q}{(Z_{\parallel}/n)}. \quad (2.10)$$

Putting in $N = 2 \times 10^{12}$, $R = 1$ km (or $I_b = 0.01529$ amp), and $\nu_\beta = 24$, one gets

$$\tau = \begin{cases} 3.3 \frac{Q}{Z_{\parallel}/n} \text{ turns} & b = 1.0 \text{ cm} \\ 130 \frac{Q}{Z_{\parallel}/n} \text{ turns} & b = 2.5 \text{ cm} \\ 2080 \frac{Q}{Z_{\parallel}/n} \text{ turns} & b = 5.0 \text{ cm} . \end{cases} \quad (2.11)$$

For $Z_{\parallel}/n = 1$ Ohm and $Q = 1$, we see that the emittance growth will be too large. Note, however, that this growth depends very sensitively on the beam-pipe radius (b^4).

C. BNS Damping

The rapid growth of the tail amplitude is a result of resonant driving of the tail by the head of the bunch. If the tail particle could be made to oscillate at a betatron frequency different from that of the head particle, this resonant interaction would be avoided and no emittance increase would occur. This idea is called BNS damping [3]. It is described by assuming that the betatron wave number is a function of length along the bunch, so that the equation of motion of the tail particle changes from Eq. (2.2) to

$$y_2''(s) + k_\beta^2(\hat{z})y_2(s) = -e \left(\frac{N\epsilon}{2} \right) \left(\frac{W_1(\hat{z})}{C} \right) \frac{1}{E} y_1(s). \quad (2.12)$$

With $k_\beta(\hat{z}) = k_\beta + \Delta k_\beta$, the evolution of the tail amplitude becomes

$$y_2(s) = \hat{y} \cos(k_\beta + \Delta k_\beta)s + \left(\frac{\Upsilon}{L\Delta k_\beta} \right) \hat{y} [\cos(k_\beta + \Delta k_\beta)s - \cos k_\beta s], \quad (2.13)$$

which is no longer linearly increasing. Furthermore, if we choose

$$\frac{\Upsilon}{L\Delta k_\beta} = -1, \quad (2.14)$$

the trailing particle will follow the leading particle but without any amplitude growth.

To achieve BNS damping, the amount of tune spread required is given by

$$\frac{\Delta \nu_\beta}{\nu_\beta} = \frac{\Delta k_\beta}{k_\beta} = \frac{\Upsilon}{Lk_\beta}. \quad (2.15)$$

In terms of the tail-amplitude doubling time τ ($\Upsilon = 1$), the required tune spread becomes

$$\frac{\Delta\nu_\beta}{\nu_\beta} = \frac{1}{\omega_0\nu_\beta\tau} . \quad (2.16)$$

If we substitute the growth time for a $N = 2 \times 10^{12}$ muon bunch in an aluminum beam pipe, we obtain

$$\frac{\Delta\nu_\beta}{\nu_\beta} = \begin{cases} 1.58 \times 10^{-4} & b = 1.0 \text{ cm} \\ 1.07 \times 10^{-5} & b = 2.5 \text{ cm} \\ 1.26 \times 10^{-6} & b = 5.0 \text{ cm} \end{cases}$$

These tune spreads are small and implementation of this idea should not be difficult. One method to provide the BNS focusing is to introduce a radio-frequency quadrupole whose strength changes as the head and the tail of the bunch pass through it. Another method is through chromaticity. The muons have a full momentum spread of $\pm 0.5\%$, so that only a small variation in energy from head to tail of the bunch would suffice to provide the required tune spread.

Instead of the two-particle model, when the whole bunch with a longitudinal distribution $\rho(z)$ is considered, the equation of motion becomes

$$y''(s, z) + [k_\beta + \Delta k_\beta(z)]^2 y(s, z) = -\frac{e^2 N}{2\pi RE} \int_z^\infty dz' \rho(z') W_1(z - z') y(s, z') , \quad (2.17)$$

where $y(s, z)$ is the transverse displacement of a point in the bunch z away from the bunch center as it passes the ring position s . The condition of BNS damping is that all particles in the bunch execute the same betatron motion, or

$$y(s, z) = \hat{y} \cos k_\beta s , \quad (2.18)$$

Substituting in Eq. (2.17), we arrive at the condition [4]

$$\frac{\Delta k_\beta}{k_\beta} = -\frac{e^2 N}{4\pi k_\beta^2 RE} \int_z^\infty dz' \rho(z') W_1(z - z') , \quad (2.19)$$

which is the equivalent of Eq. (2.15) for a bunch of particles.

III. STRONG HEAD-TAIL

In the Gallardo-Palmer design of the interaction region (IR) [5], the betatron function rises from the low-beta of $\beta^* = 3$ mm to a maximum of ~ 400 km in about 40 m along the ring. A particle executing a betatron oscillation will travel a much longer path length. In a perfectly isochronous ring, it is shown in the Appendix that a particle in a bunch with a rms normalized emittance of

$\epsilon_N = 50 \times 10^{-6} \pi$ m can lag as much as $\Delta\ell = 3.52 \times 10^{-5}$ m per turn, or 3.52 cm for a store of 1000 turns, an amount which exceeds the nominal bunch length. For this reason, the muon collider ring will only be quasi-isochronous. To avoid bunch lengthening, a synchrotron oscillation period of about $T_s = 150$ turns is suggested [6], which requires an rf voltage of $V_0 = 1.5$ GV and a momentum-compaction factor of $|\alpha| = 1.5 \times 10^{-5}$ for the collider ring.

With a synchrotron oscillation, the head and tail of the bunch interchange position and the resonant blowup of the tail can be avoided. The condition for stability can be inferred by requiring that the amplitude of the tail not double in a quarter of the synchrotron period. Putting in $L = cT_s/4$ into Eq. (2.5), we obtain the threshold

$$\Upsilon = -\frac{\epsilon^2 N W_1(\hat{z}) c^2}{16 R E \omega_\beta \omega_s} < 1, \quad (3.1)$$

where $\omega_\beta/2\pi$ and $\omega_s/2\pi$ are, respectively, the betatron and synchrotron frequencies. Physically, the two macro-particles oscillate coherently in the σ and π modes with distinct frequencies. At threshold, the two modes merge into one and become unstable. This instability has been referred to as a strong head-tail in the literature.

By putting in the wall impedance or the broad-band impedance given by Eqs. (2.6) and (2.9), Eq. (3.1) can be used as a check for instability. We can also infer the result from the calculations made in Sect. II. With the synchrotron period of $T_s = 150$ turns, stability against a strong head-tail interaction is equivalent to requiring that the tail amplitude double in a time longer than $T_s/4$, or 37.5 turns. From Table I and II as well as Eq. (2.11), we can see immediately that the beam will be stable if the beam pipe radius is not less than 2.5 cm.

IV. LONGITUDINAL MICROWAVE

The limit for longitudinal microwave instability is given by the Boussard-modified Keil-Schnell [7] or Krinsky-Wang [8] criterion:

$$\frac{|Z_{||}|}{n} < \frac{2\pi|\eta|(E/e)}{I_p} \delta^2, \quad (4.1)$$

where $I_p = \epsilon N / (\sqrt{2\pi} \sigma_\tau) = 1.28 \times 10^4$ Amp is the local peak current of the bunch having a rms length of $\sigma_\tau = 10$ ps or 3 mm. The rms momentum spread is $\delta \approx 0.0025$. With a phase-slip factor of $|\eta| = 1.5 \times 10^{-5}$, we find that for stability $|Z_{||}|/n$ must be less than 0.092 Ohms. Because of the short bunch length, the muons see a frequency around $\omega_r/2\pi = 10$ GHz. Unfortunately, the impedance per harmonic does remain constant up to and over 10 GHz

provided that all the detailed contributions from the vacuum chamber are summed carefully [9]. Therefore, this microwave self-bunching may become a serious impediment to generating stable muon beams.

The Boussard modified Keil-Schnell criterion of Eq. (4.1) can be rewritten in the form

$$\omega_r \sqrt{\frac{|\eta| I_p |Z_{\parallel}|/n}{2\pi E}} < \omega_r |\eta| \delta. \quad (4.2)$$

The left side represents the growth rate without Landau damping and equals $\tau^{-1} = 7.76 \times 10^3 \text{ s}^{-1}$ if $|Z_{\parallel}|/n$ is taken as 1 Ohm. The right side of Eq. (4.2) denotes the spread in frequency of the 10 GHz self-bunching buckets, which leads to Landau damping. At this moment, this spread amounts to only $2.36 \times 10^3 \text{ s}^{-1}$, which is, of course, not enough to counteract resonant emittance growth.

To safeguard against microwave instability, we suggest that the vacuum chamber be made as smooth as possible so that the the broad-band impedance near 10 GHz can be kept well under, for example, $|Z_{\parallel}|/n \approx 0.1 \text{ Ohm}$.

The Boussard-modified Keil-Schnell criterion as stated in Eq. (4.1) does not take into account whether the machine is operated below or above transition, or, in other words, the sign of the phase-slip factor η . In reality, the bell-shaped stability curve shows clearly that a machine should be more stable below transition. Recently, this fact has been demonstrated both analytically and numerically by Fang *et al* [10] and Ng [11]. As a result, the muon collider should be designed to have an imaginary transition gamma or a negative momentum-compaction factor [12].

V. CONCLUSION

Using current muon-collider specifications, the issue of beam breakup has been investigated under the assumption of a perfect isochronous ring. The only way to circumvent beam breakup may be to utilize the BNS focusing method. In this paper the path-length lag across the IR was computed and it was found that synchrotron-oscillation focusing must be implemented to avoid bunch lengthening. In other words, the ring must be quasi-isochronous. Also, although it was computed that the muon beam would be below the threshold of a strong head-tail instability, it may, however, be above the threshold for longitudinal microwave instability. To ensure a stable muon beam, it is suggested here that the coupling impedance be controlled accurately and that the collider ring be designed with a negative momentum-compaction factor.

APPENDIX

Let us start from a particle in a ring with uniform focusing channel. The transverse displacement is

$$y = a \sin k_\beta s, \quad (\text{A.1})$$

where a is the maximum transverse displacement. For quarter of the betatron wavelength, $s = \pi/(2k_\beta)$, the path length along the sine curve is

$$\ell = \frac{\sqrt{1 + a^2 k_\beta^2}}{k_\beta} E \left(\frac{a k_\beta}{\sqrt{1 + a^2 k_\beta^2}} \right), \quad (\text{A.2})$$

where

$$E(\mu) = \frac{\pi}{2} \left(1 - \frac{\mu^2}{4} - \frac{3\mu^4}{64} + \dots \right) \quad (\text{A.3})$$

is the complete elliptic function of the second kind. The fractional increase in path length is therefore

$$\frac{\Delta\ell}{\ell_0} \approx 1 + \frac{1}{4} a^2 k_\beta^2. \quad (\text{A.4})$$

For a normalized emittance of $\epsilon_N = 50 \times 10^{-6} \pi$ m and a maximum betatron function of $\beta \sim 100$ m, a 2 TeV muon with one-sigma offset will have a maximum transverse displacement of $a \sim 0.5$ mm. Taking a tune of $\nu_\beta \sim 24$ and a ring radius of $R = 1$ km, we obtain

$$\frac{\Delta\ell}{\ell_0} \approx 0.75 \times 10^{-10}, \quad (\text{A.5})$$

or for a turn, $\Delta\ell = 4.90 \times 10^{-7}$ m.

The actual muon collider contains two interaction regions (IR), each with high-beta point both upstream and downstream of the interaction point (IP). For the $\beta^* = 3$ mm IR [5], the betatron function increases from β^* to ~ 400 km in about $\ell_0 = 40$ m. The path-length difference in executing betatron oscillation in this region will contribute significantly. Let us assume that the betatron function increases to its highest value according to

$$\beta(s) = \beta^* + \frac{s^2}{\beta^*}. \quad (\text{A.6})$$

In reality, in the situation when $\beta_x^* = \beta_y^*$, the first quadrupole tries to bend down one of the betatron functions while pushing the other betatron function to increase at a faster rate. Therefore, Eq. (A.6) is actually an underestimate.

Consider a particle at the edge of the unnormalized emittance ϵ passing through the IR. Its displacement can be written as

$$y = \sqrt{\frac{\beta\epsilon}{\pi}} \sin(\psi + \varphi), \quad (\text{A.7})$$

where ψ is the phase advance, which is set to zero right at the IP, and φ is a random phase. According to Eq. (A.6),

$$\psi(s) = \int_0^s \frac{ds}{\beta(s)} = \tan^{-1} \frac{s}{\beta^*}. \quad (\text{A.8})$$

From this, the slope can be easily derived,

$$\frac{dy}{ds} = \left(\frac{\epsilon}{\beta^* \pi} \right)^{1/2} \cos \varphi, \quad (\text{A.9})$$

which turns out to be s independent. For an advance of ℓ_0 along the ideal closed orbit from the IP to the high-beta point, the path length along the betatron-oscillation trajectory is

$$\ell = \left(1 + \frac{\epsilon}{\beta^* \pi} \cos^2 \varphi \right)^{1/2} \ell_0. \quad (\text{A.10})$$

Averaging over the random phase φ , we obtain

$$\ell = \frac{2}{\pi} \left(1 + \frac{\epsilon}{\beta^* \pi} \right)^{1/2} \ell_0 E \left(\sqrt{\frac{\epsilon/(\beta^* \pi)}{1 + \epsilon/(\beta^* \pi)}} \right), \quad (\text{A.11})$$

or a fractional increase of

$$\frac{\Delta \ell}{\ell_0} \approx \frac{\epsilon}{4\beta^* \pi}. \quad (\text{A.12})$$

There are two IR's in the ring. The path-length increase will become $\Delta \ell = 3.52 \times 10^{-5}$ m per turn for the 3 mm low-beta and $\ell_0 = 40$ m. Thus the contribution from the IR's dominates. To avoid bunch lengthening, synchrotron oscillation has to be introduced. The rule of thumb is that during half a synchrotron period, the amount lengthened must be less than the rms bunch length. Thus, for a rms bunch length of 3 mm, the synchrotron period has to be less than 170 turns. In another design [13] with $\beta^* = 1$ cm, $\ell_0 \approx 17$ m. Therefore, $\Delta \ell = 4.49 \times 10^{-6}$ m per turn and the synchrotron period can be made much longer.

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