General Relativity and Gravitation (2025) 57:39
https://doi.org/10.1007/s10714-025-03361-w

PERSPECTIVE

Testing the nature of compact objects and the black hole paradigm

Mariafelicia De Laurentis[1] · Paolo Pani[2]

Received: 8 November 2024 / Accepted: 9 January 2025

Abstract
Do compact objects other than black holes and neutron stars exist in the universe? Do all black holes conform with the predictions of Einstein's General Relativity? Do classical black holes exist at all? Future gravitational-wave observations and black-hole imaging might shed light on these foundational questions and deepen our understanding of the dark cosmos.

Contents

1 Motivation & current status
  1.1 Do compact objects other than BHs and neutron stars exist in the universe?
  1.2 Do all BHs in the universe conform with the GR prediction?
  1.3 Do classical BHs exist at all?
2 Outlook
References

Keywords Black holes · Gravitational waves · Gravity

1 Motivation & current status

A common mantra in astrophysics is that *compact objects* – self-gravitating bodies with mass M and radius R such that their compactness $GM/(c^2R) \sim 1$ – must be either neutron stars or black holes (BHs). This is predicted by stellar evolution and by the universality of gravitational collapse in Einstein's General Relativity (GR) [1–6].

There are strong motivations to challenge this paradigm. On the one hand, the behavior of matter inside ultradense stars is unknown and might involve new degrees

✉ Paolo Pani
paolo.pani@uniroma1.it

1 Dipartimento di Fisica, Universitá di Napoli "Federico II" & INFN Sezione di Napoli, Compl. Univ. di Monte S. Angelo, Edificio G, Via Cinthia, I-80126 Naples, Italy

2 Dipartimento di Fisica, Sapienza Università di Roma & INFN, Sezione di Roma, Piazzale Aldo Moro 5, 00185 Rome, Italy

Published online: 08 February 2025

of freedom. At least some of them might be associated with the dark matter comprising about 85% of the total matter content of the universe, of which little is known, aside from its gravitational interactions. On the other hand, BHs, once considered bizarre solutions to GR, are now pillars of high-energy astrophysics, cosmology, and gravitational-wave (GW) astronomy. Their importance has grown to the point where the initial issues tied to these unique solutions might be easily overlooked. BHs connect gravity, thermodynamics, quantum mechanics and astrophysics, and the paradoxes they reveal are deep and far-reaching: their Hawking evaporation is incompatible with quantum unitarity, the microstates underlying their enormous entropy are unknown, and they conceal singularities where Einstein's theory breaks down.

Given this state of affair, and the ever-growing wealth of data made accessible by GW observations, BH imaging, and other electromagnetic probes, it is compulsory to *test the nature of compact objects* with an agnostic attitude. In addition to GWs and BH imaging, X-ray reflection features in BH spectra provide another robust avenue for testing the compact object hypothesis. [7–9]. Our reflections will revolve around three related questions, which become progressively more radical:

- *Do compact objects other than BHs and neutron stars exist in the universe?*
- *Do all BHs in the universe conform with the GR prediction?*
- *Do classical BHs exist at all?*

We will provide a bird's-eye view on these problems and discuss the many opportunities for this field in the years to come.

1.1 Do compact objects other than BHs and neutron stars exist in the universe?

In GR, any compact object with a mass exceeding a few times that of the Sun must be a BH. Observations contradicting this would imply either new physics beyond GR or new exotic matter fields beyond the Standard Model. Such discoveries could also offer insights into the mysterious properties of BHs (see [10, 11] for some reviews).

From a phenomenological standpoint, BHs and neutron stars might just be two "species" within a broader category including *exotic compact objects* (ECOs). These objects could possess unique characteristics that enable precision searches using current and future experiments. Notably, some GW events are compatible with objects in the so-called lower-mass and upper-mass gap forbidden for standard stellar-origin BHs [12–14], and do not rule out more exotic interpretations [15].

A helpful guide for exploring the ECO landscape comes from Buchdhal's theorem [16], which states that, under certain conditions, the maximum compactness of a self-gravitating object is $GM/(c^2R) = 4/9$. This result rules out the existence of ECOs with compactness arbitrarily close to that of a BH. However, relaxing some of these conditions offers a way around the theorem and provides a framework for classifying ECOs [10]. In addition to some technical assumptions, Buchdahl's theorem relies on GR, spherical symmetry, and the assumption that the matter sector consists of a single fluid that is, at most, mildly anisotropic. A quite common feature of ECOs is the presence of strong tangential stresses to support very compact self-gravitating configurations. This is the case, for instance, of boson stars [17, 18], gravastars [19–22], fermion-boson stars [23], ultracompact anisotropic stars [24, 25], elastic stars [26–28], and wormholes [29–31]. Only some of the ECOs models are embedded in consistent field theories coupled to gravity (although the number of consistent models is steadily growing), in which case they are prone to be studied in their full glory, including their formation and nonlinear dynamics.

1.2 Do all BHs in the universe conform with the GR prediction?

If GR deviations are relevant at the scale of compact objects (namely when $GM/(c^2R) \sim 1$), the properties of BHs (and neutron stars) could differ from those predicted by GR. Specifically, GR uniqueness theorems suggest that all properties of a BH are ultimately determined solely in terms of its mass and angular momentum, offering multiple complementary approaches to test and potentially falsify the GR BH hypothesis. Indeed, the entire quasinormal mode (QNM) spectrum, ringdown, multipolar structure, tidal deformability, but also all geodesics properties such as the innermost stable circular orbits and photon rings, are known functions of the BH mass and spin.

Strong-field tests of gravity largely rely on (dis)proving these predictions (see [32, 33] for some reviews). In the context of testing the BH paradigm, it is useful to highlight some analogies:

- several tests of GR (especially the model-agnostic ones [34, 35]) can be directly mapped into – or easily adapted to – tests for ECOs, since also in this case one expects a deviations from the standard GR BH predictions;
- most gravity theories beyond GR introduce extra scales and dimensionful coupling constants, so they might introduce observable deviations only for BHs in given mass ranges. This means that supermassive BHs can be practically indistinguishable from GR ones while stellar-mass are not (as in the case of theories with higher-curvature corrections) or viceversa (as in the case of massive gravity and other theories with ultralight bosonic degrees of freedom);
- Some models of ECOs require modified gravity either for their existence or to have a stronger theoretical motivation. For example, ordinary matter supporting wormhole solutions beyond GR might not need to violate energy conditions [36]. In these models testing ECOs is another flavor of testing GR.

Finally, it is worth noting that standard tests of gravity typically assume *vacuum*. Dirty astrophysics and environmental effects can cause false GR violations [37]. For the ordinary environments expected around BHs, however, such violations are small [38, 39]. Exceptions might be orbital migration in accretion disks [38, 40, 41], or more exotic scenarios such as large dark-matter spikes around intermediate-mass BHs [42] and ultralight-boson condensates grown around BHs due to superradiance [43]. As for all environmental effects, deviations would be source dependent. Therefore, by detecting multiple sources with similar mass and spin, it should be relatively easy to disentangle environmental effects from beyond-GR ones [37, 38]. On the other hand, distinguishing whether a given source is a BH with strong environmental effects or an ECO can be more challenging and likely requires a detailed model selection.

1.3 Do classical BHs exist at all?

The BH hosted in M87, featured in the very first image taken by the Event Horizon Telescope (EHT) [44], and the remnant of GW150814, the first binary BH merger ever detected by LIGO/Virgo [45], have entropies of the order 10^{95} and 10^{80}, respectively, and hence an enormous number of states. The same holds true for SgrA*, the supermassive BH at the center of our galaxy, observed with the EHT [46] and by the GRAVITY collaboration [47], which provided unprecedented precision in tracking the orbits of stars around this object. Like M87*, SgrA* provides crucial insights into the nature of event horizons and strengthens the case for the classical BH paradigm as predicted by GR [48]. The consistency of both images, despite the difference in scale between M87* and SgrA*, offers compelling observational evidence that GR successfully describes supermassive BHs across various mass ranges [49]. GR explains all BH observations so far, but does not offer any clue about the microscopic origin of the gigantic entropy of astrophysical BHs. On the contrary, it predicts a huge discrepancy: for a given mass, angular momentum and charge, a BH is unique [50]. Furthermore, GR predicts that every BH hosts a singularity in its interior, where classical physics breaks down. Finally, BHs evaporate by emitting entangled Hawking particles in a process that violates quantum unitarity [51, 52]. Thus, despite the recent observational breakthroughs in GW astronomy and BH images, these *entropy, singularity, and information-loss problems* remain dramatic manifestations of the incompleteness of our fundamental laws of nature. A very general argument in quantum information theory shows that if BHs have vacuum at the horizon and "normal" local physics (as GR predicts), quantum unitarity is violated [53–55]. No small deviation from GR or effective-field-theory arguments can come to rescue [53], so a resolution to this theoretical problem might require a radical change of the GR paradigms. In particular, the information loss paradox might be resolved by postulating the existence of structure at the scale of the horizon [53, 54]. This finds a concrete and particularly appealing realization in the *fuzzball* program of string theory, which aims at describing the classical BH horizon as a coarse-grained description of a superposition of regular quantum states [53, 56, 57]. The horizon-scale structure is provided by *microstate geometries*: solitons with the same mass and angular momentum as a BH, but where the horizon is replaced by a smooth horizonless cap [58–62]. Besides potentially resolving all theoretical problems associated with BHs, microstate geometries are possibly the only motivated BH mimicker because, just like BHs, they: i) have a general (albeit complicated [63, 64]) formation mechanism, ii) can exist with any mass, iii) can be more compact than neutron stars without collapsing. The very existence of these solutions and their unique properties hinge on nonperturbative string theory effects, the presence of nontrivial topologies and fluxes, and on the fact that these solutions are intrinsically higher dimensional. Although they are perfectly regular in higher dimensions, from a four-dimensional perspective they appear to have pathologies.

The lesson to learn here is that, in order to solve the paradoxes associated to BHs, one needs *radically new effects* that cannot be captured by educated phenomenological models or ordinary parametrizations. Testing the BH paradigm would then require computing observables in these highly nontrivial theories from scratch.

2 Outlook

The previous considerations suggest that, as is often the case, to test the nature of compact objects it is advisable developing *both* model-agnostic searches and top-down predictions in well-motivated theories. The former approach is broad but potentially inaccurate, while the latter is tailored to a specific theory and hence more precise. Either ways, it is essential to model the signatures in these scenarios as accurately as possible, since current detection strategies rely heavily on matched-filtering techniques and Bayesian model comparison.

Below we list the most promising observables and smoking guns to test the nature of compact objects:

- **Kerr bound:** While the mass M of a BH in GR is arbitrary, its angular momentum J is not and in fact limited by the Kerr bound, $J < GM^2/c$. This bound is easily (and in fact enormously!) exceeded in everyday life, e.g. by a spinning top. It would therefore be interesting to devise agnostic tests of the Kerr bound in compact objects. This is a challenging task in the strong-field regime, since quantities associated to J are typically model dependent and oftentimes models, templates, and parameter estimation pipelines *assume* the Kerr bound. Extreme mass-ratio inspirals detectable by LISA provide an appealing opportunity to measure the spin of the secondary object in a model-agnostic fashion [65], but such measurement will be very challenging due to parameter correlations [66].
- **Multipolar structure:** Due to their axial and equatorial symmetry, GR BHs have a very rigid and highly constrained multipolar structure. The difference between model-agnostic and model-specific approaches is particularly clear in this case. The model-agnostic approach is based on assuming the same symmetries as Kerr, introducing parametrized deviations from the Kerr multipole moments, in particular the mass quadrupole [34, 67, 68]. However, motivated by certain concrete models of microstate geometries [69–72], one might also consider multipole moments that are vanishing in the Kerr case (e.g., current dipoles [73]), and even breaking axial symmetry [74].
  Multipole moments affect the inspiral waveforms at different post-Newtonian order [75]. While the model-agnostic approach can be directly implemented in existing waveforms [34, 67], the absence of symmetries in specific models makes the computation much more involved due to intrinsic precession [74]. However, these effects break parameter degeneracies and can drastically improve measurability [76].
- **Extra charges/dipoles:** No-hair theorems [50] guarantee that Kerr BHs in GR cannot be endowed with a long list of extra matter fields. Some exceptions exist, for example in case of oscillating bosonic fields [77, 78] or electroweak hair [79] (see [80] for a review). Furthermore, various extensions of GR predict BHs with extra massless hair. Finally, although astrophysical BHs are expected to be electrically neutral, they can have extra $U(1)$ charges in beyond-Standard Model theories, e.g. in the presence of dark photons [81]. In addition to modifying the BH GR solution, if compact objects are endowed with light degrees of freedom one expects extra dissipative channels during the inspiral, and non-gravitational modes during

the ringdown. This can be studied in a model-agnostic way within post-Newtonian theory [82], in extreme mass-ratio inspirals for a large class of theories [83], and for ringdown [84].
Finally, even if globally neutral, in various scenarios compact objects can have a nonvanishing dipole moment, which can also be modelled generically in some scenarios [85].

- **Geodesics & integrability:** Due to special symmetries of the Kerr metric, geodesic motion around a Kerr BH is fully integrable. This property is broken by essentially any other object, even if axisymmetric. Examples include neutron stars, boson stars, BHs in modified gravity. In fact, beside GR BHs, non-integrable geodesic motion is the rule rather than the exception.
This fact would impact imaging, accretion, and extreme mass-ratio inspirals. In general, geodesic motion beyond GR BHs can also be chaotic [86–89]. This is hard to model generically, but a qualitative signature would be the existence of resonances, glitches, or islands were orbital dephasing is particularly large.
- **Tidal properties:** If at least one of the two objects in a binary system does not have a horizon, the GW emission will be different, due to finite-size effects [90–92]. Compact objects without an event horizon, such as neutron stars or ECOs like boson stars, exhibit different tidal characteristics compared to classical BHs. Unlike BHs, objects with a solid surface or internal structure can undergo significant tidal deformation. This effect would cause deviations in the waveform during the inspiral phase, especially in the late stages when the objects are in close proximity. Such deviations could provide key signatures that distinguish horizonless objects from standard BHs [90]. In addition, horizonless objects would exhibit negligible tidal heating or dissipation, significantly altering the GW signal from comparable-mass binaries [93] and extreme mass-ratio inspirals [91, 92, 94]. The detection of these tidal imprints is particularly relevant for future GW observatories like LISA and the next generation of ground-based detectors, which will have the sensitivity to probe these subtle effects.
- **Ringdown, QNMs, echoes:**
The ringdown phase marks the final stage of a BH merger, during which the newly formed BH settles into a stable state by emitting GWs mostly in the form of QNMs. While the amplitudes and phases of these modes are set by the binary parameters, their frequencies and damping times are uniquely determined by the remnant's mass and angular momentum, as predicted by GR. These modes allow for precise tests of the no-hair theorem (see [95] for a recent detailed analysis). If the remnant is a horizonless compact object or one that carries additional degrees of freedom, the ringdown signal will exhibit deviations from the GR prediction. These could manifest as deviations to the BH QNMs, different QNM excitation factors [96] or excitation of new non-gravitational modes [84], or also as post-merger GW echoes, delayed and weaker signals caused by wave reflections near the surface or by the internal structure of the object [97]. The presence of such echoes would indicate the breakdown of the classical event horizon and provide strong evidence for new physics beyond GR. Testing the nature of the remnant through a multitude of (both agnostic and theory-specific) ringdown tests is a major focus of current theoretical work and observational efforts [34, 95], as they could offer key insights

into the nature of compact objects, quantum gravity effects [98], or the existence of new fundamental fields and interactions.

- **Accretion:** Accretion flows around compact objects offer critical observational opportunities to study their nature (see, e.g., [99]). In the case of horizonless objects, infalling material would not disappear across an event horizon but could instead interact with a surface, leading to distinct signatures in the dynamics of the accretion disk. These interactions could manifest as increased luminosity from the surface [100], different thermal emission properties, or variability in the radiation due to the accumulation of matter [49]. Accretion-induced signals in such scenarios might reveal deviations from the classical BH predictions, where matter is absorbed without observable effects near the horizon, due to the infinite redshift. The presence of a surface would alter the energy dissipation process, potentially producing additional observational features, such as enhanced emission from the innermost regions of the accretion disk or even secondary signals reflecting the interaction of the infalling matter with a solid boundary.
  The recent multi-wavelength campaigns, including the EHT's continued observation of SgrA* and M87*, are already beginning to constrain accretion models, offering a direct test of whether the observed objects conform to classical BHs or exhibit signs of some structure at the horizon scale.
  In this context, it would be important to improve accretion models onto ECOs, to include realistic dissipation effects.
- **Merger counterparts:** The absence of an event horizon or the presence of additional degrees of freedom can lead to the generation of observable electromagnetic or neutrino counterparts during a merger event. This contrasts sharply with a classical BH, where the event horizon prevents any significant emission of such radiation during or after the merger. For horizonless objects, instead, matter that would otherwise be absorbed by a BH could interact with the object's surface or internal structure. The interaction between the merger debris and the surface of these exotic objects could lead to a variety of post-merger emissions, potentially akin to the case of neutron-star mergers [101, 102] but occurring also for heavier mergers and potentially probing higher-curvature and higher-redshift regions. These emissions might range from prompt flashes of electromagnetic radiation to more sustained afterglows, depending on the object's specific properties, such as surface composition, compactness, and temperature. In addition, shock waves generated by the collision of the compact objects might accelerate particles to relativistic speeds, producing detectable high-energy emissions, including X-rays or gamma rays. Similarly, neutrinos could be produced in significant quantities if the dense matter interacts with the horizonless surface during or after the merger, offering a unique multi-messenger signal. Finally, novel degrees of freedom might be present or excited, resulting in dark radiation.
  The observation of such counterparts (or lack thereof) would provide critical insights into the nature of the merging objects. A detection of electromagnetic or neutrino signals in coincidence with GW events would strongly suggest the presence of a horizonless compact object or some other exotic physics beyond the standard BH paradigm. On the other hand, the absence of these signals would reinforce the classical BH interpretation, in line with the predictions of GR, or at

least constrain ECO models even further. Future multi-messenger observations, combining GWs, electromagnetic radiation, and neutrino detections, will play a key role in distinguishing between these possibilities and testing the fundamental nature of compact objects in the universe.

For all these observables, strong efforts are ongoing to develop model-agnostic tests, improve existing ones to make them more accurate, and in parallel improve the theoretical understanding of well-grounded models to compute the above observables in a motivated top-down approach, possibly looking "outside the lamppost" for more radical deviations. Eventually, for selected well-motivated theories, it is imperative to develop models and templates that are as accurate as the current ones for GR BHs, in order to perform a robust data-driven model comparison, possibly using a synergy of multiwavelength/multimessenger probes of BH candidates across various mass ranges. Finally, in case some tension with the standard paradigm is detected in future experiments, one would need to carefully examine degeneracies with model systematics, experimental errors, and astrophysical uncertainties [37].

Acknowledgements PP is partially supported by the MUR PRIN Grant 2020KR4KN2 "String Theory as a bridge between Gauge Theories and Quantum Gravity" and by the MUR FARE programme (GW-NEXT, CUP: B84I20000100001). M.D.L. acknowledges INFN Sez. di Napoli (Iniziativa Specifica TEONGRAV.

Funding Open access funding provided by Università degli Studi di Roma La Sapienza within the CRUI-CARE Agreement.

References

1. Penrose, R.: Phys. Rev. Lett. **14**, 57 (1965)
2. Hawking, S.: Phys. Rev. Lett. **15**, 689 (1965)
3. Hawking, S.: Proc. R. Soc. Lond. A **294**, 511 (1966)
4. Hawking, S.: Proc. R. Soc. Lond. A **295**, 490 (1966)
5. Hawking, S.: Proc. R. Soc. Lond. A **300**, 187 (1967)
6. Christodoulou, D.: in 12th Marcel Grossmann Meeting on General Relativity. pp. 24–34, (2008). arXiv:0805.3880 [gr-qc]
7. Riaz, S., Mirzaev, T., Abdikamalov, A.B., Bambi, C.: Eur. Phys. J. C **83**, 838 (2023). arXiv:2303.12581 [astro-ph.HE]
8. Liu, H., et al.: Astrophys. J. **951**, 145 (2023). arXiv:2303.10593 [astro-ph.HE]
9. Mall, G., Liu, H., Bambi, C., Steiner, J.F., Garcia, J.A.: Mon. Not. R. Astron. Soc. **527**, 12053 (2024). arXiv:2309.05018 [astro-ph.HE]
10. Cardoso, V., Pani, P.: Living Rev. Rel. **22**, 4 (2019). arXiv:1904.05363 [gr-qc]
11. Maggio, E., Pani, P., Raposo, G.: (2021). arXiv:2105.06410 [gr-qc]
12. Abbott, R., et al.: (LIGO Scientific, Virgo). Astrophys. J. Lett. **896**, L44 (2020). arXiv:2006.12611 [astro-ph.HE]

13. Abac, A.G., et al.: (LIGO Scientific, Virgo, KAGRA, VIRGO). Astrophys. J. Lett. **970**, L34 (2024). arXiv:2404.04248 [astro-ph.HE]
14. Abbott, R., et al.: (LIGO-Virgo). Phys. Rev. Lett. **125**, 101102 (2020). arXiv:2009.01075 [gr-qc]
15. Calderón Bustillo, J., Sanchis-Gual, N., Torres-Forné, A., Font, J.A., Vajpeyi, A., Smith, R., Herdeiro, C., Radu, E., Leong, S.H.W.: Phys. Rev. Lett. **126**, 081101 (2021). arXiv:2009.05376 [gr-qc]
16. Buchdahl, H.A.: Phys. Rev. **116**, 1027 (1959)
17. Kaup, D.J.: Phys. Rev. **172**, 1331 (1968)
18. Ruffini, R., Bonazzola, S.: Phys. Rev. **187**, 1767 (1969)
19. Mazur, P.O., Mottola, E.: (2001). arXiv:gr-qc/0109035 [gr-qc]
20. Mazur, P.O., Mottola, E.: Proc. Nat. Acad. Sci. **101**, 9545 (2004). arXiv:gr-qc/0407075 [gr-qc]
21. Mottola, E., Vaulin, R.: Phys. Rev. D **74**, 064004 (2006). arXiv:gr-qc/0604051 [gr-qc]
22. Cattoen, C., Faber, T., Visser, M.: Class. Quant. Grav. **22**, 4189 (2005). arXiv:gr-qc/0505137 [gr-qc]
23. Del Grosso, L., Franciolini, G., Pani, P., Urbano, A.: Phys. Rev. D **108**, 044024 (2023). arXiv:2301.08709 [gr-qc]
24. Bowers, R.L., Liang, E.P.T.: Astrophys. J. **188**, 657 (1974)
25. Raposo, G., Pani, P., Bezares, M., Palenzuela, C., Cardoso, V.: Phys. Rev. D **99**, 104072 (2019). arXiv:1811.07917 [gr-qc]
26. Alho, A., Natário, J., Pani, P., Raposo, G.: Phys. Rev. D **106**, L041502 (2022). arXiv:2202.00043 [gr-qc]
27. Alho, A., Natário, J., Pani, P., Raposo, G.: (2023) arXiv:2306.16584 [gr-qc]
28. Alho, A., Natário, J., Pani, P., Raposo, G.: Class. Quant. Grav. **41**, 073002 (2024). arXiv:2307.03146 [gr-qc]
29. Morris, M.S., Thorne, K.S.: Am. J. Phys. **56**, 395 (1988)
30. Visser, M.: Lorentzian wormholes: From Einstein to Hawking (1995)
31. Lemos, J.P.S., Lobo, F.S.N., Quinet de Oliveira, S.: Phys. Rev. D **68**, 064004 (2003). arXiv:gr-qc/0302049 [gr-qc]
32. Berti, E., Barausse, E., Cardoso, V., Gualtieri, L., Pani, P., et al.: Class. Quant. Grav. **32**, 243001 (2015). arXiv:1501.07274 [gr-qc]
33. Yunes, N., Siemens, X., Yagi, K.: (2024). arXiv:2408.05240 [gr-qc]
34. Abbott, R., et al.: (LIGO-Virgo-KAGRA), (2021), arXiv:2112.06861 [gr-qc]
35. Yunes, N., Pretorius, F.: Phys. Rev. D **80**, 122003 (2009). arXiv:0909.3328 [gr-qc]
36. Kanti, P., Kleihaus, B., Kunz, J.: Phys. Rev. Lett. **107**, 271101 (2011). arXiv:1108.3003 [gr-qc]
37. Gupta, A., et al.: (2024). arXiv:2405.02197 [gr-qc]
38. Barausse, E., Cardoso, V., Pani, P.: Phys. Rev. D **89**, 104059 (2014). arXiv:1404.7149 [gr-qc]
39. Barausse, E., Cardoso, V., Pani, P.: Proceedings, 10th International LISA Symposium. J. Phys. Conf. Ser. **610**, 012044 (2015). arXiv:1404.7140 [astro-ph.CO]
40. Yunes, N., Kocsis, B., Loeb, A., Haiman, Z.: Phys. Rev. Lett. **107**, 171103 (2011). arXiv:1103.4609 [astro-ph.CO]
41. Kocsis, B., Yunes, N., Loeb, A.: Phys. Rev. D **84**, 024032 (2011). arXiv:1104.2322 [astro-ph.GA]
42. Cole, P.S., Bertone, G., Coogan, A., Gaggero, D., Karydas, T., Kavanagh, B.J., Spieksma, T.F.M., Tomaselli, G.M.: Nat. Astron. **7**, 943 (2023). arXiv:2211.01362 [gr-qc]
43. Brito, R., Cardoso, V., Pani, P.: Lect. Notes Phys. **906**, 1 (2015). arXiv:1501.06570 [gr-qc]
44. Akiyama, K., et al.: (Event Horizon Telescope). Astrophys. J. Lett. **875**, L1 (2019). arXiv:1906.11238 [astro-ph.GA]
45. Abbott, B.P., et al.: (LIGO-Virgo). Phys. Rev. Lett. **116**, 061102 (2016). arXiv:1602.03837 [gr-qc]
46. Akiyama, K., et al.: Event Horizon Telescope. Astrophys. J. Lett. **930**, L12 (2022)
47. Abuter, R., et al.: (GRAVITY). Astron. Astrophys. **636**, L5 (2020). arXiv:2004.07187 [astro-ph.GA]
48. de Laurentis, M., De Martino, I., Della Monica, R.: Rept. Prog. Phys. **86**, 104901 (2023). arXiv:2211.07008 [astro-ph.GA]
49. Akiyama, K., et al.: (Event Horizon Telescope). Astrophys. J. Lett. **930**, L17 (2022). arXiv:2311.09484 [astro-ph.HE]
50. Robinson, D.: Four decades of black holes uniqueness theorems. Cambridge University Press, Cambridge (2009)
51. Hawking, S.W.: Phys. Rev. D **14**, 2460 (1976)
52. Polchinski, J.: In: Theoretical Advanced Study Institute iElementary Particle Physics: New Frontiers in Fields and Strings (2017) pp. 353–397, arXiv:1609.04036 [hep-th]
53. Mathur, S.D.: Class. Quant. Grav. **26**, 224001 (2009). arXiv:0909.1038 [hep-th]

54. Almheiri, A., Marolf, D., Polchinski, J., Sully, J.: JHEP **02**, 062 (2013). arXiv:1207.3123 [hep-th]
55. Hayden, P., Penington, G.: (2020). arXiv:2012.07861 [hep-th]
56. Bena, I., Martinec, E.J., Mathur, S.D., Warner, N.P.: (2022). arXiv:2204.13113 [hep-th]
57. Bena, I., Martinec, E.J., Mathur, S.D., Warner, N.P.: (2022), arXiv:2203.04981 [hep-th]
58. Bena, I., Wang, C.-W., Warner, N.P.: JHEP **11**, 042 (2006). arXiv:hep-th/0608217
59. Bena, I., Giusto, S., Martinec, E.J., Russo, R., Shigemori, M., Turton, D., Warner, N.P.: Phys. Rev. Lett. **117**, 201601 (2016). arXiv:1607.03908 [hep-th]
60. Bena, I., Giusto, S., Martinec, E.J., Russo, R., Shigemori, M., Turton, D., Warner, N.P.: JHEP **02**, 014 (2018). arXiv:1711.10474 [hep-th]
61. Bah, I., Heidmann, P.: JHEP **09**, 128 (2021). arXiv:2106.05118 [hep-th]
62. Bah, I., Heidmann, P., Weck, P.: JHEP **08**, 269 (2022). arXiv:2203.12625 [hep-th]
63. Kraus, P., Mathur, S.D.: Int. J. Mod. Phys. D **24**, 1543003 (2015). arXiv:1505.05078 [hep-th]
64. Bena, I., Mayerson, D.R., Puhm, A., Vercnocke, B.: JHEP **07**, 031 (2016). arXiv:1512.05376 [hep-th]
65. Piovano, G.A., Maselli, A., Pani, P.: Phys. Lett. B **811**, 135860 (2020). arXiv:2003.08448 [gr-qc]
66. Piovano, G.A., Brito, R., Maselli, A., Pani, P.: Phys. Rev. D **104**, 124019 (2021). arXiv:2105.07083 [gr-qc]
67. Krishnendu, N.V., Arun, K.G., Mishra, C.K.: Phys. Rev. Lett. **119**, 091101 (2017). arXiv:1701.06318 [gr-qc]
68. Krishnendu, N.V., Saleem, M., Samajdar, A., Arun, K.G., Del Pozzo, W., Mishra, C.K.: Phys. Rev. D **100**, 104019 (2019). arXiv:1908.02247 [gr-qc]
69. Bena, I., Mayerson, D.R.: Phys. Rev. Lett. **125**, 221602 (2020). arXiv:2006.10750 [hep-th]
70. Bianchi, M., Consoli, D., Grillo, A., Morales, J.F., Pani, P., Raposo, G.: Phys. Rev. Lett. **125**, 221601 (2020). arXiv:2007.01743 [hep-th]
71. Bena, I., Mayerson, D.R.: JHEP **03**, 114 (2021). arXiv:2007.09152 [hep-th]
72. Bianchi, M., Consoli, D., Grillo, A., Morales, J.F., Pani, P., Raposo, G.: JHEP **01**, 003 (2021). arXiv:2008.01445 [hep-th]
73. Fransen, K., Mayerson, D.R.: Phys. Rev. D **106**, 064035 (2022). arXiv:2201.03569 [gr-qc]
74. Loutrel, N., Brito, R., Maselli, A., Pani, P.: Phys. Rev. D, pRD Editors' Suggestion **105**, 124050 (2022). arXiv:2203.01725 [gr-qc]
75. Blanchet, L.: Living Rev. Rel. **17**, 2 (2014). arXiv:1310.1528 [gr-qc]
76. Loutrel, N., Brito, R., Maselli, A., Pani, P.: (2023). arXiv:2309.17404 [gr-qc]
77. Herdeiro, C.A.R., Radu, E.: Phys. Rev. Lett. **112**, 221101 (2014). arXiv:1403.2757 [gr-qc]
78. Herdeiro, C., Radu, E., Rúnarsson, H.: Class. Quant. Grav. **33**, 154001 (2016). arXiv:1603.02687 [gr-qc]
79. Gervalle, R., Volkov, M.S.: (2024). arXiv:2406.14357 [hep-th]
80. Herdeiro, C.A.R., Radu, E.: Int. J. Mod. Phys. D **24**, 1542014 (2015). arXiv:1504.08209 [gr-qc]
81. Cardoso, V., Macedo, C.F.B., Pani, P., Ferrari, V.: JCAP **05**, 054 (2016). [Erratum: JCAP 04, E01 (2020)], arXiv:1604.07845 [hep-ph]
82. Barausse, E., Yunes, N., Chamberlain, K.: Phys. Rev. Lett. **116**, 241104 (2016). arXiv:1603.04075 [gr-qc]
83. Maselli, A., Franchini, N., Gualtieri, L., Sotiriou, T.P., Barsanti, S., Pani, P.: Nat. Astron. **6**, 464 (2022). arXiv:2106.11325 [gr-qc]
84. Crescimbeni, F., Forteza, X.J., Bhagwat, S., Westerweck, J., Pani, P.: (2024). arXiv:2408.08956 [gr-qc]
85. Lestingi, J., Cannizzaro, E., Pani, P.: Phys. Rev. D **109**, 044052 (2024). arXiv:2310.07772 [gr-qc]
86. Destounis, K., Suvorov, A.G., Kokkotas, K.D.: Phys. Rev. D **102**, 064041 (2020). arXiv:2009.00028 [gr-qc]
87. Destounis, K., Suvorov, A.G., Kokkotas, K.D.: Phys. Rev. Lett. **126**, 141102 (2021). arXiv:2103.05643 [gr-qc]
88. Destounis, K., Kokkotas, K.D.: Phys. Rev. D **104**, 064023 (2021). arXiv:2108.02782 [gr-qc]
89. Destounis, K., Angeloni, F., Vaglio, M., Pani, P.: Phys. Rev. D **108**, 084062 (2023). arXiv:2305.05691 [gr-qc]
90. Cardoso, V., Franzin, E., Maselli, A., Pani, P., Raposo, G.: Phys. Rev. D **95**, 084014 (2017). [Addendum: Phys.Rev.D 95, 089901 (2017)], arXiv:1701.01116 [gr-qc]
91. Datta, S., Brito, R., Bose, S., Pani, P., Hughes, S.A.: Phys. Rev. D **101**, 044004 (2020). arXiv:1910.07841 [gr-qc]
92. Maggio, E., van de Meent, M., Pani, P.: Phys. Rev. D **104**, 104026 (2021). arXiv:2106.07195 [gr-qc]
93. Chia, H.S., Zhou, Z., Ivanov, M.M.: (2024). arXiv:2404.14641 [gr-qc]

94. Datta, S., Brito, R., Hughes, S.A., Klinger, T., Pani, P.: Phys. Rev. D **110**, 024048 (2024). arXiv:2404.04013 [gr-qc]
95. Baibhav, V., Cheung, M.H.-Y., Berti, E., Cardoso, V., Carullo, G., Cotesta, R., Del Pozzo, W., Duque, F.: Phys. Rev. D **108**, 104020 (2023). arXiv:2302.03050 [gr-qc]
96. Forteza, X.J., Bhagwat, S., Kumar, S., Pani, P.: Phys. Rev. Lett. **130**, 021001 (2023). arXiv:2205.14910 [gr-qc]
97. Cardoso, V., Pani, P.: Nat. Astron. **1**, 586 (2017). arXiv:1709.01525 [gr-qc]
98. Abedi, J., Afshordi, N., Oshita, N., Wang, Q.: Universe **6**, 43 (2020). arXiv:2001.09553 [gr-qc]
99. Zulianello, A., Carballo-Rubio, R., Liberati, S., Ansoldi, S.: Phys. Rev. D **103**, 064071 (2021). arXiv:2005.01837 [gr-qc]
100. Broderick, A.E., Narayan, R.: Class. Quant. Grav. **24**, 659 (2007). arXiv:gr-qc/0701154
101. Metzger, B.D., Berger, E.: Astrophys. J. **746**, 48 (2012). arXiv:1108.6056 [astro-ph.HE]
102. Metzger, B.D.: Living Rev. Rel. **23**, 1 (2020). arXiv:1910.01617 [astro-ph.HE]