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Branes at angles from worldvolume actions

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Abstract

We investigate possible stable configurations of two arbitrary branes at general angles using the dynamics of DBI + WZ action. The analysis naturally reveals two types of solutions which we identify as the "marginal" and "non-marginal" configurations. We characterize possible configurations of a pair of identical or non-identical branes in either of these two classes by specifying their proper intersection rules and allowed intersection angles. We also perform a partial analysis of configurations with multiple angles of a system of asymptotically flat curved branes.

1. Introduction and summary

Brane configurations consisting of intersecting BPS branes [1,2] have been the cornerstone of many recent developments and applications of string theory. Beside their role in studying black hole solutions in string theory [3], they have been of importance in providing a new perspective on supersymmetric gauge theories [4]. Phenomenologically, stable or unstable configurations of intersecting branes are of interest both for model building in particle physics [5] and for a realization of inflationary models in cosmology [6].

Configurations of intersecting BPS branes at arbitrary angles have been studied both in view of their supersymmetry properties and in terms of their interactions in string theory [7–11]. At low energies, a brane configuration is described by a classical supergravity solution with suitable isometries that encodes the physical information regarding pairwise brane intersec-

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tions [1,2]. Using this approach, several orthogonal brane configurations [12,14–17] as well as the non-orthogonal ones [19–25] made up of branes at arbitrary angles have been constructed. Brane intersections (or bound states) naturally fall in either of the two categories termed as the "marginal" and "non-marginal" configurations [12–14], depending on whether or not they remain stable for arbitrary separation of the constituent branes. In the search for general supergravity solutions corresponding to orthogonal marginal intersections of branes of arbitrary dimensions, one finds a set of consistency conditions that determine dimensions of the pairwise intersections in terms of the branes dimensions and couplings in the problem [15,25]. These relations give the "intersection rules" [15–17] of the brane system which determine the allowable intersections in a given model of supergravity. It turns out that these relations have physical interpretations in terms of the no-force conditions between pairs of p-branes [12,25].

Non-orthogonal BPS brane configurations at more general angles were first explored in Ref. [7] using both supersymmetry and worldsheet techniques. They found that generally two BPS D$(p+n)$-branes sharing p of their dimensions form a BPS configuration, if they are rotated into each other by an $SU(n)$ ($\subset SO(2n)$) rotation. Shortly afterwards, examples of supergravity solutions describing D$(p+2)$-branes at $SU(2)$ angles were explicitly constructed [19]. It turns out that in this case solutions describe marginal configurations. These are in contrast to another type of solutions, found later in [20], describing a configuration of NS5-branes at $Sp(2)$ angles, which indeed belongs to the non-marginal category. Except for these two main examples, and those related to them by dualities and some generalizations [21–24], no other solutions for branes at general angles are known. In particular, there have been no solutions that describe multi-angle intersections of a pair of identical[1] branes at $SU(n)$ angles for $n \geq 3$, or those for a pair of non-identical branes at angles other than $0, \pi/2$. Since the $SU(n)$ condition [7] is a result of the asymptotic Killing spinor equation for flat geometries of the spacetime and the branes, one is tempted to guess that such intersections may be formed between pairs of locally curved but asymptotically flat p-branes.

The purpose of this paper is to study the problem of branes at angles from the different point of view of brane's worldvolume (DBI + WZ) action. This has the advantage of directly determining allowable (i.e. stable equilibrium) configurations of branes by giving the full set of consistency conditions for the corresponding supergravity solutions without actually solving the field equations. This will give the allowed number of dimensions shared by the two branes and their possible angles of intersection. Further, it has the capability of determining the marginal or non-marginal nature of a bound state of branes, while predicting the flat or curved profile of the individual branes in the equilibrium configuration. To use this method in full generality, one should in principle take into account the coupling of each of the branes to the supergravity background due to all of the branes including that of each brane itself. For a configuration of flat BPS branes in equilibrium, however, one can use the simplification due to the fact that the individual branes are already kept in balance by their self-interaction forces. So, in effect, any of the branes in the system can be treated as a probe in the supergravity background produced *only* by the rest of the branes. In this paper we will apply this sort of probe analysis to pairwise brane intersections in models of supergravity with a metric, one (or no) dilaton field and appropriate p-form gauge fields. For simplicity and uniformity of the formulation, we will ignore the possibility of

[1] In this paper we will refer to two branes as being "identical" if they carry the same type of form-field charges, and as the "non-identical" on the contrary.

having branes with worldvolume fluxes, although the same kind of analysis can be applied to such cases as well [27].

Our main achievements of this analysis can be summarized as follows: Using a general setup for studying branes at angles, we first derive general (model independent) formulations for marginal and non-marginal stability conditions for intersections of two flat p-branes (Sec. 2). By applying these conditions to the cases of identical and non-identical branes, we then derive the conventional rules [15,25] of marginal intersections beside new rules for non-marginal intersections (Sec. 3, 4). We also find the restrictions on the angles in either of these cases. It turns out that in all cases the rule for the number of common dimensions of the two branes with general angles is the same as that in the orthogonal case. For two identical p-branes we must have $p-2$ ($p-4$) common dimensions and two $SU(2)$ (four $Sp(2)$) angles in the marginal (non-marginal) case. On the other hand, for two non-identical p_1- and p_2-branes we must have q ($q-2$) common dimensions with all right (all right but two $SU(2)$) angles in the marginal (non-marginal) case, where q is given by the conventional rule[2] of intersections. The two cases of an electromagnetic dual pair of branes and branes within branes are somehow different than the above cases and are analyzed separately (Sec. 5, 6). These are the only intersections of flat branes without fluxes at angles. We explore the possibility of more general angles between asymptotically flat curved branes and find a partial solution: when only one of the branes (the probe) is curved, a multi-angle configuration, satisfying the marginal stability condition asymptotically, is allowable (Sec. 7). Finally, by studying propagation of small oscillations of the probe around its flat (equilibrium) configuration, we find interpretations for the two stability conditions as the no-force and no-torque conditions of the two brane system (Sec. 7).

2. General formulation of the stability conditions

2.1. The supergravity and brane models

Following Ref. [2], in this paper we will work with a D-dimensional model of supergravity with an Einstein frame metric G_{MN}, a dilaton ϕ and a d-form gauge potential $C_{(d)}$ of field strength $F_{(d+1)} = dC_{(d)}$, whose action is of the form

$$S_D = \int d^D x \sqrt{-G} \left(R - \frac{1}{2} |\nabla\phi|^2 - \frac{1}{2} e^{2\alpha(d)\phi} |F_{(d+1)}|^2 \right). \tag{1}$$

This model has a classical solution for a source of the type of a flat $(d-1)$-brane:

$$\begin{aligned} ds^2 &= H^{-\tilde{d}/(D-2)} \eta_{\mu\nu} dx^\mu dx^\nu + H^{d/(D-2)} \delta_{mn} dy^m dy^n \\ e^\phi &= H^{\alpha(d)}, \\ C_{(d)} &= H^{-1} dx^0 \wedge \cdots \wedge dx^{d-1}, \end{aligned} \tag{2}$$

where the brane's worldvolume directions are along x^μ, $\mu = 0, \cdots, d-1$, and its transverse directions are along $y^{\cdot m}$, $m = 1, \cdots, \tilde{d}+2$ (with $\tilde{d} = D - d - 2$). The harmonic function $H(y)$ in the case of an asymptotically flat geometry ($\tilde{d} \geq 1$) is given by

$$H(y) = 1 + \frac{Q}{|y|^{\tilde{d}}}. \tag{3}$$

[2] In our present notation, this is $q + 1 = -2\alpha(p_1)\alpha(p_2) + (p_1+1)(p_2+1)/(D-2)$, where $\alpha(p_1)$, $\alpha(p_2)$ are the coupling constants of the dilaton and form-fields in the model under consideration, see eq. (30).

Table 1
Decomposition of the spacetime coordinates.

Coordinates	#Dimensions	Definition
y^i	$(d_2 - d_1)$	$y^i \perp d_1,\ y^i \parallel d_2$
y^r	$(d_1 - \delta)$	$y^r \parallel d,\ y^r \perp d_2$
y^a	$(D - d)$	$y^a \perp d_1,\ y^a \perp d_2$
x^ρ	δ	$x^\rho \parallel d_1,\ x^\rho \parallel d_2$
x^r	$(d_1 - \delta)$	$x^r \perp \delta,\ x^r \perp y^i,\ x^r \parallel d_2$

For eq. (2) to be a consistent solution of the action (1), $\alpha(d)$ must obey

$$\alpha^2(d) = 1 - \frac{d\tilde{d}}{2(D-2)}, \tag{4}$$

which meanwhile is the BPS (equivalent to the no-force) condition of a single $(d-1)$-brane. The DBI + WZ action for a brane of the same kind (the probe) with vanishing worldvolume fluxes in such a background is written as

$$S_d = -T_d \int d^d x e^{-\alpha(d)\phi} \sqrt{-\det P(G)_{\mu\nu}} + T_d \int P(C_{(d)}), \tag{5}$$

where the P sign stands for the pullback of the bulk fields on the worldvolume. In cases that the probe is of a different type than the source, the WZ coupling drops out of the above action.

2.2. *General setup for a pair of branes at angles*

Let us consider a system consisting of a pair of BPS branes of worldvolume dimensions d_1 and d_2 carrying form-field charges associated to d_1- and d_2-form potentials, respectively. For definiteness, we assume $d_1 \le d_2$ and take the (d_1-1)-brane as a probe moving in the background geometry defined by the (d_2-1)-brane as the source. At equilibrium, both the source's and probe's worldvolumes are flat and span hyperplanes which are generally tilted with respect to each other. We use an orthogonal coordinate system whose axes are adapted to the tangent or normal directions of these two hyperplanes as defined in Table 1. [In this table $d = d_1 + d_2 - \delta$ stands for the dimension of the hyperplane spanned by the world directions of the two branes, and D denotes the spacetime dimension. For simplicity all subspaces are denoted by the same symbol as their number of dimensions.] In this coordinate system, the source's flat worldvolume is spanned by the (x^ρ, x^r, y^i) directions.

To represent the spacetime embedding of the probe we choose $x^\alpha = (x^\rho, x^r)$ as the probe's worldvolume coordinates and express $y^A = (y^i, y^r, y^a)$ as the functions $Y^A = Y^A(x)$. The probe's action with only these scalar degrees of freedom generically has the form of

$$S_{d_1} = \int d^{d_1}x\, \mathcal{L}(Y^A, \partial_\alpha Y^A). \tag{6}$$

Its equations of motion are accordingly

$$0 = \partial_\alpha \left(\frac{\partial \mathcal{L}}{\partial Y^A_{,\alpha}} \right) - \frac{\partial \mathcal{L}}{\partial Y^A} = \partial_\alpha \partial_\beta Y^B \frac{\partial^2 \mathcal{L}}{\partial Y^B_{,\beta} \partial Y^A_{,\alpha}} + \partial_\alpha Y^B \frac{\partial \mathcal{L}}{\partial Y^B \partial Y^A_{,\alpha}} - \frac{\partial \mathcal{L}}{\partial Y^A}, \tag{7}$$

where $Y^A_{,\alpha} = \partial_\alpha Y^A$ and we have used the chain rule of differentiation in the second line. The second form of the equations of motion will prove useful when we deal with a probe of flat worldvolume geometry. The embedding of such a flat probe is described by a simple linear ansatz

$$Y^A(x) = \omega^A_\alpha x^\alpha + y^A_0, \tag{8}$$

where ω^A_α represents constant slopes or velocities and y^A_0 are some constant shifts. This embedding can be obtained by a rotation/ boost on a probe who shares all of its worldvolume directions with those of a static flat source. From the definitions in Table 1 it is clear that

$$\omega^i_\alpha = \omega^a_\alpha = \omega^A_\rho = 0, \tag{9}$$

which means that Y^i and Y^a are constants (independent of x^α) and none of Y^A's depend on the directions x^ρ shared by the source and the probe. If we demand that eq. (8) is a solution to the probe's equation of motion (7), then using $\partial_\alpha Y^B = \omega^B_\alpha$ and $\partial_\alpha \partial_\beta Y^B = 0$ we find that

$$\frac{\partial}{\partial Y^B}\left(\omega^B_\alpha \frac{\partial \mathcal{L}}{\partial \omega^A_\alpha} - \delta^B_A \mathcal{L}\right) \equiv 0, \tag{10}$$

which must hold as an identity for all values of the independent variables x^r. Generally, for this to be the case, ω^A_α's must be constrained in a very specific way. Note that in eq. (10) $\mathcal{L}$ must be considered as the function $\mathcal{L}(Y^A; \omega^A_\alpha)$ with a Y-dependence of the following form

$$\mathcal{L} = \mathcal{L}\left(H(|Y|)\,;\, \omega^A_\alpha\right), \qquad |Y| = (Y^m Y^m)^{1/2}, \tag{11}$$

where $H(|Y|)$ is a harmonic function (satisfying $\nabla^2 H = 0$) in the directions $Y^m = (Y^r, Y^a)$ transverse to the source's worldvolume. Taking the $A = a, r$ components of eq. (10), owing to eq. (9) and that Y^r's are independent variables like x^r's themselves, one finds the following two conditions

$$\frac{\partial \mathcal{L}}{\partial H} Y^a \equiv 0 \qquad (\forall H), \tag{12}$$

$$\frac{\partial}{\partial H}\left(\omega^r_\alpha \frac{\partial \mathcal{L}}{\partial \omega^s_\alpha} - \delta^r_s \mathcal{L}\right) \equiv 0 \qquad (\forall H). \tag{13}$$

The component $A = i$ of eq. (10) is trivially satisfied for the consistent choice of $Y^i = 0$. We will often refer to eqs. (12), (13) as the 'no-force' and 'no-torque' conditions of the two brane system, respectively. The reason will become clear in section 7 by studying small oscillations of the probe around its flat equilibrium configuration.

2.3. The marginal and non-marginal bound states

In an equilibrium configuration, both eqs. (12) and (13) must hold identically for all values of the independent variable H. This strictly constrains possible choices of the parameters ω^A_α. To analyze their consequences, we first note that eq. (12) implies that either $\partial\mathcal{L}/\partial H = 0$ or $Y^a = 0$. In the first case (with $Y^a = y^a_0$ arbitrary constants) $\mathcal{L}$ must be a constant and then eq. (13) will be satisfied automatically. In the second case (with $Y^a = 0$), eq. (12) is trivial and only eq. (13) must be examined for being satisfied identically. Thus we are naturally led to the two classes of solutions termed as the "marginal" and "non-marginal" configurations [14].

A marginal intersection (or actually an overlapping) is one in which the two branes have an arbitrary separation vector y^a_0 in their overall transverse space and is exclusively described by the condition

$$\mathcal{L}(H;\omega) = \text{const.} \equiv \mathcal{L}_0(\omega) \quad (\forall H), \tag{14}$$

showing that the total interaction energy between the two branes has a constant value. On the other hand, a non-marginal intersection is one that can only be formed at zero separations of the two branes, $Y^a = 0$, in which case the branes actually intersect each other. Such a configuration is exclusively described by the condition

$$\omega^r_\alpha \frac{\partial \mathcal{L}}{\partial \omega^s_\alpha} - \delta^r_s \mathcal{L} = \text{const.} \equiv C^r_s(\omega) \quad (\forall H). \tag{15}$$

In the following three sections, we will analyze each of the eqs. (14), (15) with the aim of classifying several marginal and non-marginal bound states that arise in cases with an identical, non-identical and electromagnetic dual pair of branes. In what follows we will mainly use notations introduced in Refs. [25,26].

3. Bound states of two identical branes

The DBI + WZ lagrangian for a $(d-1)$-brane probe moving in the presence of a $(d-1)$-brane source of the same type is obtained by plugging the background (2) in the action (5), leading to [26]

$$\mathcal{L} = H^{-1}(Y)\left[\sqrt{-\det(\eta_{\alpha\beta} + H(Y)\partial_\alpha Y^m \partial_\beta Y^m)} - 1\right], \tag{16}$$

where we have ignored an irrelevant overall constant including a minus sign. Using the ansatz (8) in this expression the function $\mathcal{L}(H;\omega)$ in this case takes the form:

$$\mathcal{L} = H^{-1}[\det{}^{1/2}(\mathbf{1} + H\mathbf{\Theta}) - 1], \tag{17}$$

where Θ^α_β is defined as follows:

$$\Theta^\alpha_\beta = \omega^{r\alpha}\omega^r_\beta. \tag{18}$$

This matrix contains all the information regarding the relative orientation of the two branes. Here and below the indices of the type of α are raised and lowered by the flat metric $\eta_{\alpha\beta} = \text{diag}(-1, +1, \cdots, +1)$. The matrix Θ^α_β can be diagonalized by a Lorentz transformation in the (x^r, y^r) directions in a way that the relative orientation of the worldvolumes of the two branes is described only by a set of $d-1$ rotation angles and a single boost parameter. In such a coordinate system eq. (8) reads $Y^r = x^r \tan\theta_r$, $Y^0 = vx^0$, $Y^a = y^a_0$ and Θ^α_β hence is simply written as:

$$\Theta^\alpha_\beta = \text{diag}(-v^2, \tan^2\theta_1, \cdots, \tan^2\theta_{d-1}) =: \delta^\alpha_\beta \Theta_\alpha, \tag{19}$$

where v is the magnitude of the probe's transverse velocity relative to the source and θ_r's are angles of $d-1$ commuting rotations needed to go from the source to the probe orientation. Obviously, a number δ of these parameters are vanishing when the two branes admit same number of common worldvolume dimensions.

3.1. The marginal case

Inserting the expression (17) for $\mathcal{L}$ into the 'marginal stability' condition, eq. (14), one obtains the identity

$$\det(\mathbf{1} + H\boldsymbol{\Theta}) \equiv (1 + \mathcal{L}_0 H)^2 \qquad (\forall H). \tag{20}$$

In the basis of eq. (19) for $\boldsymbol{\Theta}$, this identity becomes

$$(1 - v^2 H)(1 + \tan^2\theta_1 H)\cdots(1 + \tan^2\theta_{d-1} H) \equiv (1 + \mathcal{L}_0 H)^2. \tag{21}$$

The only possibility for this to hold for all H is to take two of the Θ_α's equal while to set the others to zero. This requires

$$\begin{aligned} &\theta_1 = \pm\theta_2 =: \theta, \\ &v = \theta_3 = \cdots = \theta_{d-1} = 0. \end{aligned} \tag{22}$$

That is the only marginal bound state of two intersecting/overlapping identical branes is the one of static p-branes ($p = d-1$) with $p-2$ common directions and two non-vanishing angles in an abelian subgroup of the $SU(2)$ rotations. This is a $1/4$ BPS state with a well-known supergravity solution [19,25]. The constant value of $\mathcal{L}$ in this case is $\mathcal{L}_0 = \tan^2\theta$. Another possibility for a system of two identical branes is the brane–anti-brane system. This differs from the above case only in a minus sign behind the WZ term of the probe's action. As a result, the constant term -1 in the brackets in eq. (17) is replaced by $+1$ which in effect leads to replacing the positive constant $\mathcal{L}_0$ in eq. (21) by the negative one $-\mathcal{L}_0$. Obviously, then eq. (21) cannot be satisfied for any real values of v and θ's. This corresponds to the fact that a brane–anti-brane pair with relative boost or rotations (or a combination of them) can never form a marginally stable configuration.

3.2. The non-marginal case

For two identical branes the 'non-marginal stability' condition eq. (15) takes the following form:

$$\sqrt{|\Omega|}(\Omega^{-1})^{\alpha\beta}\omega^r_\alpha\omega^s_\beta - \delta^{rs}H^{-1}(\sqrt{|\Omega|} - 1) = C^{rs}, \tag{23}$$

where $\Omega^\alpha_\beta = \delta^\alpha_\beta + H\Theta^\alpha_\beta$, $|\Omega| = \det\Omega$ and (C^{rs}) are a set of angle-dependent constants. In the basis (19) with diagonal Θ, the above equation is also diagonal and reduces to

$$\prod_\alpha(1 + H\Theta_\alpha) \equiv (1 + H\Theta_r)^2(1 + HC_r)^2 \qquad (\forall H,\ \forall r,\ \ \Theta_r \neq 0, \infty), \tag{24}$$

where C_r's are the diagonal elements $C^{rs} =: \delta^{rs}C_r$. It is important to note that the above identity must hold only for those values of the index r for which Θ_r is neither zero nor infinity. For this to be an identity for all H, one needs four of Θ_α's to be pairwise equal while the rest of them are vanishing. This requires

$$\begin{aligned} &\theta_1 = \pm\theta_3,\ \ \theta_2 = \pm\theta_4, \\ &v = \theta_5 = \cdots = \theta_{d-1} = 0. \end{aligned} \tag{25}$$

That is the only non-marginal bound states of two identical branes are the static ones with four angles defined by two independent $SU(2)$ rotations, or equivalently by $Sp(2)$ angles.

It is easy to check that, unless one of the (θ_1, θ_2) vanishes, in this case $\mathcal{L}$ is not a constant, but is a linear function of H. Orthogonal configurations of the intersections with four angles like this one have been identified earlier via their supergravity solutions [17]. The most famous examples in this class include the configurations $NS5 \cap NS5 = 1$ in both type II A and II B theories in $D = 10$ dimensions, and $D5 \cap D5 = 1$ in type II B, and $M5 \cap M5 = 1$ in M-theory in

$D = 11$ [17]. In Ref. [20], a generalization of such configurations for type II A NS5-branes with $Sp(2)$ angles has been specified directly by solving the supergravity and Killing spinor equations, showing that it is at least a $3/32$ BPS state (see however [8,9,11,21]).

4. Bound states of two non-identical branes

Two non-identical branes are those that carry different form-field charges. As a result, the form-field produced by the source brane has no coupling to the probe brane. This has the effect of having no WZ term in the probe action. The DBI dynamics for a $(d_1 - 1)$-probe in a $(d_2 - 1)$-source background $(d_1 \leq d_2)$ [26] then similar to eq. (16) gives an expression for $\mathcal{L}(H; \omega)$ of the form

$$\mathcal{L} = H^{-m/2} \det{}^{1/2}(\mathbf{1} + H\mathbf{\Theta}), \tag{26}$$

where $\mathbf{\Theta}$ is given by eq. (18) and m is a function of dimensions defined by

$$m(d_1, d_2) = 2\alpha(d_1)\alpha(d_2) + \frac{d_1\tilde{d}_2}{D-2}. \tag{27}$$

For marginal bound states, m gives the number of right angles between the two branes (see [25] and below).

4.1. The marginal case

The marginal stability condition, eq. (14), in this case gives

$$\prod_{\alpha}(1 + H\Theta_\alpha) \equiv \mathcal{L}_0^2 H^m \qquad (\forall H). \tag{28}$$

Obviously, this identity can hold only when $m \in \mathbf{Z}^+$, and only in the limit that m of Θ_α's go to infinity while the others are vanishing. This implies that

$$\begin{aligned} &\theta_1 = \cdots = \theta_m = \pi/2, \\ &\theta_{m+1} = \cdots = \theta_{d_1-1} = v = 0. \end{aligned} \tag{29}$$

The number of common directions is then $\delta = d_1 - m$. Therefore, the only marginal bound states of a pair of non-identical $(d_1 - 1, d_2 - 1)$-branes are the static orthogonal intersections in which the two branes share $(\delta - 1)$ of their directions, with δ given by

$$\delta = -2\alpha(d_1)\alpha(d_2) + \frac{d_1 d_2}{D-2}. \tag{30}$$

This is the so called 'intersection rule' of intersecting brane systems, which originally was found as a consistency condition for constructing their supergravity solutions [15,16,25]. Here in agreement with Ref. [25] we find an interpretation for it as a no-force condition. As an application of this rule, for type II theories in $D = 10$, with $\alpha(d) = (4-d)/4$, eq. (30) gives $\delta = (d_1 + d_2)/2 - 2$ for two D-branes. For example, in the type IIA, IIB cases this gives marginal intersections of the forms $D4 \cap D6 = 3$, $D5 \cap D7 = 4$ each with only one angle equal to $\pi/2$.

4.2. The non-marginal case

The non-marginal stability condition, eq. (15), in this case gives the counterpart of eq. (23) for a pair of non-identical branes as

$$H^{-m/2}\sqrt{|\Omega|}\{H(\Omega^{-1})^{\alpha\beta}\omega^r_\alpha\omega^s_\beta - \delta^{rs}\} = C^{rs}, \tag{31}$$

which in the diagonal $\mathbf{\Theta}$ basis (eq. (19)) takes the form of the identity

$$\prod_\alpha(1+H\Theta_a) = C_r^2 H^m (1+H\Theta_r)^2 \qquad (\forall H,\ \ \forall r,\ \ \Theta_r \neq 0,\infty). \tag{32}$$

This is the analogue of eq. (24) for non-identical branes. This identity holds, if and only if $m \in \mathbf{Z}^+$, and further m of Θ_α's go to infinity while two of them have finite equal values and the rest are vanishing. Therefore,

$$\begin{aligned}&\theta_1 = \pm\theta_2,\ \theta_3 = \cdots = \theta_{m+2} = \pi/2,\\ &\theta_{m+3} = \cdots = \theta_{d_1-1} = v = 0.\end{aligned} \tag{33}$$

So we have the result that the only non-marginal bound states of a pair of non-identical (d_1-1, d_2-1)-branes are the static intersections with two $SU(2)$ angles, $(d_1-\delta-2)$ right angles and $(\delta-1)$ common directions, where δ is determined by the relation

$$\delta + 2 = -2\alpha(d_1)\alpha(d_2) + \frac{d_1 d_2}{D-2}. \tag{34}$$

This reduces δ by 2 as compared to that given by eq. (30). This result has been previously obtained in Ref. [17] as a rule for specifying 'localized' brane intersections. By the above derivation, however, it is reinterpreted as a rule determining non-marginal intersections. Again applying this rule to D-branes in $D=10$ type II theories gives $\delta = (d_1+d_2)/2-4$. Examples in the type IIA, IIB cases are the non-marginal intersections of the forms $D4\cap D6 = 1,\ \ D5\cap D7 = 2$ with two $SU(2)$ angles beside one angle equal to $\pi/2$.

5. Bound state of an electromagnetic dual pair of branes

An electromagnetic dual pair of branes, a priori, cannot be placed in either of the two categories studied in sections 3, 4. This is because for an electromagnetic dual pair, like $(d-1, \tilde{d}-1)$-branes with $\tilde{d} = D-d-2$, the two branes both source and couple to a single d-form potential $C_{(d)}$. To display the difference, let us take $(\tilde{d}-1)$-brane as the source and $(d-1)$-brane as the probe (assuming $d \leq \tilde{d}$ to adapt conventions of section 2). The magnetic d-form potential generated by $(\tilde{d}-1)$-brane couples to the equation of motion of $(d-1)$-brane via a term of the form

$$\frac{1}{d!}\epsilon^{\alpha_1\cdots\alpha_d}F_{mn_1\cdots n_d}\partial_{\alpha_1}Y^{n_1}\cdots\partial_{\alpha_d}Y^{n_d}, \tag{35}$$

where indices of the type m refer to the coordinates $y^m = (y^r, y^a)$ and the non-vanishing components of the $(d+1)$-form field strength $F_{(d+1)} = dC_{(d)}$ are given by

$$F_{n_1\cdots n_{d+1}} = (-1)^{\tilde{d}}\,\epsilon^m{}_{n_1\cdots n_{d+1}}\partial_m H. \tag{36}$$

On a solution like eq. (8), the term in eq. (35) reduces to

$$\frac{(-1)^{\tilde{d}}}{d!}\epsilon^{\alpha_1\cdots\alpha_d}\epsilon_{mnr_1\cdots r_d}\omega^{r_1}_{\alpha_1}\cdots\omega^{r_d}_{\alpha_d}\partial_n H, \tag{37}$$

where we have used $\omega^a_\alpha = 0$. If $\delta \geq 1$, then $\#(x^r) = d - \delta$ is less than d so that $\epsilon_{mnr_1\cdots r_d} = 0$ in the above equation, hence giving no contribution from eq. (35) to the probe's equation of motion. As such, one can use the results of section 4 for non-identical branes with $(d_1, d_2) = (d, \tilde{d})$ for an electromagnetic dual pair. In particular, the rule (30) with $\alpha(d) = -\alpha(\tilde{d})$ implies that $\delta = 2$. We conclude that the only marginal bound state of an electromagnetic dual pair in the static case is a configuration in which the two branes orthogonally intersect over a string. Examples in the type IIA, IIB theories are given by orthogonal and parallel intersections of the form $D2 \cap D4 = 1$ and $D1 \cap D5 = 1$.

On the other hand, a non-marginal bound state should obey the rule (34), which in this case implies that $\delta = 0$. This is in obvious contradiction with being static asserted above the rule (34) which means that such bound states of $(d-1, \tilde{d}-1)$-branes do not exist.

6. Non-marginal bound states of the form of branes within branes

The results of the previous sections regarding the non-marginal bound states, in particular their intersection rule (eq. (34)), are based on the no-torque condition (eq. (13)) which severely relies on the existence of at least one pair of coordinates (x^r, y^r) in Table 1. Since for a pair of parallel branes (i.e. a configuration of the form $d_1 \subseteq d_2$ for which $\delta = d_1$) there are no such coordinates, the stability condition for such configurations must be of a different form. It is easy, however, to derive it by setting $\omega^A_\alpha = 0$ and rewriting the equation of motion (eq. (10)) for this case. This leads to the simple equation

$$\partial\mathcal{L}\left(H(Y^A); \omega^A_\alpha = 0\right)/\partial Y^A = 0, \tag{38}$$

instead of eq. (13). For a single center $(d_2 - 1)$-brane, the harmonic function $H(y^A)$ only depends on y^a (there is no y^r) through the radial distance $r := \sqrt{y^a y^a}$. We must set $r = 0$ for a non-marginal bound state. Hence the above condition is further simplified to

$$\frac{\partial}{\partial r}\mathcal{L}\left(H(r); \omega^A_\alpha = 0\right)|_{r=0} = 0. \tag{39}$$

For a pair of non-identical branes, the function $V(r) := \mathcal{L}\left(H(r); \omega^A_\alpha = 0\right)$ has the specific from (see eq. (26)):

$$V(r) = (H(r))^{-m/2} = \left(1 + \frac{Q_2}{r^{\tilde{d}_2}}\right)^{-m/2}, \tag{40}$$

where Q_2 is (proportional to) the form field charge of (d_2-1)-brane and m is defined by eq. (27). (We have assumed that $\tilde{d}_2 > 0$.) It is obvious that $V(r)$ is proportional to the potential energy between the two branes at a separation r and eq. (39) indicates that their mutual forces tend to balance each other at zero separation $r \to 0$. The above condition then means that the function $V(r)$ must be regular near $r = 0$ at least up to its first order derivative, which is the case when $m\tilde{d}_2 > 2$, or equivalently

$$\alpha(d_1)\alpha(d_2) > \frac{2(D-2) - d_1\tilde{d}_2^2}{2\tilde{d}_2(D-2)}. \tag{41}$$

This inequality together with $d_1 \leq d_2$ and $\tilde{d}_2 > 0$ in turn specifies all the non-marginal bound states of the form of branes within branes. Special cases of such configurations are those with a self dual pair ($d_1 = d,\ d_2 = \tilde{d}$). The above condition in this case (along with $d \leq \tilde{d}$) yields $3 \leq d \leq (D-2)/2$, showing that such configurations are possible only in $D \geq 8$ dimensions. Famous examples of such configurations are $M2 \subset M5$ in $D = 11$ and the dyonic membrane (bound state of an electric and a magnetic 2-brane) in $D = 8$ dimensions, which were known through their supergravity solutions [18].

7. Multi-angle intersections of asymptotically flat curved branes

The results of the previous sections indicate that, except for the static bound state of identical branes at $SU(2)$ angles, no other bound state of two flat p-branes with several angles or a relative boost is marginally stable. Since both the string scattering [11] and supersymmetry computations [7–10] suggest that such more general intersections do exist, it is puzzling that why such configurations did not appear among our marginal solutions of sections 3, 4. This apparent discrepancy is simply resolved if we recall that both the D-brane scattering computations and supersymmetry techniques essentially depend on the existence of asymptotic states in which both p-branes are like flat hypersurfaces in a Minkowski space. In string perturbation theory, scattering amplitudes describing brane interactions are reliable only at weak coupling where the gravity and other fields appear only at linearized order. This restricts the domain of validity of such computations to a region in spacetime in which both of the branes look like flat hypersurfaces in a Minkowski background. On the other hand, supersymmetry computations in the literature do not involve solving the full Killing spinor equation in the general case of branes at angles. To determine the amount of supersymmetry preserved by a brane configuration, only the asymptotic (algebraic) form of the Killing spinor equation would suffice to be solved [7–9], which *only* involved the asymptotic form of the spacetime metric and of the geometry of the branes. Therefore, there may be BPS states of *curved* p-branes which look asymptotically like marginal multi-angle intersections of *flat* p-branes. It turns out that the asymptotic form of the no-force condition for such configurations to first order exactly reproduces the $SU(n)$ condition for $n \leq 3$ angles required by supersymmetry preservation [7–11]. However, both the no-force and no-torque conditions cannot be satisfied simultaneously unless for the marginal configurations specified in sections 3, 4. This implies, firstly, that general marginal multi-angle intersections cannot be formed simply from flat p-branes. Secondly, even in the case of an asymptotically marginal bound state, the equilibrium condition between the two branes can be easily lost by the action of twisting forces (or toques) that vary the relative orientation of the two branes.

Generally, the two conditions (14), (15) can be traded for two infinite sets of algebraic constraints among the parameters ω^r_α. This can be easily seen by setting $H = 1 + h$ and expanding $\mathcal{L}(H;\omega)$ in powers of h as follows

$$\mathcal{L}(1+h;\omega) = \sum_{k=0}^{\infty} \frac{1}{k!} \mathcal{L}_k(\omega) h^k. \tag{42}$$

Using this expansion in eqs. (14), (15), they will reduce to the two sets of equations

$$\mathcal{L}_k(\omega) = 0, \tag{43}$$

$$\omega^r_\alpha \frac{\partial \mathcal{L}_k}{\partial \omega^s_\alpha} - \delta^r_s \mathcal{L}_k = 0, \tag{44}$$

for $k = 1, 2, \cdots$. For a marginal bound state with a *flat* probe brane, these two sets of equations for *all* k restrict possible configurations to those of sections 3, 4. For a configuration with an asymptotically flat *curved* probe, we *define* the marginal stability property by demanding that $\mathcal{L}(H; \omega)$ is nearly constant only up to first order in $h = H - 1$ as $h \to 0$. Such asymptotically marginal configurations then will be specified by a single condition on their angles of the form $\mathcal{L}_1(\omega) = 0$. This condition, though provides a mean of translational stability of the system to first order, does *not* insure its rotational stability to this order, which is guaranteed by eq. (44) for $k = 1$. These two conditions together restrict possible marginal bound states to those obtained in sections 3, 4. We now examine explicit form of these conditions and their consequences in the previous cases.

7.1. Identical branes

It is easy to see, by expanding eq. (17) to $\mathcal{O}(h)$, that in this case

$$\mathcal{L}_0(\Theta) = \det{}^{1/2}(\mathbf{1} + \mathbf{\Theta}) - 1,$$
$$\mathcal{L}_1(\Theta) = \det{}^{1/2}(\mathbf{1} + \mathbf{\Theta}) \left\{ \frac{1}{2} \operatorname{tr} \left(\frac{\mathbf{\Theta}}{\mathbf{1} + \Theta} \right) - 1 \right\} + 1. \tag{45}$$

The ω dependences of these quantities come from expressing $\mathbf{\Theta}$ as in eq. (18). (The expression for $\mathcal{L}_0$ is given here only for later reference.) In the basis (19), the $\mathcal{L}_1 = 0$ condition then is equivalent to

$$V(\{\theta_\alpha\}) := \prod_{\alpha=0}^{d-1} \cos\theta_\alpha + \frac{1}{2} \sum_{\alpha=0}^{d-1} \sin^2\theta_\alpha - 1 = 0, \tag{46}$$

where, for convenience, we have included the velocity parameter v in θ_α by defining $\theta_0 = \tan^{-1}(iv)$. It is easy to check that eq. (46) is equivalent to the $SU(n)$ condition for $n \leq 3$, where $n = d - \delta$ is the number of non-vanishing angles:

$$\theta_1 \pm \cdots \pm \theta_n = 0 \ (\mathrm{mod}\ 2\pi). \tag{47}$$

This implies that all the asymptotically marginal configurations with $n \leq 3$ angles necessarily preserve some amount of supersymmetry [7–11]. On the other hand, by solving eq. (46) for v one finds that configurations with a boost and several rotations are not allowed unless there are at least three non-vanishing angles. If, in addition, one requires rotational stability of the configuration to first order, eq. (44) for $k = 1$ along with the $\mathcal{L}_1 = 0$ condition gives

$$\frac{\partial V}{\partial \theta_\alpha} = -\sin\theta_\alpha \left(\prod_{\beta \neq \alpha} \cos\theta_\beta - \cos\theta_\alpha \right) = 0. \tag{48}$$

The only simultaneous solution of eqs. (46), (48), when at least two θ_α's exist, is given by

$$\theta_1 = \pm\theta_2, \ \ \theta_0 = \theta_3 = \cdots = \theta_{d-1} = 0 \tag{49}$$

and all its permutations for θ_α's with $\alpha \neq 0$. This again shows that the only rotationally stable marginal bound states are those with two $SU(2)$ angles which preserve some amount of supersymmetry. This also establishes that except for this $SU(2)$ case, which consists of totally flat p-branes, all other cases with $n \geq 3$ non-vanishing angles must be made of curved p-branes. The case of curved branes with three angles is special in that it also satisfies the supersymmetry preservation $SU(3)$ condition.

7.2. Non-identical branes

In this case eq. (26) gives

$$\begin{aligned}\mathcal{L}_0(\Theta) &= \det{}^{1/2}(\mathbf{1}+\boldsymbol{\Theta})\\ \mathcal{L}_1(\Theta) &= \frac{1}{2}\det{}^{1/2}(\mathbf{1}+\Theta)\left\{\mathrm{tr}\left(\frac{\boldsymbol{\Theta}}{\mathbf{1}+\boldsymbol{\Theta}}\right)-m\right\}\end{aligned} \tag{50}$$

In the basis (19), the condition $\mathcal{L}_1 = 0$ then gives in this case

$$\begin{aligned}&V(\{\theta_\alpha\}) \equiv \sum_{\alpha=0}^{d_1-1}\sin^2\theta_\alpha - m = 0 \quad \Rightarrow\\ &-2\alpha(d_1)\alpha(d_2) + \frac{d_1 d_2}{D-2} = \sum_{\alpha=0}^{d_1-1}\cos^2\theta_\alpha\end{aligned} \tag{51}$$

This gives, in fact, a modification of the usual intersection rule, eq. (30), to the general case involving arbitrary boost and angles between the two branes. Obviously, an orthogonal static configuration, with θ_α's equal to $0, \pi/2$, exists only in cases with $m \in \mathbf{Z}^+$. That is, for an orthogonal intersection, the rule (30) must hold and in such a case m counts the number of $\theta_\alpha = \pi/2$ angles. Despite eq. (46), this eq. (51) allows for the possibility of combinations of a boost with any number of angles. Specially, when $m \leq 0$, one can find boosted configurations of two asymptotically parallel branes having a relative transverse velocity $v = \sqrt{\frac{m}{m-1}}$. However, configurations defined by eq. (51) are not rotationally stable, unless we have

$$\frac{\partial V}{\partial \theta_\alpha} = 2\sin\theta_\alpha\cos\theta_\alpha = 0, \tag{52}$$

which means that θ_α's must be $0, \pi/2$. Therefore, the rotationally stable marginal bound states are the static ones with orthogonal branes obeying the rule (30).

7.3. Small oscillations of the probe

So far we have analyzed the basic conditions (14), (15) to determine possible equilibrium configurations of a pair of branes with flat worldvolume geometries. To find physical interpretations of these conditions, we slightly perturb the probe's worldvolume around a flat configuration, while the source is kept fixed. We focus on the probe's configurations that are asymptotically flat in the region far from the source, but are slightly curved in the region close to it. The suitable small quantity for expansion around a flat configuration is the harmonic function $h(Y) = H(Y) - 1$ which also measures deviations from flatness of the supergravity background. This function goes to zero asymptotically at $|Y(x)| \to \infty$ where the source and probe have a large separation. In this limit the lagrangian of the probe reduces to that of minimal hypersurfaces in a flat Minkowski space, which is denoted by $\mathcal{L}_0$. The general lagrangian then has an expansion

$$\mathcal{L}(Y^A, \partial_\alpha Y^A) = \sum_{k=0}^{\infty}\frac{1}{k!}h^k(Y^A)\mathcal{L}_k\left(\partial_\alpha Y^A\right). \tag{53}$$

Clearly, $\mathcal{L}_0$ has classical solutions which are in the form of flat hypersurfaces described by eq. (8), which plays the role of an unperturbed solution $Y_0^A(x)$. This choice for $Y^A(x)$ replaces

$\mathcal{L}_k\left(\partial_\alpha Y^A\right)$'s with the constant coefficients $\mathcal{L}_k\left(\omega^A_\alpha\right)$ in eq. (53). Writing the equation of motion of $Y^A(x)$ from this lagrangian and expanding its curved probe solution in powers of h as $Y^A(x) = Y^A_0(x) + Y^A_1(x) + \cdots$, we will find non-homogeneous linear equations for $Y^A_n(x)$ at different orders of h. For the lowest order perturbation $Y^A_1(x)$ we find explicitly

$$\frac{\partial^2 \mathcal{L}_0}{\partial\omega^A_\alpha \partial\omega^B_\beta}\partial_\alpha\partial_\beta Y^B_1 + \left(\omega^A_\alpha \frac{\partial\mathcal{L}_1}{\partial\omega^B_\alpha} - \delta_{AB}\mathcal{L}_1\right)\partial_B h(Y_0) = 0. \tag{54}$$

This constitutes a system of linear second order PDE's with the *constant* coefficients $\frac{\partial^2\mathcal{L}_0}{\partial\omega^A_\alpha\partial\omega^B_\beta}$, and with source terms which are linear combinations of $\partial_B h(Y_0)$. Using either of the expressions (45) or (50) for $\mathcal{L}_0(\Theta)$, this equation can be further simplified to

$$I_{AB}\Omega^{\alpha\beta}\partial_\alpha\partial_\beta Y^B_1 + \frac{1}{\mathcal{L}_0}\left(\omega^A_\alpha \frac{\partial\mathcal{L}_1}{\partial\omega^B_\alpha} - \delta_{AB}\mathcal{L}_1\right)\partial_B h(Y_0) = 0, \tag{55}$$

where the coefficients $\Omega_{\alpha\beta}$ and I_{AB} are defined by

$$\begin{aligned}\Omega_{\alpha\beta} &= [(1+\Theta)^{-1}]_{\alpha\beta},\\ I_{AB} &= \delta_{AB} - \Omega^{\gamma\delta}\omega^A_\gamma\omega^B_\delta.\end{aligned} \tag{56}$$

The component equations of eq. (55) for $A = i, a, r$ give the following uncoupled equations for the perturbations in these directions

$$\Omega^{\alpha\beta}\partial_\alpha\partial_\beta Y^i_1 = 0, \tag{57}$$

$$\Omega^{\alpha\beta}\partial_\alpha\partial_\beta Y^a_1 - \frac{\mathcal{L}_1}{\mathcal{L}_0}\partial_a h(Y_0) = 0, \tag{58}$$

$$I_{rs}\Omega^{\alpha\beta}\partial_\alpha\partial_\beta Y^s_1 + \frac{1}{\mathcal{L}_0}\left(\omega^r_\alpha \frac{\partial\mathcal{L}_1}{\partial\omega^s_\alpha} - \delta_{rs}\mathcal{L}_1\right)\partial_s h(Y_0) = 0. \tag{59}$$

The linear operator $\Omega^{\alpha\beta}\partial_\alpha\partial_\beta$ appearing in these equations is indeed the wave operator along the probe's worldvolume directions, as can be seen easily using the basis (19),

$$\Omega^{\alpha\beta}\partial_\alpha\partial_\beta = -\frac{1}{1-v^2}\partial_0^2 + \cos^2\theta_1\partial_1^2 + \cdots + \cos^2\theta_{d_1-1}\partial^2_{d_1-1}. \tag{60}$$

Thus the set of eqs. (57)–(59) are indeed the equations for the propagation of perturbations at the speed of light along the probe's worldvolume directions. These perturbations are forced by the gravity and other interaction effects of the source brane as reflected in the function $h(Y_0)$. According to eq. (57), perturbations in the y^i directions propagate as free waves, as expected from the homogeneity of the space in these directions. Eq. (58) for the transverse (y^a) oscillations plays the role of a force equation for transverse relative motions while eq. (59) for the longitudinal (y^r) oscillations has the role of a torque equation. In the equilibrium conditions with $Y^A_1 = 0$, eqs. (58), (59), respectively, reduce to eqs. (43), (44) for $k = 1$. The first order perturbations around equilibrium configuration thus propagate as free oscillations in all directions. These same conditions for higher order Y^A_k's reproduce eqs. (43), (44) for higher order k and the full set of perturbation equations for all k in the equilibrium case reproduce the conditions (14), (15). As a result, our perturbative approach provides physical interpretations for eqs. (14), (15) as the balance conditions for the relative total force and torque in the two brane system, respectively.

Remarkably, eqs. (58), (59) admit static solutions because they involve time independent sources. General solutions are then obtained by superimposing freely propagating waves with these static solutions. It is convenient to construct static solutions in the basis (19) where I_{rs} has the diagonal form $I_{rs} = \mathrm{diag}(\cos^2\theta_1, \cdots, \cos^2\theta_{d_1-1})$. Defining the coordinates $\bar{x}^\alpha \equiv x^\alpha/\cos\theta_\alpha$ along the probe's worldvolume directions, then eqs. (58), (59) in the static case (with $\partial_0 Y^r = \partial_0 Y^a = 0$) reduce to the two Poisson equations

$$\bar{\nabla}^2 Y_1^a = \frac{\mathcal{L}_1}{\mathcal{L}_0}\partial_a h(Y_0(\bar{x})), \tag{61}$$

$$\bar{\nabla}^2 Y_1^r = -\frac{1}{\mathcal{L}_0\cos^2\theta_r}\left(\tan\theta_r\frac{\partial\mathcal{L}_1}{\partial\tan\theta_r} - \mathcal{L}_1\right)\partial_r h(Y_0(\bar{x})), \tag{62}$$

with $\bar{\nabla}^2 \equiv \sum_{r=1}^{d_1-1}(\partial/\partial\bar{x}^r)^2$ denoting the flat probe's Laplacian operator. Here $h(Y_0(\bar{x}))$ is the harmonic function evaluated at the location of the probe in terms of the $\bar{x}^r$ coordinates, namely $h(Y_0(\bar{x})) = Q/[\sum_a (y_0^a)^2 + \sum_r (\bar{x}^r)^2\sin^2\theta_r]^{\tilde{d}_2/2}$. The solutions to eqs. (61), (62) with asymptotic flat boundary conditions for the probe ($Y_1^a(\bar{x}), Y_1^r(\bar{x}) \to 0$ as $|\bar{x}^r| \to \infty$) then give the embedding functions up to first order:

$$Y^a(\bar{x}) = y_0^a + y_0^a\frac{\mathcal{L}_1}{\mathcal{L}_0}\int d^{d_1-1}\bar{x}'\rho(\bar{x}')G(\bar{x}-\bar{x}') + \cdots, \tag{63}$$

$$Y^r(\bar{x}) = \bar{x}^r\sin\theta_r - \frac{1}{\cos^2\theta_r\mathcal{L}_0}\left(\tan\theta_r\frac{\partial\mathcal{L}_1}{\partial\tan\theta_r} - \mathcal{L}_1\right)\int d^{d_1-1}\bar{x}'\bar{x}'^r\rho(\bar{x}')G(\bar{x}-\bar{x}') + \cdots, \tag{64}$$

where the effective source density $\rho(\bar{x}')$ and the Green's function of the Laplacian operator $G(\bar{x}-\bar{x}')$ are given by

$$\rho(\bar{x}') = \frac{\tilde{d}_2 Q}{[\sum_a (y_0^a)^2 + \sum_r (\bar{x}'^r)^2\sin^2\theta_r]^{(\tilde{d}_2+2)/2}}, \tag{65}$$

$$G(\bar{x}-\bar{x}') = \frac{A_{d_1}}{[\sum_r (\bar{x}^r - \bar{x}'^r)^2]^{(d_1-3)/2}}. \tag{66}$$

The above solutions generally describe a curved equilibrium configuration of the probe with asymptotically flat boundary conditions. They are generally valid without need to any additional constraints on the angles θ_r or the dimension of intersection. Such constraints arise only when we restrict to flat or near flat (asymptotically marginal) equilibrium configurations, as discussed at the beginning of this section. As eqs. (63), (64) indicate, asymptotic flatness up to next to leading order in all y^a directions needs $\mathcal{L}_1 = 0$ while that in any of the y^r directions requires $\tan\theta_r\frac{\partial\mathcal{L}_1}{\partial\tan\theta_r} - \mathcal{L}_1 = 0$. The two conditions together for all transverse directions to the source exclusively characterize globally flat configurations of the probe (as we have seen in subsections 7.1, 7.2).

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