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The Upgraded Planck System of Units That Reaches from the Known Planck Scale All the Way Down to Subatomic Scales

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Simple Summary: We connect the Planck scale to the subatomic world without assuming a priori that such a connection exists; we deduce the physical meaning of the famous dimensionless fine-structure constant 1/137 (known simply as 137); we relate the masses of quarks, leptons, and vector bosons to the Higgs mass, i.e., we derive scaling relations for the entire mass ladder of the subatomic world; we deduce from first principles the origin and physical significance of Koide's $K = 2/3$ enigmatic constant and analogous quark and vector boson constants; and we calculate the gauge coupling factors of quarks, leptons, bosons, and the weak interaction in terms of only the Higgs field and Koide's constant.

Abstract: Natural systems of units $\{U_i\}$ need to be overhauled to include the dimensionless coupling constants $\{\alpha_{U_i}\}$ of the natural forces. Otherwise, they cannot quantify all the forces of nature in a unified manner. Thus, each force must furnish a system of units with at least one dimensional and one dimensionless constant. We revisit three natural systems of units (atomic, cosmological, and Planck). The Planck system is easier to rectify, and we do so in this work. The atomic system discounts $\{G, \alpha_G\}$, thus it cannot account for gravitation. The cosmological system discounts $\{\hbar, \alpha_\hbar\}$, thus it cannot account for quantum physics. Here, the symbols have their usual meanings; in particular, α_G is the gravitational coupling constant and $\alpha_\hbar$ is Dirac's fine-structure constant. The speed of light c and the impedance of free space Z_0 are resistive properties imposed by the vacuum itself; thus, they must be present in all systems of units. The upgraded Planck system with fundamental units UPS $:= \{c, Z_0, G, \alpha_G, \hbar, \alpha_\hbar, \ldots\}$ describes all physical scales in the universe—it is nature's system of units. As such, it reveals a number of properties, most of which have been encountered previously in seemingly disjoint parts of physics and some of which have been designated as mere coincidences. Based on the UPS results, which relate (sub)atomic scales to the Planck scale and the fine-structure constant to the Higgs field, we can state with confidence that no observed or measured physical properties are coincidental in this universe. Furthermore, we derive from first principles Koide's $K = 2/3$ enigmatic constant and additional analogous quark and vector boson constants. These are formal mathematical proofs that justify a posteriori the use of geometric means in deriving the quark/boson mass ladder. This ladder allows us to also calculate the Higgs couplings to the vector bosons and the Weinberg angle in terms of K only, and many of the "free" parameters of the Standard Model of particle physics were previously expected to be determined only from experiments.

Keywords: atomic processes; cosmological parameters; cosmology: theory; early universe; elementary particles; galaxies: kinematics and dynamics; gravitation

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1. Introduction and Motivation

1.1. Three Fundamental Systems of Units Under Consideration

In a recent paper [1], we used a cosmological system of units based on the speed of light c, Newton's gravitational constant G, and MOND's characteristic acceleration

a_0 [2–4]. Since G is a building block of this system, then a_0 can substitute for the universal MOND unit, the mysterious constant $\mathcal{A}_0 \equiv a_0 G$. Besides $\mathcal{A}_0$, pairs of the fundamental units produced two more de facto important dynamical units: the surface density $\sigma_0 = a_0/G$ and the force $F_0 = c^4/G$. Notice a disparity implicit in unit F_0: inverting G produces $c^4 G$, which (very much unlike c^4/G) is a composite unit of no particular interest with dimensions of $[\text{M}]^{-1}[\text{L}]^7[\text{T}]^{-6} = c^8/F_0$. This disparity singles out the unit of force $F_0 = c^4/G$ as an important component of the cosmological system (along with the pair of units $\mathcal{A}_0$ and σ_0), but there is not much more one can do with it at this point, besides noting that the same unit of force appears in the famous Planck system of units as well and that F_0 is a geometry-independent quantity since both c and G do not carry an imprint of (dependence on) the geometry of our four-dimensional spacetime.

The cosmological system of units does not use Planck's constant h [5,6], which turns out to be a derived unit of no particular interest, but current thinking forgives the oversight—after all, this is a cosmological system designed for measurements on universal scales. Nonetheless, we were drawn into comparisons with the Planck system, which now uses Dirac's $\hbar = h/(2\pi)$ as a fundamental unit [7–9]; and, soon enough, we also added Hartree's atomic system of units [10], which paradoxically does not use c (or G, for that matter) as a fundamental unit. The immediate implication is that the speed of light is not a preset limit in the atomic world, where the unit of speed is $\alpha_{\hbar}\, c \ll c$, where $\alpha_{\hbar} = 1/137.036$ is the fine-structure constant. Under these circumstances, we ended up juggling three different fundamental systems of units, comparing and contrasting their building blocks and the assumptions that have gone into their architectures.

1.2. Dirac's Problematic Constant $\hbar$ and the Three Widely-Used Atomic Radii

The modern atomic and Planck systems use Dirac's $\hbar = h/(2\pi)$ [7] instead of Planck's original and purely physical constant h [5]. This modification is not trivial because the 2π carries the "unit" of radians, which, although not a physical unit, alerts us to the presence of 2-D geometry. The descriptive unit of radian has been dropped by many authors and also by the SI system of units, leading to a widespread misunderstanding that $\hbar$ simply absorbs a numerical factor of 2π with no further ramifications. The inconsistency was noted by Bunker et al. [11], who inserted the unit of radian in the definition of $\hbar$ and the unit of cycle in the definition of h. The SI system must reinstate at least the radian/steradian "units" as descriptive words because they alert us to the presence of geometry (see below). The same holds for trigonometric functions, whose arguments must always be in radians—although this is such common knowledge that the radian is no longer mentioned. On the other hand, the radian is not dropped from the unit of angular velocity, which has always been radians/sec, where "radians" is a descriptive term and "sec" is the only physical unit.

Dirac believed that $\hbar$ is the true universal constant, and we can only guess the reason why: the 2π in $\hbar$ has introduced 2-D geometry into the constant, so, unlike Planck's h, the constant $\hbar$ is not purely physical; it is a composite constant. This fact was effectively proven by Leblanc et al. [12] who showed that the Compton radius r_c (where $r_c \propto \hbar$) also includes a geometric component. The seminal results presented in Refs. [11,12] have important consequences in physics that become detectable when we write side-by-side the three famous electronic radii of the atomic world (where m_e is the electron mass and e is the fundamental positive charge):

$$\begin{aligned} \text{Classical radius}: &\quad r_\text{e} = e^2/(\not\epsilon_0 m_\text{e} c^2) = r_\text{c}\,\alpha_{\hbar} \\ \text{Compton radius}: &\quad r_\text{c} = \hbar/(m_\text{e} c) \\ \text{Bohr radius}: &\quad r_\text{b} = \not\epsilon_0 \hbar^2/(m_\text{e} e^2) = r_\text{c}/\alpha_{\hbar} \end{aligned} \quad . \tag{1}$$

Here, $\alpha_{\hbar}$ is the fine-structure constant and $\not\epsilon_0$ is the reduced vacuum permittivity defined by $\not\epsilon_0 \equiv 4\pi\epsilon_0$, an equation that shows how the stereometry of space modifies the physical unit ϵ_0 of the vacuum. Therefore, we have an SI unit problem here too, just as Bunker et al. [11] discovered for $\hbar$. The vacuum is a three-dimensional space, hence the stereometric term of 4π; thus, the units of $\not\epsilon_0$ must also include the descriptive unit steradians[1]. Now, these geometric considerations show why three different radii do exist in atomic physics: although they have the same physical dimension of length [L], they capture entirely different geometries; the electrons in the atoms venture in 3-D space (their orbitals are 3-D structures, hence, the $1/(4\pi)$ in r_e); the emitted photons only "see" two dimensions (see Note 1); and the electrons in the Bohr model of the atom are quantized and they see only discrete sectors embedded in 3-D space (hence, the $1/\pi$ factor in r_b). The factor of $1/4$ "missing" from the $1/\pi$ is, however, applied to the energy levels because this factor is included in the Rydberg energy E_R (see below).

Deriving the geometric pattern of the quantized radii r_n of the Bohr model is a little harder, yet within our grasp[2]. In any case, the factor of $1/4$ is necessarily missing from r_b, so that the quantized angular momentum $\mathcal{L}_n$ is truly a 2-D quantity ($\mathcal{L}_n \propto \not{h} \propto 1/(2\pi)$), and the associated Rydberg energy E_R is independent of geometry (although the abolished geometry contributes a unitless constant of $1/4$, i.e., $E_R \propto 1/(\not\epsilon_0 \not{h})^2 \propto 1/[4\pi/(2\pi)]^2 = 1/4$, the same factor as that "missing" from r_b).

Lastly, the Bohr radius is the fundamental unit of length in the atomic system [10], but we argue that the Compton radius is actually the most important unit because its definition in Equation (1) does not contain the fine-structure constant $\alpha_{\not{h}}$. Furthermore, there is more circumstantial evidence that r_c is important among the three radii shown in Equation (1): r_c is the geometric mean of r_e and r_b (i.e., $r_c = \sqrt{r_e r_b}$), and this implies that the Compton radius r_c is also the geometric mean of all three length scales combined together, viz.

$$r_c = \sqrt[3]{r_e r_c r_b} \,. \tag{2}$$

Thus, r_c is singled out among the three electronic radii for further duty in all systems of units (but probably without the 2π term in order to remove the artificially inserted 2-D geometry). Furthermore, it may not be as obvious yet, but geometric averaging plays a huge role in nature, as was first discovered in Ref. [1]. The above geometric means (hereafter denoted as G-Ms, to avoid confusion with the famous heliocentric constant "GM") are only a prelude to their ubiquitous appearances in many G-M combinations of natural constants and physical quantities as well. That many G-Ms appear in physics equations is an empirical observation for which we do not presently have a full explanation; the properties of G-M averaging, in comparison to those of arithmetic averaging, provide some hints, which we discuss in Section 4.3 below.

1.3. Dimensionless Constants

The general notion about constructing a system of units is that one is free to choose any units to be the building blocks. Dimensionless constants do not have units to offer, so they are not chosen as building blocks. They remain as passive invariants in any adopted system of units, and they serve mostly as cross-checks of the various calculations performed between dimensional quantities. The current thinking is summarized in the following excerpt from Zeidler [14]:

> *"A special role is played by those physical quantities that are dimensionless in the SI system. We expect that such quantities are related to important physical effects. The experience of physicists confirms this."*

Therefore, we suspect that such constants are important in physics, but we do not really know what to do with them beyond their ascertained invariance, simply because they lack units.

The general notion about constructing a physical system of units is wrong on two counts: (1) Although unitless constants do not have units to offer, they must be actively included in systems of units because they introduce the natural forces that cause the important physical effects mentioned by Zeidler [14]. (2) We are not free to choose any dimensional units as building blocks; we must choose wisely the units that measure the fundamental forces and, in addition, those units dictated by the vacuum itself. In particular, choosing a favorite particle to supply its properties for building blocks could be a bad idea[3], and the reason is that such favoritism could violate a principle of "fairness" in this world. As will be seen below, nature does not at all favor or neglect any particle or force field, not even the "very small" ones against the "very large" ones, and vice versa.

One or both of the above defects have crept into our systems of units, where they selectively impaired or eliminated entirely some fundamental forces of nature. A natural force is impaired when its dimensional or dimensionless constant is not included as a building block of a system of units, and a force is eliminated entirely when both of its defining constants are not included in a system of units.

1.4. Outline

In this work, we construct a self-consistent system of units that does not suffer from the above defects and that includes gravity, electromagnetism, and the weak interaction (Section 2). The coupling constant of the strong interaction is not included yet because massive particles at the TeV energy scale have not been discovered. We are going to test this system's performance on the atomic and subatomic scales, as well as on the Planck and macroscopic scales (Section 3).

It turns out that the Planck system is easier to upgrade because it already includes the appropriate dimensional constants $\{c, Z_0, G, \hbar\}$, although the impedance of free space Z_0 has so far been sidelined. Therefore, what we need to do for the upgrade is to activate the unitless coupling constants α_G (gravitational coupling constant) and $\alpha_{\hbar}$ (fine-structure constant) and to repair the damage that $\hbar$ has caused by inadvertently introducing geometry in them (see Note 9 for details), besides the well-intended quantum forces.

In Section 2, we describe the building blocks of the upgraded Planck system of units. In Section 3, we collect the new results concerning masses, charges, and lengths in the new system. In Section 4, we discuss the results, and in Section 5, we summarize potential issues still lingering in this system of units as well as some future research prospects.

Finally, in Section 6, we list the most important highlights of our investigation, including the results obtained in the Appendices. In Appendix A, we derive from first principles the long-sought physical significance of Koide's lepton constant $K = 2/3$ [15] of atomic physics; the Higgs couplings to the vector bosons [16] and the bottom quark; and the Weinberg angle [16] in terms of K. In Appendix B, we discuss the universality of the Tully–Fisher/Faber–Jackson relation [17,18] discovered in spiral and elliptical galaxies, respectively (see also Ref. [1]). This fundamental relation that relates the fourth power of a kinetic scalar to a quantity with units of surface density signifies a new universal law of nature that has manifestations in several other parts of physical science besides astrophysics.

2. The Building Blocks of the Upgraded Planck System

The upgraded Planck system (UPS) includes the following building blocks:

$$\text{UPS} := \{c, Z_0, G, \alpha_G, \hbar, \alpha_{\hbar}\}, \tag{3}$$

where $\{c, Z_0, G, \hbar\}$ are the usual dimensional units and the coupling constants $\{\alpha_G, \alpha_{\hbar}\}$ are dimensionless units. We use a slash to indicate the presence of geometric units, which are undesirable. This is a problem we have to contend with throughout this work. In Section 2.2, we will be ready to replace the units $\{\hbar, \alpha_{\hbar}\}$ with Planck's original units $\{h, \alpha_h\}$ in the UPS in order to eliminate the 2-D geometry introduced to Dirac's constants by the 2π term (see also Note 28 below).

2.1. Dimensional Units

For future reference, we need to recall and emphasize a gem of natural units: the fundamental dimensional relation between gravity (supplying G) and electromagnetic (EM) forces (supplying also the vacuum's constant $(4\pi\epsilon_0)^{-1}$). This is obtained by equating the dimensions of the forces in Newton's gravitational law and Coulomb's law for two electrons. We find, in dimensional form, that

$$G\, m_\mathrm{e}^2 \;\sim\; (4\pi\epsilon_0)^{-1}\, e^2, \tag{4}$$

where m_e is the mass of the electron and e is the elementary positive charge. Here, we explicitly write down the vacuum permittivity $\not\epsilon_0$ as $4\pi\epsilon_0$ to ensure that its geometric content (the 4π factor in the EM term) is clearly noticeable.

Each side of Equation (4) becomes unitless when divided by $\hbar c$, as is done separately in the definitions of the two fundamental coupling constants $\alpha_{\hbar}$ and α_G. Unfortunately, the $\hbar$ introduces additional geometry into the gravitational part and eliminates geometry from the EM part, clearly altering the original geometrical characteristics of the two coupling constants (see Section 2.1.2 below). The unit of $[\mathrm{rad}]^{-1}$ has been dropped from $\hbar$ by international agreement, so this intrusion of geometry is no longer visible [11]; going as far back as Schrödinger [19], our community is under the impression that it is only a pure numerical factor of 2π which has been absorbed in the definition of $\hbar$. Not keeping track of pure geometric factors, such as the 2π in Schrödinger's equation and the Bohr model of the atom (or the 4π in the electric field), was, in hindsight, a miscue that set us back during the past 100 years [4].

2.1.1. Constants Imposed by the Vacuum

In order to dictate the speed of EM waves, it also sets an upper limit to the motion of material objects possessing mass. The vacuum does that incidentally by providing the smallest possible natural resistance to any kind of motion. The magnitude of c is set by the G-M of two inverse properties of the vacuum [13], viz.

$$c = \sqrt{\not\epsilon_0^{\,-1} \not\mu_0^{\,-1}}\,. \tag{5}$$

The SI value of c is $c = 2.9979 \times 10^8$ m s^{-1}, and its dimensions are $[\mathrm{length}][\mathrm{time}]^{-1}$ [23].

In Equation (5), the reduced values of vacuum permittivity and vacuum permeability combine in a way that removes geometric constraints from this speed (see also Note 1); the maximum permitted velocity of a combined EM wave or a massive object must be the same in any direction. In contrast, it is understood that a static electric field in a vacuum must adjust to the geometric constraint imposed by $\not\epsilon_0^{\,-1}$, and this is why the vacuum's inverse permittivity appears in Coulomb's law. In fact, $\not\epsilon_0^{\,-1}$ is the slope between the electric field $\mathcal{E}$ and the charge surface density e/r^2 [1], where r represents distance (see also Appendix B for the role that various surface densities play in disjoint parts of physics).

Since Equation (5) can be written in the equivalent form

$$c = \sqrt{\epsilon_0^{-1} \mu_0^{-1}}\,, \tag{6}$$

We can surmise that the physical quantities ϵ_0 and μ_0 are geometry-free. Indeed, after some manipulations involving the geometry-free fine-structure constant $\alpha_{\hbar}$ (as this was inadvertently defined long ago using Dirac's $\hbar$), we find that

$$\epsilon_0^{-1} = c\left(\frac{2h\alpha_{\hbar}}{e^2}\right), \tag{7}$$

and

$$\mu_0^{-1} = c\left(\frac{e^2}{2h\alpha_{\hbar}}\right). \tag{8}$$

There is no geometric influence on the right-hand sides of these equations. The quantity that is inverted from one equation to the other, $h/e^2 = \mu_0 c/(2\alpha_{\hbar})$, is proportional to the impedance of free space $Z_0 = \sqrt{\mu_0/\epsilon_0} = 376.730\ \Omega$, which is the G-M of μ_0 and ϵ_0^{-1}; thus, h/e^2 has dimensions of [ohmic resistance] (see Ref. [24] and Note 22 below). Thus, ϵ_0^{-1} and μ_0^{-1} can effectively be expressed as G-Ms involving the squares of c and Z_0 (i.e., $\sqrt{c^2 Z_0^2}$ and $\sqrt{c^2(1/Z_0^2)}$, respectively); the first G-M involves a direct multiplication of the two constants involved, whereas the second G-M uses the Lie-type inversion of one of the two constants [13]. We will pick up this important inference in again Section 4.2 below.

2.1.2. Dirac's Constant $\hbar = h/(2\pi)$

Dirac's constant is the slope between the energy E carried by a single photon and its angular frequency ω, viz.

$$E = \hbar\,\omega\,. \tag{9}$$

Its SI value is $\hbar = 1.0546 \times 10^{-34}$ J s rad^{-1} [11,23], and its dimensions are [action][rad]$^{-1}$ or, equivalently, [moment of inertia][second]$^{-1}$[rad]$^{-1}$. Here, the physical unit is J s, while the descriptive unit [rad]$^{-1}$ alerts us to the presence of 2-D geometry.

This new awareness that inertia is built into $\hbar$ (and Planck's h) may be the spark we need to theorize that the weak equivalence principle [25] is embedded into the microcosm as well, where gravity is not important. Action integrals [26], in particular, may be viewed as carrying the physical units of [moment of inertia][second]$^{-1}$, thus each action is a measure of the rate of change of moment of inertia at all scales of the universe, large and small.

In the spirit of Equations (7) and (8), Planck's reduced constant may also be split into a product of two G-Ms, viz.

$$\hbar \;=\; \sqrt{\hbar(\not\epsilon_0 c)}\,\sqrt{\hbar\left(\frac{1}{\not\epsilon_0 c}\right)} \;=\; e\left(\frac{\hbar}{e}\right); \tag{10}$$

the first G-M (=e) on the right-hand side is geometry-independent; the next G-M (=$\hbar/e$) is influenced by 2-D geometry since it is directly proportional to

$$\sqrt{\hbar(\not\epsilon_0^{-1})} \propto \sqrt{(4\pi^2)^{-1}(2\epsilon_0)^{-1}} \propto (2\pi)^{-1}.$$

This G-M that reduces to $(\hbar/e)$ has dimensions of [magnetic flux] = [magnetic field][area][5]. It is understood from the G-M decomposition (10) that the vacuum quantity $\not\epsilon_0 c = 4\pi/Z_0$ can couple to $\hbar$ and thus influence quantum phenomena, and it does so in the definition of the fine-structure constant (Section 2.2.1).

2.1.3. Newton's Gravitational Constant G

Newton's gravitational constant G is the slope between the gravitational field $a(r)$ (i.e., acceleration) and the surface mass density $\sigma(r) \equiv M(r)/r^2$ [1] on the surface of a sphere of radius r enclosing a total mass of $M(r)$, viz.

$$a(r) = G\,\sigma(r)\,. \tag{11}$$

Its SI value is $G = 6.67430 \times 10^{-11}$ m^3 kg^{-1} s^{-2} [23], with dimensions of [acceleration] [surface density]$^{-1}$ [1].

In the spirit of Equation (10), Newton's gravitational constant can also be split into a product of two G-Ms, viz.

$$G \;=\; \sqrt{G\,M^2}\,\sqrt{G\left(\frac{1}{M^2}\right)} \;=\; \sqrt{\hbar c}\left(\frac{G}{\sqrt{\hbar c}}\right), \tag{12}$$

which shows the potential of mass M to couple to G, and thus influence gravitation. The M^2 term does that in the definition of the gravitational coupling constant (Section 2.2.2). According to Equation (4), the first G-M reduces to $e/\sqrt{\not\epsilon_0} = \sqrt{\hbar c}$, and the second G-M reduces to $G(\sqrt{\not\epsilon_0}/e) = G/\sqrt{\hbar c}$, as shown in the second equality of Equation (12).

2.2. Dimensionless Units

We now come to the operations and properties of the unitless coupling constants $\{\alpha_G, \alpha_\hbar\}$. We show how these units rectify the Planck system of units and make it functional over all scales of the universe, including atomic and subatomic scales as well.

2.2.1. Fine-Structure Constant $\alpha_\hbar$

The fine-structure constant has been defined as

$$\alpha_\hbar = \frac{e^2}{\not\epsilon_0 \hbar c} \,. \tag{13}$$

Its value has been measured [27] to be very close to $(137.036)^{-1}$ (or $\alpha_h = (861.022576)^{-1}$ for the wiser choice $h \to \hbar$ in the definition). Other than that, $\alpha_\hbar$ brings no geometry and no units into the system of units. In particular, the geometry embedded in the electric field (and carried on by $\not\epsilon_0$) has been inadvertently eliminated by the insertion of $\hbar$ in the modern definition (13).

Nevertheless, definition (13) provides a powerful tool (Section 2.2.3), which we have not taken advantage of in the past: being a measurable constant, $\alpha_\hbar$ may serve as the reference UPS unit against which we can quantify all the other unitless coupling constants. For instance, the gravitational coupling constant α_G, which we describe next, acquires a quantitative meaning by comparison to $\alpha_\hbar$ in the ratio $(\alpha_G/\alpha_\hbar)$; most importantly, $\hbar$ and its artificial dependence on geometry drop out of this comparative ratio, which is another strong hint that $\hbar$ should not have replaced h in Equation (13).

2.2.2. Gravitational Coupling Constant α_G

Using G and the electron mass m_e, the gravitational coupling constant has been defined as

$$\alpha_G = \frac{G m_e^2}{\hbar c} \,. \tag{14}$$

Its value is 1.7518×10^{-45} (or $\alpha_G = 2.7881 \times 10^{-46}$, the geometry-free value obtained for $h \to \hbar$ in the definition), as determined by calculation.

Comparing the definitions (13) and (14), we see that α_G is, unfortunately, geometry-dependent. This problem did not exist during Max Planck's heydays, when h was in use and $\hbar$ did not exist. In general, the problem with the modern definitions of constants and variables is that $\hbar$ necessarily introduces 2-D geometry and a [rad] measure, in addition to the intended physical constant h. We must pronounce this Dirac's miscue [7,8].

The geometry dependence so artificially inserted in α_G will be taken out entirely in the calculations that follow. We must emphasize up front that reinstating the physical nature of α_G (and $\alpha_\hbar$) is necessary for the successful repair of the modern Planck system, and it leads to the determination of natural scales of mass, length, and charge for the chosen mass-to-charge ratio (m_e/e) of the electron (Section 2.2.3) or any other chosen particle for that matter (see Note 10 for details).

2.2.3. Relative Strength of Gravitational Coupling β_G

Leaving aside the descriptive [rad] unit in the above coupling constants, we come now to the only known method of actively using such dimensionless (pure) numbers. Being pure numbers, these constants have absolutely no meaning or practical use, but they are useful in ratios, in which their strengths are compared against other dimensionless constants; in such comparisons, these ratios acquire quantitative meaning, and then their relative strengths are, for all practical purposes, measurements of the same stature and importance as dimensional quantities (which, incidentally, are also measured by comparisons to international standards). One unitless coupling constant should, however, be included in the system in absolute terms in order to provide the reference value for the comparisons [6].

For the UPS, we choose α_h (let $h \to \hbar$ in Equation (13); Ref. [28]) for this duty because it has been measured by experiment [27], and its physical meaning has now become clear (see Section 3.1 and Table A1 below): the factor $\sqrt{\alpha_h} \simeq 1/30$ is a fundamental scale used by the Higgs field to couple to the bottom quark, and then on to all lower particle masses.

In this study, we assume that the coupling constants do not vary at the low energies (<246.22 GeV; Ref. [29]) of the subatomic particles. Furthermore, we calculate the UPS unit

$$\beta_G \equiv \frac{\alpha_G}{\alpha_\hbar} = \frac{\not{\epsilon}_0 G m_e{}^2}{e^2} = 2.4006 \times 10^{-43}\,, \tag{15}$$

a pure comparative number that is independent of $\hbar$ and h (and c, for that matter—as would be expected, the vacuum does not at all contribute to such ratios of forces). This "measurement" of β_G represents the strength of gravitational coupling relative to that of the EM coupling obtained from electrons (see Note 10 for considering protons instead, and constructing another UPS with different scales, but with the same elementary particles).

Compared to the relation between units shown in Equation (4), β_G carries a lot more weight because it can be used in quantitative calculations (although it was Equation (4) that gave us a reason to define β_G). The numerical value obtained in Equation (15) does not tell us that gravity is weak and the EM force is strong [7]; it only tells us about the relative couplings of these forces in the particular system of units that includes β_G as a building block. Gravity is attractive and has always had a chance to grow past the other forces in extraordinarily massive [8]. settings ($M \gg m_e$)—something that is actively occurring in many places in the present universe. In contrast, the Coulomb force cannot do the same because its attraction brings together charges of opposite signs that cancel each other out [30].

Furthermore, Equation (15) does not tell us that, in the distant past, gravity could have been much stronger in the atomic world, and it got weaker going forward in time [8,9] because of the expansion of the universe. The gravitational force has always been weak in the atomic world because the characteristic atomic masses are too small (much smaller than the Planck mass $M_p = 3 \times 10^{19}$ GeV/c^2). Therefore, instead of Dirac's "large numbers hypothesis", a safer assumption is probably that the gravitational constant G does not vary in time or that some meaningful physical reason must be found to the contrary rather than relying on pure speculation [1,8,9].

2.3. Determining a New Atomic Mass Scale

Definitions (13) and (14) have both incorporated $\hbar$, thus the coupling constants have been defined in the microcosm. Here, we use the above results to establish a new atomic mass scale after correcting for the unintended insertion of planar geometry into the coupling constants.

Looking at Equation (14), we see two problems that need to be addressed: (a) Despite the apparent lack of units (not entirely true, since $\hbar$ also carries radians), α_G is not influenced by EM coupling (there is no e in the definition, only mass m_e, and the two long-range forces are not linked to one another, although they are in the real world). (b) The unfortunate use of $\hbar$ has had the unforeseen consequence of arbitrarily adding spurious geometry into the dimensionless mix [9].

We can solve both problems by adopting Equation (15) to help us define a new atomic mass scale M_A in the UPS. The relative ratio β_G carries both forces, and the composite unit $\hbar$, which was not appropriate in the first place, has been eliminated (correcting thus a century-old oversight). One unavoidable conclusion is that the geometry of the vacuum (the $\not{\epsilon}_0$) is still present in β_G. This comes from the geometric dependence of the electric field, which will now overtly influence the new mass scale $M_{\rm A}$ (see also Note 9 and Section 5 below).

Based on these considerations, we return to Equation (14), and we rewrite this definition by making the following substitutions: $\beta_G \to \alpha_G$, $h \to \hbar$, and $M_{\rm A} \to m_{\rm e}$. We find a new equation, viz.

$$\beta_G = GM_{\rm A}^2/(hc)\,, \tag{16}$$

in which both sides are dimensionless comparative ratios, and substituting for β_G from Equation (15), we obtain the new atomic mass scale

$$M_{\rm A} = \sqrt{\epsilon_0 hc}\left(\frac{m_{\rm e}}{e}\right) = 2.6730 \times 10^{-29}\ {\rm kg}\,. \tag{17}$$

We reiterate here the ingredients that form the physical basis for this mass: (i) the unitless ratio β_G in Equation (15) has no dependence on $\hbar$, or h, or c; (ii) the substitution $h \to \hbar$ produces a truly unitless Equation (16); there are no loose radians in this equation, covertly suppressed by SI conventional practices (although the descriptive unit "cycle" [11] has indeed been suppressed in h, since it does not signify insertion of geometry); and (iii) the ratio of electric charges $e/\sqrt{\epsilon_0 hc} = 1/30$ is the same deflation factor described above (e.g., in Note 8) and in Section 3.1 below.

It is quite interesting that only the ratio $(m_{\rm e}/e)$ of the characteristic parameters of the electron ends up being a building block of the new mass scale $M_{\rm A}$. The reciprocal ratio, i.e., $e/m_{\rm e} = 1.7588 \times 10^{11}\ {\rm C\,kg^{-1}}$, was first measured by J. J. Thomson [31], years before the electronic charge itself was finally measured by experiment (see also Section 4.2).

The presence of ϵ_0 (coupled to h, as shown in the Lie-type G-Ms given in Section 2.1.2) in the new mass scale $M_{\rm A}$ is necessary (the vacuum's ϵ_0 is a building block of the electrostatic field); after some algebraic manipulations, we recast Equation (17) (or Equation (16)) to the equivalent form

$$M_{\rm A} = \sqrt{\left(\frac{hc}{G}\right)\left(\frac{E_{\rm grav}}{E_{\rm elec}}\right)}\,, \tag{18}$$

where the comparative ratio of energies,

$$\frac{E_{\rm grav}}{E_{\rm elec}} \equiv \beta_G\,,$$

was determined from the corresponding forces acting between two interacting electrons [10]. In dimensional analysis, this ratio is 1 (see Equation (4) above), but here, β_G plays an important quantitative role: the unitless factor

$$\sqrt{\beta_G} = 4.900 \times 10^{-22}, \tag{19}$$

scales the original Planck mass [5] ($M_{\rm p} = \sqrt{hc/G} = 5.4555 \times 10^{-8}$ kg) down to the atomic world. This scaling is a significant result of our work, as it connects the original Planck mass scale with the $M_{\rm A}$ scale of the atomic world [11], viz.

$$M_{\rm A} = M_{\rm p}\sqrt{\beta_G}\,. \tag{20}$$

We note that $M_{\rm A}$ and $M_{\rm p}$ are mass scales related by this equation; as such, they do not correspond to any actual particle or object in nature (see also Section 4.3).

We return now to the complete absence of Dirac's $\hbar$ from Equations (15)–(18). The only geometric dependence entering these equations is that which is imposed by the vacuum on the electrostatic field (hence, $M_{\rm A} \propto \sqrt{\epsilon_0} \propto 2\sqrt{\pi}$). The $\sqrt{\pi}$ does not carry angular units since $E_{\rm elec} \sim e^2/\epsilon_0 \sim$ [Joule] in Equation (18)—just like the $1/\pi$ in the Bohr radius and the factor of $1/4$ in the Rydberg energy (see the analysis following Equation (1) in Section 1.2). Therefore, besides introducing the speed of light, the vacuum also manages to imprint $M_{\rm A}$ with a unitless, purely numerical constant [12] (see also Note 5).

3. Results within the UPS Realm

3.1. Subatomic Masses

This new UPS mass scale (17) corresponds to a value of

$$M_{\rm A} = 15.0\ {\rm MeV}/c^2, \tag{21}$$

thus, it lands near the subatomic world of the low-mass up-and-down quarks, with corresponding masses $m_{\rm u} = 2.16\ {\rm MeV}/c^2$ and $m_{\rm d} = 4.67\ {\rm MeV}/c^2$ [32]; and it is smaller than the G-M defined for the hydrogen atom

$$\sqrt{m_{\rm e} m_{\rm p}} = 21.9\ {\rm MeV}/c^2, \tag{22}$$

where $m_{\rm p}$ is the proton mass.

The new mass scale $M_{\rm A}$ appears to be important for the Standard Model of particle physics, and it should be investigated further theoretically (there is no elementary particle corresponding to this energy). So far, we have derived the following empirical relations (sufficient to lead us to a clear physical interpretation of Koide's enigmatic constant and other constants in the Standard Model; for details, see Appendix A):

(1) The mismatch between $M_{\rm A}$ and $\sqrt{m_{\rm e} m_{\rm p}}$ may be related to Koide's K-constant $K = 2/3$ [15], viz.

$$M_{\rm A}/\sqrt{m_{\rm e} m_{\rm p}} = 0.6850\,, \tag{23}$$

connecting thus the masses of leptons to the atomic constants $M_{\rm A}$ and $m_{\rm p}$.

(2) Using the above values of first-generation quark masses and the mass of the strange quark, $m_{\rm s} = 93.4\ {\rm MeV}/c^2$ [32], we find that

$$\sqrt{m_{\rm u} m_{\rm s}}/M_{\rm A} \simeq 0.95\,, \tag{24}$$

and

$$\sqrt{m_{\rm d} m_{\rm s}}/\sqrt{m_{\rm e} m_{\rm p}} \simeq 0.95\,, \tag{25}$$

showing only a 5% deviation of both quark G-Ms from the two atomic mass constants. The results indicate that the mass of the second-generation strange quark is connected to both $M_{\rm A}$ and the masses of the first-generation quarks. Thus, a connection should exist for the charm quark too,[13] and so on for the third generation of quarks as well.

(3) It certainly appears that there exists a ladder-type mechanism that uses G-Ms (and some scaling coefficients) to relate various particle masses (see also Table A1 in Appendix A below). Some examples (and their corresponding deviations from experiment) are:

$$m_{\rm s} = \sqrt{m_{\rm d} m_\tau} \quad (2.5\%)\,, \tag{26}$$

where $m_\tau = 1.777\ {\rm GeV}/c^2$ is the tauon mass;

$$m_{\rm s} = \sqrt{m_{\rm u} m_{\rm b}} \quad (1.7\%)\,; \tag{27}$$

$$m_{\rm c} = \sqrt{m_{\rm p} m_\tau} \quad (1.7\%)\,; \tag{28}$$

$$m_{\rm c} = \sqrt{2 m_{\rm d} m_{\rm t}} \quad (0.054\%)\,, \tag{29}$$

where $m_{\rm t} = 172.5\ {\rm GeV}/c^2$ is the top quark mass;

$$m_{\rm u} = \sqrt{2 m_{\rm d} m_{\rm e}} \quad (1.1\%)\,; \tag{30}$$

$$m_{\rm p} = \sqrt{2 m_\mu m_{\rm b}} \quad (0.17\%)\,, \tag{31}$$

where $m_\mu = 105.66\ \mathrm{MeV}/c^2$ is the muon mass;

$$m_\mathrm{b} = \sqrt{m_\mu m_\mathrm{t}} \quad (2.1\%)\,; \tag{32}$$

and

$$M_\mathrm{A} = \sqrt{m_\mu m_\mathrm{u}} \quad (0.71\%)\,, \tag{33}$$

$$M_\mathrm{A} = \sqrt{\sqrt{m_\mathrm{e} m_\mu}\,\sqrt{m_\mathrm{e} m_\tau}} \quad (0.80\%)\,. \tag{34}$$

(4) The Higgs boson ($m_\mathrm{H} = 125.25\ \mathrm{GeV}/c^2$) is certainly special, although unavoidably a part of the mass ladder. This is the only particle that is not involved in simple G-Ms with the low-mass particles. Two of its complex relations are the following:

$$m_\mathrm{b} = \sqrt{m_\mathrm{s}\,(m_\mathrm{H}/K)} \quad (0.21\%)\,, \tag{35}$$

where $K = 2/3$ [15]; and

$$\frac{m_\mathrm{H}}{m_\mathrm{b}} = 30.0 \simeq \frac{M_\mathrm{A}}{m_\mathrm{e}} \quad (2.0\%)\,. \tag{36}$$

This relation shows how the Higgs boson manages to assign mass to the much lower-mass bottom quark by using a novel mechanism not related to a G-M or Koide's scale factor (see below).

(5) The vacuum expectation value (VEV) of the Higgs field is $v = 246.22\ \mathrm{GeV}/c^2$ [29]. To within a deviation of 1.8%, we find for the compact [14] triplet H-t-v that

$$m_\mathrm{t} = \sqrt{m_\mathrm{H}\, v}\,, \tag{37}$$

which shows exactly where the most massive quark is located at the top of the mass ladder. Furthermore, the Higgs mass is the G-M of the top quark mass and the mass of the Z^0 boson $m_{\mathrm{Z}^0} = 91.1876\ \mathrm{GeV}/c^2$ (a deviation of only 0.13%), viz.

$$m_\mathrm{H} = \sqrt{m_\mathrm{t}\, m_{\mathrm{Z}^0}}\,. \tag{38}$$

Obviously, the top quark receives its mass from the Higgs mechanism, and then it participates in the G-Ms that define the masses of the other particles (see Table A1 in Appendix A). The high-mass geometric sequence Z^0-H-t-v appears to be very compact indeed (Note 14), and its common ratio is about 1.38[15]. We note that $\mathrm{W}^\pm$ (mass $m_{\mathrm{W}^\pm} = 80.377\ \mathrm{GeV}/c^2$) is not a member of this sequence since $m_{\mathrm{Z}^0}/m_{\mathrm{W}^\pm} = K^{-1/4} \simeq 1.11$[16]. This relation provides another definition of Koide's K in terms of the decay products of the Higgs boson (deviation 2.5%), viz.

$$K^{1/4} = \frac{m_{\mathrm{W}^\pm}}{m_{\mathrm{Z}^0}} \equiv \cos\theta_\mathrm{w}, \tag{39}$$

where θ_w is the Weinberg angle [16,20] (deviation 2.8 degrees; see also Appendix A.3.1).

(6) On the other hand, the G-M of m_H and $m_{\mathrm{W}^\pm}$ is 10% larger than m_{Z^0}, but using empirically Koide's constant, we find that

$$m_{\mathrm{Z}^0} = \sqrt{\left(K^{1/2}\, m_\mathrm{H}\right) m_{\mathrm{W}^\pm}}\,, \tag{40}$$

an important relation with a deviation of the G-M from the measured m_{Z^0} value of only 0.57%. Furthermore, the relation $m_{\mathrm{W}^\pm} = \sqrt{(K\, m_\mathrm{H})\, m_{\mathrm{W}^\pm}}$ also appears to hold (1.9% deviation), which then implies that

$$m_{\mathrm{W}^\pm} = K m_\mathrm{H}\,. \tag{41}$$

This relation helps us understand the important role of the exact constant $K = 2/3$ [15]: K is a numerical scale factor that relates some close pairs of particle masses. Here, the Higgs field connects to Z^0 by an inverse-mapping G-M[17], viz.

$$m_{Z^0} = \sqrt{m_H^3\,(1/v)}\,,$$

and to $W^\pm$ by the simple scale factor K, as seen in Equation (41). In hindsight, the Higgs mechanism could not assign two different (but comparable) masses to Z^0 and $W^\pm$, both by using G-M averages, so it used two different couplings involving K and $1/v$, respectively.

(7) Returning now to Equation (35), we see the Higgs mass is scaled by $1/K$ to participate in a G-M with m_s and m_b. Although we have only a limited view of the dynamics of the Higgs mechanism in the above equations, it is apparent that this mechanism uses a set of scaling rules in the various coupling factors that appear in the Lagrangians. The origin of these scaling rules is unknown to us at this moment, but we feel confident that we have made a step in the right direction with this analysis (see Appendix A.3 for calculations of the free parameters of the Standard Model of particle physics).

(8) The next and considerably more difficult step concerns the assignment of mass to the bottom quark, whose mass is much lower than the Higgs mass and the masses of its decay products. We were surprised to find yet another method being used by the Higgs mechanism for this coupling (no G-M can reach down to m_b because the barrier set by the Higgs VEV is not too high): the only way that we could find for this coupling was the deflation factor of $1/30$, which we discuss below.

Notice the unitless factor of 30.0 in Equation (36). This equation suggests that the mass scale M_A and the electron mass m_e are related to the mass ratio m_H/m_b. However, m_e is not a mass scale and M_A is not a particle mass, so the proportion in Equation (36) involving the ratio M_A/m_e would be at least obscure if it were not for similar mass and charge ratios presented in Note 8 and in item (iii) following Equation (17). Using Equation (59) derived below and the equations in Section 2.3, we can rewrite proportion (36) in a physical form, viz.

$$\frac{m_b}{m_H} = \frac{1}{30} \simeq \sqrt{\alpha_h} \quad (2.2\%)\,, \tag{42}$$

where $\alpha_h = (861.022576)^{-1}$ is given by Equation (13) after the corrective substitution $h \to \hbar$ that restores Planck's constant h [5,6] in the definition of the fine-structure constant.

Thus, the mass of the bottom quark m_b, which is 30 times lower than m_H, is determined self-consistently from this scaling equation by effectively using the ratio of scales M_p/M_A in the intermediate steps and the Planckian fine-structure constant [18] $\alpha_h = e^2/(\not{\epsilon}_0 hc)$ in the final step. This is the third method employed by the Higgs boson to couple with other particles. In particular, it uses this $\sqrt{\alpha_h} \simeq 1/30$ scaling to get down to the bottom quark and, then, into the regime of the lower particle masses (see Table A1 below). If the m_b coupling also involves the W^- boson (which carries Koide's scale factor K) to deliver charge to the bottom quark, then Equations (41) and (42) combine to show that $m_b = (m_{W^-})(\sqrt{\alpha_h}/K) \simeq 0.05(m_{W^-})$. The physical significance of Koide's scale for the high-mass quarks (c, b, t) and the vector bosons is discussed in detail in Appendix A.2.

3.2. The Planck Charge

The Planck charge q_p is a prime example of the state of confusion in the field: not understanding the meddling of geometry in the modern Planck units, people adopted different definitions of q_p by arbitrarily choosing between $\not{\epsilon}_0$ and ϵ_0 and between $\hbar$ and h. In the end, this unit, along with the Planck units of magnetic flux $[\hbar/(\not{\epsilon}_0 c)]^{1/2}$ and ohmic resistance $(\not{\epsilon}_0 c)^{-1}$, fell out of favor [19].

Now, we know better. The definition of the Planck charge $q_{\rm P}$ must be geometry-free, viz.

$$q_{\rm P} \equiv \sqrt{\not{\epsilon}_0 \not{h} c} \; = \; \sqrt{2\epsilon_0 h c}\,. \tag{43}$$

Absence of geometry is required, first because this is a unit of charge, and second because $q_{\rm P}$ provides an alternative definition of the fine-structure constant (which is geometry-independent in its current definition (13)), viz.

$$\alpha_{\not{h}} \; = \; \left(\frac{e}{q_{\rm P}}\right)^2. \tag{44}$$

We find that $q_{\rm P} = 1.8755 \times 10^{-18}$ C $= 11.7062e$ (where $11.7062 = \sqrt{137.036}$). Once again, nature shows us here her principle of fairness (or impartiality). As in the case of the electron mass $m_{\rm e}$, the elementary charge e here is not related to the fundamental unit of charge $q_{\rm P}$ by a rational numerical factor; instead, $q_{\rm P}$ is chosen as the UPS scale of charge, a scale that does not correspond to a charge multiple of any specific particle or field.

3.3. A New Atomic Length Scale

Equation (1) can help us determine a new length scale for the UPS, a scale that certainly does not correspond to any of the three atomic radii in Equation (1); based on nature's apparent principle of fairness, we understand that none of the known electronic radii can be the fundamental unit of length. We know that scale values generally fall between particle values and vice versa. To proceed, we use the G-M of $r_{\rm e}$ and $r_{\rm c}$ to determine a new atomic length scale, $L_{\rm A}$ [20].

The G-M of $r_{\rm e}$ and $r_{\rm c}$ gives

$$L_{\rm A} = r_{\rm c}\sqrt{\alpha_{\not{h}}} \; = \; \sqrt{\frac{\not{h}}{\not{\epsilon}_0 c^3}}\left(\frac{e}{m_{\rm e}}\right), \tag{45}$$

and $L_{\rm A} = 3.2987 \times 10^{-14}$ m $= r_{\rm c}/11.7062$. The numerical value 11.7062 is the same as that found for the ratio $q_{\rm P}/e$ (Equation (44)) because

$$\alpha_{\not{h}} = (L_{\rm A}/r_{\rm c})^2, \tag{46}$$

and then the following proportion (cross-multiplied) holds exactly:

$$L_{\rm A} q_{\rm P} \; = \; r_{\rm c} e\,. \tag{47}$$

This relation implies that the G-M of the new scales $L_{\rm A}$ and $q_{\rm P}$ is equal to the G-M of the traditional and widely-used electronic constants $r_{\rm c}$ and e, and it brings to light a previously unused combination of units with dimensions of [length][charge]. These dimensions are equivalent to

$$\frac{[\text{momentum flux}]}{[\text{magnetic flux}]} = \frac{[\text{momentum}]}{[\text{magnetic field}]} = \frac{[\text{energy}]}{[\text{electric field}]}\,;$$

these interesting units compare mass flows ("matter waves") to EM waves ("energy flows") and energy/momentum to EM field components. These quotients also indicate a close correspondence between the relativistic energy-momentum (E-p) equation

$$E = c\,p\,, \tag{48}$$

and Maxwell's EM amplitudes ($\mathcal{E}_0$, $\mathcal{B}_0$; [36]) relation

$$\mathcal{E}_0 = c\mathcal{B}_0\,. \tag{49}$$

The above dimensional ratios of units are obtained easily by dividing these two equations. We see then that $\mathcal{B}_0$ (current flow) is to EM waves what momentum p (mass flow) is to dynamics, and similarly for amplitude $\mathcal{E}_0$ and energy E.

Length $L_{\rm A}$ is much larger than the modern Planck length $L_{\rm p} = \sqrt{\hbar G/c^3} = 1.6163 \times 10^{-35}$ m. (The modern definition of $L_{\rm p}$ must be used here because $r_{\rm c}$ in Equation (45) brought its 2-D geometry into $L_{\rm A}$, and $\alpha_{\hbar}$ is accidentally geometry-free.) In this case, $L_{\rm A}$ must be scaled down to produce $L_{\rm p}$; thus, we find that $L_{\rm p} = L_{\rm A}\sqrt{\beta_G}$. This scaling-down of $L_{\rm A}$ should be contrasted to the scaling-up of $M_{\rm A}$ to produce the original Planck mass $M_{\rm p}$ (i.e., $M_{\rm p} = M_{\rm A}/\sqrt{\beta_G}$; see Section 2.3).

3.4. Cosmological Scales and Some Ambivalent Superatomic Particles

In Sections 2.3 and 3.3 above, we rescaled the fundamental scales of the UPS to obtain the corresponding Planck scales. These "A" and "p" values do not describe any specific particle or object in the universe. Now, we can extend both scales into the macrocosm by running the G-Ms toward larger masses and lengths.

(a) Cosmological Mass Scales. We evaluate a geometric progression that starts with scales $M_{\rm A}$ and $M_{\rm p}$ and moves on to larger mass scales:

$$\{M_{\rm B}, M_{\rm C}, M_{\rm D}\} = \{1.113 \times 10^{14},\ 2.271 \times 10^{35},\ 4.633 \times 10^{56}\}\ \text{kg}. \tag{50}$$

Mass scale $M_{\rm D}$ is 2–3 orders of magnitude larger than the current estimates of the mass of the universe [1], so we can halt the sequence at $M_{\rm D}$. The common ratio of the geometric progression is $M_{\rm p}/M_{\rm A} = 1/\sqrt{\beta_G} = 2.041 \times 10^{21}$. The G-M of $M_{\rm B}$ and $M_{\rm C}$ is equal to 0.84 earth masses, and the G-M of $M_{\rm C}$ and $M_{\rm D}$ is 5×10^{15} solar masses, which identifies universal structures much larger than individual galaxies (e.g., galaxy clusters).

(b) Cosmological Length Scales.—We evaluate a geometric progression that starts with scales $L_{\rm p}$ and $L_{\rm A}$ and moves on to longer length scales:

$$\{L_{\rm B}, L_{\rm C}\} = \{6.730 \times 10^{7},\ 1.373 \times 10^{29}\}\ \text{m}. \tag{51}$$

Length scale $L_{\rm C}$ is 2–3 orders of magnitude larger than the current estimates of the size of the universe [1], so we can halt the sequence at $L_{\rm C}$. The common ratio of this geometric progression is $L_{\rm A}/L_{\rm p} = 1/\sqrt{\beta_G} = 2.041 \times 10^{21}$, the same as the common mass ratio given in item (a) above. The G-M of $L_{\rm B}$ and $L_{\rm C}$ is equal to 98.5 parsecs, a value typical of giant molecular cloud complexes in spiral galaxies.

(c) Cosmic Microwave Background (CMB). We convert the temperature of the CMB, $T_{\rm CMB} = 2.7255$ K, to an equivalent mass, $m_{\rm CMB} = 3.52 \times 10^{-10}$ MeV$/c^2$ (see also Ref. [37]). Since $m_{\rm CMB} \ll M_{\rm A}$, we need to extend the geometric progression of mass scales to much lower masses as well. At the low-mass end of the geometric sequence $\{M_0, M_{\rm A}, M_{\rm p}\}$, the tiny mass scale M_0 is found to be $M_0 = 7.35 \times 10^{-21}$ MeV$/c^2$. Furthermore, the G-M relation

$$m_{\rm CMB} = \sqrt{M_0 M_{\rm A}}\,, \tag{52}$$

holds to within a 5.7% deviation between the two sides. This deviation is relatively small, given the enormous difference in scales (by 21.3 orders of magnitude) involved on the right-hand side of Equation (52).

(d) A Superatomic Particle Near the Planck Mass? The equivalent mass of the CMB photons is so low that, by extending the geometric sequence of $\{m_{\rm CMB} and m_{\rm H}\}$ to higher masses, we obtain a potential particle mass of $M_{\rm S} = m_{\rm H}^2/m_{\rm CMB} = 4.453 \times 10^{16}$ GeV$/c^2 \simeq 1.455 \times 10^{-3} M_{\rm p}$, which is at the scales where the strong force supposedly joins in with the other forces [38]. Since the Higgs mass is $m_{\rm H} = 125.25$ GeV$/c^2$, then the energy ratio $\sqrt{\beta_W}$ (analogous to $\sqrt{\beta_G}$ in Section 2.3) that scales the strong interaction down to the weak interaction is

$$\sqrt{\beta_W} = \sqrt{\frac{E_{\rm W}}{E_{\rm S}}} = \sqrt{\frac{m_{\rm H}}{M_{\rm S}}} = 5.30 \times 10^{-8}. \tag{53}$$

This value is smaller by a factor of 20 compared with the usually quoted coupling constant ratio of the weak to the strong interaction. One reason is that the quoted estimates of this ratio in particle physics depend on microphysics [38]; these values are not really constants since they show a secular dependence on particle energy [39,40]. In any case, it is doubtful that the Higgs field can assign masses above its VEV of 246.22 GeV/c^2 [41]; a phase transition from the Higgs VEV up to the mass $m = 10^{18}$ GeV/c^2 (Note 8) may be necessary, in which case there would be no particles in this mass range.

(e) Sub-TeV Particles? In the atomic world, the Higgs VEV appears to be a barrier against growing more massive nuclei and particles [21]. Nevertheless, researchers are searching the TeV scales in hopes of discovering such particles [43]. If there is a way to jump across the Higgs VEV (which we do not currently see; see also Ref. [41]), then the next few particle slots generated by the high-mass geometric progression Z^0-H-t-v... will have rest-mass energies of 0.351, 0.502, 0.716, and 1.022 TeV.

4. Discussion

4.1. Pairs of Fundamental Dimensional Units

Equation (46) shows that two lengths are needed to produce the fine-structure constant $\alpha_{\hbar\!\!\!/}$ in any system of units: the fundamental scale $L_{\rm A}$ and a Compton-type scale such as $r_{\rm c}$. This subsidiary scale cannot be defined by using the fundamental mass scale (then, one gets $\alpha_{\hbar\!\!\!/} = 1$). Therefore, Equation (46) defines $r_{\rm c}$ independently of mass $M_{\rm A}$. In our case, this definition is obtained easier from Equation (47): $r_{\rm c} = L_{\rm A}(q_{\rm p}/e)$. Using the definition of the fine-structure constant is an integral part of the above derivation of $r_{\rm c}$, and this example justifies our statement that all systems besides the UPS are incomplete, missing at least the unitless coupling constants, and thus incapable of describing all scales and forces in the universe.

Next, we consider Planck's original set of dimensional units $\{c, G, h\}$, with h in place of $\hbar\!\!\!/$ to avoid misunderstandings from the introduction of geometry into the units. The speed of light barrier is applicable to all systems of units, but h is not fundamental in the cosmological system and G is not fundamental in the atomic system for "obvious" (now obviously wrong) reasons: "negligibly weak influences should not be building blocks at the core of a system." We believe that all three constants are necessary building blocks and that the vacuum-force pairs $\{c, G\}$ and $\{c, h\}$ serve two different complementary functions within the UPS:

(a) The pair of constants $\{c, G\}$ with its universal unit of force [22]. $F_0 = c^4/G$, and the corresponding unit imprint of the famous Tully-Fisher/Faber-Jackson relation [17,18] $c^4 = GMa_0$ (where $F_0 = Ma_0$; [2–4]) was analyzed previously [1] within the cosmological system of units. (We discuss the universality of this relation in Appendix B.) Combined with Newton's G, powers of c define units whose purpose is to monitor the effectiveness of forces F in producing motion (speed V). Some of these units are very well-known: $c^2/G \sim F/V^2 = M/R_{\rm S}$, $c^3/G \sim F/V = Z_{\rm m}$, $c^4/G \sim F$, and $c^5/G \sim FV = P$. Here, M is mass, $R_{\rm S}$ is (Schwarzschild) radius, $Z_{\rm m}$ is mechanical impedance, and P is power.

(b) With the notable exceptions of $\sqrt{h(\epsilon_0 c)} \sim q$ (charge) and its Lie-type inversion $\sqrt{h/(\epsilon_0 c)} \sim \Phi_{\rm B}$ (magnetic flux) (Section 2.1.2), the pair of constants $\{c, h\}$ can only generate composite units, which cannot be viewed as fundamental units in the physical world, although these units do afford some interesting symmetries. For instance, examine the sequence of units $hc \to [E][L]$, $h \to [E][T]$, $h/c \to [M][L]$, and $h/c^2 \to [M][T]$, before the next powers of c generate some lower-level subsidiary units, e.g., $h/c^3 \to [M][a]^{-1}$. Combining powers of c with Planck's h, these units are designed to monitor the action integral $\mathcal{S}$ (i.e., energy integrated over time) during motion, although they are not as well-known: $h/c^3 \sim \mathcal{S}/V^3$, $h/c^2 \sim \mathcal{S}/V^2$, $h/c \sim \mathcal{S}/V$, $h \sim \mathcal{S}$, and $hc \sim \mathcal{S}V$. Since action $\mathcal{S}$ determines both speed V and acceleration a, this sequence of units can also be interpreted as: $h/c^3 \sim (E/V^2)/a = M/a$, $h/c^2 \sim (E/V)/a = p/a$, $h/c \sim E/a$, $h \sim (EV)/a$, and $hc \sim (EV^2)/a$, where E represents energy and p represents momentum.

The above symmetries are naturally propagated to derivative units. As a typical case, we discuss the sequence of composite units M/T^n (for integer n) generated by the widely-used pair of units of mass and time $\{M, T\}$, because this sequence holds some surprises. These units apparently measure resistive properties in the material world:

$$\begin{array}{llll} M/T & \sim F/(L/T) & = Z_{\rm m} & [\text{mechanical impedance}] \\ M/T^2 & \sim F/L & = S_{\rm m} & [\text{mechanical stiffness}] \\ M/T^3 & \sim F/(LT) & = \sigma_P & [\text{power}][\text{area}]^{-1} \end{array} , \tag{54}$$

where L represents length and subscript P represents power. It is surprising that the unit M/T (of the ubiquitous "mdot" in accretion physics) turns out to be a resistive property of inflowing matter. It is also quite surprising that the "power surface density" σ_P is a member of this sequence of units that describe the various types of mechanical resistance. In Appendix B, we find that power surface density is a universal dynamical quantity, although it appears prominently only in the Stefan–Boltzmann law [44,45]. Its resistive character becomes apparent when we rewrite it in terms of force F and moment of inertia I, viz.

$$\sigma_P = F^2/(I/T)\,, \tag{55}$$

where (I/T) represents resistance due to the rate of change of the moment of inertia. In this equation, we recognize the importance of the force squared F^2 in $\sigma_P \sim M/T^3$. Coming full circle to expressing the resistances in terms of F^2, we find for the impedance and the stiffness that $Z_{\rm m} = F^2/P$ and $S_{\rm m} = F^2/E$, respectively, where E represents energy. Therefore, the magnitude of F^2 appears to be regulated by power in impedance, by energy in stiffness, and by inertial changes in power surface density.

Furthermore, the inertial magnitude itself appears in the next term of the sequence (54), i.e., $M/T^4 \sim F^2/I$, and the integrated quantity (IT) appears next in $M/T^5 \sim F^2/(IT)$. Obviously, then, the units of the sequence M/T^n describe resistive properties in which F^2 is regulated by the temporal variations of inertia according to the formula $M/T^n = F^2/(I/T^{4-n})$[23].

4.2. The Varied Contributions of the Vacuum

The free space known as the vacuum is described by four interdependent constants (ϵ_0, μ_0, $c = 1/\sqrt{\epsilon_0\mu_0}$, $Z_0 = \sqrt{\mu_0/\epsilon_0}$). When the vacuum wishes to also affix geometry in some parts of the natural world, then it introduces either $\not\epsilon_0 \equiv 4\pi\epsilon_0$ or $\not\mu_0 \equiv \mu_0/(4\pi)$ or both, provided they are not introduced in a product (there is no geometry in $\not\epsilon_0\not\mu_0 = 1/c^2$).

From the nongeometric vacuum quantities ϵ_0 and μ_0, only two additional purely physical quantities can be constructed by simple G-Ms: the speed of light c and the impedance of free space Z_0 (Section 2.1.1). They both represent upper limits [24] in nature, the only known upper limits communicated by the vacuum to all scales and in all directions within the universe. Their origin is the least (but nonzero) resistance that the vacuum mounts passively against all motions in the material world (see also Section 5 below).

Next, we wish to track down the geometry that is affixed selectively by the vacuum, so we rewrite the fundamental G-Ms discussed in Section 2.1.1 as follows:

$$\sqrt{\not\epsilon_0^{\,-1}\,\not\mu_0^{\,-1}} = c\,, \tag{56}$$

and

$$\sqrt{\not\epsilon_0^{\,-1}\left(\frac{1}{\not\mu_0^{\,-1}}\right)} = \frac{Z_0}{4\pi}\,. \tag{57}$$

The G-M (56) is clearly geometry-free, whereas G-M (57) attaches the 4π of 3-D space to the geometry-free impedance of free space Z_0. This is an important conclusion: when $\not\epsilon_0^{\,-1}$ or $\not\mu_0^{\,-1}$ appear in equations, or they both appear in a combination other than their product (56), then they carry 3-D geometry with them. These composite vacuum constants show us how free space manages to interfere in the construction and evolution of additional (ready-to-interact with one another) physical entities, such as mass and electric charge, that characterise the underlying force fields.

We emphasize here that mass and charge are not actually fundamental quantities, as is widely believed; they can only be derived and clearly understood if the contributions of the vacuum and the unitless coupling constants are also taken into account. We demonstrate this point here, with exact calculations:

(a) Consider, first, Equation (13). Solving for the charge e, we obtain a scaled-down G-M relation of the form

$$e \;=\; \alpha_{\hbar}^{1/2}\,\sqrt{h(2\epsilon_0 c)} \;=\; \alpha_{\hbar}^{1/2} q_{\rm p}\,. \tag{58}$$

Therefore, Planck's physical constant h and the vacuum's combination of $(2\epsilon_0 c)$ determine e as a geometry-free, G-M quantity. From this point of view, we can also see how dimensionless constants resize properties of the material world: this G-M is scaled down by the geometry-free factor $\alpha_{\hbar}^{1/2} \simeq 1/\sqrt{137} \simeq 1/11.7062$ (see also Section 3.2).

(b) Consider, next, Equation (14). Solving for the mass $m_{\rm e}$, we obtain a G-M relation of the form

$$m_{\rm e} \;=\; \left(\frac{\alpha_G}{2\pi}\right)^{1/2} \sqrt{\left(\frac{h}{G}\right)c} \;=\; \left(\frac{\alpha_G}{2\pi}\right)^{1/2} M_{\rm p}\,. \tag{59}$$

In this case, $m_{\rm e}$ is determined by the G-M of the composite physical constant h/G and the vacuum's c. (G participates because a mass is determined here.) The G-M is scaled down by a factor of $[\alpha_G/(2\pi)]^{1/2} = 1.670 \times 10^{-23}$ relative to $M_{\rm p}$. Due to the inclusion of 2π, this factor is geometry-free, and so is $m_{\rm e}$ (since the original Planck mass $M_{\rm p}$ is also geometry-free).

(c) By dividing Equations (58) and (59) and neglecting for the moment the dimensionless, geometry-free factor $(4\pi/\beta_G)^{1/2} = 7.235 \times 10^{21}$, we obtain a geometry-independent G-M for the electron's charge-to-mass ratio, viz.

$$\frac{e}{m_{\rm e}} \;\propto\; \sqrt{\epsilon_0 G}\,. \tag{60}$$

Thus, the ratio $e/m_{\rm e}$ is determined by the G-M of the nongeometric constants ϵ_0 and G (vacuum and gravity, respectively), and the neglected scale factor carries the relative strength of the two unitless coupling constants ($\sqrt{4\pi/\beta_G} = \sqrt{4\pi\,\alpha_{\hbar}/\alpha_G}$) with the geometry due to the electrostatic field eliminated by the 4π term.

4.3. Geometric-Mean Averaging and Particle-Mass Deflation in Nature

We think we understand why virtually all pairs of constants and units (U_1, U_2) combine in G-Ms[25], involving the direct form $U_1 U_2$ or the inversion form $U_1 U_2^{-1}$ (or $U_1^{-1} U_2$). Physically, two basic (lowest-power) G-M quantities can be derived from each pair of units. Mathematically, these two operations result in mappings that are always "smooth" since they involve constants; thus, the units of a system of units always form a Lie group [13], and the associated Lie algebra can be carried out with ease.

One remaining question is why there are also square roots on top of the basic unit combinations, thus establishing G-Ms. We fall back to what is already known about G-Ms: compared with the commonly-used arithmetic means, G-Ms place significantly more weight on the smaller of the two values. Thus, the most obvious property of the geometric averages $\sqrt{U_1 U_2^{\pm 1}}$ is that they help smaller physical constants leave their indelible marks when they combine with larger constants. In a sense, by not letting small constants become negligible (or dominant) in combinations with large constants [26], nature seems to subscribe to a principle of fairness or impartiality at all scales of the universe. The degree of support for the small constants can be quite dramatic for much differing scales, as Equations (15) and (19) vividly demonstrate: the G-M $\sqrt{\beta_G}$ gains 21.3 orders of magnitude relative to the pure ratio β_G in connecting the Planck scale with the atomic world. Current thinking, on the other hand, seems to be at ease with the assumption that both β_G and $\sqrt{\beta_G}$ are practically zero on atomic scales. Comparing the two practices, we must now realize that nature is telling us that our assumption is wrong [27].

Consider next the subatomic particles discussed in Section 3.1. Nature did not make a particle in each individual G-M slot. The mass spectrum is mostly empty, and only a few actual particles have materialized on the subatomic scales of the universe [38]. Therefore, there are additional selection criteria (scaling rules) on top of the G-Ms that regulate the creation of particles. Besides the factors of 2 and $\sqrt{2}$ in the equations of Section 3.1, we have seen that the Higgs boson does not rely on pure G-Ms to reach down to lower masses; it uses, in addition, two different scale factors, Koide's $K = 2/3$ and $\sqrt{\alpha_h} \simeq 1/30$ (Equation (42)), to bypass many available particle slots (see also Table A1 below). In particular, the dramatic drop by 121 GeV$/c^2$ from the Higgs mass to the mass of the bottom quark can only be described as a deflation of particle mass that bypasses 10 G-M particle slots intervening between m_{Z^0} and m_b. In Appendix A.3, the deflation factor of 1/30 is identified with the coupling constant of the weak interactions, $\alpha_\mathrm{w} = g^2/(4\pi)$, where $g = 0.653$ is the weak isospin g-factor [38].

5. Lingering Issues, Future Prospects, and a Brief Summary

The UPS was summarized in Equation (3). The system is not flawless yet, and several issues must be investigated and resolved in the future (see, e.g., Note 28). These issues can be traced to Dirac's introduction of $\hbar = h/(2\pi)$ in place of Planck's h.

It is certainly true that in quantum mechanics, Dirac's composite constant $h/(2\pi)$ always appears in form, and this also prompted Schrödinger [11,19] to absorb the 2π into a convenient new constant K. This tactic tells us that Schrödinger was not aware that he was including geometry in his constant K. Dirac [7–9], on the other hand, believed that $\hbar =$ K is the true constant (not h), so we can guess that he sensed that the two constants are fundamentally different in their makeup (see Section 1.2 for more details).

Dirac's reform has modified quite substantially the systems of units that have adopted $\hbar$, but this modification came with a heavy price. Planck's purely physical constant h cannot be dropped so nimbly because then we introduce errors in the definitions of the coupling constants. Dimensionless coupling constants should not include geometric dependencies other than $\not{\epsilon}_0$ or $\not{\mu}_0$ (and these enter only via EM terms); geometry would give the constants an additional descriptive unit of [rad] and it would alter their nature. On the dimensional side of vacuum-asserted units, c and Z_0 (Section 4.2) are also geometry-free constants for a good reason: they represent upper limits set by the vacuum to be applicable in any direction of space, irrespective of the dimensionality of space.

We note another issue concerning $\hbar$: In the dimensional part of the UPS, the constant $\hbar$ is the only fundamental dimensional unit that introduces geometry in the physical units. This is an unusual and singular property. Although we were inclined to adopt Planck's h in place of $\hbar$, we did not do so because we did not know how to choose between the two constants. It seems from the calculations above that the use of h in the definitions of scales (Planck units, coupling constants) is mandatory, but then $\hbar$ may be more appropriate to be retained for particles and fields, as Dirac [7–9] also thought. Perhaps both constants should be retained in a modified UPS, along with α_h and β_G (see the UPS as described in Note 28).

Examining now the definitions of the dimensionless units that we summarized in Section 2.2 (Equations (13) and (14)), we see that $\alpha_{\hbar}$ is indeed geometry-free ($\not{\epsilon}_0\hbar = 2\epsilon_0 h$), but α_G is not ($\alpha_G \propto 1/\hbar \propto 2\pi$). We think this is an enormous oversight flying undercover, at least since Dirac [8] introduced his "large numbers hypothesis"; and it has prevented physicists from defining an atomic mass scale in the modern Planck system, thus creating an insurmountable obstacle to force unification. The state of confusion can best be seen in the widespread misconception "that G carries units into the action of general relativity, thus gravity is not like the other forces of nature", taught to thousands upon thousands of physics students for nearly a century. We now understand that gravity is just like the other forces, and it enters the "ring" with one dimensionless (α_G) and one dimensional (G) constant, just as the EM forces and the short-range forces do too.

Owing to the omnidirectional nature of the gravitational force, both of its constants should be geometry-free. For this reason, we tried to bypass the problem with the definition (14) of α_G (it effectively carries a descriptive unit of [rad], thus it cannot be utilized) and to define new consistent atomic units within the UPS. First, we created a dimensionless ratio $\beta_G = \alpha_G/\alpha_{\hbar}$ of the coupling constants that describes their relative strength; the $\hbar$ does not partake in this ratio, and the only geometric influence left comes from the EM field. However, this does not affect the makeup of the relative strength β_G, since β_G is expressed as a ratio of energies [28].

Next, we created a dimensionless geometry-free combination of fundamental units to attach to β_G, viz.

$$\beta_G = GM_{\rm A}^2/(hc) = \left(M_{\rm A}/M_{\rm p}\right)^2 ,$$

where $M_{\rm p}$ is the mass scale of the original Planck [5] system of units. Finally, the new atomic mass scale $M_{\rm A}$ was derived from the known values of β_G and $M_{\rm p}$, viz.

$$M_{\rm A} = M_{\rm p}\sqrt{\beta_G} .$$

The interpretation of this relation is straightforward: the ratio of the two widely different mass scales $M_{\rm A}/M_{\rm p} = 4.9 \times 10^{-22}$ is precisely equal to the square root of the relative ratio of the two coupling constants $\beta_G = \alpha_G/\alpha_{\hbar} = 2.4 \times 10^{-43}$.

In Section 3, we tested the influence of this mass scale in the atomic and subatomic worlds, and the results appear to be strong. The mass constant $M_{\rm A}$ has no trouble meddling in the G-Ms (Section 3.1) along with particle (sub)atomic masses that have been measured by experiment [23,32]; but see also Note 10 for UPS′, an alternative system of units based on the proton's parameters. We worked out elements of UPS′ to show that it does not matter which particle is chosen in the definitions of the various scale factors. In the process, we also clarified the confusion surrounding the so-called Planck charge (Section 3.2), and we also derived a new atomic length scale that had no trouble meshing in G-M calculations with the already-known atomic radii (Sections 1.2 and 3.3).

In Sections 3.1 and 3.4, we calculated both mass scales and actual particle masses at practically all scales of the universe. The Higgs mechanism uses a multitude of scalings and couplings to distribute masses to (sub)atomic particles. This diversity of methods is, in part, responsible for hindering progress in the effort to unify the four fundamental forces of nature. The other part concerns the role of the vacuum (Sections 2.1.1 and 4.2). The behavior of the vacuum is not at all what our books describe (e.g., [20,33,38]). As far as we can see, the vacuum is not subject to forcing of any kind, and it seems to be impervious to quantum fluctuations, which occur exclusively in fields. By and large, the vacuum appears to be a passive, independent entity with no intrinsic properties of its own that imposes implicitly certain rules (by resisting) on the material world that all inhabitants must necessarily observe and obey (to within the bounds of the uncertainty principle, of course; see also Appendix B.2). In these circumstances, there is no back reaction from the material world on to the vacuum itself. In hindsight, this conclusion makes sense—how can anything tangible manage to tangle up that which is the epitome of nothingness?

6. Highlights

6.1. Conclusions

(1) Current systems of units are incomplete and incapable of describing all aspects of this universe. They do not include some of the fundamental dimensional constants, the dimensionless coupling constants, and all the restrictions installed by the vacuum itself on the material world.

(2) Each force of nature must be represented in a system of units with a dimensional and a dimensionless coupling constant. If Planck's h is dropped, then the system cannot measure quantities related to quantum phenomena. If Newton's G is dropped,

then the system does not include gravity. The vacuum also comes in with any two of its four interdependent constants $\{c, Z_0, \epsilon_0, \mu_0\}$, and it inserts a stamp of the 3-D geometry of space to the electric charge (the 4π term in $e^2/\not{\epsilon}_0$), but not to the mass.

(3a) The fine-structure constant $\alpha_h = (861.022576)^{-1}$, not multiplied by 2π, is the only coupling constant that must be included in absolute terms. It has been measured by experiment, and it provides the scale factor $\sqrt{\alpha_h} \simeq 1/30$ used by the Higgs mechanism to deflate and couple to the bottom quark, and then to reach down to all the other lower-mass particles (Table A1 below). Furthermore, the Higgs mass is apparently related to the masses of vector bosons, quarks, and leptons by G-M averaging and Koide's scale of 2/3 in various incarnations. The above scales should be present in the coupling constants of the various fields (see Appendix A.3).

(3b) All other unitless constants must be included in relative terms because only ratios of coupling constants have physical meaning—such ratios provide relative strengths, just like the ratios of dimensional quantities do too.

(3c) The modern definitions of the unitless coupling constants are incorrect because $\hbar$ was used instead of Planck's physical constant h. Dirac's $\hbar$ is a composite constant that also carries planar 2-D geometry and a descriptive unit of $[\text{rad}]^{-1}$; the 2π term in $\hbar$ has inadvertently reversed the influence of geometry on the coupling constants.

(4) The vacuum is a passive entity impervious to forcing of any kind by the material world. By providing the least (but nonzero) resistance to all motions that occur in its domain, the vacuum installs upper limits on the material world (c and Z_0 in nearly perfect dielectrics), which must then be included in systems of units as well. These two geometry-free constants also bring the composite constants $4\pi\epsilon_0$ and $\mu_0/(4\pi)$ with them, in which the influence of 3-D geometry (the 4π term) is apparent. (Here, the vacuum's ϵ_0 and μ_0 are both lower limits.) In unit combinations, such as $4\pi\epsilon_0\hbar$ and $\mu_0/(4\pi\hbar)$, geometry inadvertently cancels out, leaving behind unitless numerical imprints in the equations (see the three atomic radii in Section 1.2).

(5a) There exists a new atomic mass scale $M_\text{A} = 15.0\ \text{MeV}/c^2$ that can be determined by deflating the original Planck mass M_p by $\sqrt{\beta_G} = 4.900 \times 10^{-22}$, where β_G is the relative ratio of the coupling constants of gravity and fine structure. Of course, in our expanding universe, the event took place in reverse ($M_\text{A}/\sqrt{\beta_G} \to M_\text{p}$). This inflation of scale accounted for 21.3 orders of magnitude in mass and explains how the Planck scale is connected to the atomic world. (At the same time, the atomic scale of length was deflated by the same factor to produce the tiny Planck length.)

(5b) No (sub)atomic particle is found to occupy a scale value, and the measured masses in the atomic world are connected mostly by G-M averaging. By using G-M averaging, nature (a) remains impartial to designating any particle as being more significant than any other, and (b) assigns more weight to the smaller participant in the G-M, thereby assisting smaller entities in leaving their marks on the universe.

(5c) We can relate characteristic atomic constants (charge e, mass m_e, the G-M $\sqrt{m_\text{e} m_\text{p}}$, the Compton radius r_c) to scale values ($q_\text{p}, M_\text{p}, M_\text{A}, L_\text{A}$, respectively), but this is not how these physical entities were created; they were created by the Higgs scalings (1/30 and 2/3) and by G-M averaging of other nearby physical entities.

(6) Leptons, quarks, and bosons get their masses from the Higgs field. The boson-quark-lepton mass ladder is shown in Table A1 below. How the Higgs field acquires its mass m_H and its vacuum expectation value v remains a mystery; the only hints in the known masses [32] are that $m_\text{H} \simeq v/2$ (to within a deviation of 1.7%) and the G-M $m_\text{t} = \sqrt{m_\text{H}\, v}$ (Equation (37), deviation 1.8%). The EM fine-structure constant α_h ought to play a prominent role as well: in Appendix A.3, we find that it is likely related to the coupling constant of weak interaction α_w, viz. $\alpha_\text{w} = \sqrt{\alpha_h} \simeq 1/30$.

(7) Koide's lepton constant $K = 2/3$ is one of the scaling constants used by the Higgs field and its decay products in couplings to other particles. We derived it from the first principles in Appendix A, and the same value is also applicable to the heavy quark triplet c-b-t. We also derived two additional Koide-type constants: $J = 4/7$ (for the

light quark triplet u-d-s) and $B = 0.336$ (for the vector bosons $W^{\pm}$-Z^0-H). Constant B is barely 0.8% larger than the absolute minimum value of 1/3 that occurs for three equal masses.

(8) In Appendix B, we pointed out four instances of a universal law that has the general form

$$\text{(a surface density)} \propto \text{(a kinetic scalar quantity)}^4\,,$$

in which the power of 4 is the sum of the 3 spatial degrees of freedom and 1 additional degree of freedom for the scale of the underlying scalar quantity. The three types of surface density involved describe force F, power P, and moment of inertia I, all divided by surface area A. Pressure F/A appears in the Higgs field and the Casimir effect; intensity P/A appears in the Stefan–Boltzmann law; and mass I/A appears in the Tully–Fisher/Faber–Jackson relation in spiral/elliptical galaxies. It certainly appears that the dynamics of the present universe are driven by the surface densities of various fundamental quantities (see also Appendix B.2).

6.2. Critical Questions and Answers

(Q1) *How does Planck mass relate to the atomic world?*
—The atomic mass scale $M_A = 15$ MeV$/c^2$ inflates precisely to the Planck mass, i.e., $M_A/\sqrt{\beta_G} \to M_p$, where ratio $\sqrt{\beta_G} = 4.900 \times 10^{-22}$ is a comparative dimensionless quantity (i.e., the ratio of two dimensionless constants).

(Q2) *What is the physical meaning of the number 137?*
Number $137 = 861/(2\pi)$, where the 2π is a geometric term carrying the descriptive unit of radian [11]; so, 137 is a composite constant, and this is the reason that we did not figure out its physical significance in the past 100 years. The actual physical constant is 861, and the scale factor $\sqrt{861} \simeq 30$ is used by the Higgs boson to assign masses dynamically to much lighter particles, starting with the bottom quark and moving on down the mass ladder (Table A1). Thus, the "deflation scale" $\sqrt{\alpha_h} = 1/30$ (or weak coupling constant α_w) should appear in the Higgs couplings of the lower-mass particles.

(Q3) *What is the physical meaning of Koide's constant?*
Koide's $K = 2/3$ is another scale factor used in the Higgs couplings to assign masses to lighter vector bosons (the particles $W^{\pm}$ and Z^0). Koide's formula holds exactly for the leptons e-μ-τ and for the heavy quarks c-b-t (corresponding proofs are given in Appendix A.2).

(Q4) *How does the top quark get its mass?*
The top quark mass is the geometric mean of the Higgs mass and the Higgs vacuum expectation value $v = 246.22$ GeV$/c^2$, so that $m_t = \sqrt{m_H v}$ to within a deviation of 1.8% from the experimentally measured m_t value [32].

(Q5) *How do Higgs vector bosons get their masses?*
By two different mechanisms (couplings): In the ordered compact high-mass triplet, Z^0-H-t, the Higgs mass is the geometric mean of the Z^0 mass and the top quark mass, viz. $m_{Z^0} = m_H^2/m_t$. In contrast, the $W^{\pm}$ coupling involves Koide's scale since $W^{\pm} = K m_H$, where $K = 2/3$.

(Q6) *How does the bottom quark get its mass?*
By a third coupling mechanism: The Higgs mass is scaled down by the weak coupling constant $\alpha_w = 1/30$, so that $m_b = m_H/30$. We have tried empirically several other scalings and G-Ms, but none of these patterns approached the experimental value of m_b, which is much lower than m_H (the rest-energy gap is 121 GeV; Table A1). The assignment of mass to the bottom quark is becoming a major issue to resolve in the future by any theory that purports to describe mass assignments to lower-mass quarks.

(Q7) *Is Dirac's $\hbar$, rather than Planck's h, the true universal constant?*
They both are, but h is a pure physical constant, whereas $\hbar = h/(2\pi)$ is composite and includes also the 2-D geometric term 2π and the descriptive unit of [rad]. Due to this geometric content, a miscue was committed in the post-Planckian era when

$\hbar$ was adopted for the modern definitions of the fine-structure constant $\alpha_{\hbar}$ and the gravitational coupling constant α_G: the 3-D geometry of the electric field in $\alpha_{\hbar}$ was eliminated, and α_G acquired 2-D geometry against its generic 3-D nature.

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Abbreviations

The following abbreviations are used in this manuscript:

EM	Electromagnetic
G-M	Geometric-Mean
MOND	Modified Newtonian Dynamics
UPS	An Upgraded Planck System Based on Electron Mass
UPS′	An Upgraded Planck System Based on Proton Mass
VEV	Vacuum Expectation Value

Appendix A. Physical Meaning of Koide's Lepton Constant and Similar Constants

Koide's constant of 2/3 involves the three lepton masses, and it is a puzzle in particle physics [15]. This constant is not a mere numerical coincidence; it is a fundamental scale that the Higgs boson uses to create the $W^{\pm}$ bosons, and then it is propagated to lower quark masses according to the empirical relations given in Section 3.1 (and in Table A1 below).

Koide's constant is defined for the three leptons e-μ-τ as

$$(m_e + m_\mu + m_\tau)/\left(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}\right)^2 \equiv 2/3\,. \tag{A1}$$

Mathematically, this relation is equivalent to an equation involving arithmetic means and G-Ms of paired quantities, viz.

$$f \equiv (m_e + m_\mu + m_\tau)/\left(\sqrt{m_e m_\mu} + \sqrt{m_e m_\tau} + \sqrt{m_\mu m_\tau}\right) = 4\,, \tag{A2}$$

where the numerator should be viewed as the triple sum of arithmetic means of paired quantities, viz.

$$m_e + m_\mu + m_\tau = \frac{1}{2}(m_e + m_\mu) + \frac{1}{2}(m_e + m_\tau) + \frac{1}{2}(m_\mu + m_\tau)\,.$$

Equation (A2) reveals a special relationship between the three arithmetic means in the numerator and the three G-Ms in the denominator: the ratio of their averages f must be equal to 4; then, Equation (A1) is an identity, as Koide [15] discovered. This special relationship is recovered from the equations in Section 3.1. We carried out several reductions [29] by evaluating lower masses starting from the Higgs mass and the masses of $m_{W^{\pm}}$, m_{Z^0}, and m_t. The endpoint is a pair of inversion G-M relations between lepton masses, viz.

$$\sqrt{m_\mu/m_e} = 22K\,, \tag{A3}$$

and

$$\sqrt{m_\tau/m_e} = 40K^{-1}\,. \tag{A4}$$

Their product is independent of K, and its value (880) carries an error of 4% as compared with the experimental value of 848. The coefficients in these ratios were rounded off to obtain a 2% accuracy in each individual ratio (not rounded; the coefficient in Equation (A4) has a deviation of 5%, sufficiently large to produce a higher K-value of 0.70). Naturally, the coefficients in Equations (A3) and (A4) are approximations to the factors 21 and 41, where $21 \times 41 = 861$. Thus, the deviations in the experimental product (848) and in the theoretical product (880) from 861 are -1.5% and $+2.2\%$, respectively. These comparisons validate Equations (A3) and (A4) and point to their physical significance; analogous equations cannot be written for Dirac's $\hbar$-based model because 137 is a prime number.

Substituting the above ratios into Equation (A2), we find a fourth-order polynomial equation of the form

$$484K^4 - 88K^3 - 3519K^2 - 160K + 1600 = 0\,, \tag{A5}$$

The solutions of the fourth-order equation can be obtained analytically, and they are all real. The two positive roots are

$$K_1 = 0.66641 \quad \text{and} \quad K_2 = 2.7283\,. \tag{A6}$$

Root K_1 is Koide's constant; it determines the lepton mass ratios, and they, in turn, satisfy Equation (A1) to within an error of 0.04% (Koide's $K = 2/3$ is an extremely robust physical constant, as experimenters have discovered).

Root K_2 is rejected based on the experimental results. Substituting K_2 into the equations of the lepton mass ratios reverses the two values, resulting in $m_\mu > m_\tau$. This solution is obviously incompatible with the measured masses of these two particles [32]. Despite the rejection, K_2 is of some theoretical interest: the ratio $K_2/K_1 = 4$, the same 4 that appears in Equation (A2). Furthermore, we see that the $f = 4$ factor in Equation (A2) is the raw physical constant, and Koide's K is derived from it:

$$K = \frac{f}{f+2} = \frac{2}{3}\,. \tag{A7}$$

This relation is derived by substituting Equation (A2) into Equation (A1).

Appendix A.1. Physical Interpretation

The physical interpretation of nature's choice of $f = 4$ is deduced from Equation (A2), rewritten in the accessible form

$$\frac{1}{3}(m_e + m_\mu + m_\tau) = \frac{4}{3}\left(\sqrt{m_e m_\mu} + \sqrt{m_e m_\tau} + \sqrt{m_\mu m_\tau}\right). \tag{A8}$$

The factor of $1/3$ indicates that the left-hand side is the arithmetic mean of the lepton masses. The factor of $4/3$ on the right-hand side (also the G-M $\sqrt{K_1 K_2}$ to within 1%) is C_F, the quadratic Casimir charge of the SU(3) fundamental representation [33]. It seems then that the assignment of masses to the leptons is also constrained by the delivery of charge, and this is why

$$f = 3C_F\,, \tag{A9}$$

appears in the right-hand side of Equation (A2). This last equation is a special case of the general formula (Section 4.5 in Ref. [33]) of quantum chromodynamics, viz.

$$\frac{N_A}{2} = NC_F\,, \tag{A10}$$

as applied to SU(3), where $N = 3$ dimensions, the indices $1, 2, \cdots, N_A$ label the $N_A \times N_A$ color generators in the "octet" quark-antiquark state, $N_A = N^2 - 1 = 8$, and $f = N_A/2$; thus, we find that $f = 4$ and $C_F = 4/3$ in SU(3). Finally, Koide's constant turns out to depend only on the SU(3) octet number N_A, viz.

$$K = \frac{N_A}{N_A + 4}, \tag{A11}$$

or, equivalently, on the dimensionality $N = 3$ of space, i.e., $K = (N^2 - 1)/(N^2 + 3)$, in which case $f = (N^2 - 1)/2$ as well. This result does not support higher-dimensional theories of space, such as strings and their variants. For as long as $K = 2/3$, space appears to be three-dimensional, and there are no additional hidden dimensions (such as those described in Ref. [50] and many other similar textbooks).

Table A1. Boson-Quark mass ladder in terms of the Higgs mass $m_{\rm H} = 125.25$ GeV/c^2 [32]. Two scales are used, Koide's $K = 2/3$ and the $\alpha_{\rm w} = \sqrt{\alpha_h} = 1/30$ deflation of $m_{\rm H}$ down to the bottom quark mass $m_{\rm b}$. Lepton masses and proton mass are also shown in terms of $m_{\rm H}$ for a comparison of scales.

Particle	Mass-Energy (MeV)	Mass Relation $^{(a)}$	Deviation $^{(b)}$ (%)
VECTOR BOSONS			
Z^0	9.1188×10^4	$m_{Z^0} = K^{3/4} m_{\rm H}$	+1.3
$W^{\pm}$	8.0377×10^4	$m_{W^{\pm}} = K m_{\rm H}$	+3.9
QUARKS			
top	1.725×10^5	$m_{\rm t} = K^{-3/4} m_{\rm H}$	−1.6
bottom	4.180×10^3	$m_{\rm b} = m_{\rm H}/30 = \alpha_{\rm w} m_{\rm H}$	−0.12
charm	1.270×10^3	$m_{\rm c} = 2\sqrt{K/30^3}\, m_{\rm H}$	−2.0
strange	93.4	$m_{\rm s} = (K/30^2)\, m_{\rm H}$	−0.67
down	4.67	$m_{\rm d} = 2\left(K^{7/4}/30^3\right) m_{\rm H}$	−2.3
up	2.16	$m_{\rm u} = (K^2/30^3)\, m_{\rm H}$	−4.5
LEPTONS & PROTON			
electron	0.511	$m_{\rm e} = \sqrt{\frac{K}{2\times 30^7}}\, m_{\rm H}$	−4.3
muon	105.66	$m_{\mu} = 22^2 \frac{K^{5/2}}{\sqrt{2\times 30^7}}\, m_{\rm H}$	−0.45
tauon	1.777×10^3	$m_{\tau} = 40^2 \frac{K^{-3/2}}{\sqrt{2\times 30^7}}\, m_{\rm H}$	−0.93
&			
proton	938.272	$m_{\rm p} = 22\left(\frac{2K^5}{30^9}\right)^{1/4} m_{\rm H}$	−0.12

Notes: $^{(a)}$ The top three masses do not depend on the deflation scale $\sqrt{\alpha_h} = 1/30$, whereas the mass of the bottom quark $m_{\rm b}$ is the only one that does not depend on Koide's scale $K = 2/3$. $^{(b)}$ Deviation = [(right-to-left side) −1] × 100%; left side is taken from Ref. [32].

Appendix A.2. Additional Koide-Type Constants

The G-M relations in Section 3.1 may help us make physical sense of various other combinations involving three particle masses. Here, we summarize the calculations for three such triplets, the quarks c-b-t and u-d-s [51], and the bosons $W^{\pm}$-Z^0-H:

Heavy quarks c-b-t. Based on experimental masses, Equation (A1) with c-b-t values in place of e-μ-τ values produces a constant of 0.669 on the right-hand side, only 0.35% higher than $K = 2/3$. This is a solid physical result. From the equations of Section 3.1, we find that $m_{\rm t} = 30 m_{\rm b} K^{-3/4}$, $m_{\rm c} = 2 m_{\rm b}\sqrt{K/30}$, and a corresponding constant of 0.668 with a deviation of 0.20% from $K = 2/3$.

Light quarks u-d-s. Koide's K is not produced by the masses of the u-d-s triplet. Based on their experimental masses, Equation (A1) with u-d-s in place of e-μ-τ produces the constant $J = 0.567$ on the right-hand side, probably a value of no interest to numerology.

We, on the other hand, have derived this constant analytically by utilizing the G-M relations of Section 3.1 and by expressing the u-d-s quark masses in terms of the Higgs scales $m_{\rm b}/m_{\rm H} = 1/30$ (deflation) and $K = 2/3$ (Koide), used in assignments of masses of light quarks. It turns out that the entire boson-quark mass ladder has to be calculated in the process. The results of our calculations are listed in Table A1. Using the values obtained for the u-d-s masses at the bottom of the quark mass ladder, we find that

$$(m_{\rm u} + m_{\rm d} + m_{\rm s})/(\sqrt{m_{\rm u}} + \sqrt{m_{\rm d}} + \sqrt{m_{\rm s}})^2 = 0.570\,, \tag{A12}$$

a constant that deviates only by 0.53% from the experimental value of $J = 0.567$. In this case, we find that $0.570 \approx 4/7$, $f = 2C_F = N_A/N = 8/3$, and $J = N_A/(N_A + 2N) = 4/7$ in SU(3) (in place of Equations (A9)–(A11)).

Higgs bosons $W^{\pm}$-Z^0-H.—Based on experimental masses, Equation (A1) with $W^{\pm}$-Z^0-H masses in place of e-μ-τ masses produces a constant of $B = 0.336$ on the right-hand side, only 0.80% higher than the lowest attainable value of 1/3 obtained in the generic case of three equal masses. From Equations (40) and (41), we find that $m_{Z^0} = K^{3/4} m_{\rm H}$ and $m_{W^{\pm}} = K m_{\rm H}$ (see also Table A1), and then, Equation (A1) for the $W^{\pm}$-Z^0-H triplet is transformed to

$$1 + K^{3/4} + K = 0.336\left(1 + K^{3/8} + K^{1/2}\right)^2, \tag{A13}$$

with an accepted root at $K_1 = 0.662$ (deviation 0.70% from $K = 2/3$) and a rejected root at $K_2 = 1.541$. We conclude that K_1 is Koide's constant, in which case, $f = 1.012$ on the right-hand side of Equation (A2) and $B = f/(f + 2)$ (let $B \to K$ in Equation (A7)). The quark-antiquark color octet number N_A and the quadratic Casimir charge C_F are not involved in these calculations (Equations (A9)–(A11) are not applicable to vector bosons).

Appendix A.3. Determination of Various Coupling Factors and Constants of the Standard Model

Although approximate, the empirical equations for the mass ladder listed in Table A1 constitute a scaling model, and we can use them to calculate theoretical values for various constants of the Standard Model, thereby eliminating many of the free gauge couplings and particle masses from the model (currently being considered out of reach and expected to be measured by experiment). We describe the determinations of several such constants below, and we collect the results in Table A2 for comparisons with the corresponding experimental measurements.

Appendix A.3.1. The Weinberg Angle

The weak mixing angle, or Weinberg angle $\theta_{\rm w}$, is usually defined by [16,20]

$$\cos\theta_{\rm w} \equiv \frac{m_{W^{\pm}}}{m_{Z^0}}\,. \tag{A14}$$

Its experimental value is 0.492 radians. Substituting the boson masses of the mass ladder (Table A1), we find that

$$\cos\theta_{\rm w} = K^{1/4}\,, \tag{A15}$$

that gives the theoretical value of 0.443 radians (effectively 2.8 degrees smaller). CODATA [23] tabulates the value of $\sin^2\theta_{\rm w} = 0.223$, whereas our $\theta_{\rm w} = 0.443$ gives $\sin^2\theta_{\rm w} = 0.184$. The Weinberg angle appears in several of the equations that follow.

Appendix A.3.2. Vector-Boson *g*-Factors and Electric Charge

We determine first an equation for the Higgs VEV: using Equation (37) and the top-quark relation in the mass ladder (Table A1), we find that

$$v = K^{-3/2}\, m_{\rm H}\,. \tag{A16}$$

This relation gives the ratio $m_{\rm H}/v = 0.54$, whereas its experimental value is 0.51 [23].

The masses of the vector bosons are given by [16,20]

$$m_{W^\pm} = g\,(v/2)\,, \tag{A17}$$

and

$$m_{Z^0} = \sqrt{g^2 + g'^2}\,(v/2)\,, \tag{A18}$$

where g is the SU(2) weak isospin coupling and g' is the U(1) weak hypercharge coupling.

Substituting the boson masses of the mass ladder and the VEV of Equation (A16), we find that

$$g = 2\,K^{5/2}\,, \tag{A19}$$

and

$$g' = 2\,K^{9/4}\sqrt{1-\sqrt{K}}\,. \tag{A20}$$

These relations give $g = 0.726$ and $g' = 0.344$. The experimental values are 0.653 and 0.350, respectively. The calculated g-factor shows a larger deviation (+11%) because of the relatively large (+3.9%) deviation of the calculated ratio $m_{W^\pm}/m_H$ in Table A1.

The electric charge in particle physics is defined in natural units as [30]

$$e \equiv \sqrt{4\pi\alpha_{\hbar\!\!/}} = 0.303\,. \tag{A21}$$

This is a constant of the EM interaction obtained in the Standard Model from the equation [16,20]

$$e = g\,\sin\theta_w = g'\,\cos\theta_w = 0.308\,. \tag{A22}$$

Using the equations of the mass ladder, we find that

$$e = 2\,K^{5/2}\sqrt{1-\sqrt{K}} = 0.311\,. \tag{A23}$$

Just as the constant of EM interaction e is defined in terms of the fine-structure constant $\alpha_{\hbar\!\!/}$ in Equation (A21), so is the weak isospin g-factor in terms of the intrinsic strength of the weak interaction α_w [38], viz.

$$g \equiv \sqrt{4\pi\alpha_w} = 0.653\,, \tag{A24}$$

where the value of g was calculated from the reduced Fermi constant G_F^0 [23] and the mass of the W boson [32], i.e., $g = 2\,m_W(\sqrt{2}\,G_F^0)^{1/2} = 2\,m_W/v$, where, in the Standard Model, $v = (\sqrt{2}\,G_F^0)^{-1/2} = 246.22$ GeV. From the second equality in Equation (A24), we get

$$\alpha_w = 0.0339 \simeq \frac{1}{30}\,, \tag{A25}$$

which is effectively the deflation scale $\sqrt{\alpha_\hbar}$ that we also found for $m_\star/M_p$ in Note 8 and for m_e/M_A in Equation (36) above. Therefore, the deflation scale appears to be the strength of the weak interaction (i.e., the weak coupling constant α_w). Furthermore, these coincidences involving the scale 1/30 suggest that a particle of mass $m_\star = M_p/30 \simeq 1.8 \times 10^{-9}$ kg could exist at the unification scale of 10^{18} GeV (i.e., 30 times below the Planck mass), just as the electron does at 30 times below the new subatomic scale of 15 MeV (Equation (21)).

Table A2. Free parameters of the Standard Model: Comparison of empirical values with the corresponding experimental measurements. Details and references are provided in Appendix A.3.

Parameter	Equation	Determined Value	Measured Value
Weingberg angle (θ_W) [degrees]	(A15)	25.4	28.2
Higgs mass to VEV ratio (m_H/v)	(A16)	0.54	0.51
Weak isospin coupling (g)	(A19)	0.726	0.653
Weak hypercharge coupling (g')	(A20)	0.344	0.350
Electric charge (e) [natural units]	(A23)	0.311	0.308
Weak coupling constant (α_W)	(42), (A24)	0.0341	0.0339
Electron Yukawa coupling (y_e)	(A27)	3.005×10^{-6}	2.935×10^{-6}
t-quark Yukawa coupling (y_t)	(A28)	1.043	0.991
u-quark coupling factor (g_u)	(A31)	1.267×10^{-5}	1.241×10^{-5}
d-quark coupling factor (g_d)	(A32)	2.805×10^{-5}	2.682×10^{-5}
b-quark coupling factor (g_b)	(A34)	2.566×10^{-2}	2.401×10^{-2}

Appendix A.3.3. Electron and Top-Quark Yukawa Coupling Factors

The Higgs Yukawa coupling to the electron y_e [20,52] determines the mass of the electron, viz.

$$m_e = y_e\left(v/\sqrt{2}\right). \tag{A26}$$

Its experimental value is $y_e = 2.935 \times 10^{-6}$. Using the mass relation for the electron (Table A1) and Equation (A16) for VEV, we find that

$$y_e = \frac{K^2}{30^{7/2}} = 3.005 \times 10^{-6}. \tag{A27}$$

On the opposite end of the mass spectrum, the Higgs Yukawa coupling to the top quark, $y_t = \sqrt{2}(m_t/v) = 0.991$ [20,32], is the largest y-factor and one of the main targets of ongoing experiments [53,54]. Using the mass relation for the top quark (Table A1) and Equation (A16) for VEV, we find that

$$y_t = \sqrt{2}K^{3/4} = 1.043, \tag{A28}$$

independent of the deflation scale of the weak interaction $1/30$ (Equation (A25)), which explains its large numerical value.

Appendix A.3.4. Quark g-Factors

In the Glashow–Weinberg–Salam theory of electroweak interactions [20], the lowest-mass quarks (u and d) have masses

$$m_u = g_u\left(v/\sqrt{2}\right), \tag{A29}$$

and

$$m_d = g_d\left(v/\sqrt{2}\right), \tag{A30}$$

where g_u and g_d are the coupling constants of the corresponding Higgs interactions with experimental values of $g_u = 1.241 \times 10^{-5}$ and $g_d = 2.682 \times 10^{-5}$. Using the corresponding mass relations (Table A1) and Equation (A16), we find that

$$g_u = \frac{\sqrt{2}K^{7/2}}{30^3} = 1.267 \times 10^{-5}, \tag{A31}$$

and

$$g_d = \frac{2\sqrt{2}K^{13/4}}{30^3} = 2.805 \times 10^{-5}. \tag{A32}$$

Finally, we use the coupling equation for the mass of the bottom quark, viz.

$$m_{\rm b} = g_{\rm b}\left(v/\sqrt{2}\right), \tag{A33}$$

where $g_{\rm b}$ is the coupling constant of the Higgs interaction with an experimental value of $g_{\rm b} = 2.401 \times 10^{-2}$. Using the mass relation for the bottom quark (Table A1) and Equation (A16), we find that

$$g_{\rm b} = \frac{\sqrt{2}K^{3/2}}{30} = 2.566 \times 10^{-2}. \tag{A34}$$

Appendix B. A Universal Natural Law Discovered in Widely Separated Scales

The work that we presented in this paper was triggered by the realization that the unit of force $F_0 = c^4/G$ is precisely the same in the cosmological and the Planck systems of units. Furthermore, this unit displays the unusual form [speed]^4/[Newton's G] also found in the Tully–Fisher/Faber–Jackson relation [17,18] discovered in spiral/elliptical galaxies, respectively (Section 4.1). This is astonishing, given the enormous difference in scales between the two systems of units.

The implication is that similar fundamental relations between variables ought to exist on the Planck scale and on the microcosmic scale as well. Indeed, the first such relation dates back to Stefan [44], who discovered the famous [temperature]^4 dependence of an emitting blackbody's intensity (or "power surface density") with units [power][area]$^{-1}$. We searched and found that such a universal law has been discovered in all of the above scales, but comparisons were not previously made because the corresponding subfields of physics have always been disjoint.

The universal law involves the surface densities of various fundamental quantities. (Such surface-density dynamical variables have become of primary importance in the work presented in Ref. [1]). These surface densities are all related to the fourth power of kinetic terms (such as speed and temperature), which are limited by the various essential resistances imposed by the vacuum. Specifically:

(1) In quantum gravity, the energy-density shift of the Higgs field $U_{\rm H}$ resulting from spontaneous symmetry breaking (that prevents ultraviolet divergence) is $U_{\rm H} \propto v^4$, where v is the Higgs vacuum expectation value [20,55]. This relation is equivalent to

$$\sigma_F \propto v^4, \tag{A35}$$

where the "force surface density" $\sigma_F \equiv F/A$ (where F is force, A is area, and $U_{\rm H} = F/A$ has dimensions of [pressure]).

(2) In the macroscopic realization of the Casimir effect, the same force per unit area is proportional to the fourth power of the reciprocal of distance D between parallel plates [56,57], viz.

$$\sigma_F \propto (1/D)^4. \tag{A36}$$

The units agree in the last two relations since the VEV v has dimensions of [length]$^{-1}$ in natural units (see Note 30 and Ref. [20]).

(3) In atomic physics, the celebrated Stefan-Boltzmann law [44,45] takes the equivalent form

$$\sigma_P \propto \Theta^4, \tag{A37}$$

where the power surface density (or intensity) $\sigma_P \equiv P/A$, P is power, and Θ is the mean temperature of the source of radiation.

(4) In astrophysics, galaxies obey the relation $M \propto V^4$, where M is mass and V is rotational speed or stellar velocity dispersion in spiral [17,58–60] and elliptical [18,61,62] galaxies, respectively. This relation is equivalent to

$$\sigma_I \propto V^4 \,, \tag{A38}$$

where the "moment-of-inertia surface density" $\sigma_I \equiv I/A$ (where I is the moment of inertia and I/A has dimensions of [mass]).

Appendix B.1. Dimensional Analysis of Surface Densities

Dimensional analysis can help us understand the physics of these surface densities, but not by reducing their definitions to the fundamental units of the UPS. We have to search a little deeper to find common properties. We begin with the power surface density (wave intensity) σ_P that assumes the simplest form among the surface densities and has a resistive character (Equation (55) in Section 4.1):

$$\sigma_P = \frac{F^2}{(I/T)} \,. \tag{A39}$$

In EM interactions, the rate of change of moment of inertia can be replaced by

$$I/T = q^2 Z_0 \,, \tag{A40}$$

where q is charge and $Z_0 = \sqrt{\mu_0/\epsilon_0}$ is vacuum impedance; we find that

$$\sigma_P = Z_0^{-1} \mathcal{E}^2 \,, \tag{A41}$$

where the electric field is given by $\mathcal{E} = F/q$. In these equations, the terms (I/T) and Z_0 express vacuum resistance to forces and fields.

For gravitational power, the force in Equation (A39) is also contains vacuum resistance that couples to Newton's G: rewriting Equation (A39) in terms of the gravitational field (i.e., acceleration) a, we find that

$$\sigma_P = \left(\frac{c}{G}\right) a^2 \,. \tag{A42}$$

In units, force is $F = P/c$ in terms of power P, and the force surface density σ_F takes the corresponding forms

$$\sigma_F = \frac{\sigma_P}{c} = \epsilon_0 \mathcal{E}^2 = G^{-1} a^2 \,, \tag{A43}$$

where the vacuum (c) drops out from gravity's σ_F. This is a fundamental difference as compared with the EM field's σ_F, in which the vacuum (ϵ_0) is permanently attached to the electric field.

Finally, as was probably expected, the moment-of-inertia surface density $\sigma_I = M$ does not quite conform to the above picture. We find that

$$\sigma_I = \frac{F^2}{(I/T^4)} = \frac{\mathcal{J}^2}{(I/T^2)} \,, \tag{A44}$$

where $\mathcal{J}$ is impulse and I/T^2 is energy. Mass is already built with inertia, so it is not surprising that it does not scale as $(I/T)^{-1}$, as the other densities do. To find out how force squared and impulse squared are regulated, we rewrite the terms in the denominators. It turns out that power P has the form

$$P = I/T^3 \,, \tag{A45}$$

so that Equation (A44) can be recast in the equivalent form

$$\sigma_I = \frac{F^2}{(P/T)} = \frac{\mathcal{J}^2}{PT}\,, \tag{A46}$$

where the integrated quantity PT represents energy E. Thus, the F^2 term is regulated by the rate of change of power P/T (see also Note 23, and impulse squared $\mathcal{J}^2$ is regulated by integrated power (or energy) PT, both of which are limited by the speed of light.

Appendix B.2. Physical Properties of the Various Surface Densities

We conclude with a summary of the properties of the above surface densities:

(a) The densities σ_P and σ_F (wave intensity and pressure, respectively) are both regulated by the rate of change of inertia (I/T) at all scales (Equations (A39) and (A43)).

(b) Density σ_I (i.e., mass) is not regulated by inertia; mass already possesses inertia; instead, we can say that mass is force squared F^2 regulated by the rate of change of power (P/T) or impulse squared $\mathcal{J}^2$ regulated by energy E (Equation (A46)), where E could also be viewed as the rate of change of the action integral, i.e., $(\mathcal{S}/T)$.

(c) Vacuum constants are explicitly present in the σ_P Equations (A41) and (A42), when σ_P is written in terms of the force fields squared ($\mathcal{E}^2$ and a^2, respectively).

(d) The vacuum remains present in the σ_F of the EM field, but it drops out from the σ_F of the gravitational field (both distinct behaviors are shown in Equation (A43)).

(e) Force surface density σ_F (Equation (A43)) represents the conventional energy density of the force fields, whereas σ_P (Equation (A39)) shows that vacuum inertial resistance (I/T) is present during the action of all forces; and this inertial resistance appears also in Equations (A41) and (A42) (Z_0^{-1} and c, respectively).

(f) Both sides of Equation (A40) have dimensions of Planck's constant $[h]$, thus $I/T \sim q^2 Z_0 = [\text{action}]$. Higher powers of T in I/T^n ($n = 2, 3$), i.e., higher-order derivatives, are also physically important: $I/T^2 = [\text{energy}]$ and $I/T^3 = [\text{power}]$. Equation (A45) for I/T^3 then implies that power stems from the third time derivative of the moment of inertia, a property that is fundamental to the emission of gravitational waves. The same relation, applied to EM waves, produces ohmic power (divide Equation (A40) by T^2) with dimensions of [electric current]^2[ohmic resistance].

(g) Equations (A35)–(A38) all have the characteristic form

$$[\text{surface density } X/A] = [\text{constant } C] \times [\text{kinetic scalar quantity } Y]^4\,,$$

in which A is area and the power of 4 represents $N+1$ degrees of freedom, with $N = 3$ for the spatial dimensions, plus 1 degree of freedom for the scale of the underlying scalar Y. This form implies the differential equation $dX/dA = CY^4$, relating X to Y.

Notes

1. Similarly, $\not{\mu}_0 \equiv \mu_0/(4\pi)$ is the reduced vacuum permeability, and then, $\not{\epsilon}_0\not{\mu}_0 = 1/c^2$. The stereometric 4π terms cancel out nicely to produce the "definition" of c, which is a purely physical quantity. Furthermore, Dirac's 2π term in $\hbar$ tells us that Planck's free photons only "see" two dimensions, no matter how they move in stereometric (3-D) space (in curves, or circles, or ellipses, etc.). We also learn that the fundamental natural constant c is produced by the vacuum itself, and it is the geometric mean of two smooth inverse Lie mappings [13] of $\not{\epsilon}_0$ and $\not{\mu}_0$ (i.e., the geometric mean of $1/\not{\epsilon}_0$ and $1/\not{\mu}_0$).
2. In Bohr's model, the (nongeometric) number-parts of energies E_n and radii r_n are related by $E_n \propto 1/r_n = 1/n^2$. Therefore, for pure numbers, we see that $\sqrt{r_n} = n$, and the coefficients of the quantized radii are essentially produced by geometric averaging, viz., $r_{n+1} = \sqrt{r_n r_{n+2}} + 1$. The +1 extends the sequence back to $n = 0$, $r_0 = 0$.
3. Unless the calculations can be repeated successfully within another system of units in which another particle is chosen as a building block (see Note 10 below for the case at hand).
4. Going as far back as 1948, some fundamental equations signaled that the introduction of 2π in $\alpha_{\hbar}$ may not be appropriate, but the warning was brushed off as a mere simplification between pure numbers. The electron vertex functions and the corrections to the form factors (Ref. [20], pp. 194–196) all show the coefficient $\alpha_{\hbar}/(2\pi)$, in which the visible (2π) term eliminates the 2-D

geometry from $\alpha_{\hbar}$, and the correction to the g-factor of the electron becomes $a_e \equiv (g-2)/g = \alpha_h = 1/861.0$ (equation (6.59) in Ref. [20]). This result was first obtained by Schwinger in 1948 [21], and it was confirmed by experiments to 8 significant figures, the most accurate ones using an ingenious technique developed by Van Dyck et al. in 1987 [22]. Therefore, unknowingly, these experiments were measuring α_h by mesuring the correction a_e to the g-factor of the electron.

5 The unit of [area] justifies the deduced geometric factor $1/(2\pi)$ in the second G-M of Equation (10). The additional factor of 2 attached to ϵ_0 is a unitless imprint, but it has a geometric origin. This type of imprinting is difficult to track down in the various equations of physics when they are presented in reduced, simplified form (see also the discussion in Section 1.2 about the numerical factor of 1/4 imprinted by geometry to the Rydberg energy).

6 The reference unitless constant (α_h here) plays the exact same role that the standard 1-meter ruler and the standard 1-kg cylinder play in the SI system of units for length and mass, respectively.

7 Had we used the mass and the charge of a supermassive black hole (e.g., [30]) in Equation (15), we would have obtained a relative strength of couplings $\beta_G \gg 1$ and a different system of units, which would be hard to relate to the Planck scale and even harder to use in the atomic world.

8 In fact, $\beta_G = 1$ near the Planck scale, for a particle of mass $m_\star = M_p/30 = 1.0 \times 10^{18}$ GeV$/c^2$, where $M_p = \sqrt{hc/G}$ is the original Planck mass. The deflation factor of 1/30 is also used by the Higgs boson to couple to the bottom quark (see Section 3.1 below)

9 We point out again that using $\hbar$ in definitions (13) and (14) reverses what nature intended. It makes $\alpha_{\hbar}$ be a geometry-free value, although the geometry should have been that of $\not\epsilon_0$ coming from the electric field; and α_G ends up with 2π radians, although it should have been geometry-free. This setup reveals that mass elements know they exist in a 3-D space, but elementary charges do not know, and they are taught accordingly by the vacuum's insertion of $\not\epsilon_0 \propto 4\pi$ and/or $\not\mu_0 \propto 1/4\pi$ into the equations of the EM field.

10 An alternative choice, such as of two interacting protons with masses m_p, leads to another complete system of units, say UPS$'$. In this case, we find that $M'_A = 27.5$ GeV$/c^2$ and $(\beta'_G)^{1/2} = 9.00 \times 10^{-19}$, but Equation (18) is still valid, and connects M'_A with the original Planck mass $M_p = \sqrt{hc/G}$. Furthermore, the scaling $M'_A/M_A = m_p/m_e$ holds precisely between the two systems of units; and the relation $M_A M'_A = (m_e m_p)/\alpha_h$ is exact as well. Finally, referring to the upcoming UPS results in Section 3.1 below, the relation $M'_A = \sqrt{m_t m_b}$ holds to within 2.5% in the UPS$'$, where m_t and m_b are the masses of the top and bottom quarks, respectively; thus, M'_A actively participates in the mass ladder of the UPS$'$, just as M_A does in the UPS mass ladder of Section 3.1 and Appendix A.

11 We knew that a rescaling of the Planck mass $M_p = \sqrt{hc/G}$ by some power of β_G would produce an atomic mass. However, we did not know which power is appropriate to use. Here, we have shown that the appropriate coefficient of M_p is $\sqrt{\beta_G}$, the G-M of E_{grav} and $(E_{elec})^{-1}$ when scaling M_p down to lower masses. If we are scaling up, then the -1 exponent naturally moves on top of E_{grav} in the G-M (see Section 3.4 below). In retrospect, these two Lie-type G-M averages make sense in a "fair" universe that uses such impartial averaging to combine pairs of interacting physical quantities and units.

12 The realization that the vacuum also leaves unitless numerical imprints (in addition to its dimensional constants $\not\epsilon_0$, $\not\mu_0$, c, Z_0) is new, unexpected, and it may prove important in future work. In the future, we will have to investigate such imprints of the vacuum to the nuclear world, especially in the strong interactions and the so-called beta functions [20].

13 The charm and bottom quarks have masses of $m_c = 1270$ MeV$/c^2$ and $m_b = 4180$ MeV$/c^2$, respectively [32]. At such high masses, something must be changing in the dynamics: for the ordered by mass triplet s-c-b, we find, to within a 1.6% accuracy, that $m_c = 2\sqrt{m_s m_b}$. We also find that the charm quark participates rather "reluctantly" in just one pure/unscaled G-M (Equation (28), referring to the compact triplet p-c-τ); and even that one is unusual, as it involves the proton mass m_p.

14 No other available particle slots in the domain.

15 It will become apparent in Appendix A that the ratio 1.38 approximates $C_F = 4/3$ (to within a deviation of 3.5%), where C_F is the quadratic Casimir charge of the SU(3) fundamental representation of the quark potential (equation (4.45) in Ref. [33]).

16 Eliminate m_H between Equations (40) and (41).

17 Eliminate m_t between Equations (37) and (38).

18 We cannot help but wonder—if A. Sommerfeld, W. Pauli, C. Jung, R. Feynman, and many others [34] became familiar with this result, would they show the same fascination for number 861 as with 137? The particle-to-scale mass and charge ratios discussed above strongly indicate that we should turn our attention to the physics behind 861 rather than trying to find the same physics in the geometry-dependent composite ratios $137 = 861/(2\pi)$ and $\hbar = h/(2\pi)$.

19 See Ref. [35] and article https://en.wikipedia.org/wiki/Planck_units in Wikipedia (accessed on 20 May 2023).

20 The remaining choice, the G-M of r_b and r_c, would give an equivalent result, scaled by a different power of $\alpha_{\hbar}$ ($L'_A = L_A/\alpha_{\hbar}$), such that the $r_c = \sqrt{L_A L'_A}$.

21 For comparison, the atomic rest-energies [42] of naturally occurring primordial uranium U^{238}_{92} and synthetic fermium Fm^{257}_{100} are 90% and 97% of the Higgs VEV, respectively.

[22] Besides combining with G to produce the units of force and power in the cosmological and Planck systems, c does something else that is notable: it combines with ϵ_0 or μ_0 to produce a surprise unit for ohmic resistance: $\mu_0 c = 1/(\epsilon_0 c) = Z_0 = h/e^2$ (see Section 2.1.1).

[23] Mass M is a special case for $n = 0$ in which $M = F^2/(I/T^4)$ (see also Appendix B). This relation sketches the complex G-M interaction between mass and inertial change that regulates the kinematics of an object: $dp/dt = \sqrt{M\,(d^4I/dt^4)}$, where p is momentum, or equivalently, $d^4I/dt^4 = Ma^2$.

[24] In a nearly perfect dielectric, the wave impedance is $Z = Z_0/(1+\chi_e) < Z_0$, where $\chi_e > 0$ is the electric susceptibility.

[25] Note that even actual planetary orbits [46] and also theoretical orbits in the virtual Hooke potential [47] show G-M averaging in many of their properties [48,49]. The two types of elliptical orbits have fundamentally different centers and forces, but this is not enough to suppress or modify the ubiquitous geometric averaging that is so obvious in the parameters of the two sets of ellipses.

[26] Arithmetic averaging would favor the large constant, whereas harmonic averaging would turn the tables and clearly favor the small constant. Compared to G-Ms, either one of these extreme averages treats "unfairly" one or the other participant.

[27] Dirac [8,9] was the first physicist to come to this realization, although he chose to find a way to instill many more orders of magnitude into G in the early universe, rather than accept its miniscule value and investigate its properties.

[28] We also timidly attempted a preliminary calculation of the scaling between weak and strong interactions, as a ratio of energies β_W (Equation (53). It seems that such energy ratios/comparisons are the way to incorporate consistently the dimensionless constants into the UPS. We can then imagine a complete UPS := $\{c, Z_0, G, h, \alpha_h, \beta_G, \beta_W, \ldots; \not{\epsilon}_0, \not{\mu}_0, \not{h}\}$ that includes geometry-free units and a set of geometry-dependent units, along with relative β-ratios $\ll 1$ of unitless constants.

[29] From (35) and (36), we get $m_b = 30m_s/K$ (#1). From (32), (36), (38)–(40), we get $m_b = 30K^{-3/4}m_\mu$ (#2). From (#1) and (#2), we get $m_s = K^{1/4}m_\mu$ (#3). From (29), (#3), and (#2), we get $m_\mu = 30K^{-5/4}m_u$ (#4). From (26), (#3), (#4), and (30), we get Equation (A4). Finally, from (34), (36), and (A4), we get Equation (A3). Equations (28), (31), (33) and (37) were not used.

[30] In natural units, $\not{h} = c = \epsilon_0 = 1$. By suppressing the 2-D geometry present in $\not{h}$ and by setting $\epsilon_0 = 1$ (not $4\pi\epsilon_0 = 1$, as in Gaussian units [36]), the 3-D geometry (the $\sqrt{4\pi}$ dividing e) imprinted on to the electric charge by the vacuum remains present in definition (A21).

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