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Bound–Free and Free–Free Pair Production Channels in Forward Delbrück Scattering

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Abstract: We present a theoretical study of forward-angle Delbrück scattering of light by the Coulomb field of a target nucleus. Special attention is paid to the Coulomb corrections, which take into account the interaction of the emerging virtual electron–positron pairs with the nucleus to higher orders of αZ. We compare the results from three different computation methods: the direct all-order evaluation of the Delbrück amplitude, the computation from the pair production cross section with the optical theorem and the low-energy limit. We find that the values obtained from the optical theorem are in very good agreement with the all-order calculations and can be used as benchmark data. Moreover, both methods agree with the low-energy limit for photon energies $\omega << m_e c^2$ when correctly accounting for the bound–free pair production cross section in the optical theorem calculations, and the discrepancy found in the literature originates from neglecting this contribution.

Keywords: quantum electrodynamics; elastic scattering; pair production; optical theorem

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1. Introduction

Delbrück scattering is the process in which a photon is scattered by the Coulomb field of an atomic nucleus via virtual electron–positron pair production. Out of all non-linear quantum electrodynamical (QED) processes, Delbrück scattering has relatively large cross sections, and hence is the easiest to measure [1,2]. For this reason, many experimental and theoretical works have been devoted to this process in the past [3–5]. Most theoretical studies evaluate the Delbrück amplitude in an approximate way, for example, using the lowest-order Born approximation which neglects the so-called Coulomb corrections that go beyond the lowest order in αZ, where $\alpha \approx 1/137$ is the fine-structure constant and Z is the nuclear charge [6,7]. However, these Coulomb corrections are of particular interest, since they are known to significantly alter the elastic scattering cross section in the high-Z regime where Delbrück scattering is most pronounced. We have recently reported the first all-order calculations of the Coulomb corrections of Delbrück scattering without any further approximation to the coupling between the electron–positron pair and the nucleus for photon energies below and above the pair production threshold [8–10].

In this contribution, we present an independent check of these results using calculations of one-photon pair production and the optical theorem, a method that was first applied to Delbrück scattering in the early 1950s and has been regularly used ever since [11–13]. Moreover, we explicitly investigate the contribution from bound–free pair production and confirm the numerical value of the low-energy limit of the Coulomb corrections calculated by Kirilin and Terekhov [14]. We show that the disagreement between the calculations in ref. [14] and the optical theorem originates from neglecting the bound–free pair production cross section.

2. Theoretical Background

Within the framework of QED, Delbrück scattering can be described by the Feynman diagram in Figure 1a to lowest order in α and all orders in αZ. Using the standard Feynman correspondence rules [15], we obtain the Delbrück amplitude

$$M^D = \frac{i\alpha}{2\pi} \int_{-\infty}^{\infty} \mathrm{d}z \int_{-\infty}^{\infty} \mathrm{d}z'\, \delta(\omega + z - z') \\ \times \int \mathrm{d}^3\boldsymbol{r}_1 \int \mathrm{d}^3\boldsymbol{r}_2\, \mathrm{Tr}\Big[\hat{R}(\boldsymbol{r}_1, \boldsymbol{k}_i, \boldsymbol{\epsilon}_i) G(\boldsymbol{r}_1, \boldsymbol{r}_2, z) \hat{R}^\dagger(\boldsymbol{r}_2, \boldsymbol{k}_f, \boldsymbol{\epsilon}_f) G(\boldsymbol{r}_2, \boldsymbol{r}_1, z')\Big], \quad (1)$$

where $\hat{R}(\boldsymbol{r}, \boldsymbol{k}, \boldsymbol{\epsilon})$ is the electron–photon interaction operator and $G(\boldsymbol{r}_1, \boldsymbol{r}_2, z)$ is the Dirac-Coulomb Green's function that describes the propagation of an electron/positron to all orders in the Coulomb interaction. Moreover, z and z' are the energies of the two electron propagators and ω is the energy of the incoming/outgoing photon. The numerical evaluation of this amplitude without any further approximations is a difficult task due to the numerical difficulties associated with the multidimensional integral that needs to be solved and we refer to our recent publications for further details [8–10].

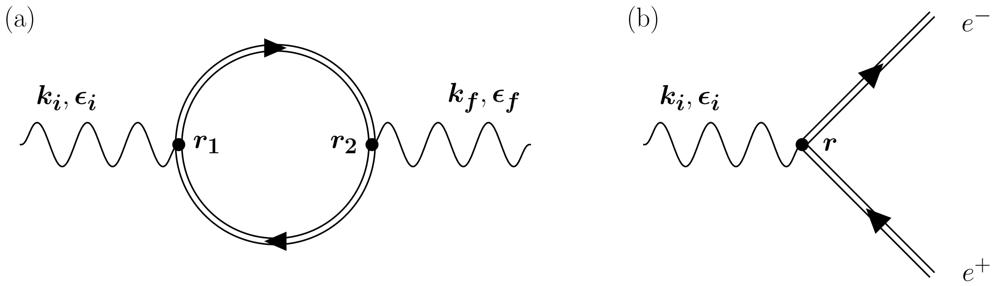

Figure 1. Leading order in α and all orders in αZ Feynman diagrams for Delbrück scattering (**a**) and one-photon pair production in the presence of a Coulomb field (**b**). The wavy lines represent photons and the double lines display electrons/positrons propagating in the Coulomb field of an atomic nucleus.

Delbrück scattering is related to one-photon pair production in the presence of a Coulomb field via the optical theorem [11–13]. This process, represented by the Feynman diagram in Figure 1b, can be evaluated relatively easily, and cross sections that are exact in all orders of αZ have been calculated a long time ago. For forward scattering, the imaginary part of the Delbrück amplitude is given by

$$Im[M^D(\theta = 0)] = \frac{\omega}{4\pi} \sigma_{PP}(\omega)\,, \quad (2)$$

where σ_{PP} is the total one-photon pair production cross section [11]. Using Chauchy's theorem, the dispersion relation for the real part of the amplitude can be found

$$Re[M^D(\theta = 0)] = \frac{\omega^2}{2\pi^2}\, \mathrm{P} \int d\omega' \, \frac{\sigma_{PP}(\omega')}{\omega'^2 - \omega^2}\,, \quad (3)$$

where P denotes the principal value of the integral; see ref. [11] for further details.

It is important to note that σ_{PP} in Equations (2) and (3) is the total cross section for one-photon pair production. In the presence of a target nucleus, this process can proceed via two channels

$$\sigma_{PP} = \sigma_{FF} + \sigma_{BF} \, , \tag{4}$$

where σ_{FF} is the total cross section for pair production in which both the electron and positron are free after the process, and σ_{BF} is the cross section for bound–free pair production where the electron is captured into a bound state. The cross sections σ_{FF} and σ_{BF} can be calculated from the pair production amplitude

$$M^{PP} = \int \mathrm{d}^3\boldsymbol{r}\, \psi^{\dagger}_{e^-}(\boldsymbol{r}) \hat{R}(\boldsymbol{r}, \boldsymbol{k_i}, \boldsymbol{\epsilon_i}) \psi_{e^+}(\boldsymbol{r}) \, , \tag{5}$$

where $\psi_{e^-}(\boldsymbol{r})$ and $\psi_{e^+}(\boldsymbol{r})$ are the electron and positron wave functions. Here, the electron wave function can either be a continuum- or bound-state wave function for the free–free or bound–free pair production, respectively. By squaring this amplitude and integrating over all emission angles and final electron/positron energies, as well as summing over all possible bound states in which the electron can be captured, one obtains the total pair production cross section σ_{PP}. In practice, we calculate the first Born approximation and the Coulomb corrections to σ_{FF} using the equations from refs. [12,16,17] and the method used in ref. [18] to compute σ_{BF}. Calculating the Delbrück amplitude from Equations (2) and (3) with the Born and all-order results for the pair production cross section and subtracting the two values yields the Coulomb corrections to forward Delbrück scattering.

3. Numerical Results

In Figure 2, we present the numerical results for the relative Coulomb corrections $M_{\mathrm{CC}}/M_{\mathrm{Born}}$, where M_{Born} is the lowest-order (in αZ) contribution to the Delbrück amplitude, and M_{CC} is the difference between the lowest-order and all-order amplitudes, to the real and imaginary parts of the amplitude for forward Delbrück scattering off bare lead ions. We compare the values obtained from a direct evaluation of the Delbrück amplitude (1) to those obtained from Equations (2) and (3) including either only σ_{FF} or the sum of σ_{FF} and σ_{BF} for the capture into different atomic shells. Moreover, we indicate the value of the relative Coulomb corrections in the low-energy limit as obtained by Kirilin and Terekhov [14]. The all-order calculations were taken from refs. [8,10] with additional computations performed for $\omega = 1.0\, m_e c^2, 1.5\, m_e c^2, 21.53\, m_e c^2$ to compare the results for a wider range of photon energies. For the high-energy computation at $\omega = 21.53\, m_e c^2$, we only include the data for the imaginary part of the amplitude since the numerical convergence is more difficult for the real part and achieving a sufficient accuracy requires too much computational resources for this energy; see ref. [10].

As seen from the figure, the all-order calculations are in excellent agreement with the calculations using the optical theorem when including bound–free pair production with capture into the K-, L- and M-shell, and both methods agree well with the low-energy limit for $\omega < m_e c^2$. Bound–free pair production has a very large contribution to the Coulomb corrections for photon energies close to and below the pair-production threshold $\omega = 2\, m_e c^2$ and is negligible for large energies. The contribution of the capture into higher atomic shells drops off rapidly and including the K- and L-shell is already enough to achieve a 5% precision. The calculations also show that the disagreement observed in ref. [14] between the low-energy limit and the optical theorem is not due to the numerical cancelation problems proposed by the authors, but from the omission of the contribution from bound–free pair production. Therefore, obtaining the forward Delbrück amplitude from Equations (2) and (3) serves as an independent test of the other computation methods and verifies the numerical values obtained from the full all-order and low-energy calculations.

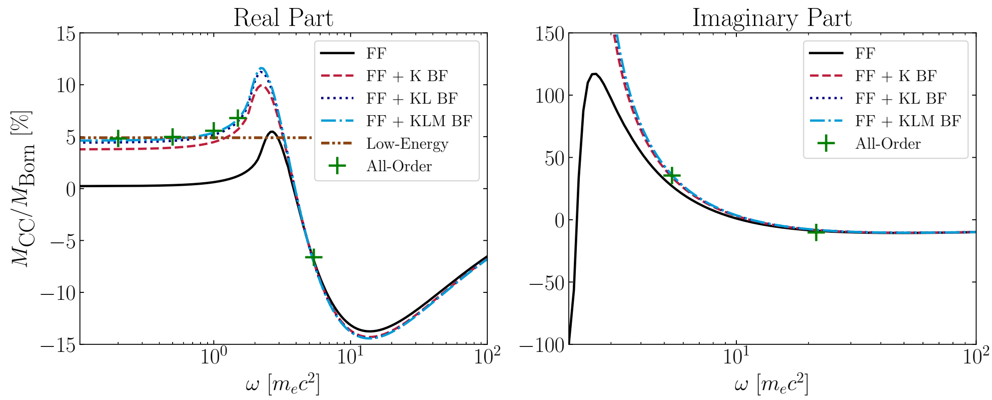

Figure 2. Real and imaginary part of the relative Coulomb corrections to the Delbrück amplitude for the forward scattering of photons with energy ω off bare lead ions. We show the results obtained from Equations (2) and (3) using just the cross section for free–free pair production (black solid line) and when adding bound–free pair production into the K-shell (red dashed line), the K- and L-shell (dark blue dotted line) and the K-, L- and M-shell (light blue dash-dotted line). Additionally, we show the results obtained from an all-order evaluation of the Delbrück amplitude (green crosses) and the low-energy limit from ref. [14] (brown dash-dot-dotted line).

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