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ENHANCEMENT OF SYNCHROTRON RADIATION BY BEAM MODULATION

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The beam density in storage rings like SPEAR (or PEP) at SLAC is high enough that a significant amount of synchrotron radiation can be emitted coherently if the beam density is appropriately modulated. Modulation can occur, for example, by passing the beam through laser light of frequency ω along a phase-matched orbit (as in an undulator). The device will enhance radiation by several decades even at frequencies greater than ω, (unlike a single-particle undulator), and would greatly improve the performance of an earlier proposed X-ray laser to be pumped by synchrotron radiation.

I INTRODUCTION

When a relativistic electron passes through a magnetic field, it can emit electromagnetic radiation which is often called synchrotron radiation. In most cases of interest, the intensity spectrum of this radiation can be calculated classically;[1] the agreement between the so calculated spectrum and experimental observation is excellent.†

When not one, but N relativistic electrons pass through the magnetic field, then all N of them can emit electromagnetic radiation. In most cases of interest, the intensity spectrum of the total radiation produced by all N electrons is simply N times the intensity spectrum of the radiation emitted by a single electron. This demonstrates that in such a calculation of intensity it is usually sufficient to add the radiation emitted by the several electrons incoherently. All intensity spectra of synchrotron radiation emitted by the various storage rings have been calculated in this manner† and are in agreement with observation.

The purpose of this note is to show that it is now possible to arrange things so that the synchrotron radiation emitted by many electrons in a relativistic beam will add coherently. If that is done, then calculations based on the usual simple incoherent addition formula will give incorrect results, and for wavelengths in the optical and X-ray region the true radiation intensity will be several decades higher than the result calculated under the usual assumption that intensities can be added incoherently. This higher intensity would be helpful in all experiments in which higher counting rates are desirable, but it is crucial for those experiments in which a certain threshold photon intensity is required. For these the higher intensity can make the difference between an impossible and a feasible experiment. An experiment of the latter type is the suggested method of coherent X-ray production by pumping a suitable target with synchrotron radiation[2]: When synchrotron radiation is produced incoherently (between electrons) in the SPEAR storage ring at Stanford, then the photon intensity is close to the required critical value. But with the enhancement due to coherent (by several electrons)‡ production, the photon intensity can be much more than that critical value.[3]

This intensity enhancement will occur if two conditions are fulfilled. First, the density of the circulating relativistic particles has to be high enough for coherence effects to be significant.

† For example, H. Winick, SLAC-PUB-1439, June 1974.

‡ We do not mean here that all emitted photons are coherent (i.e., in phase) with each other as in a laser. We mean that several electrons emit essentially in phase with each other thereby mutually enhancing the radiation emitted; this radiation may have a wide frequency spectrum.

Second, the density variation along the particle beam must have short-wavelength components. We will show that in storage rings like SPEAR, the beam density in the interaction region is high enough to meet the first condition. The second condition can also be fulfilled by letting the beam move along an oscillating path while interacting with a suitable laser beam of frequency ω, as in an undulator. The device is not a single-particle undulator, however, because it will radiate at several frequencies, and enhance the synchrotron radiation spectrum even at frequencies higher than ω.

In Section II of this paper we discuss coherent radiation and in Section III we discuss realization of coherent radiation in a modulated beam.

II COHERENT RADIATION BY DENSE ELECTRON BEAMS

To avoid unessential complications, we assume that only one beam is circulating in the ring. We assume that the beam circulates in the (x, z) plane. The z axis is chosen to be tangential to the beam velocity at the point of interest, and the x axis is radial at that point. The x, y, z form a right-handed coordinate frame. We assume that the beam consists of bunches which, in the laboratory rest frame K_L, have a length l, width a, and height b. At first we assume that the electron density within the bunch is uniform, and that it abruptly falls to zero at the edges of the bunches. Although this last assumption is unrealistic, it will not seriously affect our results, and later we will modify it. Let the number of electrons in the bunch be N.

As measured in the laboratory frame K_L, the electron density inside the bunch is

$$\rho = \frac{N}{abl}. \tag{1}$$

If the bunch is moving with a velocity v along z in the laboratory, then the electron density in the bunch, as measured in the average restframe of the bunch, K', will be

$$\rho' = \rho\gamma^{-1}, \tag{2}$$

where $\gamma = [1 - (v^2/c^2)]^{-1/2}$ and quantities measured in K' are denoted by a prime.

Imagine an electron gas of density ρ' located inside a sphere of radius r' and moving with average velocity $v' = 0$ as measured in K'. If the electrons in this gas undergo acceleration along the $\hat{x}$ direction, the electrons will radiate. Consider that part of the radiation which has wavelength λ' in K'. If $r' \leq \lambda'/8$, then at all points in space the electric and magnetic fields emitted by any one electron will increase (not decrease) the absolute value of the electric and magnetic fields produced by the other electrons located within the sphere, because the largest distance between any two electrons inside the sphere is $\leq \lambda'/4$. In other words, all electrons within the sphere will radiate essentially coherently. When $r = \lambda'/8$, then the number of these electrons is

$$n = \rho' \frac{4\pi}{3}(r')^3 = \frac{\pi}{384}\rho\gamma^{-1}(\lambda')^3. \tag{3}$$

As an example, let us assume that $N = 7 \cdot 10^{11}$, $l = 3$ cm, $a = 3 \cdot 10^{-2}$ cm, $b = 10^{-3}$ cm. These are parameters similar to those which are expected for a single beam in the interaction region of SPEAR at Stanford. Then $\rho = 7.78 \cdot 10^{15}$ cm^{-3} and $n = 6.36 \cdot 10^{13}\gamma^{-1} (\lambda'\{\text{cm}\})^3$, where $\lambda'\{\text{cm}\}$ is the wavelength measured in centimeters. At $\gamma = 10^4$ and $\lambda' = 2.10^{-2}$ cm, $n = 1.5 \cdot 10^4$. Thus an electron gas of $n = 5 \cdot 10^4$ electrons confined inside a sphere of radius $r' = \lambda'/8$ would emit radiation of wavelength in the laboratory down to $\lambda \approx \lambda'\gamma^{-1} = 2 \cdot 10^{-6}$ cm coherently, with an intensity which is n^2 times as high as that emitted by one single electron, and n times higher than if all electrons radiated incoherently. Therefore, under these circumstances the intensity will be enhanced due to coherence by at least a factor of about $1.5 \cdot 10^4$, for wavelengths down to $\lambda \approx 2 \cdot 10^{-6}$ cm as measured in the laboratory.

Assume for a moment that as seen from K' the beam in the interaction region of the storage ring consists of a series of spheres of radius $r \leq \lambda'/8$, filled with an electron gas of density ρ'. If the position of these spheres along the orbit is random, then the radiation emitted by all the spheres can be added incoherently. On the other hand, if the position of the various spheres follows some regular pattern, then there will be directions in which the radiation emitted by several spheres will add coherently, resulting in a "coherence maximum," i.e. an intensity peak in that direction. For example, suppose that the spheres are located at points 0, d', ± 2d', ± 3d', ..., along an infinite straight line. Assume also, that $r' \ll \lambda'$, that d $\gg r'$, that each sphere oscillates harmonically with a frequency $\omega' = 2\pi c/\lambda'$ along x and that in K' their velocity component along x satisfies $v'_x \ll 1$. Then each sphere will emit predominantly an electromagnetic wave of wavelength λ'. If all the spheres oscillate in phase in K', all the waves emitted by them will add coherently in the direction perpendicular to the straight line. In addition, the total radiated intensity will have coherence maxima of integer order $M \geq 1$ in directions which make an angle θ'_M (measured in K') with the straight line, when $\cos\theta'_M = M\lambda'/\mathrm{d}' \leq 1$. If the straight string of harmonically oscillating spheres oscillates in such a manner that viewed from K' the phase of oscillation is not the same for all spheres at any one moment, but the phase travels along the string with a constant velocity v', then waves emitted by each sphere will add coherently at those angles θ'_M for which $\cos\theta'_M = c/v' \pm M\lambda'/\mathrm{d}' \leq 1$. In fact, the radiation pattern produced by such a string as a function of θ' will be just that which would be produced by a grating of slit spacing d$'$ illuminated by a plane wave incident at an angle $\theta'_0 = \arcsin c/v'$ from the normal.

At present, it is not possible to produce an electron beam in which the electrons are bunched in a series of spheres as just described. But it is possible to approximate that situation. To see how, let us restate the above simple results in another language:

Consider a bunch of electrons, each of which is on the average at rest in K'. The electrons are all located inside a rectangular volume with length l', width a', and height b'. As a function of x' and y', the density of electrons is constant inside this volume, and $a', b' \ll \lambda'$. As a function of z', the density of electrons inside the volume is seen from K' to be

$$\rho'(z') = \sum_{k'_j} \rho_{k'_j} e^{ik'_j z'}. \tag{4}$$

Reality requires $\rho'_{k'_j} = \rho'^{*}_{-k'_j}$. A plane electromagnetic wave is incident on the electrons. The phase velocity of the wave is v' along the axis of the electron bunch. The electric field E, is polarized along $\hat{x}$, and

$$E'_x = A'_i \cos\left[\omega'\left(t' - \frac{z'}{v'}\right)\right]. \tag{5}$$

Assume that in K' the electrons are nonrelativistic at all times. Then only electric forces act on them, and they will oscillate harmonically with frequency ω'. Each electron will then radiate according to the usual nonrelativistic expression, emitting a wave of frequency ω' with an angular distribution given by the well known non-relativistic form factor $f(\theta', \varphi')$, when the radiation is emitted in a direction which makes an angle θ' and φ' with the z' and x' axis respectively. The total radiated field will then also oscillate with frequency ω'. Its amplitude at some point that is at a distance $R' \gg l'$ from the electron bunch in the direction (θ', φ'), is obtained by integrating over the contributions emitted by all electrons:

$$\begin{aligned} A'_f(\theta', \varphi') &= \frac{1}{R'} \int_{l'} \mathrm{d}z' f(\theta', \varphi') \rho'(z') \\ &\quad \times \cos\left[\omega'\left(t' - \frac{z'}{v'}\right)\right] \delta\left[t' - \frac{z'}{c}\cos\theta'\right] \\ &= \frac{1}{R'} f(\theta', \varphi') \sum_{k'_j} \rho_{k'_j} \int_{l} \mathrm{d}z' e^{ik'_j z'} \\ &\quad \times \cos\left[\omega'\left(\frac{\cos\theta'}{c} - \frac{1}{v'}\right) z'\right]. \end{aligned} \tag{6}$$

The delta function in the first integral insures that the contribution from the individual electrons is added with the correct phase. An overall multiplicative constant is of no interest, and is absorbed into $f(\theta', \varphi')$. When l' is large, then due to the oscillating integrand, the value of the integral will be small, unless

$$\pm k'_j = k'_f - k'_i \tag{7}$$

where

$$k'_f \equiv \frac{\omega'}{c}\cos\theta', \tag{7a}$$

$$k'_i \equiv \frac{\omega'}{v'}. \tag{7b}$$

The physical interpretation of Eq. (7) is transparent: an interaction of the initial wave of wave number k'_i (measured along the z axis) with a periodic structure of wavenumber $\pm k'_j$ along z, induces a change in wave number by $\pm k'_j$, stepping up (or down) the wavenumber k'_i by that amount. The k'_j occur with both signs as required by the reality of ρ'.

Corresponding to each Fourier component in Eq. (4) there is one θ' for which the intensity is a maximum. For an infinitely long periodic $\rho'(z')$, the position of these maxima is fixed if the periodicity is left unchanged. The shape of the $\rho'(z')$ merely changes the amplitude of the various Fourier components and thus of the corresponding intensity maximum. In the special case when $\rho'(z')$ has the shape of a series of delta function-like barriers located at $z' = 0, \pm\mathrm{d}', \pm 2\mathrm{d}', \ldots$, then the Fourier components in Eq. (6) have precisely the amplitudes proportional to the various intensity maxima produced by a slit grating. The constant term gives the zeroth-order maximum, the next Fourier component the first order maximum, etc. When l' is not infinite and is decreased, then the value of the integral is small and increasing in the neighborhood of the maxima: the maxima get broader. An increase in the width of the barriers clearly cannot change the position of the maxima unless the periodicity is changed. Thus the string of oscillating spheres discussed earlier is a simple special case of the present one. When $|k'_f| > \omega'/c$, the wave is virtual, and cannot propagate. No Fourier component with $|k'_j| > 2(\omega'/c)$ can ever contribute to the radiation pattern for large R', since by Eqs. (7) $|k'_i|, |k'_f| \leq \omega'/c$.

Consider now the case when the electron bunch, as seen from the laboratory frame K_L, is moving along z with velocity v in a magnetic field which points along the y axis. Transforming to K', the magnetic field appears as a (virtual) electromagnetic field moving with velocity $-v$, the electric and magnetic fields being polarized along $\hat{x}'$ and $\hat{y}'$ respectively. This field forces the electrons to undergo a complicated motion[4] in K' and radiate. We focus our attention on the motion of one electron parallel to the x axis and Fourier analyze it. Consider that Fourier component which has frequency ω' and whose wave number along z is k'_i. We evaluate the radiation due to this Fourier component of the electron motion, then evaluate it for all electrons, and sum over all electrons. The resultant radiation will also have frequency ω' and its amplitude as a function of θ' and φ' can be calculated as in Eqs. (6) and (7). We find that the amplitude will have maxima at those values of θ' which satisfy Eq. (7). The oscillation velocity v_y of the electrons may also have a Fourier component with ω' and k'_i. Adding the two, one gets the total radiation due to the Fourier components with ω' and k'_i. Summation over all i gives the total radiation.†

Consider again the radiation due to the Fourier component with ω' and k'_i only. If the electron bunch is infinitely long, then the intensity maxima will be infinitely sharp. In general, the fwhm angular width of the intensity maxima will be of order $\Delta\theta' \approx \lambda'/l'$. The amplitude at the maximum will be N times the amplitude produced by one electron only. Therefore, the intensity at the maximum will be

$$I = N^2 I_1, \tag{8}$$

where I_1 is the intensity due to one electron only. If this radiation were spread out over the complete 4π solid angle, then it would have everywhere a value (setting $\sin\theta' = \frac{1}{2}$ for purposes of this estimate)

$$\tilde{I} = N^2 I_1 \frac{2\pi \sin\theta'}{4\pi} \Delta\theta' = \frac{1}{4} N^2 I_1 \frac{\lambda'}{l'} = n^2 \frac{l'}{\lambda'} I_1, \tag{9}$$

where $n \equiv \frac{1}{2}N\lambda'/l'$, is the number of electrons per half wavelength $\lambda'/2$ in the bunch. This shows that the total intensity can be obtained approximately by adding coherently the amplitudes due to all the electrons within sections of length $\lambda'/2$, and then summing over every second such section.

We have thus obtained in more precise terms the result expected on the basis of our earlier simple-minded argument related to slit gratings. If the $\rho'(z)$ has a piece in which high- and low-density regions of dimension $\approx\lambda'/2$ alternate, then all the electrons within any one of the dense regions radiate approximately coherently.

Note that coherence between electrons within one dense region increases the total radiated intensity, and is of central interest in this paper. On the other hand, coherence between the several regularly placed dense regions does not increase the total radiated intensity, it merely confines the radiation to the vicinity of the intensity maxima. This may become important if one wishes to confine the radiation within narrow angles or suppress certain frequencies in K_L, but is of less interest to us here.

III MODULATION OF THE BEAM DENSITY

We will describe the electron motion in K_L. The electron moves along the z axis on the average, and describes oscillations around it in the (x, z) plane, as shown in Figure 1. If $\gamma \gg 1$, then the electron speed is close to c, and the velocity components are

$$v_x \quad \text{and} \quad v_z \approx c(1 - v_x^2/c^2)^{1/2}. \tag{10}$$

We assume that $|v_x| \ll |v_z|$ at all times and that the amplitude A_e and wavelength λ_e of the wavy path

† A Lorentz transformation of this spectrum into K_L, in the special case when $\rho(z)$ is a constant in z, and when the magnetic field is uniform in K_L, gives the usual synchrotron radiation spectrum.

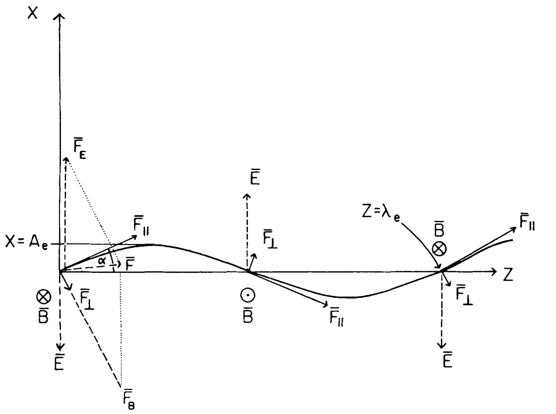

FIGURE 1 An electron travels (on the average) along the z axis, and oscillates in the (x, z) plane. Its path resembles a sine wave curve of amplitude A_e and wavelength λ_e as shown in the figure by the solid wavy line. The slope of the path at z is tg $\alpha(z)$. The direction of the electric field $\bar{E}$ and magnetic field $\bar{B}$ are shown at three positions of the electron. The longitudinal and transverse components of the electromagnetic force $\bar{F}$ acting on the electron are $\bar{F}_{\parallel}$ and $\bar{F}_{\perp}$. The electric and magnetic forces $\bar{F}_E$ and $\bar{F}_B$ are shown only for one position.

satisfy $A_e \ll \lambda_e$. We may then approximate one half wavelength of the path $x(z)$ by a power series, dropping third and higher order terms in z. In this approximation there is no difference between a section of a parabola, a circle, or one half wavelength of a sine wave. Therefore, in this approximation, we may produce such a path by producing a path of successive circular segments, for example, by placing a series of dipole magnets along the z axis, each $\lambda e/2$ long, and each producing a magnetic field alternately in the $+y$ or $-y$ direction. For a sinusoidal path, the slope of the $x(z)$ path is

$$\alpha(z) = \frac{d}{dz}x(z) = A_e\frac{2\pi}{\lambda_e}\cos\frac{2\pi}{\lambda_e}z \tag{11}$$

whose maximum value is $\alpha_M = 2\pi A_e/\lambda_e$.

An electromagnetic wave travels along the z axis. The electric vector $\bar{E}$ has amplitude E_0, and is polarized along the x axis (see Figure 1). Its frequency is ω, and its wavelength in vacuum $\lambda = 2\pi c/\omega$. The electric force is always along $\hat{x}$, while the magnetic force can have both an x and a z component. Parametrize the position of the electron with the z coordinate, and assume that

$$z \approx \tilde{v}_z(t - t_0) + z_0 \tag{12}$$

is a good approximation, where $\tilde{v}_z$ is the average (over t) of v_z. The total electromagnetic force acting on an electron at z, is

$$\bar{F}(z) = eE_0\left[\hat{x}\left(1 - \frac{v_z}{c}\right) + \hat{z}\frac{v_x}{c}\right]\cos\left[\omega\left(\frac{1}{\tilde{v}_z} - \frac{1}{c}\right)z + \omega\left(t_0 - \frac{z_0}{\tilde{v}_z}\right) + \varphi_0\right]. \tag{13}$$

Its longitudinal and transverse components (with respect to the instantaneous electron velocity) are:

$$F_{\parallel} = \frac{F_x\alpha|dz| + F_z|dz|}{|dz|} = eE_0\left[\left(1 - \frac{\tilde{v}_z}{c}\right)\alpha + \frac{v_x}{c}\right]\cos\left[\left(\frac{\omega}{\tilde{v}_z} - \frac{2\pi}{\lambda}\right)z + \omega\left(t_0 - \frac{z_0}{\tilde{v}_z}\right) + \varphi_0\right] \tag{14a}$$

$$F_{\perp} = \frac{F_x|dz| - F_z\alpha|dz|}{|dz|} = eE_0\left[\left(1 - \frac{\tilde{v}_z}{c}\right) - \frac{v_x}{c}\alpha\right]\cos\left[\left(\frac{\omega}{\tilde{v}_z} - \frac{2\pi}{\lambda}\right)z + \omega\left(t_0 - \frac{0}{\tilde{v}_z}\right) + \varphi_0\right] \tag{14b}$$

We neglect $\bar{F}_{\perp}$ because for small α it is proportional to α^2. Due to the action of $\bar{F}_{\parallel}$, the energy gain of the electron is

$$\begin{aligned} W = \int_{z_0}^{z_0+L} dz F_{\parallel}(z) = \int_{z_0}^{z_0+L} dz \tfrac{1}{2}eE_0 A_e\frac{2\pi}{\lambda_e} \\ \times\left(1 - \frac{\tilde{v}_z}{c} + \frac{v_z}{c}\right)\left\{\cos\left[\left(\frac{2\pi}{\lambda_e} - \omega\left(\frac{1}{\tilde{v}_z} - \frac{1}{c}\right)\right)z + \omega\left(t_0 - \frac{z_0}{\tilde{v}_z}\right) + \varphi_0\right] \right. \\ \left. + \cos\left[\left(\frac{2\pi}{\lambda_0} + \omega\left(\frac{1}{\tilde{v}_z} - \frac{1}{c}\right)\right)z + \omega\left(t_0 - \frac{z_0}{\tilde{v}_z}\right) + \varphi_0\right]\right\}. \end{aligned} \tag{15}$$

If λ_e satisfies

$$\lambda_e = 2\pi\left[\omega\left(\frac{1}{\tilde{v}_z} - \frac{1}{c}\right)\right]^{-1} = \lambda\left(\frac{c}{\tilde{v}_z} - 1\right)^{-1}, \tag{16}$$

then the first cosine in the curly bracket of Eq. (15) will be independent of z, and its integral monotonically increases with z. The second cosine in the curly bracket will oscillate, so that its average contribution will be small compared with that of the first, and we therefore neglect it. For small α, $\tilde{v}_z \approx v_z$ at all times, so we set approximately $1 - \tilde{v}_{z/c} + v_{z/c} \approx 1$, and obtain

$$W = eE_0 L \tfrac{1}{2} \alpha_M \cos \Phi_0, \tag{17}$$

where

$$\alpha_M \equiv \frac{2\pi A_e}{\lambda_e}$$

and

$$\Phi_0 \equiv \omega\left(t_0 - \frac{z_0}{\tilde{v}_z}\right) + \varphi_0$$

is the phase of the electric field at the time t_0 when the electron is injected into the wave at position z_0. When Eq. (12) holds and $\alpha_M \gg \gamma^{-1}$, Eq. (16) can be rewritten as

$$\lambda_e \approx \lambda_0 \alpha_M^{-2}. \tag{16a}$$

If the wavy path of the electron is produced by a section-wise stationary magnetic field B whose direction is the same in every second section, as described above, then the radius of orbit curvature for relativistic electrons is $R = pc/eB$ in each section, where e and p are the electron charge and momentum. When $\alpha_M(z) \ll 1$, simple geometrical considerations give

$$\frac{A_e}{\lambda_e} = \frac{\lambda_e}{R}, \tag{18}$$

and together with Eq. (16)

$$\alpha_M = (\pi B\{\text{kilogauss}\}\lambda_0\{\text{cm}\}/3\gamma)^{1/3}, \tag{19}$$

where each quantity is to be measured in the units which follow it in curly brackets.

Equation (17) shows that the energy gained by an electron varies linearly with L, and, therefore is a monotonic function of the time t. The energy gain W does not oscillate as it would if the plane wave were incident on an electron moving along a straight line instead of an oscillating path. The value of $W(L)$ depends on Φ_0. The electron continuously gains energy if $\cos \Phi_0 > 0$; otherwise its energy keeps decreasing, except when $\cos \Phi_0 = 0$, in which case its energy is unchanged. As a result, electrons lying within every second short section $\lambda/2$ long will start moving parallel to z with respect to the center of mass of the electron bunch, while the rest of the electrons will start moving in the opposite direction. This suggests the possibility of bunching.

To estimate the capabilities of this method, let us consider an example. The electron energy is 5 GeV, corresponding to $\gamma = 10^4$. The laser delivers light of wavelength $\lambda = 10^{-4}$ cm in 100-J bursts lasting $1.6 \cdot 10^{-10}$ sec each. The light focuses on an elliptical spot with horizontal and vertical diameters respectively $a_\gamma = 6 \cdot 10^{-1}$ cm, and $b_\gamma = 3 \cdot 10^{-1}$ cm. Then due to diffraction, the beam cross section of the laser beam will start changing appreciably at about 10^2 cm from the focus in either direction. The maximum electric force exerted on an electron by the laser light near the focus will be $eE_0 = 1.41 \cdot 10^7$ eV/cm. Choose $\alpha_M = 5 \cdot 10^{-3}$, which requires $\lambda_e = 3.8$ cm, $A_e = 3 \cdot 10^{-3}$ cm, and $B = 12.4$ kG. The bore in the magnet must have horizontal and vertical diameter $a_B > a_\gamma$, $b_B > b_\gamma$. Equation (17) now becomes

$$W\{\text{MeV}\} = 5 \cdot L\{\text{m}\} \cos \Phi_0. \tag{20}$$

When the magnet length is $L = 3$ m, then electrons with $\cos \Phi_0 = 1$ will gain 33.3 MeV energy from the laser beam during one passage through the device. We call this device the "energy modulator."

As a result, these electrons will tend to move ahead of the rest of the electrons. The increase in their velocity is $\Delta v \approx c\Delta\gamma/\gamma^3 = 2$ cm/sec, so that in order to move ahead a distance of $\lambda/2$, they would have to travel about $2.5 \cdot 10^{-5}$ sec, or cover a distance of 7.5 km, an unreasonable distance. On the other hand, if the electrons were to be led along a curved path instead of a straight one after they emerge from the interaction region with the laser beam, then bunching would soon result. Indeed, in a uniform magnetic field the electrons travels along a circle of radius $R \sim \gamma$, those with higher γ would move with velocity $v \approx c$ along a circle with larger radius, so they would tend to fall behind, while those with smaller γ would move ahead. In this manner, bunches of dimension $\lambda/2$ would result almost instantly for even small $\Delta\gamma/\gamma$ and originally monoenergetic electrons. In practice the energy spread $\Delta\mathscr{E}_e$ of the original electrons will wash out any bunching unless

$$W > \Delta\mathscr{E}_e. \tag{21}$$

Assume that at time $t = 0$ all electrons are relativistic, are located at the same point A, have velocities which are all parallel to each other, and perpendicular to the uniform, constant magnetic field in which the electrons are moving. During a time interval Δt, all electrons will travel along circular arcs of length $l \approx c\Delta t$. The radii of the various arcs are $R_i \sim \gamma_i^{-1}$. For the ith electron $\gamma_i = \gamma + \Delta\gamma_i$, where γ is the average electron energy. It is shown in Appendix 2 that when $\Delta\gamma_i/\gamma \ll 1$, and $R_i \gg c\Delta t$, then during the time Δt, the electron will fall behind an electron with average energy by a distance.

$$\Delta l_i \approx l\frac{1}{4}\left(\frac{l}{R}\frac{\Delta\gamma_i}{\gamma}\right)^2 = l\frac{1}{4}\left(\varphi\frac{\Delta\gamma}{\gamma}\right)^2, \qquad (21)$$

where l and R refer to average-energy electrons.

When the parameters are chosen as before Eq. (20), then $R = 1.21 \cdot 10^3$ cm, and while travelling a distance $l = \lambda_e$, $\varphi \approx 2\alpha_M = 10^{-3}$ rad. After passing through the laser field $\Delta\gamma_i/\gamma \leq 6.66 \cdot 10^{-3}$. Assume $\Delta\gamma_i/\gamma = 6.7 \cdot 10^{-3}$ for some electron, and let that electron travel a distance $l = \lambda_e$ in this magnetic field. Equation (21) shows that for this electron $\Delta l_i \approx 1.16 \cdot 10^{-10}$ cm. Since there are 79 sections of length λ_e in the energy modulator, during one passage through all of them the electron would fall behind by $\Delta l_{\text{tot}} \approx 9.2 \cdot 10^{-9}$ cm, a negligible distance on the scale of λ_0. Actually, the Δl_{tot} is lower, since in reality the electrons enter the energy modulator with $\Delta\gamma_i = 0$, and can acquire the here-assumed $\Delta\gamma_i$ only the instant before exiting. Clearly, no significant bunching will be caused by the energy modulator.

To induce bunching, the beam is led through a section referred to in the following as the "buncher." Our aim here is to demonstrate feasibility, as opposed to selecting the parameters best suited for any particular ring (such as SPEAR or PEP). Therefore, we discuss a particularly simple buncher: constant and section-wise uniform magnetic field whose direction is perpendicular to the velocity of the entering electrons. The magnitude B of the magnetic field is the same in all sections, but its direction is reversed between adjacent sections. To simplify our discussion, we assume that there are only two sections each 6.9 m long, and that B is relatively low; 1.24 kG, implying $R = 1.24 \cdot 10^4$ cm. Now $l/R \approx 5.7 \cdot 10^{-2}$, so that substitution in Eq. (21) gives $\Delta l \approx \frac{1}{4}10^{-4}$ cm $= \lambda_0/4$. As we will see, this is more than sufficient for our purposes.

In practice, one would like to reduce R. That can be done by increasing B, but at higher B values, synchrotron radiation is more intense and may interfere with the bunching process (see below). For such large B values this effect has to be taken into account, requiring a somewhat more sophisticated calculation which we do not intend to carry out here.

Next, we calculate the beam density distribution after bunching. From Eqs. (17) and (21), when t_0 is so chosen that $\omega t_0 + \varphi_0 = 0$, one finds

$$\Delta l \approx \frac{l}{4}\left(\frac{l}{R}\right)^2 \gamma^{-2}(eE_0L\tfrac{1}{2}\alpha_M)^2 \cos^2\frac{\omega}{\tilde{v}_z}z_0$$

$$\equiv \Delta l_0 \cos^2\frac{\omega}{\tilde{v}_z}z_0. \qquad (22)$$

Define the point $z_{00} \equiv (c/\omega)(\pi/4)$, and write $z_0 = z_{00} + \Delta z_0$. For $z_0 \ll \lambda_0$,

$$\cos^2\frac{\omega}{\tilde{v}_z}z_0 \approx \frac{1}{2} - \frac{1}{2}\frac{\omega}{\tilde{v}_z}\Delta z_0 + \cdots. \qquad (23)$$

While passing through the buncher, all electrons in the vicinity of z_{00} are deplaced by $\frac{1}{2}\Delta l_0$, and in addition, by an amount which is proportional, but has opposite sign, to Δz_0. Therefore, apart from an overall "background shift" of $\frac{1}{2}\Delta l_0$, all electrons in the vicinity of z_{00} are moved towards z_{00}. In fact, if

$$\Delta l_0 \frac{1}{2}\frac{\omega}{\tilde{v}_z} = 1, \qquad (24)$$

then all electrons in the vicinity of z_{00} will be concentrated at z_{00}, within the accuracy of our approximation, i.e., up to a term

$$\left(\frac{2}{3}\frac{\omega}{\tilde{v}_z}\Delta z_0\right)^3 \xrightarrow[\tilde{v}_z \to c]{} \left(\frac{4\pi}{3}\frac{\Delta z_0}{\lambda_0}\right)^3.$$

Thus, all electrons for which $\Delta z_0/\lambda_0 \leq 2.7 \cdot 10^{-2}$ will be concentrated within a distance $10^{-3}\lambda_0$ of z_{00}, resulting in a 27-fold increase in density there. Similarly, all electrons with $\Delta z_0/\lambda_0 \lesssim 5.5 \cdot 10^{-2}$, will congregate to within a distance $10^{-2}\lambda_0$ of z_{00}, implying a 5.5-fold density increase.

One consequence of the nonlinear density increase near z_{00} is that even if the initial beam density is low, after bunching it may become high enough for significant coherent effects to appear. The achievable densities depend on the properties of the buncher. A full study of those properties cannot be carried out in this paper. Nevertheless, it should be recognized that Eq. (21) is not universally true; different bunchers would lead to different equations. Adjustment of the buncher will change the emitted spectrum, a great convenience in actual applications..

A second consequence of the nonlinear density increase near z_{00} is that the beam density will include Fourier components with wavelength $\lambda \ll \lambda_0$. Therefore, as the beam passes through a magnetic field, significant coherent synchrotron radiation will be emitted with wavelength $\lambda \ll \lambda_0$, even for λ_0/λ of the order of 10^2 or 10^3.

So far we have neglected the reaction of the emitted synchrotron radiation photons on the electron beam. This reaction will tend to counteract the development of bunches. The question arises: Will it noticeably reduce the effectiveness of the bunching mechanism here proposed? First of all, note that it cannot have any effect on the electrons in the energy modulator, since we have shown that no significant bunching occurs in that device, so that coherent emission of synchrotron radiation cannot take place there. One can suppress such radiation from the buncher, by bunching a low-density (as in a high-beta section) beam. Furthermore, synchrotron radiation is proportional to R^{-2}, so that it can be further reduced by increasing R. After the buncher, the beam cross section is reduced (as in a low-beta section) and the particles enter the radiator section where coherent synchrotron radiation is emitted. Here the reaction force due to coherently emitted photons will be unavoidably high. Nevertheless, debunching can, in principle, be made arbitrarily small, since by Eq. (21), Δl will be arbitrarily small for any $\Delta\gamma$ if l/R is small enough.

As an example, let the magnetic field in the radiator section be constant and sectionwise homogeneous, with magnitude B, its direction being reversed between neighboring sections.† Let each section be 1 cm long, and choose $\gamma = 10^4$, $B = 10$ kG, so that $R = 15$ m and $l/R = 6.7 \cdot 10^{-4}$. Assume that the horizontal and vertical beam diameters are $a = 3 \cdot 10^{-2}$ cm, and $b = 10^{-3}$ cm, respectively, and $\rho = 7.8 \cdot 10^{15}$ cm^{-3}. Consider radiation with $\lambda = 10^{-6}$ cm, i.e. with $\lambda' = 10^{-2}$ cm. The number of electrons within a sphere of radius $\lambda'/8$ around z_{00}, after bunching will be

$$n \approx a\pi(\lambda'/8)^2 \rho\gamma^{-1} \cdot 5.8 = 2.8 \cdot 10^3; (\lambda' = 10^{-6}\,\text{cm}), \tag{25}$$

where 5.8 is the increase in density due to bunching, as discussed after Eq. (24). Calculating in the same way, $n \approx 5.9 \cdot 10^4$ for $\lambda = 10^{-5}$ cm, and $n \approx 5 \cdot 10^6$ for $\lambda = 10^{-4}$ cm. The beam density has no Fourier components with $\lambda > 10^{-4}$ cm and no coherence effects can occur for such wave lengths. Clearly, the main contribution to the reactive force (due to the emission of photons) will come from photons with $\lambda \approx 10^{-4}$ eV. In Appendix 1, an expression is given for an upper limit on this force: $5 \cdot 10^6$ times the force caused by photons with wavelengths 10^{-5} cm $\leq \lambda \leq 10^{-4}$ cm, radiated noncoherently. The spectrum of noncoherent synchrotron radiation emitted by 5 GeV photons in a 10 kG magnetic field shows that† force to be about 10^{-3} times the total reactive force acting on the electrons. The total reactive force in SPEAR causes approximately 1 MeV energy loss per turn. Thus, due to coherent emission, the reactive force will be less than about $5 \cdot 10^6 \cdot 10^{-3} = 5 \cdot 10^3$ times the usual force; the electrons will lose less than 0.5 MeV/cm in the radiator. Even if the electrons travel a total of 30 cm in the radiator, their final change in energy due to the reaction force will cause $\Delta_r\gamma/\gamma < 3 \cdot 10^{-3}$, and from Eq. (21) $\Delta l \lesssim 2.7 \cdot 10^{-8}$ cm, a negligible amount. The point to remember is that Δl depends sensitively on the geometry of the path and for suitably chosen orbits it can be very small.

In practice, the electrons in a storage ring have a non-zero energy spread $\Delta\mathscr{E}_e$. In SPEAR, at 5 GeV, the (rms) $\Delta\mathscr{E}_e \lesssim 6$ MeV, considerably less than W given by Eq. (20). Thus, we expect a background density term of less than 50%, (of the total density) caused by this spread. It would be desirable to calculate more carefully the collective effects exerted by the electrons on the beam, but here we will not do so.

The mechanism by which an electron gains (or loses) energy in the energy modulator is precisely that which is employed in the so-called "undulator" devices. These play an important role in electronics[5] and could in principle be used to generate coherent (between photons) radiation, or to build accelerators.[6,7] They have so far not been found useful in connection with high-energy electrons, because at this time only relatively small energy gain (or loss) $\Delta\gamma/\gamma$ can be imparted in this manner to high-energy electrons. The device suggested in this paper does not require high $|\Delta\gamma/\gamma|$; or very high power lasers.‡ It makes use of the short-

† At the present time, there is no magnet in the low beta section of SPEAR but one could be placed there.

† See Winick, SLAC-1439, June, 1974.

‡ Large lasers today are able to produce bursts whose energy is at least two orders of magnitude higher than the value used here.

wavelength coherent waves available in lasers to induce the small-scale beam-density variations necessary for coherent enhancement of the radiation emitted by high-energy electrons. The increased radiation emitted is caused by these short scale beam density variations. By contrast, in single-particle undulators, for fixed λ_e, the wave length λ of the enhanced radiation is always equal to the wave length λ_0 of the electromagnetic wave in the undulator† (except when the electron orbit is not sinusoidal). The device described in this paper is capable of enhancing the radiation intensity by several decades, even for $\lambda \ll \lambda_0$. At the end of Ref (8), this device was already alluded to.‡

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Appendix 1

The following simple argument gives an upper limit on the force exerted by the emitted synchrotron-radiation photons on the emitting electron beam.

First consider a single particle of charge e accelerating in K'. By energy conservation, the energy ΔW_γ of all photons emitted by it is equal to ΔW_r, the kinetic energy lost by the emitting particle. Since $\Delta W_\gamma \sim e^2$,

$$\Delta W_r \sim e^2. \tag{A1-1}$$

Next, consider N particles each with charge e, all located within a sphere of radius $r' \ll \lambda'$. The kinetic energy lost by the 1st particle, $\Delta_1 W_r$, is the sum of N terms: $\Delta_{11} W_r$ is due to the field of photons emitted by the 1st particle itself, $\Delta_{12} W_r$ is caused by the field emitted by the 2nd particle, ..., $\Delta_{1N} W_r$ caused by the Nth particle. Similarly, the kinetic energy lost by the kth particle as a result of emitting synchrotron radiation will be (in analogous notation)

$$\Delta_k W_r = \sum_{i=1}^{N} \Delta_{ki} W_r, \tag{A1-2}$$

and the total energy loss of all the particles

$$\Delta_{\text{tot}} W_r = \sum_{k,i=1}^{N} \Delta_{ki} W_r. \tag{A1-3}$$

In the limit when $r' \to 0$, the field due to any particle at the position of any other particle will obviously be equal: $\Delta_{ki} W_r = \Delta_{lj} W_r = \Delta W_r$ for all k, i, l and j, so that

$$\Delta_{\text{tot}} W_r \to N^2 \Delta W_r \sim (Ne)^2, \tag{A1-4}$$

$$\Delta_k W_r \to N \Delta W_r \sim Ne^2. \tag{A1-5}$$

Equation (A1-4) was expected, since when $r' \to 0$, all particles are located at the same point, the collection of N particles can be considered to be a single particle of charge Ne.

Equation (A1-5) gives an upper limit on the kinetic energy lost by the kth particle, caused by the emission of synchrotron radiation photons by the whole collection of particles. The limit is reached when $r' \to 0$, and all fields are in phase. In practice, $r' \neq 0$, the fields are not all in phase, and

$$\Delta_k W_r < N \Delta W_r. \tag{A1-6}$$

The energy ΔW_r is known. An accurate calculation of $\Delta_k W_r$ for all k would be a tedious task, but in this paper we do not need it; a knowledge of the upper limit stated by Eq. (A1-6) will suffice.

† It is possible to imagine a device with a short, almost delta-function like magnetic field, which would then necessarily have short wavelength components, but its very shortness would make it inefficient from many points of view.

‡ After submission of this paper for publication, I learned that Skrinsky and Vinokurov made calculations along similar lines, but seemed to have obtained different results. I did not yet have the opportunity to read their work: Skrinsky and Vinokurov, "The Optical Klystron Using Ultra Relativistic Electrons," Novosibirsk, 1977.

Appendix 2

One should resist the temptation of claiming that after one of the electrons travelled a distance l, the relative displacement between electrons, Δl, will satisfy $\Delta l/l = \Delta R/R = \Delta\gamma/\gamma$. Actually, Δl can be calculated as follows: Let the first and second electron have $\gamma_1 \gg 1$, and $\gamma_2 \gg 1$, respectively. In a uniform magnetic field, both electrons will travel along circles with radius R_1 and R_2 respectively. Clearly, $R_1/R_2 = \gamma_1/\gamma_2$. Let both electrons start out from the same point A at time $t = 0$, moving parallel to each other with velocities v_1 and v_2 respectively. After a time Δt, the first electron will have travelled to point B, along a circular arc of length l_1. The arc spans an angle $\varphi_1 = l_1/R_1$, as seen from the center of the circle. During the same time, the second electron will travel to point C along a circular arc of essentially the same length (since $v_1 \approx v_2 \approx c$), spanning an angle $\varphi_2 \approx l_1/R_2$ as seen from the center of that circle. Now $\varphi_1/\varphi_2 = \gamma_2/\gamma_1$. At time Δt, the angle between the momenta of the two electrons will be $\Delta\varphi = \varphi_2 - \varphi_1$. The angle between the vectors $\overline{AB}$ and $\overline{AC}$ is $\Delta\varphi_2$. Denote by h_1 the length of $\overline{AB}$, and by h_2 that of $\overline{AC}$. When $\varphi_1 \ll 1$ and $\varphi_2 \ll 1$, one may write up to second order in φ inclusive:

$$h_i = l_i(1 - \tfrac{1}{8}\varphi_i^2); \qquad i = 1, 2, \qquad \text{(A2-1)}$$

and, since $l_1 \approx l_2$,

$$\Delta h \equiv h_2 - h_1 \approx l_1\tfrac{1}{4}\Delta\varphi(\varphi_1 + \tfrac{1}{2}\Delta\varphi). \quad \text{(A2-2)}$$

Denote the perpendicular projection of point B onto the arc AC by D. Let D' be the intersection point of the straight line going through B and D, with the straight line AC. We want to calculate the distance CD. For small φ_2, this distance is equal to the distance CD', up to terms of order φ_2^2. Thus, for small φ_2, it suffices to evaluate CD'. Let E be the perpendicular projection of B onto the straight line AC. The distance CD' is the sum of two terms. The first is the distance $D'E$, the second is the distance EC. To third order in angles.

$$D'E \approx h_1\tfrac{1}{4}\varphi_2\Delta\varphi, \qquad \text{(A2-3)}$$

$$EC \approx h_1\tfrac{1}{8}(\Delta\varphi)^2 - \Delta h, \qquad \text{(A2-4)}$$

so that with Eq. (A2-2), to lowest order

$$D'C \approx l_1\tfrac{1}{4}(\Delta\varphi)^2 = R_1\varphi_1\tfrac{1}{4}\left(\varphi_1\frac{\Delta\gamma}{\gamma}\right)^2: \quad \text{(A2-5)}$$

REFERENCES

1. J. Schwinger, *Phys. Rev.*, **75**, 1912 (1949); D. H. Tomboulian and D. E. Bedo, *J. Appl. Phys.*, **29**, 804 (1953); D. H. Tomboulian and P. L. Hartman, *Phys. Rev.*, **102**, 1423 (1956).
2. Paul L. Csonka and Bernd Crasemann, Possibility of Coherent X-Ray Production by X-Ray Pumping, University of Oregon, I.T.S. preprint, N.T.054/74 (1974), *Phys. Rev. A*, **12**, 611 (1975); Paul L. Csonka, A Suggested Method of Coherent X-Ray Production by Combined X-Ray and Low Frequency Photon Pumping, University of Oregon, I.T.S. preprint, N.T. 055/74 (1974), *Phys. Rev. A*, **13**, 405 (1976).
3. Paul L. Csonka, Enhancement of Synchrotron Radiation by Beam Modulation, University of Oregon, I.T.S. Preprint N.T. 059/75 (1975); Some Coherence Effects with Wigglers and Their Applications, in "Wiggler Magnets" ed. H. Winick and T. Knight, SSRL Report 77/05 (1977) (Material presented at the Wiggler Workshop, SLAC, March 21–23, 1977).
4. E. M. McMillan, *Phys. Rev.*, **79**, 498 (1950).
5. R. M. Phillips, I.R.E. Transactions on Electron Devices 231 (1960). The undulator is named "ubitron" in this paper.
6. A. A. Kolomenskii and A. N. Lebedev, *Zh. Eksp. Teor. Fiz.*, **44**, 261 (1963) [*Sov. Phys.-JETP*, **17** (1963)].
7. R. B. Palmer, *J. Appl. Phys.*, **43**, 3014 (1972).
8. Paul L. Csonka, *Particle Accelerators*, **7**, 225 (1976).