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About the probability of close collisions during stochastic deflection of positively charged particles by a bent crystal

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ABSTRACT

The probability of close interactions of high-energy positively charged particle with atoms in a bent crystal was considered as a function of the angle between the initial particle momentum and the bending plane. The results of simulation of particle motion presented in the article show the great efficiency of high-energy positively charged particle deflection by a bent crystal due to the stochastic deflection mechanism and strong reduction of the probability of close collisions during the stochastic deflection in comparison to the planar channeling in a bent crystal.

1. Introduction

When a high-energy charged particle impinges on a crystal with a small angle ψ^{in} with respect to one of the main crystallographic axes (z-axis), correlations between consecutive collisions of the particle with lattice atoms appear. As a result of these correlations, the particle motion in the crystal is basically defined by the continuous potential of atomic strings parallel to the z-axis. Due to strong intra-crystalline fields, the direction of a high-energy charged particle motion can be changed within quite small distances. The passage of high-energy charged particles through a bent crystal is of particular interest because, in this case, it is possible to deflect the particle beam direction with a small-sized crystal. There are several mechanisms of deflection of high-energy charged particles by a bent crystal connected with finite (channeling) and infinite (above barrier) motion in relation to bent atomic strings or bent crystal atomic planes. These mechanisms are realized in the planar and axial channeling in a bent crystal, in the stochastic mechanism of beam deflection (connected with multiple scattering by bent crystal atomic strings) and volume reflection from bent crystal atomic planes.

The stochastic mechanism of beam deflection, that is connected with the phenomenon of dynamical chaos [1] in particle scattering on crystal atomic strings, was predicted in [2] and experimentally confirmed in [3] for positively and in [4] for negatively charged particles. This mechanism allows to deflect both positively and negatively charged particles on angles much higher than the critical angle of axial channeling $\psi_c = \sqrt{4Z|qe|/pvd}$, where e is the electron charge, $Z|e|$ is the crystal atomic charge, q, p and v are the particle charge, momentum and velocity respectively and d is the distance between neighboring atoms in the atomic string parallel to the z-axis. In [5] it was shown that by means of stochastic deflection it is possible to deflect particles up to the angle of

$$\alpha_{cr} = \frac{2R\psi_c^2}{l_0}, \tag{1}$$

where R is the radius of crystal curvature, $l_0 = 4/(\pi^2 n d R_a \psi_c)$, n is the concentration of atoms in the crystal, R_a is the atomic screening radius. This condition was found without an account of incoherent processes in particle scattering, that is why the maximum deflection angle in (1) depends only on particle energy and crystal radius of curvature and does not depend on crystal thickness. The most of incoherent processes take place when particle

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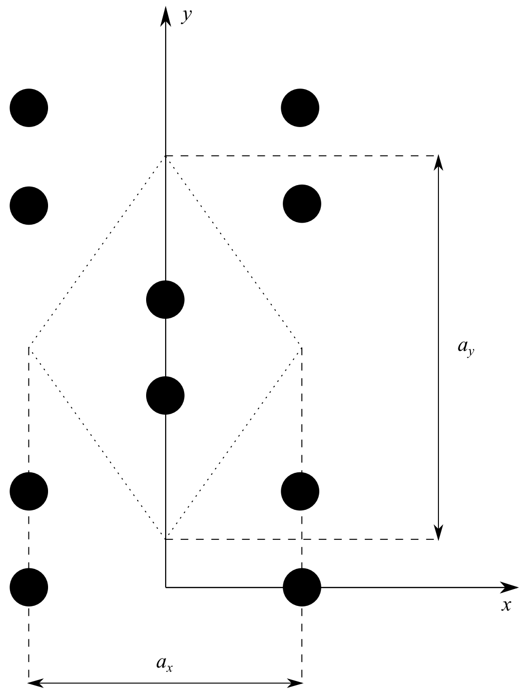

Fig. 1. Positions of crystal atomic strings that are located parallel to ⟨110⟩ crystal axis.

come close to the atomic nuclei. Recent experiments [6,7] showed that in the case of planar channeling in a bent crystal the probability of incoherent scattering on atomic nuclei is smaller than in the case of amorphous crystal orientation. In this article we study the probability of incoherent processes connected with close collisions for the case of stochastic deflection of positively charged particles by a bent crystal.

2. Probability of close collisions

Let us carry out the consideration of the probability of close collisions for positively charged high-energy particles in a bent crystal on the example of particles impinging on bent Si crystal oriented near ⟨110⟩ crystal axis. Axis ⟨110⟩ was chosen for consideration because this axis has the greatest value of ψ_c among other axes of Si crystal, which allows to obtain high possible particle deflection angle (1). As the x-axis we assume the axis lying in the crystal plane (001). Thus, y-axis will lye in the initial position of the $(1\bar{1}0)$ crystal plane. Fig. 1 shows positions of crystal atomic strings that are located parallel to ⟨110⟩ crystal axis. Dotted line shows the edge of elementary cell in this orientation, $a_x = a\sqrt{2}/2$, $a_y = a$, where a is the lattice constant (5.431 Å for Si crystal). The direction of crystal bending coincides with the direction of x-axis.

Moving in the field of straight crystal atomic strings particle has orthogonal energy

$$\varepsilon_\perp = \frac{E\psi^2}{2} + U(x, y), \tag{2}$$

where ψ is the angle between particle momentum and the z-axis, E is particle energy, $U(x, y)$ is particle potential energy. Without an account of incoherent scattering $\varepsilon_\perp$ is the integral of motion. In this case the area of particle motion is defined by the initial orthogonal energy. The boundary of this area is defined by the relation $U(x_b, y_b) = \varepsilon_\perp$. The closest possible distance $\rho_{\min}$ between particle and crystal atomic string is thus $\rho_{\min} = \min\sqrt{(x_b - x_s)^2 + (y_b - y_s)^2}$, where (x_s, y_s) are the coordinates of the nearest to (x_b, y_b) crystal atomic string. If $\rho_{\min}$ is less than the rms atomic thermal vibration amplitude in one direction r_T then particle has high probability to be incoherently scattered by atomic nuclei. If $\rho_{\min} > r_T$ then the probability of close collisions and thus incoherent scattering on atomic nuclei is much less.

When particles impinge on a crystal with $\psi^{in} = 0$ (that corresponds to the initial conditions of the stochastic deflection in the case of a bent crystal) their orthogonal energy is equal to the potential energy in the field of crystal atomic strings. Thus the probability of close collisions in crystal is high only for particles which initial distance to the closest atomic string was less than r_T. If impinging particles are uniformly distributed in the elementary cell described in Fig. 1 the ratio between particles with initial distance to the closest atomic string less than r_T and the total number of particles is equal to the ratio between the area of two circles with radius r_T and the area of the elementary cell

$$w_a = \frac{4\pi r_T^2}{a_x a_y} = 4\sqrt{2}\pi r_T^2/a^2 \approx 3.39 * 10^{-3}. \tag{3}$$

Now let us consider the case of planar channeling in the plane $(1\bar{1}0)$. The ratio between particles with high probability of close collisions and the total number of particles in this case is

$$w_p = \frac{4r_T}{a_x} = 4\sqrt{2}r_T/a \approx 78.12 * 10^{-3}. \tag{4}$$

Comparing (3) and (4) we see that in the axial case the probability of incoherent scattering connected with close collisions is much lower than in the planar case.

For the case of a bent crystal the orthogonal energy of particle in the reference system connected with bent crystal axis is

$$\varepsilon_\perp = \frac{E\psi^2}{2} + U(x, y) + E\left(\frac{\mathfrak{R}}{R} - 1\right), \tag{5}$$

where $\mathfrak{R}$ is the distance between particle and the center of crystal curvature, ψ is the angle between particle momentum and current direction of z-axis. The only distinction of (5) in comparison with (2) is in the last summand which is centrifugal energy. This term in both axial and planar cases will increase the probability of close collisions. However, in the case of a bent crystal the calculation of ratios (3) and (4) is not as easy as in the case of a straight crystal because not for all particles $\varepsilon_\perp$ is a constant in a bent crystal. To make a comparison between the probability of close collisions in the case of stochastic deflection and planar channeling in a bent crystal we carried out numerical simulation of high-energy charged particles motion through the bent crystal on the example of 270 GeV/c protons passing 5 mm of Si crystal with radius of curvature 50 m. For the considering particle energy the critical angle of axial channeling is 28 µrad. Therefore, for the considering case using Eq. (1) we obtain that the maximum angle α_{cr} up to which most of the beam particles will be deflected by the bent crystal is about 450 µrad which is more than four times greater than the angle of crystal curvature. Thus, from simulation we expected deflection of most of the beam particles to the crystal bending angle of 100 µrad. The simulation was carried out using the same method as in [8]. In the simulation code we considered incoherent scattering connected with atomic thermal oscillations and scattering on electronic subsystem of the crystal.

First let us consider the angular distribution of particles after passing the crystal that is shown in Fig. 2. Colors in this figure show the beam intensity distribution in logarithmic scale. We see that angular distribution is rather complicated. After passing the crystal the beam splits into several parts. The main part is deflected at the crystal bending angle. This part is good collimated in both x and y directions. These are particles that are deflected due

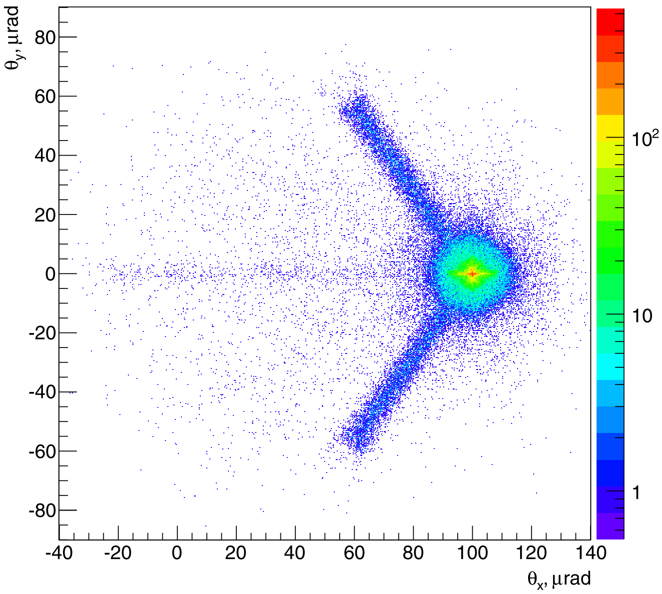

Fig. 2. 270 GeV/c protons angular distribution after passing 5 mm of Si crystal with radius of curvature 50 m. (For interpretation of the colors in this figure, the reader is referred to the web version of this article.)

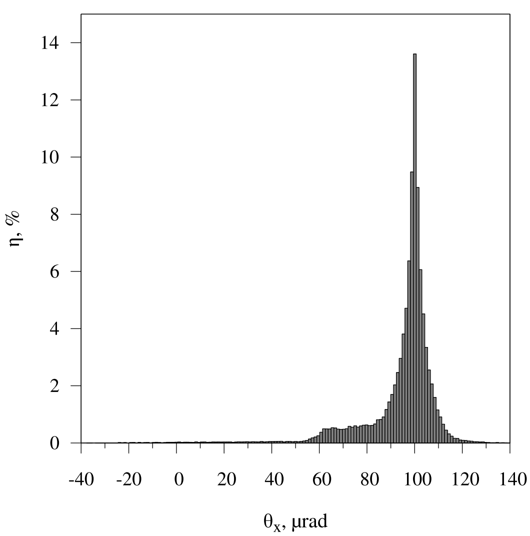

Fig. 3. Profile of the angular distribution shown in Fig. 2.

to the stochastic deflection mechanism. Another group of particles are particles that were moving in crystal in the regime of planar channeling in skew (i.e. not vertical) planes. The main skew planes for considered crystal orientation are $(1\bar{1}1)$ and $(\bar{1}11)$. In Fig. 3 one can see the horizontal profile of the angular distribution shown in Fig. 2. This profile shows how many percent of beam particles were scattered by the bent crystal in the direction of $(\theta_x, \theta_x + \Delta\theta)$, where $\Delta\theta$ is the step of diagram (for Fig. 3 $\Delta\theta = 8.3$ μrad). From this beam profile we can see that collimation of the deflection particles is close to ψ_c and the number of particles in skew planes is around 15%. Also from Figs. 2 and 3 we see that almost all particles were deflected by the bent crystal in the same direction (in the direction of crystal bend).

Now let us consider the simulation results of the probability of close collisions of protons in the bent crystal. For this in simulation we integrated the nuclear reaction probability along particle trajectories considering nuclear reactions to be proportional to the

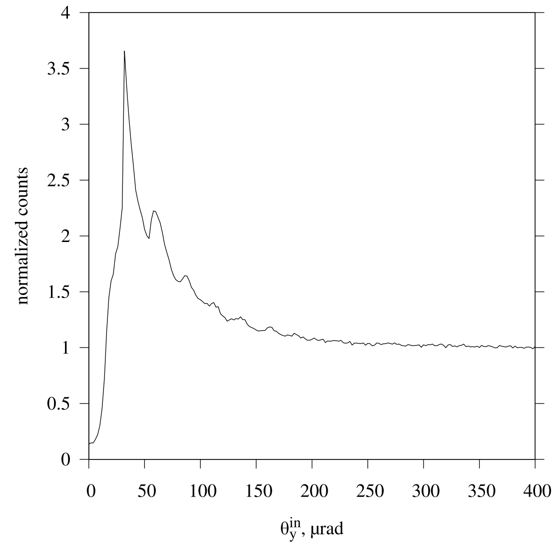

Fig. 4. The probability of incoherent scattering of protons in the crystal.

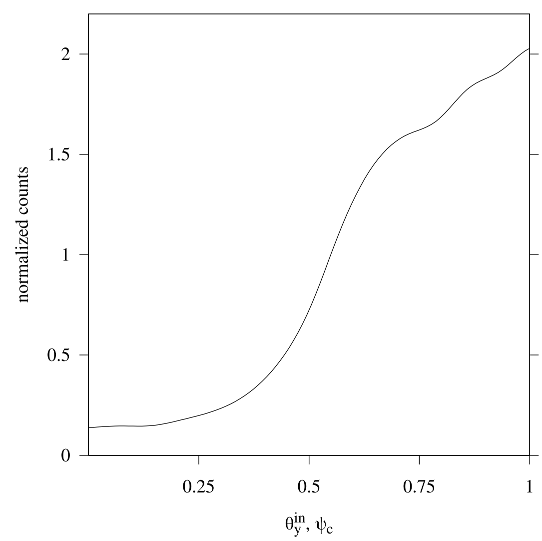

Fig. 5. The probability of incoherent scattering of protons in the crystal in the angular region $(0, \psi_c)$.

probability of atomic nuclei location in each point of the trajectory. We used the model of normal distribution of atomic nucleus location near the lattice site

$$w_n(\vec{r}) = \frac{1}{\sqrt{2\pi r_T^2}} \exp\left(-\frac{(\vec{r} - \vec{r}_n)^2}{2r_T^2}\right),$$

where r_T is the rms atomic thermal vibration amplitude in one direction and vector $\vec{r}_n$ gives the coordinates of the lattice site. Then we summed the obtained time over all beam particles. If the angle ψ^{in} between particle initial momentum and the z-axis in simulation was equal to zero the obtained results would have corresponded to the stochastic deflection mechanism, while if the angle between particle initial momentum and the plane $(1\bar{1}0)$ θ_x^{in} was equal to zero and the angle between particle initial momentum and the plane (001) θ_y^{in} was much greater than ψ_c the obtained results would have corresponded to the planar channeling in the bent crystal. Fig. 4 shows the results of simulation of the probability of close collisions of protons in the bent crystal for different angles θ_y^{in} while angle θ_x^{in} was equal to zero. In this figure we normalized the probability to the probability in the case of planar channeling. We see that in the case of stochastic deflection the probability of close collisions is several times smaller than in the case of planar channeling. The grows of probability in the region near 50 μrad is because the increase of particle orthogonal energy leads to the decrease of the area that is forbidden for particle motion. However, the increase of particle orthogonal energy also leads to the capture of particles in the bent planar channel $(1\bar{1}0)$ and thus to the reduction of the probability of close collisions. Because of this starting approximately from 50 μrad the probability starts to decrease and at $\theta_y \approx 10\psi_c$ becomes a constant.

Fig. 5 shows the same normalized probability of close collisions as Fig. 4 for θ_y^{in} in $(0, \psi_c)$. We see that in the angular region $(0, \psi_c/2)$ the probability of close collisions is smaller than in the case of planar channeling.

3. Conclusions

Simulation results presented in the article show two interesting features of the stochastic particle deflection. The first one is the great efficiency of high-energy positively charged particle deflection by a bent crystal due to the stochastic deflection mechanism. The second is the strong reduction of the probability of close collisions during the stochastic deflection in comparison to the planar channeling in a bent crystal. These two facts indicate that stochastic deflection can be used for beam extraction or cleaning of a beam halo in accelerators.

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