

10 HIGGSLESS MODELS

10.1 Introduction

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Recent developments in string theory have definitely had an impact of phenomenological model building. The possible existence of branes in large extra dimensions has opened up new classes of theories, especially in the area of electroweak symmetry breaking. The anti-de Sitter/conformal field theory (AdS/CFT) correspondence led to the Randall-Sundrum (RS) model [1], see Section 9, which allows for a new approach to the hierarchy problem. Thus discovering inverse TeV sized extra dimensions at the LHC has become a tantalizing possibility.

The existence of inverse TeV sized dimensions themselves allow for a completely new way to break electroweak symmetry: boundary conditions in the extra dimension [2]. Since this mechanism is intrinsically extra dimensional it leads to a very different phenomenology from the standard Higgs mechanism. In fact, the Dirichlet boundary condition required to break the gauge symmetry can be thought of as arising through the limit of a Higgs with an infinite VEV. Since the Higgs mass is of the order of its VEV, we see that boundary condition breaking is effectively a class of Higgsless models for electroweak symmetry breaking.

At asymptotically high energies it can be shown that the terms in the WW scattering amplitude that grow with energy are cancelled by the exchange of W Kaluza-Klein (KK) modes [2, 3]. Quark and lepton masses can also arise through boundary conditions [4]. In a warped AdS background (like the RS model) a custodial symmetry can ensure the correct ratio for the W and Z masses [5]. Most corrections to precision electroweak measurements and Z' couplings can be suppressed if the quarks and leptons are approximately uniformly spread out in the extra dimension.

However not everything is rosy in Higgsless models. If the W' and Z' resonances are too heavy (roughly > 1 TeV) then WW scattering becomes strongly coupled [6–8]. Also implementing a mechanism to produce the top quark mass without messing up the $Zb\bar{b}$ coupling is quite difficult.

The most obvious implication of this model for colliders is that no physical Higgs state will be found. However, there are many new positive signals that can be searched for. In particular the Kaluza-Klein states of the gauge bosons will be easily visible in the Drell-Yan and dijet channels [8], and analysis of longitudinal gauge boson scattering can directly probe the Higgsless mechanism of electroweak symmetry breaking [9].

In the following we will review the requirements for maintaining perturbative unitarity, the constraints imposed by precision electroweak measurements, and the collider signatures.

10.1.1 KK mode couplings

We will mainly focus on the most interesting case of a warped extra dimension. We will use the conformally flat metric

$$ds^2 = h(z)^2 \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2 \right) \quad (10.1)$$

where the extra spatial dimension z is on the interval $[R, R']$. A flat extra dimension can be recovered by taking $h(z) = \text{constant}$, while AdS is obtained by taking $h(z) = R/z$. If one is uncomfortable with a non-renormalizable 5D theory, one can always deconstruct the theory [10, 11] which provides a renormalizable 4D ultraviolet completion of the theory with the same low-energy (TeV) predictions.

The 5D gauge boson decomposes into a 4D gauge boson A_μ^a and a 4D scalar A_5^a in the adjoint representation. Since there is a quadratic term mixing A_μ and A_5 we need to add a gauge fixing term that eliminates this cross term. Thus (using $\sqrt{-g} = h^5(z)$) we write the action after gauge fixing in R_ξ

gauge as

$$\mathcal{S} = \int d^4x \int_R^{R'} dz h(z) \left(-\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} - \frac{1}{2} F_{5\nu}^a F^{a5\nu} - \frac{1}{2\xi} (\partial_\mu A^{a\mu} - \xi \partial_5 A_5^a)^2 \right), \quad (10.2)$$

where $F_{MN}^a = \partial_M A_N^a - \partial_N A_M^a + g_5 f^{abc} A_M^b A_N^c$, and the f^{abc} 's are the structure constants of the gauge group. The gauge fixing term is chosen such that (as usual) the cross terms between the 4D gauge fields A_μ^a and the 4D scalars A_5^a cancel (see also [12]). Taking $\xi \rightarrow \infty$ will result in the unitary gauge, where all the KK modes of the scalars fields A_5^a are unphysical (they become the longitudinal modes of the 4D gauge bosons), except if there is a zero mode for the A_5 's. We will assume that every A_5^a mode is massive, and thus that all the A_5 's are eliminated in unitary gauge.

The variation of the action (10.2) leads, as usual after integration by parts, to the bulk equations of motion as well as to boundary terms (we denote by $[F]$ the boundary quantity $F(R') - F(R)$):

$$\begin{aligned} \delta\mathcal{S} = & \int d^4x dz \left(\partial_M h F^{aM\nu} - g_5 f^{abc} h F^{bM\nu} A_M^c + \frac{h}{\xi} \partial^\nu \partial^\sigma A_\sigma^a - h \partial^\nu \partial_5 A_5^a \right) \delta A_\nu^a \\ & - \int d^4x dz \left(h \partial^\sigma F_{\sigma 5}^a - g_5 f^{abc} h F_{\sigma 5}^b A^{c\sigma} + \partial_5 h \partial_\sigma A^{a\sigma} - \xi \partial_5 h \partial_5 A_5^a \right) \delta A_5^a \\ & + \int d^4x ([h F_{5\nu}^a \delta A^{a\nu}] + [h(\partial_\sigma A^{a\sigma} - \xi \partial_5 A_5^a) \delta A_5^a]). \end{aligned} \quad (10.3)$$

The bulk terms will give rise to the usual bulk equations of motion:

$$\begin{aligned} \partial_M h F^{aM\nu} - g_5 f^{abc} h F^{bM\nu} A_M^c + \frac{h}{\xi} \partial^\nu \partial^\sigma A_\sigma^a - h \partial^\nu \partial_5 A_5^a &= 0, \\ h \partial^\sigma F_{\sigma 5}^a - g_5 f^{abc} h F_{\sigma 5}^b A^{c\sigma} + \partial_5 h \partial_\sigma A^{a\sigma} - \xi \partial_5 h \partial_5 A_5^a &= 0. \end{aligned} \quad (10.4)$$

However, one has to ensure that the variation of the boundary pieces vanish as well. This will lead to the requirements

$$h F_{\nu 5}^a \delta A^{a\nu}|_{R,R'} = 0, \quad (10.5)$$

$$h(\partial_\sigma A^{a\sigma} - \xi \partial_5 A_5^a) \delta A_5^a|_{R,R'} = 0. \quad (10.6)$$

The boundary conditions (BCs) have to be such that the above equations be satisfied.

For the case of a scalar (Higgs) with a VEV localized at the endpoint the generic form of the BC for the gauge fields (in unitary gauge) will be of the form

$$\partial_5 A_\mu^a|_{R,R'} = V|_{R,R'}^{ab} A_\mu^b h|_{R,R'}, \quad (10.7)$$

where $V|_R^{ab}$ and $V|_{R'}^{ab}$ are proportional to the VEV's squared at R and R' . The BCs in (10.7) are mixed BCs that still ensure the hermiticity (self-adjointness) of the Hamiltonian. In the limit $V^{ab} \rightarrow 0$ the mixed BC reduces to a Neumann BC, while the limit $V^{ab} \rightarrow \infty$ the mixed BC reduces to a Dirichlet BC.

Finding the KK decomposition of the gauge field reduces to solving a Sturm–Liouville problem with Neumann or Dirichlet BCs, or in the case of boundary scalars with mixed BCs. Those general BCs lead to a Kaluza–Klein expansion of the gauge fields of the form

$$A_\mu^a(x, z) = \sum_n \epsilon_\mu \psi_n^a(z) e^{ip_n x}, \quad (10.8)$$

where $p_n^2 = M_n^2$ and ϵ_μ is a polarization vector. These wavefunctions then satisfy the equation:

$$h(z) \psi_n^{a''}(z) + h'(z) \psi_n^{a'}(z) + M_n^2 h(z) \psi_n^a(z) = 0, \quad \psi_n^{a'}|_{R,R'} = h V|_{R,R'}^{ab} \psi_n^b|_{R,R'}. \quad (10.9)$$

The KK mode wavefunctions can be normalized by requiring

$$\int dz h(z) (\psi_n^a(z))^2 = 1. \quad (10.10)$$

The couplings between the different KK modes can then be obtained by substituting this expression into the Lagrangian (10.2) and integrating over the extra dimension. The resulting couplings are then the usual 4D Yang-Mills couplings, with the gauge coupling g_4 in the cubic and gauge coupling square in the quartic vertices replaced by the effective couplings involving the integrals of the wave functions of the KK modes over the extra dimension:

$$g_{cubic} \rightarrow g_{mnk}^{abc} = g_5 \int dz h(z) \psi_m^a(z) \psi_n^b(z) \psi_k^c(z), \quad (10.11)$$

$$g_{quartic}^2 \rightarrow g_{mnkl}^{2abcd} = g_5^2 \int dz h(z) \psi_m^a(z) \psi_n^b(z) \psi_k^c(z) \psi_l^d(z). \quad (10.12)$$

Here a, b, c, d refer to the gauge index of the gauge bosons and m, n, k, l to the KK number.

10.1.2 The elastic scattering amplitude

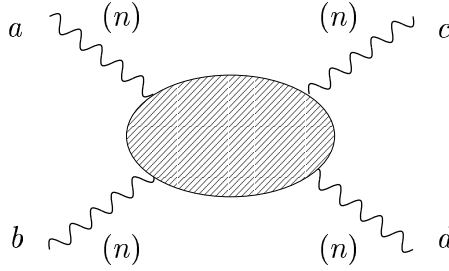


Fig. 10.1: Elastic scattering of longitudinal modes of KK gauge bosons, $n + n \rightarrow n + n$, with the gauge index structure $a + b \rightarrow c + d$.

We want to calculate the energy dependence of the amplitude of the elastic scattering of the longitudinal modes of the KK gauge bosons $n + n \rightarrow n + n$ with gauge index structure $a + b \rightarrow c + d$ (see Fig. 10.1), where this process involves both exchange of the k 'th KK mode from the cubic vertex, and the direct contribution from the quartic vertex. There are four diagrams as shown in Fig. 10.2: the s, t and u-channel exchange of the KK modes, and the contribution of the quartic vertex. The kinematics assumed for this elastic scattering is in the center of mass frame, where the incoming momentum vectors are $p_\mu = (E, 0, 0, \pm\sqrt{E^2 - M_n^2})$, while the outgoing momenta are $(E, \pm\sqrt{E^2 - M_n^2} \sin \theta, 0, \pm\sqrt{E^2 - M_n^2} \cos \theta)$. E is the incoming energy, and θ the scattering angle with forward scattering for $\theta = 0$. The longitudinal polarization vectors are as usual $\epsilon_\mu = (\frac{|\vec{p}|}{M}, \frac{E}{M} \frac{\vec{p}}{|\vec{p}|})$ and accordingly the contribution of each diagram can be as bad as E^4/M_n^4 . It is straightforward to evaluate the full scattering amplitude, and extract the leading behavior for large values energies of this amplitude. The general structure of the expansion in energy contains three terms:

$$\mathcal{A} = A^{(4)} \frac{E^4}{M_n^4} + A^{(2)} \frac{E^2}{M_n^2} + A^{(0)}. \quad (10.13)$$

We would like to understand under what circumstances will $A^{(4)}$ and $A^{(2)}$ vanish, this does not imply that the conventional unitarity bounds on the finite amplitudes have to be satisfied for all processes.

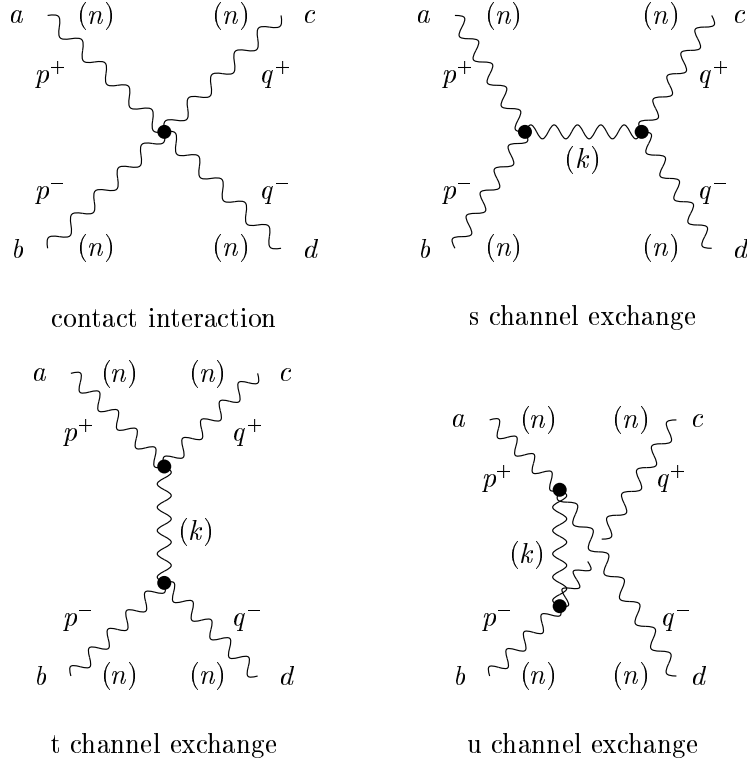


Fig. 10.2: The four gauge diagrams contributing at tree level to the gauge boson elastic scattering amplitude.

The term growing with E^4 depends only on the effective couplings and not on the mass spectrum. The condition for cancelling the coefficient of this term is:

$$g_{nnnn}^2 = \sum_k g_{nnk}^2. \quad (10.14)$$

Using this relation the condition for the E^2 terms to cancel can be simplified to:

$$4g_{nnnn}^2 M_n^2 = 3 \sum_k g_{nnk}^2 M_k^2. \quad (10.15)$$

The goal of the remainder of this section is to examine under what circumstances the terms that grow with energy actually cancel. Consider first the E^4 term. According to (10.14) the requirement for cancellation is

$$\int_R^{R'} dz h(z) \psi_n^4(z) = \sum_k \int_R^{R'} dy \int_R^{R'} dz h(y) h(z) \psi_n^2(y) \psi_n^2(z) \psi_k(y) \psi_k(z). \quad (10.16)$$

One can easily see that this equation is in fact satisfied no matter what BC one is imposing, as long as that BC still maintains hermiticity of the kinetic operator

$$h \partial_z^2 + (\partial_z h) \partial_z. \quad (10.17)$$

In this case one can explicitly check that

$$\int_R^{R'} h \psi_n^* \psi_m' + h' \psi_n^* \psi_m' = \int_R^{R'} h \psi_n^{*''} \psi_m + h' \psi_n^{*'} \psi_m. \quad (10.18)$$

For such hermitian operators one is guaranteed to get an orthonormal complete set of solutions $\psi_k(y)$, thus from the completeness it follows that

$$\sum_k \psi_k(y) \psi_k(z) = \frac{1}{h(z)} \delta(y - z), \quad (10.19)$$

which immediately implies (10.16).

The condition for the cancellation of the E^2 terms is as in (10.15)

$$3 \sum_k M_k^2 \int_R^{R'} dy \int_R^{R'} dz h(y) h(z) \psi_n^2(y) \psi_n^2(z) \psi_k(y) \psi_k(z) = 4 M_n^2 \int_R^{R'} dz h(z) \psi_n^4(z). \quad (10.20)$$

By integration by parts we find (we denote again by $[F]$ the boundary quantity $F(R') - F(R)$)

$$\begin{aligned} \sum_k M_k^2 \left(\int dz h(z) \psi_n^2(z) \psi_k(z) \right)^2 &= \frac{4}{3} M_n^2 \int dz h(z) \psi_n^4(z) - \frac{2}{3} [h \psi_n^3 \psi'_n] \\ &\quad - \sum_k [h \psi_n^2 \psi'_k] \int dz h(z) \psi_n^2(z) \psi_k(z) + 2 \sum_k [h \psi_n \psi'_n \psi_k] \int dz h(z) \psi_n^2(z) \psi_k(z). \end{aligned} \quad (10.21)$$

Thus one can see that for arbitrary BCs the E^2 terms do not cancel. However, if one has pure Dirichlet or Neumann BCs for all modes then all the extra boundary terms will vanish, and thus the cancellation of the E^2 terms goes through [2]. The fact that in the absence of a Higgs VEV, or any other source of gauge symmetry breaking, (i.e. the Neumann BC) there is no problem with unitarity is not really surprising. It is somewhat surprising that with an infinite Higgs VEV (the Dirichlet BC) there is also no problem. To understand what is happening it is useful to recall what actually happens in the general mixed case. Even with mixed BCs there is no problem with unitarity once one includes the diagrams corresponding to the exchange of the Higgs on the boundary. These diagrams cancel the E^2 terms just as they do in 4D. However in the limit that Higgs VEV is large the gauge boson wavefunctions are repelled from the boundary since it costs a lot of energy to reside there. A simple calculation shows that the product of the Higgs VEV times the gauge boson wavefunction squared evaluated on the boundary goes to zero in the large VEV limit. Thus the Higgs decouples in the infinite VEV limit and unitarity is preserved without any need for a physical Higgs boson.

10.1.3 Electroweak gauge bosons

We will now apply the Higgsless idea to electroweak symmetry breaking in a mostly realistic RS type model. We denote by A_M^{Ra} , A_M^{La} and B_M the gauge bosons of $SU(2)_R$, $SU(2)_L$ and $U(1)_{B-L}$ respectively; g_5 is the gauge coupling of the two $SU(2)$'s and \tilde{g}_5 , the gauge coupling of $U(1)_{B-L}$. We impose the following BCs:

$$\text{at } z = R' : \quad \begin{cases} \partial_z(A_\mu^{La} + A_\mu^{Ra}) = 0, & A_\mu^{La} - A_\mu^{Ra} = 0, & \partial_z B_\mu = 0, \\ (A_5^{La} + A_5^{Ra}) = 0, & \partial_z(A_5^{La} - A_5^{Ra}) = 0, & B_5 = 0. \end{cases} \quad (10.22)$$

$$\text{at } z = R : \quad \begin{cases} \partial_5 A_\mu^{La} = 0, & A_\mu^{R1,2} = 0, \\ \partial_z(g_5 B_\mu + \tilde{g}_5 A_\mu^{R3}) = 0, & \tilde{g}_5 B_\mu - g_5 A_\mu^{R3} = 0, \\ A_5^{La} = 0, & A_5^{Ra} = 0, & B_5 = 0. \end{cases} \quad (10.23)$$

The BCs break $SU(2)_R \times U(1)_{B-L}$ down to $U(1)_Y$ on the Planck brane ($z = R$) and break $SU(2)_L \times SU(2)_R$ down to a diagonal $SU(2)$ on the TeV brane ($z = R'$).

The Euclidean bulk equation of motion satisfied by spin-1 fields in AdS space is

$$(\partial_z^2 - \frac{1}{z} \partial_z + q^2) \psi(z) = 0, \quad (10.24)$$

where the solutions in the bulk are assumed to be of the form $A_\mu(q)e^{-iqx}\psi(z)$. The KK mode expansion is given by the solutions to this equation which are of the form

$$\psi_k^{(A)}(z) = z \left(a_k^{(A)} J_1(q_k z) + b_k^{(A)} Y_1(q_k z) \right), \quad (10.25)$$

where A labels the corresponding gauge boson.

To leading order in $1/R$ and for $\log(R'/R) \gg 1$, the lightest solution for eigenvalue equation for the mass of the W^\pm 's is

$$M_W^2 = \frac{1}{R'^2 \log\left(\frac{R'}{R}\right)}, \quad (10.26)$$

while the lowest mass in the Z tower is approximately given by

$$M_Z^2 = \frac{g_5^2 + 2\tilde{g}_5^2}{g_5^2 + \tilde{g}_5^2} \frac{1}{R'^2 \log\left(\frac{R'}{R}\right)}. \quad (10.27)$$

The correct mass ratios (a small T parameter) are guaranteed by the unbroken diagonal $SU(2)$ symmetry on the TeV brane which acts as a custodial symmetry [5].

From the expansion for small arguments of the Bessel functions appearing in (10.25), the wavefunction of a mode with mass $M \ll 1/R'$ can be written as [13]:

$$\psi^{(A)}(z) \approx c_0^{(A)} + M_A^2 z^2 \left(c_1^{(A)} - \frac{c_0^{(A)}}{2} \log(z/R) \right) + \mathcal{O}(M_A^4 z^4), \quad (10.28)$$

with $c_0^{(A)}$ at most of order one, $c_1^{(A)} \sim \mathcal{O}(\log(R'/R))$, and $M^2 \sim \mathcal{O}(1/\log(R'/R))$.

The boundary conditions on the bulk gauge fields give the following results for the leading and next-to-leading log terms in the wavefunction for the lightest charged gauge bosons

$$c_0^{(L\pm)} = c_\pm, \quad c_0^{(R\pm)} \approx 0, \quad (10.29)$$

$$c_1^{(L\pm)} \approx 0, \quad c_1^{(R\pm)} \approx \frac{c_\pm}{2} \log\left(\frac{R'}{R}\right), \quad (10.30)$$

while for the neutral gauge bosons we find in the same approximation

$$c_0^{(L3)} \approx c, \quad c_0^{(R3)} \approx -c \frac{\tilde{g}_5^2}{g_5^2 + \tilde{g}_5^2}, \quad c_0^{(B)} \approx -c \frac{g_5 \tilde{g}_5}{g_5^2 + \tilde{g}_5^2}. \quad (10.31)$$

To leading log order we also have:

$$c_1^{(L3)} \approx 0, \quad c_1^{(R3)} \approx c \frac{g_5^2}{2(g_5^2 + \tilde{g}_5^2)} \log\left(\frac{R'}{R}\right), \quad c_1^{(B)} \approx -c \frac{g_5 \tilde{g}_5}{2(g_5^2 + \tilde{g}_5^2)} \log\left(\frac{R'}{R}\right). \quad (10.32)$$

10.1.4 Precision electroweak measurements

The simplest way to calculate the contributions to precision electroweak measurements is to use “equivalent vacuum polarizations” that can be extracted from the gauge boson wavefunction renormalizations:

$$Z_\gamma = 1, \quad Z_W = 1 - g^2 \Pi'_{11}, \quad Z_Z = 1 - (g^2 + g'^2) \Pi'_{33}, \quad (10.33)$$

Since the photon is massless its wavefunction is exactly flat, so requiring that the quarks and leptons have the correct charge fixes $Z_\gamma = 1$. For the W and Z however the wavefunctions have some nontrivial shape so canonically normalizing the light quark and lepton modes and requiring that their overlaps with

the W and Z reproduce the standard model couplings fixes $Z_W, Z_Z \neq 1$. Given these “equivalent vacuum polarizations” it is straightforward to compute the S, T , and U parameters.

$$\begin{aligned} S &\equiv 16\pi(\Pi'_{33} - \Pi'_{3Q}), \\ T &\equiv \frac{4\pi}{s^2 c^2 M_Z^2}(\Pi_{11}(0) - \Pi_{33}(0)), \\ U &\equiv 16\pi(\Pi'_{11} - \Pi'_{33}), \end{aligned} \quad (10.34)$$

where $\Pi_{ii}(0)$ is simply extracted from the gauge boson mass terms. For fermions localized on the Planck brane the calculation can be performed analytically and we find there is a problem with S :

$$S \approx \frac{8\pi R'^2 M_z^2}{g^2 + g'^2} \approx 1.5 \quad (10.35)$$

Such a large value is reminiscent of technicolor models.

However if we allow the quarks and leptons can have an arbitrary bulk mass then the lightest modes can be localized with a wavefunction that is arbitrary power of z . Then since it is known that in RS models with quarks and leptons on the TeV brane (at $z = R'$) S is large and negative [13], it is clear that S will vanish for some intermediate localization. Indeed when the fermions are roughly uniformly distributed through the bulk S goes through zero [10, 14–17]. The fact that S vanishes has a deeper significance, since it follows from orthogonality of wavefunctions. If the currents that couple to the W and Z have the same profile as the gauge bosons then the overlap of the current with all higher gauge boson KK modes will be exactly zero. If the current and the gauge bosons have very similar profiles in the extra dimension then the gauge boson coupling is still relatively enhanced while the coupling to higher KK modes is suppressed. Thus in the region where the precision electroweak constraints are satisfied ($S \approx 0$) because the coupling to higher KK is suppressed, the bounds from the Tevatron and LEP on such KK modes are relaxed down to a bound around 500 – 600 GeV.

The remaining problem with precision electroweak measurements is the compatibility of a large top quark mass with the observed $Zb\bar{b}$ coupling. The large top mass requires the left-handed and right-handed top quarks to have a profile which is localized toward the TeV brane. However if the left-handed top (and thus the left-handed bottom) are too close to the TeV brane then the gauge couplings of the bottom will be too different from the down and strange quarks. This problem may be avoided by separating the physics which generates the top quark mass [17, 18] or by allowing the Higgs VEV to extend slightly into the bulk [19].

10.1.5 Collider phenomenology

The most distinctive feature of the Higgsless models is, of course, the absence of a physical scalar state in the spectrum. However, other models exist in which the Higgs is unobservable at the LHC. For this reason, identification of this as the mechanism of electroweak symmetry breaking will require examination of other sectors. There are two potential types of signals that will help to identify the model: those that are related to the RS physics required to realize the Higgsless mechanism, and those that directly probe this as the mechanism of symmetry breaking and unitarity restoration. In the first class are observations of Kaluza-Klein of electroweak gauge bosons and gluons [8]. In the second class are observations of resonances in the scattering of longitudinal electroweak bosons [20].

The easiest signal to see is the first gluon Kaluza-Klein (KK) resonance. This will show up as a resonance in the dijet spectrum, as seen in Fig. 10.3. Like all low-lying KK resonances, this is localized near the IR brane. In most RS models with fermions in the bulk the need to produce a large top mass forces the right-handed top and third generation quark doublet to also be localized near the IR brane. This means that the gluon resonances will generically couple more strongly to tops and bottoms than to light quarks, and observation of this will be a clue that the model may be RS.

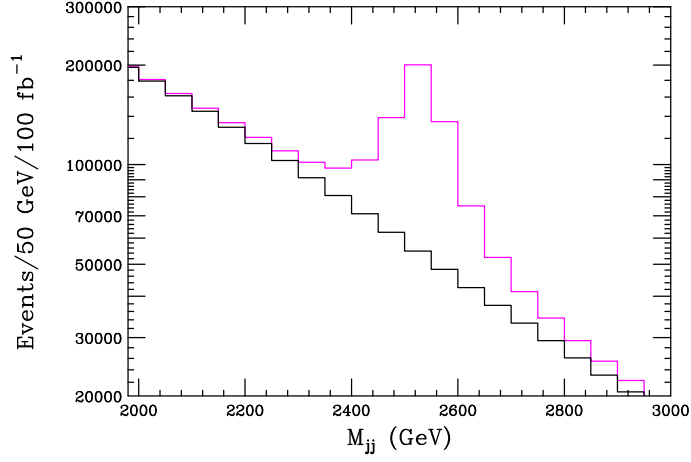


Fig. 10.3: Dijet invariant mass spectrum at the LHC showing a prominent resonance due to the first gluon Kaluza-Klein state.

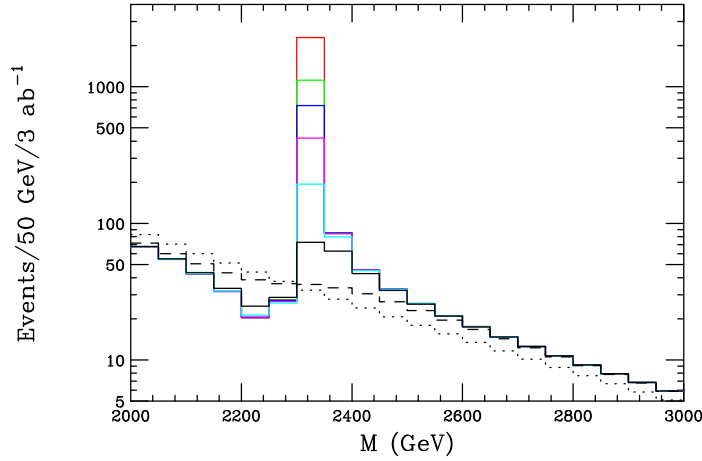


Fig. 10.4: Event rate for Drell-Yan production of the first neutral KK gauge boson, as a function of the invariant mass of the lepton pair. The dotted line is the SM background. The other histograms, from top to bottom, include the resonance with width parameter $c = (1, 2, 3, 5, 10, 25, 100)$.

More important for studies of electroweak symmetry breaking, of course, is the observation of gauge boson KK resonances. The simplest place to look is in the Drell-Yan spectrum. The couplings of these states to fermions depend on the fermion localization parameters, in particular the top localization.

Fig. 10.4 shows the Drell-Yan spectrum from a neutral KK at about 2.3 TeV. We can write the width as $\Gamma = c\Gamma_0$, where Γ_0 is what the width of the state would be if all fermions were localized to the Planck brane. The different curves in Fig. 10.4 correspond to different values of c , ranging from $c = 1$ to $c = 100$. Note that the state presented is at a high mass. In general, a successful Higgsless model is expected to have lighter states, and hence they should be more easily discoverable at the LHC.

As shown in [20] searches for the process $WZ \rightarrow WZ$ can directly probe the Higgsless mechanism for electroweak symmetry breaking. In particular, the sum rules that ensure unitarity can be directly probed by measurements of the couplings of the gauge KKs to longitudinal gauge bosons. Fig. 10.5 shows the production of the first charged KK resonance in this channel. For comparison, two resonances appearing in different technicolor-type models are also shown. As can be seen, the most striking feature of the Higgsless model is the narrow width of the resonance. Note that these searches have the additional advantage of being largely independent of the parameters in the fermion sector.

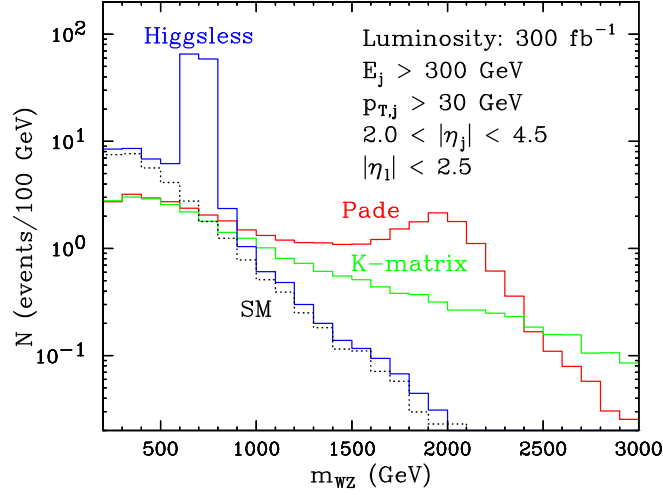


Fig. 10.5: Event rate for production of $2j + 3\ell + \nu$, corresponding to WZ production with fully leptonic decays. From ref. [9].

10.2 Quark and lepton masses

Christophe Grojean

10.2.1 Chiral fermions from a 5D theory on an interval

In the SM, quarks and leptons acquire a mass, after EWSB, through their Yukawa couplings to the Higgs. In absence of a Higgs, one cannot write any Yukawa coupling and one should expect the fermions to remain massless. However, as for the gauge fields, appropriate boundary conditions will force the fermions to acquire a momentum along the extra dimension and this is how they will become massive from the 4D point of view. We are now going to review this construction [4].

The SM fermions cannot be completely localized on the UV boundary: since the unbroken gauge group on that boundary coincides with the SM $SU(2)_L \times U(1)_Y$ symmetry, the theory on that brane would be chiral and there is no way for the chiral zero mode fermions to acquire a mass. The SM fermions cannot live on the IR brane either since the unbroken $SU(2)_D$ gauge symmetry will impose an isospin invariant spectrum and the up-type and down-type quarks will be degenerate. The only possibility is thus to embed the SM fermions into 5D fields living in the bulk and feeling the gauge symmetry breakings on both boundaries. Since the irreducible spin-1/2 representations of the 5D Lorentz group correspond to 4-component Dirac spinor, extra fermionic degrees of freedom are needed to complete the SM chiral spinors to 5D Dirac spinors and we are back to a vector-like spectrum. However, as it is well known, orbifold like projections (or equivalently appropriate boundary conditions) can get rid of half of the spectrum at the lowest KK level to actually provide a 4D effective chiral theory. This way we can embed the SM quarks and leptons into 5D Dirac spinors following Table 10.1.

10.2.2 Fermions in AdS background

In principle when one is dealing with fermions in a non-trivial background, one needs to work with the “square-root” of the metric also known as *vielbeins* and to introduce the spin connection to covariantize derivatives. Fortunately, in an *AdS* background, the spin connection drops out from the spin-1/2 action that simply reads

$$\mathcal{S} = \int d^5x \frac{R^4}{z^4} \left(-i\bar{\chi}\bar{\sigma}^\mu\partial_\mu\chi - i\psi\sigma^\mu\partial_\mu\bar{\psi} + (\psi\overleftrightarrow{\partial}_5\chi - \bar{\chi}\overleftrightarrow{\partial}_5\bar{\psi}) + \frac{c}{z}(\psi\chi + \bar{\chi}\bar{\psi}) \right) \quad (10.36)$$

Table 10.1: Embedding of the SM fermions into 5D Dirac spinors. We have indicated the quantum numbers of the different components under the bulk $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetry, the subgroup $SU(2)_L \times U(1)_Y$ that remains unbroken on the UV boundary, the subgroup $SU(2)_D \times U(1)_{B-L}$ unbroken on the IR brane and finally the electric charge. The shaded spinors are the fields with the right quantum numbers to be identified as the massless SM fermions while the other spinors correspond to partners needed to complete 5D Dirac spinors. The latter become massive by the orbifold projection/boundary conditions. Through the Dirac mass added on the IR boundary, there will be a mixing between the would be zero modes and some partners and at the end the guy that would be identified as the SM u_L is a mix of χ_{uL} and a small amount of χ_{uR} . Since this last field has wrong SM quantum numbers, we would end up with deviations in the couplings of the fermions to the gauge bosons. These deviations will be particularly sizable for the third generation due to the heaviness of the top.

Particle	bulk $L \times R \times (B-L)$	UV $L \times Y$	IR $D \times (B-L)$	Q_{em}
$\begin{pmatrix} \chi_u \\ \chi_d \end{pmatrix}_L$	$(\square, 1, 1/6)$	$(\square, 1/6)$	$(\square, 1/6)$	$\begin{pmatrix} 2/3 \\ -1/3 \end{pmatrix}$
$\begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}_L$	$(\square, 1, -1/6)$	$(\square, -1/6)$	$(\square, -1/6)$	$\begin{pmatrix} -2/3 \\ 1/3 \end{pmatrix}$
$\begin{pmatrix} \chi_u \\ \chi_d \end{pmatrix}_R$	$(1, \square, 1/6)$	$\begin{pmatrix} (1, 2/3) \\ (1, -1/3) \end{pmatrix}$	$(\square, 1/6)$	$\begin{pmatrix} 2/3 \\ -1/3 \end{pmatrix}$
$\begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}_R$	$(1, \bar{\square}, -1/6)$	$\begin{pmatrix} (1, -2/3) \\ (1, 1/3) \end{pmatrix}$	$(\bar{\square}, -1/6)$	$\begin{pmatrix} -2/3 \\ 1/3 \end{pmatrix}$

$Q_{em} = Y + T_{3L} \quad Y = (B-L) + T_{3R}$

where the coefficient $c = mR$ is the bulk Dirac mass in units of the AdS curvature (and $\overleftrightarrow{\partial}_5 = (\overrightarrow{\partial}_5 - \overleftarrow{\partial}_5)/2$). The bulk equations of motion are:

$$-i\bar{\sigma}^\mu \partial_\mu \chi - \partial_5 \bar{\psi} + \frac{c+2}{z} \bar{\psi} = 0 \quad -i\sigma^\mu \partial_\mu \bar{\psi} + \partial_5 \chi + \frac{c-2}{z} \chi = 0. \quad (10.37)$$

The KK decomposition is of the form

$$\chi = \sum_n g_n(z) \chi_n(x) \quad \text{and} \quad \bar{\psi} = \sum_n f_n(z) \bar{\psi}_n(x). \quad (10.38)$$

and the 5D Dirac equation is equivalent to the coupled first order differential equations

$$f'_n + m_n g_n - \frac{c+2}{z} f_n = 0, \quad g'_n - m_n f_n + \frac{c-2}{z} g_n = 0, \quad (10.39)$$

which can be combined into uncoupled second order differential equations

$$f''_n - \frac{4}{z} f'_n + (m_n^2 - \frac{c^2-c-6}{z^2}) f_n = 0, \quad g''_n - \frac{4}{z} g'_n + (m_n^2 - \frac{c^2+c-6}{z^2}) g_n = 0. \quad (10.40)$$

The solutions are now linear combinations of Bessel functions, as opposed to sin and cos functions for the flat case:

$$g_n(z) = z^{\frac{5}{2}} \left(A_n J_{c+\frac{1}{2}}(m_n z) + B_n Y_{c+\frac{1}{2}}(m_n z) \right) \quad (10.41)$$

$$f_n(z) = z^{\frac{5}{2}} \left(C_n J_{c-\frac{1}{2}}(m_n z) + D_n Y_{c-\frac{1}{2}}(m_n z) \right). \quad (10.42)$$

The first order bulk equations of motion (10.39) further impose that

$$A_n = C_n \text{ and } B_n = D_n. \quad (10.43)$$

The remaining undetermined coefficients are determined by the boundary conditions, and the wave function normalization.

Finally, when the boundary conditions permit, there can also be a zero mode. For instance, if $\psi|_{R,R'} = 0$, the zero mode is given by

$$g_0(z) = A_0 \left(\frac{z}{R}\right)^{2-c}, \quad f = 0. \quad (10.44)$$

The coefficient A_0 is determined by the normalization condition

$$\int_R^{R'} dz \left(\frac{R}{z}\right)^5 \frac{z}{R} A_0^2 \left(\frac{z}{R}\right)^{4-2c} = A_0^2 \int_R^{R'} \left(\frac{z}{R}\right)^{-2c} dz = 1. \quad (10.45)$$

To understand from these equations where the fermions are localized, we study the behavior of this integral as we vary the limits of integration. If we send R' to infinity, we see that the integral remains convergent if $c > 1/2$, and the fermion is then localized on the UV brane. If we send R to zero, the integral is convergent if $c < 1/2$, and the fermion is localized on the IR brane. The value of the Dirac mass determines whether the fermion is localized toward the UV or IR branes. We note that the opposite choice of boundary conditions that yields a zero mode ($\chi|_{R,R'} = 0$) results in a zero mode solution for ψ with localization at the UV brane when $c < -1/2$, and at the IR brane when $c > -1/2$. The interesting feature in the warped case is that the localization transition occurs not when the bulk mass passes through zero, but at points where $|c| = 1/2$. This is due to the curvature effects of the extra dimension. The *CFT* interpretation of the c parameter is an anomalous dimension that controls the amount of compositeness of the fermion [21].

10.2.3 Higgsless fermions masses

We have already explained how to embed SM fermions into 5D Dirac spinors. To get the zero modes we desire, the following boundary conditions have to be imposed

$$\begin{pmatrix} \chi_{u_L} \\ \bar{\psi}_{u_L} \\ \chi_{d_L} \\ \bar{\psi}_{d_L} \end{pmatrix} \begin{matrix} + & + \\ - & - \\ + & + \\ - & - \end{matrix} \quad \begin{pmatrix} \chi_{u_R} \\ \bar{\psi}_{u_R} \\ \chi_{d_R} \\ \bar{\psi}_{d_R} \end{pmatrix} \begin{matrix} - & - \\ + & + \\ - & - \\ + & + \end{matrix} \quad (10.46)$$

Where the $+$ and $-$ refer to Neumann and Dirichlet boundary conditions, the first/second sign denoting the BC on the UV/IR brane respectively. These boundary conditions give massless chiral modes that match the fermion content of the standard model. However, the u_L , d_L , u_R , and d_R are all massless at this stage, and we need to lift the zero modes to achieve the standard model mass spectrum. While simply giving certain boundary conditions for the fermions will enable us to lift these zero modes, in the following discussion, we talk about boundary operators, and the boundary conditions that these operators induce. There are some subtleties in dealing with boundary operators for fermions. These arise from the fact that the fields themselves are not always continuous in the presence of a boundary operator. This is due to the fact that the equations of motion for fermions are first order. The most straightforward approach is to enforce the boundary conditions that give the zero modes as shown in Eq. (10.46) on the real boundary at $z = R, R'$ while the boundary operators are added on a fictitious brane a distance ϵ away from it. The new boundary condition is then obtained by taking the distance ϵ to be small. This physical picture is quite helpful in understanding what the different boundary conditions will do. The details can be found in [4].

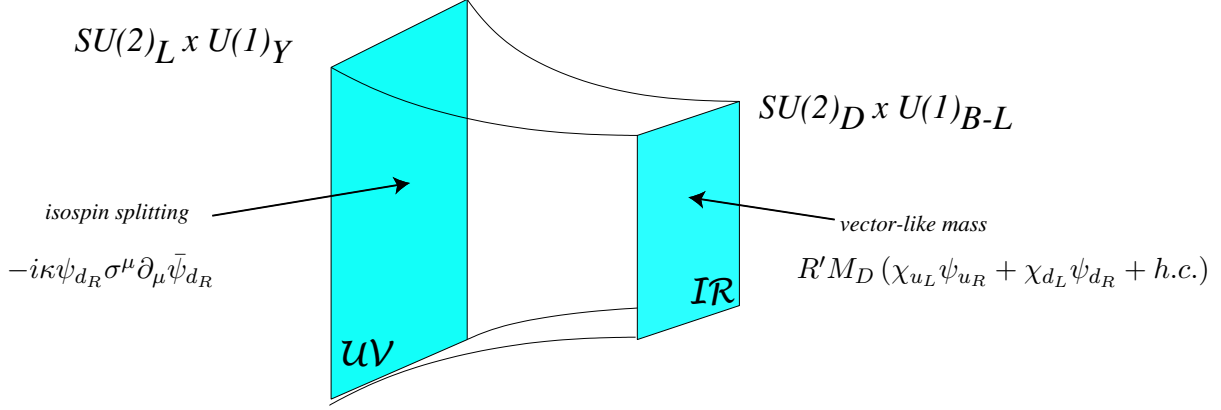


Fig. 10.6: Brane localized operators needed to lift up the masses of the SM fermions.

The IR brane being vector-like, we can now form an $SU(2)_D$ mass term that will mix the L and R SM helicities. However, this Dirac mass term has to be the same for the up and the down quarks (the mass term is isospin invariant). Fortunately, the $SU(2)_R$ invariance is broken on the UV brane and there we can introduce operators that will distinguish between u_R and d_R . Technically, the effects of the brane localized operators is to modified the BCs. Explicitly, the IR Dirac mass affect the BCs as follows

$$\begin{array}{ccccc} \chi_L & + & & & \\ \psi_L & - & \psi_{L|IR} = 0 & M_D & \text{discontinuities} \\ \chi_R & - & \chi_{R|IR} = 0 & \Rightarrow & \text{in} \\ \psi_R & + & & & \chi_L \& \psi_R \end{array} \quad \begin{array}{l} \psi_{L|IR} = -M_D R' \psi_{R|IR} \\ \chi_{R|IR} = M_D R' \chi_{L|IR} \end{array}$$

In the same way, the UV brane operator will modify the BCs as follows

$$\begin{array}{ccccc} \chi_{u_R} & - & & & \text{discontinuity} \\ \psi_{u_R} & + & \chi_{u_R|UV} = 0 & \xRightarrow{\kappa} & \text{in} \\ & & & & \psi_{u_R} \end{array} \quad \chi_{u_R|UV} = \kappa m \psi_{u_R|UV}$$

It is now easy to enforce these modified boundary conditions using the general form of the wavefunctions (10.41)–(10.42) that satisfy the bulk equations of motion. For fermions localized toward the UV brane ($c_L > 1/2$ and $c_R < -1/2$), we obtain the approximate expression

$$m \approx \frac{\sqrt{2c_L - 1}}{\sqrt{\kappa^2 - 1/(2c_R + 1)}} M_D \left(\frac{R_{UV}}{R_{IR}} \right)^{c_L - c_R - 1}. \quad (10.47)$$

10.2.4 Top mass and $Zb_L\bar{b}_L$ deviation

The spectrum of the light generations of quarks can be easily reproduced along these lines. The top mass poses a difficulty, however. Indeed, increasing M_D won't arbitrarily increase the fermion mass which will saturate: the situation is similar to what happens with a large Higgs vev localized on the boundary, the gauge boson masses remain finite even when the vev is sent to infinity. The maximum value of the fermion mass can be inferred by noticing that in the infinite M_D limit, there is a *chirality flip* in the BCs that become

$$\begin{array}{ccccc} \chi_L & + & & & \chi_L & - \\ \psi_L & - & \psi_{L|IR} = -M_D R' \psi_{R|IR} & M_D \rightarrow \infty & \psi_{R|IR} = 0 & \psi_L & + \\ \chi_R & - & \chi_{R|IR} = M_D R' \chi_{L|IR} & \Rightarrow & \chi_{L|IR} = 0 & \chi_R & + \\ \psi_R & + & & & & \psi_R & - \end{array}$$

and the corresponding mass is

$$m^2 = \frac{2}{R'^2 \log R'/R} = 2M_W^2. \quad (10.48)$$

where in the last equality, we used the expression of the W mass in terms of R and R' and we have assumed $g_{5R} = g_{5L}$. If we want to go above this saturated mass, one needs to localize the fermions toward the IR brane. However, even in this case a sizable Dirac mass term on the TeV brane is needed to obtain a heavy enough top quark. The consequence of this mass term is the boundary condition for the bottom quarks

$$\chi_{bR} = M_D R' \chi_{bL}. \quad (10.49)$$

This implies that if $M_D R' \sim 1$ then the left handed bottom quark has a sizable component also living in an $SU(2)_R$ multiplet, which however has a coupling to the Z that is different from the SM value. Thus there will be a large deviation in the $Z b_L \bar{b}_L$ coupling. Note, that the same deviation will not appear in the $Z b_R \bar{b}_R$ coupling, since the extra kinetic term introduced on the Planck brane to split top and bottom will imply that the right handed b lives mostly in the induced fermion on the Planck brane which has the correct coupling to the Z .

The only way of getting around this problem would be to raise the value of $1/R'$, and thus lower the necessary mixing on the TeV brane needed to obtain a heavy top quark. One way of raising the value of $1/R'$ is by increasing the ratio g_{5R}/g_{5L} (at the price of also making the gauge KK modes heavier and thus the theory more strongly coupled). Another possibility for raising the value of $1/R'$ is to separate the physics responsible for electroweak symmetry breaking from that responsible for the generation of the top mass. In technicolor models this is usually achieved by introducing a new strong interaction called topcolor. In the extra dimensional setup this would correspond to adding two separate AdS_5 bulks, which meet at the Planck brane [18]. One bulk would then be mostly responsible for electroweak symmetry breaking, the other for generating the top mass. The details of such models have been worked out in [18] (see also [22]). The main consequences of such models would be the necessary appearance of an isotriplet pseudo-Goldstone boson called the top-pion, and depending on the detailed implementation of the model there could also be a scalar particle (called the top-Higgs) appearing. This top-Higgs would however not be playing a major role in the unitarization of the gauge boson scattering amplitudes, but rather serve as the source for the top-mass only.

10.2.5 Fermion delocalization and EW precision tests

As already mentioned in earlier, the delocalization of SM fermions in the bulk is helpful in keeping the oblique corrections under control. In order to quantify this statement, it is sufficient to consider a toy model where all the three families of fermions are massless and have a universal delocalized profile in the bulk. When the profile of the fermion wavefunction is almost flat, $c_L \approx 1/2$, the leading contributions to S are:

$$S = \frac{2\pi}{g^2 \log \frac{R'}{R}} \left(1 + (2c_L - 1) \log \frac{R'}{R} + \mathcal{O}((2c_L - 1)^2) \right). \quad (10.50)$$

In the flat limit $c_L = 1/2$, S is already suppressed by a factor of 3 with respect to the Planck brane localization case. Moreover, the leading terms cancel out for:

$$c_L = \frac{1}{2} - \frac{1}{2 \log \frac{R'}{R}} \approx 0.487. \quad (10.51)$$

In Fig. 10.7 we have plotted the value of the NDA cut-off scale as well as the mass of the first resonance in the $(c_L - R)$ plane. Increasing R also affects the oblique corrections. However, while it is always possible to reduce S by delocalizing the fermions, T increases and puts a limit on how far R can be raised. One can also see from Fig. 10.8 that in the region where $|S| < 0.25$, the coupling of the first resonance with the light fermions is generically suppressed to less than 10% of the SM value.

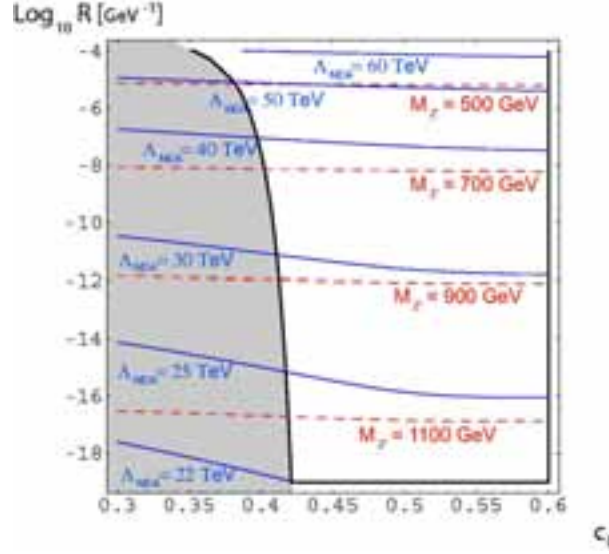


Fig. 10.7: Contour plots of Λ_{NDA} (solid blue lines) and $M_{Z(1)}$ (dashed red lines) in the parameter space c_L - R . The shaded region is excluded by direct searches of light Z' at LEP.

This means that the LEP bound of 2 TeV for SM-like Z' is also decreased by a factor of 10 at least (the correction to the differential cross section is roughly proportional to $g^2/M_{Z'}^2$). In the end, values of R as large as 10^{-7} GeV^{-1} are allowed, where the resonance masses are around 600 GeV. So, even if, following the analysis of [23], we take into account a factor of roughly 1/4 in the NDA scale, we see that the appearance of strong coupling regime can be delayed up to 10 TeV.

It is fair to say that, to date, the major challenge facing Higgsless models is really the incorporation of the third family of quarks while the oblique corrections can be kept under control, at a price of some conspiracy in the localization of the SM quarks and leptons along the extra dimension.

10.3 Higgsless electroweak symmetry breaking from moose models

Stefania De Curtis and Daniele Dominici

Higgsless models, in their "modern" version, are formulated as gauge theories in a five dimensional space-time and symmetry breaking is realized by means of field boundary conditions in the fifth dimension [2]. One of the interesting features of these schemes is the possibility to delay the unitarity violation scale via the exchange of massive (Kaluza Klein) KK modes [2, 3]. However, it is generally difficult to reconcile a delayed unitarity with the electroweak (EW) constraints. For instance in the framework of models with only ordinary fermions, it is possible to get a small or zero S parameter [24], at the expenses of having a unitarity bound as in the Standard Model (SM) without the Higgs, that is of the order of 1 TeV. A recent solution to the problem, which does not spoil the unitarity requirement at low scales, has been found by delocalizing the fermions in five dimensional theories [14, 25]. We will investigate this possibility in the context of deconstructed gauge theories which come out when the extra dimension is discretized [26]. Through discretization of the fifth dimension we get a finite set of four-dimensional gauge theories each of them acting at a particular lattice site. In this construction, any connection field along the fifth dimension, A_5 , goes naturally into the link variables $\Sigma_i = e^{-iaA_5^{i-1}}$ realizing the parallel transport between two lattice sites (here a is the lattice spacing). The link variables satisfy the condition $\Sigma\Sigma^\dagger = 1$ and can be identified with chiral fields. In this way the discretized version of the original 5-dimensional gauge theory is substituted by a collection of four-dimensional gauge theories with gauge

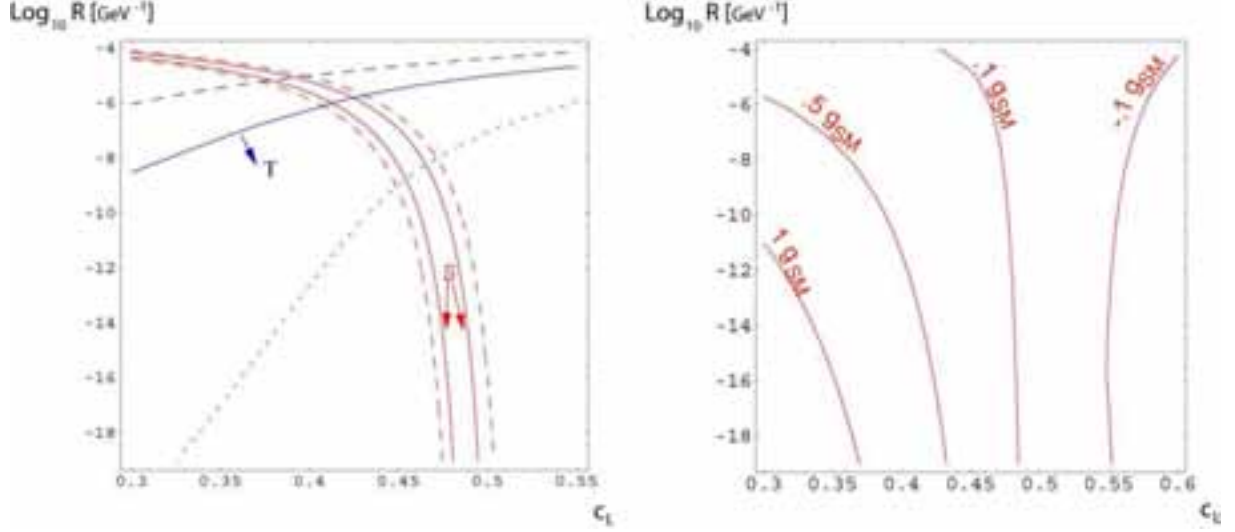


Fig. 10.8: In the left, the contours of S (red), for $|S| = 0.25$ (solid) and 0.5 (dashed) and T (blue), for $|T| = 0.1$ (dotted), 0.3 (solid) and 0.5 (dashed), as function of c_L and R are shown. On the right, contours for the generic suppression of fermion couplings to the first resonance with respect to the SM value can be seen. The region for c_L , allowed by S , is between 0.43 ± 0.5 , where the couplings are suppressed at least by a factor of 10.

interacting chiral fields Σ_i , synthetically described by a moose diagram (an example is given in Fig. 10.9). Here we consider the simplest linear moose model for the Higgsless breaking of the EW symmetry and we delocalize fermions by introducing direct couplings between ordinary left-handed fermions and the gauge vector bosons along the moose string [27].

Let us briefly review the linear moose model based on the $SU(2)$ symmetry [24, 27]. We consider $K + 1$ non linear σ -model scalar fields Σ_i , $i = 1, \dots, K + 1$, K gauge groups G_i , $i = 1, \dots, K$ and a global symmetry $G_L \otimes G_R$ as shown in Fig. 10.9. A minimal model of EW symmetry breaking is

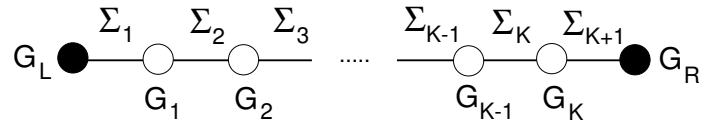


Fig. 10.9: The simplest moose diagram for the Higgsless breaking of the EW symmetry.

obtained by choosing $G_i = SU(2)$, $G_L \otimes G_R = SU(2)_L \otimes SU(2)_R$. The SM gauge group $SU(2)_L \times U(1)_Y$ is obtained by gauging a subgroup of $G_L \otimes G_R$. The Σ_i fields can be parameterized as $\Sigma_i = \exp[i/(2f_i)\vec{\pi}_i \cdot \vec{\tau}]$ where $\vec{\tau}$ are the Pauli matrices and f_i are $K + 1$ constants that we will call link couplings. The Lagrangian of the linear moose model is given by

$$\mathcal{L} = \sum_{i=1}^{K+1} f_i^2 \text{Tr}[D_\mu \Sigma_i^\dagger D^\mu \Sigma_i] - \frac{1}{2} \sum_{i=1}^K \text{Tr}[(F_{\mu\nu}^i)^2] - \frac{1}{2} \text{Tr}[(F_{\mu\nu}(\tilde{W}))^2] - \frac{1}{2} \text{Tr}[(F_{\mu\nu}(\tilde{Y}))^2], \quad (10.52)$$

with the covariant derivatives defined as follows:

$$D_\mu \Sigma_1 = \partial_\mu \Sigma_1 - i\tilde{g}\tilde{W}_\mu \Sigma_1 + i\Sigma_1 g_1 V_\mu^1 \quad (10.53)$$

$$D_\mu \Sigma_i = \partial_\mu \Sigma_i - i g_{i-1} V_\mu^{i-1} \Sigma_i + i\Sigma_i g_i V_\mu^i \quad (i = 2, \dots, K) \quad (10.54)$$

$$D_\mu \Sigma_{K+1} = \partial_\mu \Sigma_{K+1} - i g_K V_\mu^K \Sigma_{K+1} + i\tilde{g}' \Sigma_{K+1} \tilde{Y}_\mu \quad (10.55)$$

where $V_\mu^i = V_\mu^{ia} \tau^a / 2$, g_i are the gauge fields and gauge coupling constants associated to the groups G_i , $i = 1, \dots, K$, and $\tilde{W}_\mu = \tilde{W}_\mu^a \tau^a / 2$, $\tilde{Y}_\mu = \tilde{Y}_\mu \tau^3 / 2$ are the gauge fields associated to $SU(2)_L$ and $U(1)_Y$ respectively. Notice that, in the unitary gauge, all the Σ_i fields are eaten up by the gauge bosons which acquire mass, except for the photon corresponding to the unbroken $U(1)_{em}$. By identifying the lowest mass eigenvalue in the charged sector with M_W , we get at $O(\tilde{g}^2/g_i^2)$ a relation between the EW scale v (≈ 250 GeV) and the link couplings of the chain:

$$\frac{4}{v^2} \equiv \frac{1}{f^2} = \sum_{i=1}^{K+1} \frac{1}{f_i^2}. \quad (10.56)$$

Concerning fermions, we will consider only the standard model ones, that is: left-handed fermions ψ_L as $SU(2)_L$ doublets and singlet right-handed fermions ψ_R coupled to the SM gauge fields through the groups $SU(2)_L$ and $U(1)_Y$ at the ends of the chain.

10.3.1 Constraints from perturbative unitarity and EW tests

The worst high-energy behavior of the moose models arises from the scattering of longitudinal vector bosons whose calculation is simplified by using the equivalence theorem. This allows to evaluate these amplitudes in terms of the corresponding Goldstone boson ones. However this theorem holds in the approximation where the energy of the process is much higher than the mass of the vector bosons. Let us evaluate the amplitude for the SM W scattering at energies $M_W \ll E \ll M_{V_i}$. The unitary gauge for the V_i bosons is given by the choice $\Sigma_i = \exp[i f \vec{\pi} \cdot \vec{\tau} / (2f_i^2)]$ with f given in Eq. (10.56) and $\vec{\pi}$ the GB's giving mass to W and Z . The resulting four-pion amplitude is

$$A_{\pi^+\pi^-\rightarrow\pi^+\pi^-} = -\frac{f^4 u}{4} \sum_{i=1}^{K+1} \frac{1}{f_i^6} + \frac{f^4}{4} \sum_{i,j=1}^K L_{ij} \left((u-t)(s-M_2)_{ij}^{-1} + (u-s)(t-M_2)_{ij}^{-1} \right), \quad (10.57)$$

with $(M_2)_{ij}$ the square mass matrix for the gauge fields, and

$$L_{ij} = g_i g_j (f_i^{-2} + f_{i+1}^{-2})(f_j^{-2} + f_{j+1}^{-2}). \quad (10.58)$$

Note that this amplitude grows linearly with the squared energy, for every choice of f_i . This reflects the fact that in the continuum limit the theory corresponds to a 5D gauge theory with boundary conditions which are not simply Neumann or Dirichlet, see 10.3.3 and [2]. In the high-energy limit, where we can neglect the second term in Eq. (10.57), the amplitude has a minimum for all the f_i 's being equal to a common value f_c . As a consequence, the scale at which unitarity is violated by this single channel contribution is delayed by a factor $(K+1)$ with respect to the one in the SM without the Higgs: $\Lambda_{moose} = (K+1)\Lambda_{HSM}$.

However the moose model has many other longitudinal vector bosons with bad behaving scattering amplitudes. For energies much higher than all the masses of the vector bosons, we can determine the unitarity bounds by considering the eigenchannel amplitudes corresponding to all the possible four-longitudinal vector bosons. Since in the unitary gauge for all the vector bosons Σ_i are given by $\Sigma_i = \exp[i \vec{\pi} \cdot \vec{\tau} / (2f_i)]$, the amplitudes are diagonal, and the high-energy result is simply

$$A_{\pi_i \pi_i \rightarrow \pi_i \pi_i} \rightarrow -\frac{u}{4f_i^2}. \quad (10.59)$$

We see that, also in this case, the best unitarity limit is for all the link couplings being equal: $f_i = f_c$. Then: $\Lambda_{moose}^{TOT} = \sqrt{K+1} \Lambda_{HSM}$ (for similar results see ref. [23] in [27]). However, in order our approximation to be correct, we have to require $M_{V_i}^{max} \ll \Lambda_{moose}^{TOT}$. By using the explicit expression for the highest mass eigenvalue, in the case of equal couplings $g_i = g_c$, we get an upper bound $g_c \sim 5$. As we will see, this choice gives unacceptable large EW correction.

In this class of models all the corrections from new physics are "oblique" since they arise from the mixing of the SM vector bosons with the moose vector fields (we are assuming the standard couplings for the fermions to $SU(2)_L \otimes U(1)$). As well known, the oblique corrections are completely captured by the parameters S , T and U or, equivalently by the parameters ϵ_i , $i = 1, 2, 3$. For the linear moose, the existence of the custodial symmetry $SU(2)_V$ ensures that $\epsilon_1 \approx \epsilon_2 \approx 0$. On the contrary, the new physics contribution to the EW parameter ϵ_3 is sizeable and positive [24]:

$$\epsilon_3 = (\tilde{g}^2/g_i^2) \sum_{i=1}^K (1 - y_i) y_i \quad (10.60)$$

where $y_i = \sum_{j=1}^i f_j^2/f_j^2$. Since $0 \leq y_i \leq 1$ it follows $\epsilon_3 \geq 0$ (see also [28–30]). As an example, let us take equal couplings along the chain: $f_i = f_c$, $g_i = g_c$. Then $\epsilon_3 = \tilde{g}^2 K(K+2)/(6 g_c^2(K+1))$, which grows with the number of sites of the moose. The requirement of satisfying the experimental constraints ($\epsilon_3 \approx 10^{-3}$), already for $K = 1$ would imply $g_c \geq 15.8\tilde{g}$, leading to a strong interacting gauge theory in the moose sector and unitarity violation. Notice also that, insisting on a weak gauge theory would imply g_c of the order of \tilde{g} , then the natural value of ϵ_3 would be of the order $10^{-1} - 10^{-2}$, incompatible with the experimental data.

10.3.2 Effects of fermion delocalization

A way to reconcile perturbative unitarity requirements with the EW bounds is to allow for delocalized couplings of the SM fermions to the moose gauge fields and some amount of fine tuning [27]. In fact, by generalizing the procedure in [31, 32], the SM fermions can be coupled to any of the gauge fields at the lattice sites by means of Wilson lines.

Define $\chi_L^i = \Sigma_i^\dagger \Sigma_{i-1}^\dagger \cdots \Sigma_1^\dagger \psi_L$, for $i = 1, \dots, K$. Since under a gauge transformation, $\chi_L^i \rightarrow U_i \chi_L^i$, with $U_i \in G_i$, at each site we can introduce a gauge invariant coupling given by

$$b_i \bar{\chi}_L^i \gamma^\mu \left(\partial_\mu + i g_i V_\mu^i + \frac{i}{2} \tilde{g}' (B - L) \tilde{Y}_\mu \right) \chi_L^i, \quad (10.61)$$

where $B(L)$ is the barion(lepton) number and b_i are dimensionless parameters. The new fermion interactions give extra non-oblique contributions to the EW parameters. These are calculated in [27] by decoupling the V_μ^i fields and evaluating the corrections to the relevant physical quantities. To the first order in b_i and to $O(\tilde{g}^2/g_i^2)$, the ϵ_i parameters are modified as follows:

$$\epsilon_1 \approx 0, \quad \epsilon_2 \approx 0, \quad \epsilon_3 \approx \sum_{i=1}^K y_i \left(\frac{\tilde{g}^2}{g_i^2} (1 - y_i) - b_i \right). \quad (10.62)$$

This final expression suggests that the introduction of the b_i direct fermion couplings to V_i can compensate for the contribution of the tower of gauge vectors to ϵ_3 . This would reconcile the Higgsless model with the EW precision measurements by fine-tuning the direct fermion couplings.

As shown in the left panel of Fig. 10.10, in the simplest model with all $f_i = \text{const} = f_c$, $g_i = \text{const} = g_c$ and $b_i = \text{const} = b_c$, the experimental bounds from the ϵ_3 parameter can be satisfied by fine-tuning the direct fermion coupling b_c along a strip in the plane $(K b_c, \sqrt{K}/g_c)$ (we have chosen these parameters due to the scaling properties of g_c and b_c with K , see ref. [27] for details).

The expression for ϵ_3 given in Eq. (10.62) suggests also the possibility of a site-by-site cancellation, provided by:

$$b_i = \delta(\tilde{g}^2/g_i^2)(1 - y_i). \quad (10.63)$$

This choice, for small b_i , gives $\epsilon_3 \approx 0$ for $\delta = 1$. Assuming again $f_i = f_c$, $g_i = g_c$, the allowed region in the space $(\delta, \sqrt{K}/g_c)$ is shown on the right panel of Fig. 10.10.

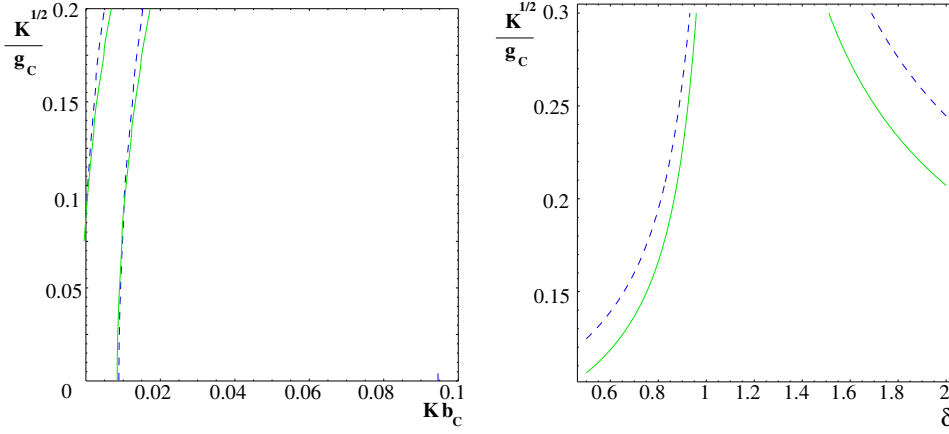


Fig. 10.10: The 95% CL bounds on the plane $(Kb_c, \sqrt{K}/g_c)$ -left panel, $(\delta, \sqrt{K}/g_c)$ -right panel, from the experimental value of ϵ_3 for $K = 1$ (solid green lines), $K = 10$ (dash blue lines). The allowed regions are between the corresponding lines.

In conclusion, by fine tuning every direct fermion coupling at each site to compensate the corresponding contribution to ϵ_3 from the moose gauge bosons (see also [15, 16]), it is possible to satisfy the EW constraints and improve the unitarity bound of the Higgsless SM at the same time.

10.3.3 Continuum limit

We would like to discuss the continuum limit of the previously discussed moose model by taking $K \rightarrow \infty$ with the condition $Ka = \pi R$, where πR is the length of the segment in the fifth dimension and $a \rightarrow 0$ is the lattice spacing. By defining

$$\lim_{a \rightarrow 0} a f_i^2 = f^2(y), \quad \lim_{a \rightarrow 0} a g_i^2 = g_5^2(y) \quad (10.64)$$

the action obtained as the continuum limit of the Lagrangian (10.52), for flat metric $g_5(y) = g_5$ and $f(y) = \bar{f}$, can be written as [11]

$$S = -\frac{1}{4} \int d^4x \int_0^{\pi R} dy \left[\frac{1}{g_5^2} (F_{MN}^a)^2 + \frac{1}{\tilde{g}^2} (F_{\mu\nu}^a)^2 \delta(y) + \frac{1}{\tilde{g}^2} (F_{\mu\nu}^3)^2 \delta(y - \pi R) \right] + S_{ferm}$$

where F_{MN} is the tensor associated to the 5D field A_N and the brane kinetic terms, arising from the left and right ends of the moose chain, are the terms which modify the otherwise linear mass spectrum of KK excitations of gauge bosons. These are necessary in order to avoid light KK excitations of the standard gauge bosons.

In the continuum limit, left- and right-handed fermions live at the opposite ends of the extra-dimension. However, in the discrete, we have introduced an interaction term invariant under all the symmetries of the model which delocalizes the left-handed fermions in the continuum limit. In fact, we have seen that the fermionic fields along the string are defined in terms of the operator

$$\Sigma_1 \Sigma_2 \cdots \Sigma_i. \quad (10.65)$$

In five-dimensions the fields Σ 's can be interpreted as the link variables along the fifth dimension. As such they can be written in terms the fifth component of the gauge fields A_N . As a consequence the operator given in Eq. (10.65) becomes a Wilson line

$$\Sigma_1 \Sigma_2 \cdots \Sigma_i \rightarrow P \left[\exp \left(-i \int_0^y dz A_5(z) \right) \right]. \quad (10.66)$$

In this way the original fermionic fields acquire a non-local interaction induced by Wilson lines. The fermion action in Eq. (10.65) is therefore given by

$$S_{ferm} = \int d^4x \int_0^{\pi R} dy \left[\delta(y) i \bar{\psi}_L \not{D} \psi_L + \delta(\pi R - y) i \bar{\psi}_R \not{D} \psi_R + b(y) i \bar{\chi}_L \not{D} \chi_L \right] \quad (10.67)$$

where

$$b(y) = \lim_{a \rightarrow 0} \frac{b_i}{a}, \quad \chi_L(y) = P \left[\exp \left(-i \int_0^y dz A_5(z) \right) \right] \psi_L \quad (10.68)$$

and

$$\not{D} \psi_L = \left(\not{D} + i \frac{\tau^a}{2} A^a(y) + i Y_L A^3(\pi R) \right) \psi_L, \quad \not{D} \psi_R = \left(\not{D} + i Y_R A^3(y) \right) \psi_R \quad (10.69)$$

$$\not{D} \chi_L = \left(\not{D} + i \frac{\tau^a}{2} A^a(y) + i Y_L A^3(\pi R) \right) \chi_L \quad (10.70)$$

with $Y_{L,R}$ the left and right hypercharges. Mass terms for the fermions can be generated by

$$\lambda^{ij} \bar{\psi}_L^i P \left[\exp(-i \int_0^{\pi R} dz A_5(z)) \right] \psi_R^j.$$

The breaking of $SU(2)$ to $U(1)_{em}$ is obtained by the following boundary conditions:

$$\partial_y A_\mu^{1,2} - \frac{g_5^2}{\tilde{g}^2} \square A_\mu^{1,2}|_{y=0} = 0, \quad A_\mu^{1,2}|_{y=\pi R} = 0, \quad (10.71)$$

$$\partial_y A_\mu^3 - \frac{g_5^2}{\tilde{g}^2} \square A_\mu^3|_{y=0} = 0, \quad \partial_y A_\mu^3 + \frac{g_5^2}{\tilde{g}^2} \square A_\mu^3|_{y=\pi R} = 0 \quad (10.72)$$

We would like to discuss what is the continuum limit for the direct fermionic couplings when we choose the b_i 's according to the Eq. (10.63) with $\delta = 1$ which corresponds to a site-by-site cancellation. By assuming $g_5(y) = g_5$, with g_5 constant, we get

$$b(y) = \frac{\tilde{g}^2}{g_5^2} \int_y^{\pi R} dt \frac{f^2}{f^2(t)}, \quad \text{with } \frac{1}{f^2} = \int_0^{\pi R} \frac{dy}{f^2(y)}. \quad (10.73)$$

From Eq. (10.73) we see that $b(0) = \tilde{g}^2/g_5^2$, $b(\pi R) = 0$. Therefore the direct fermionic coupling decreases along the fifth dimension going from the brane located at $y = 0$ to the brane at $y = \pi R$. For the case of constant $f(y) = \bar{f}$ we find

$$b(y) = \frac{\tilde{g}^2}{g_5^2} \left(1 - \frac{y}{\pi R} \right). \quad (10.74)$$

With this choice the contribution from the new delocalized fermion interactions to ϵ_3 is given by

$$\epsilon_3|_{ferm} = -\frac{1}{\pi R} \int_0^{\pi R} dy y b(y) = -\frac{\tilde{g}^2}{g_5^2} \frac{\pi R}{6} \quad (10.75)$$

which is just the opposite of the contribution to ϵ_3 in the linear moose [11, 24].

Another interesting case corresponds to a Randall-Sundrum metric along the fifth dimension [1, 33]. It corresponds to

$$f(y) = \bar{f} e^{ky} \quad (10.76)$$

and we find

$$b(y) = \frac{\tilde{g}^2}{g_5^2} \frac{e^{-2\pi k R} - e^{-2ky}}{e^{-2\pi k R} - 1}. \quad (10.77)$$

In this case we get

$$\epsilon_3|_{ferm} = - \int_0^{\pi R} dy \frac{e^{-2ky} - 1}{e^{-2\pi k R} - 1} b(y) = - \frac{\tilde{g}^2}{g_5^2} \frac{1}{4k} \frac{e^{4k\pi R} - 4k\pi R e^{2k\pi R} - 1}{(1 - e^{2k\pi R})^2} \quad (10.78)$$

which is the opposite of the contribution from the gauge bosons derived in [24].

Therefore by allowing for a fine tuning obtained with a convenient delocalization of the fermion couplings, the contribution to the S parameter coming from the gauge and fermion sectors vanishes. We are currently investigating whether a geometrical mechanism can be found in order to guarantee such a cancellation. Notice that in the previous formulation we do not assume the existence of bulk fermions; under certain hypotheses these can generate the delocalization of the standard model fermionic couplings, parameterized by b .

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