ANALYTICITY AND $\rho' \to \pi\pi$

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Abstract: Analyticity, in the form of fixed t and fixed u dispersion relations, distinguishes between the $\pi\pi$ partial wave solutions in favour of solutions with a $\rho'(1600)$ resonance with a branching ratio of 25% into $\pi\pi$.

Résumé: Analyticité, dans la forme des dispersion relations t fixé et u fixé, fait la distinction entre les solutions $\pi\pi$ ondulations partiales en faveur des solutions avec resonance $\rho'(1600)$ avec une proportion ramifieé de 25% à $\pi\pi$.

An outstanding problem in meson spectroscopy, which is particularly relevant with the advent of the new ψ particles, concerns the existence of ρ', ω', ϕ' vector mesons. To date the only experimental evidence for these mesons is for a ρ' resonance. Although the Frascati data[1] for $e^+e^- \to 2\pi(4\pi)$ is not conclusive, there is definite evidence for a $\rho'(1600)$ resonance in the FNAL photoproduction data[2] for the reaction $\gamma Be \to 2\pi(4\pi)Be$. The analysis of these latter data is not completed yet, so the branching ratio $\rho' \to \pi^+\pi^-/\rho' \to \pi^+\pi^-\pi^+\pi^-$ is not given, but there are indications that it will be small.

Independent evidence on the $\rho' \to \pi^+\pi^-$ decay mode can be obtained from the $\pi\pi$ partial wave analyses of the high statistics CERN-Munich data[3], which should lead to a reliable estimate for the $\rho' \to \pi^+\pi^-$ branching ratio. Unfortunately, these analyses are subject to discrete Barrelet-Gersten ambiguities: that is, if the $\pi^+\pi^-$ amplitude is written in the form

$$F(s,t) = F(s,o) \prod_i \left[\frac{z-z_i(s)}{1-z_i(s)}\right] \quad .$$

where $z = 1+t/2q^2$, then the observable $|F|^2$ is unchanged by $z_i \to z_i^*$. This leads to four possible solutions[3,4], which as far as the $\rho'(1600)$ is concerned† divide into two categories: (i) solutions B,D (in the notation of ref. 4) which have a relatively strong $\rho' \to \pi\pi$ coupling (elasticity 25%), and (ii) solutions A,C which show no evidence for a ρ' signal (elasticity $\lesssim 4\%$). In terms of the Barrelet zeros these two categories arise because the first zero, $z_1(s)$, to enter the physical region has Im $z_1 \approx 0$ near $\sqrt{s} \equiv M_{\pi\pi} = 1.25$ GeV which causes a branching of solutions. Solutions of type (i) and (ii) correspond to Im $z_1 > 0$ and Im $z_1 < 0$ respectively above this energy.

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†The partial wave analyses[3,4] show no evidence for $\rho'(1250)$ coupling to $\pi\pi$.

One way to distinguish between the solutions would be to obtain $\pi N \to \pi\pi N$ data in other channels besides $\pi^+\pi^\pm$ scattering. For example, data on $\pi^\pm\pi^o$ scattering or $\pi^-\pi^+ \to \pi^o\pi^o$ will be a valuable discriminant; a recent discussion is given in ref. 5.

Analyticity may also be used to select the physical $\pi\pi$ partial wave solution. The data determines $|F(s,t)|$ and each solution has a characteristic phase $\phi(s,t)$ of the amplitude relative to its phase at t=o. This is illustrated in fig. 1 at one typical energy. We see that solutions

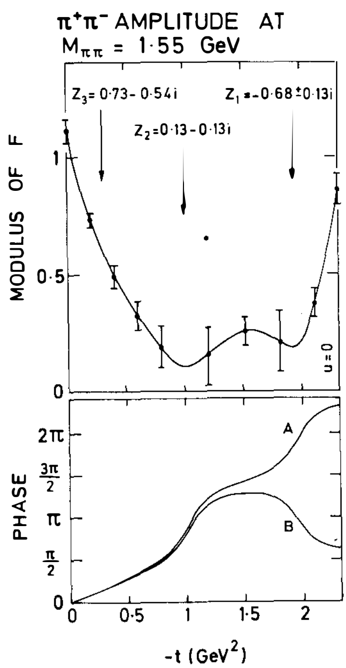

Fig. 1
The modulus and phase of the $\pi^+\pi^-$ amplitude. The modulus can be extracted directly from the data, but the phase (shown relative to the phase at t=o) depends on the partial wave solution. The Barrelet zeros are indicated and solutions A,B correspond to Im $z_1<0$, Im $z_1>0$ respectively.

A and B though of course having the same $|F|$, have quite different phases, especially near the backward direction due to their opposite signs of the imaginary part of the nearby zero, $z_1(s)$. Since analyticity inter-relates the phase and the modulus of an amplitude we expect it to discriminate between solutions. From fig. 1 we anticipate analyticity at fixed t to determine the overall phase, but not to choose between solutions, since their real and imaginary parts are very similar near the forward direction. In contrast, near the backward direction, where the phases differ most, fixed-u analyticity should distinguish solutions once the overall phase is known.

Froggatt and Petersen[6)] have studied fixed t and fixed u analyticity using a technique developed by Pietarinen. They conformally map the right (and left) hand cut s-planes into the unit circle and at each fixed t (fixed u) they parametrize the amplitude as a polynomial in the new variable and fit to the data for $|F|$. They find a solution very similar to B, that is a $\rho'(1600)$ with a 25% branching ratio into $\pi\pi$. However, there are two aspects of the procedure that could lead one to doubt the result. First, they explicitly include the ρ,f and g resonance poles in the full amplitude so as to leave a smoother function for polynomial parametrization. This would appear to bias the analysis in favour of the solution with a ρ' resonance under the g. Secondly, the expansion of an amplitude in terms of large order polynomials in the unit circle leads to large oscillations in the behaviour of the amplitude between the end of the data and $|s| = \infty$.

To avoid these difficulties we may consider conventional fixed t and fixed u dispersion relations, parametrizing the amplitude in terms of Regge pole forms beyond the region of the data. Here the procedure[7)] is to minimize the difference between the output Re F obtained from dispersion relations, by integrating over the data with a free $\phi(s)$ for $1 < \sqrt{s} < 1.8$ GeV, and Re F calculated directly from the data using the same phase. The minimization is performed using several fixed t and u values, and at several values of s with a particularly fine grid of points in the region $1.25 < \sqrt{s} < 1.5$ GeV where analyticity is expected to distinguish solutions of categories (i) and (ii), typified by solutions A and B. We show in Fig. 2 only the results for u=o; we see that

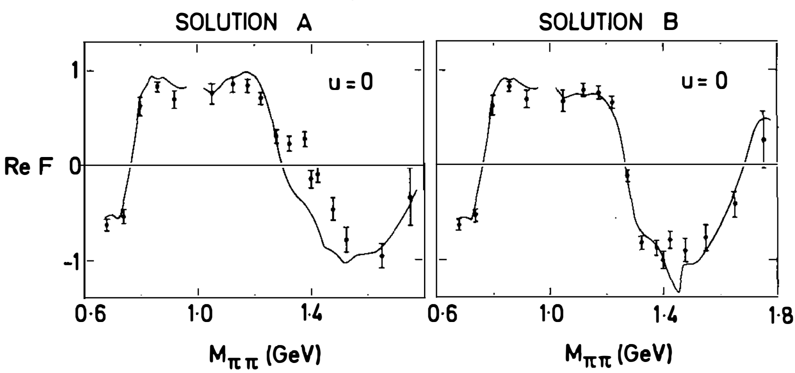

Fig. 2 The real part of the backward $\pi^+\pi^-$ amplitude as obtained from the u=o dispersion relation (the curve) and the input data (the points). Several t and u values are fitted simultaneously and the overall phase $\phi(s,t=o)$ is determined (see ref. 7).

solution B, which has a sizeable $\rho' \to \pi\pi$ coupling, satisfies analyticity very well whereas solution A, which has no ρ' signal, fails badly in the region $\sqrt{s} \sim 1.4$ GeV. As hoped analyticity discriminates between the solutions and strongly favours solutions with a $\rho'(1600)$ with a 25% branching ratio into $\pi\pi$.

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References

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