

Small- x Helicity Evolution and the Proton Spin Puzzle

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We report on the first phenomenological analysis of the world polarized deep-inelastic scattering (DIS) data incorporating small- x helicity (Kovchegov-Pitonyak-Sievert) evolution. This framework allows for one to predict the behavior of helicity parton distribution functions (PDFs) down to very low x . Consequently, one can control the uncertainties in these functions beyond the measured region and make precise calculations of the integrals needed to determine the contribution of quark and gluon spin to the proton spin. Therefore, the small- x helicity formalism will play a crucial role in using future Electron-Ion Collider data to resolve the proton spin puzzle.

KEYWORDS: small x , spin puzzle, helicity PDFs

1. Introduction

The spin $1/2$ of the proton (in units of \hbar) can be decomposed into partonic spin and orbital angular momentum (OAM), such as in the Jaffe-Manohar definition [1]:

$$\frac{1}{2} = S_q + \mathcal{L}_q + S_g + \mathcal{L}_g, \quad (1)$$

where $S_{q(g)}$ and $\mathcal{L}_{q(g)}$ are the spin and OAM of the quark (gluon), respectively. One of the outstanding puzzles in spin physics is how much each of these pieces contributes to the proton spin. One can calculate the quark and gluon spin as integrals of helicity parton distribution functions (PDFs) for the quarks $\Delta q(x)$ and gluon $\Delta g(x)$,

$$S_q = \frac{1}{2} \int_0^1 dx \left[\sum_{q=u,d,s} (\Delta q + \Delta \bar{q})(x, Q^2) \right] \equiv \frac{1}{2} \int_0^1 dx \Delta \Sigma(x, Q^2), \quad (2)$$

$$S_g = \int_0^1 dx \Delta g(x, Q^2). \quad (3)$$

Current measurements estimate that $S_q(Q^2 = 10 \text{ GeV}^2) \approx 0.15 \div 0.20$ and $S_g(Q^2 = 10 \text{ GeV}^2) \approx 0.13 \div 0.26$, with the source of the remaining proton spin thought to be from OAM. However, there are two caveats to this picture. First, the numbers just quoted are for *truncated* integrals that only go down to the lowest x , call it x_{min} , that has been accessed by experiment. For S_q , $x_{min} = 0.001$ and for S_g , $x_{min} = 0.05$. Second, the extractions of $\Delta \Sigma(x)$ and $\Delta g(x)$ have large uncertainties at small x *even when data from the Electron-Ion Collider is included*. The reason for this is standard helicity PDF fits parametrize the x dependence and use DGLAP evolution to evolve in Q^2 . Once the extracted helicity PDFs go below x_{min} , they just become an extrapolation. Instead, what is needed to control uncertainties at low x are evolution equations that evolve in x instead of Q^2 that can then *predict* the behavior of helicity PDFs down to small x . This was worked out in a series of papers [2–8], and herein will be referred to as KPS evolution. We will discuss the results, reported in Ref. [9], of applying KPS evolution to a fit of small- x polarized DIS data and give an outlook for future studies.

2. Theoretical Background

The fundamental quantity of interest for all (singlet) small- x helicity quantities is the so-called polarized dipole amplitude [2, 3]. How this dipole enters polarized DIS is schematically depicted in Fig. 1. The exchanged virtual photon fluctuates into a $q\bar{q}$ dipole of transverse size r_{10} , with β the fractional energy carried by the less energetic parton in the dipole. The spin-dependent scattering amplitude of the dipole on the polarized nucleon N is described by $G_q(r_{10}^2, \beta s)$, producing an asymmetry between the cross sections for positive and negative helicity leptons ($s = Q^2(1-x)/x$ is the invariant mass squared of the virtual photon-nucleon system). The polarized dipole amplitude involves the insertion of one non-eikonal scattering with the target, which carries spin information, among the usual eikonal propagation. In the large- N_c limit the polarized dipole amplitude obeys the evolution

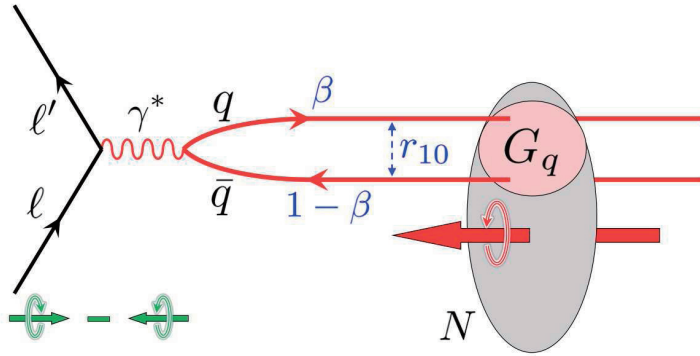


Fig. 1. Polarized DIS at small x .

equations [2–4],

$$G_q(s_{10}, \eta) = G_q^{(0)}(s_{10}, \eta) + \int_{s_{10}}^{\eta} d\eta' \int_{s_{10}}^{\eta'} ds_{21} [\Gamma_q(s_{10}, s_{21}, \eta') + 3 G_q(s_{21}, \eta')], \quad (4)$$

$$\Gamma_q(s_{10}, s_{21}, \eta') = G_q^{(0)}(s_{10}, \eta') + \int_{s_{10}}^{\eta'} d\eta'' \int_{s_{10}}^{s_{32}^{\min}} ds_{32} [\Gamma_q(s_{10}, s_{32}, \eta'') + 3 G_q(s_{32}, \eta'')], \quad (5)$$

where $\eta = \sqrt{\frac{\alpha_s N_c}{2\pi}} \ln \frac{\beta s}{\Lambda^2}$ and $s_{10} = \sqrt{\frac{\alpha_s N_c}{2\pi}} \ln \frac{1}{r_{10}^2 \Lambda^2}$. The quantity $\Gamma_q(s_{10}, s_{21}, \eta')$ is the so-called neighbor dipole, which is an auxiliary function needed in the evolution of $G_q(s_{10}, \eta)$ but ultimately does not enter physical observables.

3. Phenomenological Results

3.1 Constraints from Existing Data

We now turn our focus to analyzing small- x polarized DIS data on the asymmetries A_{\parallel} and A_1 , which are both proportional to the ratio of the g_1 and F_1 structure functions, $A_{\parallel} \sim A_1 \sim g_1/F_1$. The g_1 structure function at leading order is given by

$$g_1(x, Q^2) = \frac{1}{2} \sum_q e_q^2 \Delta q^+(x, Q^2), \quad (6)$$

where $\Delta q^+ \equiv \Delta q + \Delta \bar{q}$. This combination of helicity PDFs can be written in terms of the polarized dipole amplitude,

$$\Delta q^+(x, Q^2) = \frac{N_c}{2\pi^3} \int_{\Lambda^2/s}^1 \frac{d\beta}{\beta} \int_{1/\beta s}^{r_{\max}^2} \frac{dr_{10}^2}{r_{10}^2} G_q(r_{10}^2, \beta s), \quad (7)$$

where $r_{\max}^2 = \min\{1/\Lambda^2, 1/(\beta Q^2)\}$. We note that using only DIS data does not allow for a flavor separation of the helicity PDFs (unless one uses SU(3) symmetry), so instead we extract the proton and neutron g_1 structure functions, g_1^p and g_1^n . The main unknown in the KPS evolution equations (4), (5) is the initial condition $G_q^{(0)}(s_{10}, \eta)$, which we parametrize using a Born-level inspired ansatz: $G_q^{(0)}(s_{10}, \eta) = a_q \eta + b_q s_{10} + c_q$. The parameters a_q, b_q, c_q are fit to the data. Another aspect of KPS evolution that is not known *a priori* is where to begin the evolution in x , which is also the cut $x < x_0$ we place on the data. We found that we are able to maintain $\chi^2/N_{\text{pts}} \sim 1$ up to $x_0 = 0.2$, and in the following, we use $x_0 = 0.1$. The fit was carried out in the Jefferson Lab Angular Momentum (JAM) Monte Carlo framework, and we refer to our results as JAMsmallx [9].

The extracted g_1^p structure function is shown in Fig. 2. This includes the result from existing polarized DIS data (light red band) as well as with Electron-Ion Collider (EIC) pseudodata (dark red band). For comparison we include g_1^p from the DSSV fit to existing data [10, 11] (light blue band) and with EIC pseudodata [12] (light purple band). The inset gives the relative uncertainty $\delta g_1^p/g_1^p$ for each fit at small x . We note two important features of this plot: 1) we have a robust *prediction*, based

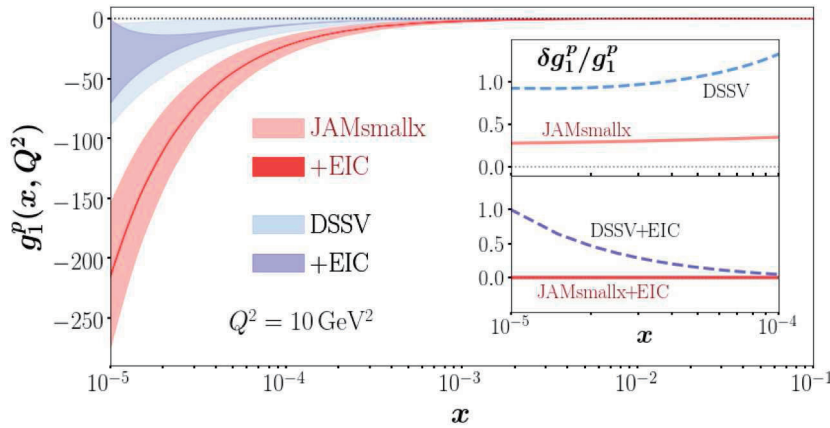


Fig. 2. Plot of the g_1^p structure function from JAMsmallx [9] compared to a DGLAP-based fit [10–12].

on existing polarized DIS measurements, that g_1^p goes negative at small x ; 2) we are able to maintain well-controlled uncertainties *even beyond the lowest x value probed by experimental data*. This is in contrast to standard DGLAP-based fits, where the behavior of g_1^p beyond the experimentally covered region is simply an extrapolation, and consequently the uncertainties increase significantly *even when including EIC data*.

3.2 Impact from EIC Data

The EIC will provide more precise measurements of polarized DIS down to even lower x values ($\sim 10^{-4}$), which will be crucial data needed to resolve the spin puzzle [13]. In Fig. 2 we already observed the drastic impact EIC pseudodata has on the extraction of g_1^p (dark red band). Another

opportunity at the EIC will also be to measure parity-violating DIS (PVDIS). This in principle also allows for a flavor separation of the helicity PDFs without the need of another observable, like semi-inclusive DIS. In order to provide an example of extracting helicity PDFs in our small- x formalism, we utilize EIC pseudodata on polarized DIS and PVDIS, as well as the existing polarized DIS data, to extract $\Delta q^+(x)$ for $q = u, d, s$. The results are displayed in Fig. 3 along with $\Delta\Sigma(x) \equiv \sum_q \Delta q^+(x)$. We especially notice in the bottom plot the same crucial feature of the small- x helicity framework as

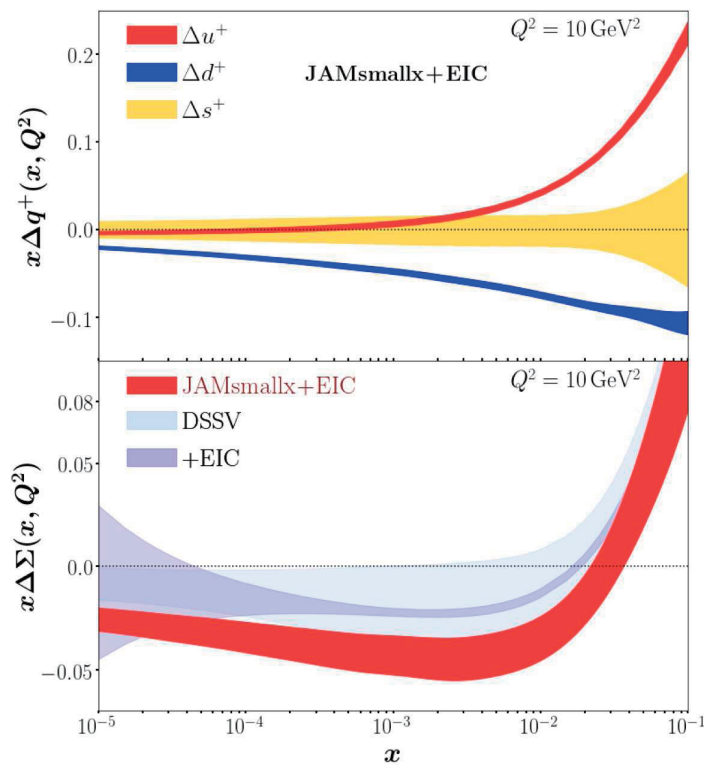


Fig. 3. (Top) Extracted helicity PDFs $x\Delta q^+(x)$ from JAMsmallx. (Bottom) The result for $x\Delta\Sigma(x)$ from JAMsmallx (red) compared with that from the DSSV analysis with [12] (light purple) and without [10,11] (light blue) EIC pseudodata.

before: well-controlled uncertainties *even beyond where there is experimental data*, which is due to the fact that KPS evolution *predicts* the behavior of helicity PDFs at small x . On the other hand, the DGLAP-based fit experiences a significant increase in uncertainties at small x *even when EIC data is included*. Therefore, if one ultimately wants to accurately calculate the integrals in Eqs. (2), (3) down to $x = 0$, the use of KPS evolution in extractions of helicity PDFs is essential.

4. Conclusion and Outlook

We have shown the JAMsmallx framework, utilizing KPS evolution, can fit the world polarized DIS data at $x < 0.1$, with significantly reduced uncertainties (compared to standard DGLAP fits) as one extends into the unmeasured (small- x) region. In the future we will include polarized SIDIS data to perform a flavor separation and make a genuine prediction for the spin carried by quarks at small x as well as explore polarized proton-proton collisions to access $\Delta g(x)$. Additional aspects to build into the framework include single-log corrections [14] and using solutions for the large- N_c &

$-N_f$ KPS evolution equations [15]. The approach we have pioneered here will allow us to achieve well-controlled uncertainties as one extends into the unmeasured small- x region (beyond what even the EIC can reach), a feature that ultimately will be crucial to understanding the partonic origin of the proton spin.

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