Factorisation and Local Subtraction of Infrared Divergences for QCD processes

C Signorile-Signorile

Dipartimento di Fisica and Arnold-Regge Center, Università di Torino and INFN, Sezione di Torino

E-mail: signoril@to.infn.it

Abstract. We review a recently proposed subtraction procedure at next-to- and next-to-next-to-leading order in quantum chromodynamics (QCD), based on the introduction of a set of minimal, local and analytically integrable counterterms. We also provide an operator definition for the counterterms by investigating the factorisation properties of a generic massless virtual amplitude. Our local, analytic scheme is designed for massless final state radiation only, but its main features are extensible to initial state radiation.

1. Introduction

In the era of high precision data from the Large Hadron Collider, the next-to-next-to-leading perturbative order (NNLO) in QCD is the unavoidable standard for fixed-order predictions. In order to achieve this level of accuracy, analytical computations have been superseded by sophisticated numerical algorithms for relevant observables. These powerful tools require to get rid of infrared divergences coming from virtual corrections and real radiation before the numerical evaluation. With this aim, in the past decades, many different subtraction schemes have been proposed and largely applied to a wide set of infrared-safe observables of scattering processes at next-to-leading order (NLO) and some of them have been extended to NNLO. At NLO the mostly used methods were presented by Frixione-Kunszt-Singer [1, 2] (FKS) and Catani-Seymour (CS) [3, 4] in the '90s. At NNLO, various schemes can be found in the literature, among them the antenna subtraction [5, 6], the colorful subtraction [7], the projection-to-Born [8] and the sector-improved residue subtraction [9, 10]. Despite this considerable variety of elaborate methods, many of them are based on demanding numerical calculations or involved analytical integrations. The intrinsic complexity of the problem encourages further investigation with a view to implementing a local, analytic, physically transparent and numerical efficient scheme, that could be adapted to higher orders. For this purpose, we present a new subtraction method that combines the main strengths of the FKS and the CS schemes and that conjugates a minimal subtraction structure with an efficient integration strategy, applied for the moment to processes with partons in the final state only.

2. Subtraction pattern at NLO

In order to present the subtraction procedure at NLO we start by introducing the generic differential cross section with respect to an IR-safe observable X

$$\frac{d\sigma^{\text{NLO}}}{dX} = \lim_{d\to 4}\left\{ \int d\Phi_n\, V_n\, \delta_n + \int d\Phi_{n+1}\, R_{n+1}\, \delta_{n+1} \right\}, \tag{1}$$

where $d = 4-2\epsilon$ identifies the number of space-time dimensions, $d\Phi_i$ is the i-particle phase space, V_n is the UV-renormalised one loop correction and R_{n+1} is the single real radiation squared amplitude at tree level. Finally, $\delta_i \equiv \delta(X - X_i)$ sets the observable X to be computed in the i-body kinematics. The virtual contribution features up to a double pole in the dimensional regulator ϵ, while R is finite for $\epsilon \to 0$, but manifests up to two singular limits in the radiation phase space. Although the sum on the *r.h.s.* of Eq.1 is finite in $d = 4$ thanks to the KLN theorem [11] [12], its evaluation is practically unfeasible in this form. For typical collider processes, the complexity of the relevant processes prevents any possibility of computing the distributions without numerical tools. This requires to implement the singularities cancellation between the real and the virtual contributions before performing the phase space integration. The idea of subtraction is to add and subtract a *counterterm* capable of reproducing the same infrared behaviour of R, but at the same time being simple enough to be integrated analytically in the unresolved phase space. Denoting the counterterm and its integrated counterpart with

$$\frac{d\sigma^{\text{NLO}}_{ct}}{dX} = \int \Phi_{n+1}\, K_{n+1}\,, \qquad I_n = \int d\Phi_{\text{rad}}\, K_{n+1}\,, \tag{2}$$

the subtracted differential cross section can be recast in the following form

$$\frac{d\sigma^{\text{NLO}}}{dX} = \int d\Phi_n \Big[V_n + I_n\Big]\, \delta_n + \int d\Phi_{n+1} \Big[R_{n+1}\, \delta_{n+1} - K_{n+1}\, \delta_n\Big], \tag{3}$$

where both terms in squared brackets are separately finite and integrable in $d = 4$.

3. The main aspects of the subtraction scheme

In order to define the counterterm K_{n+1} in the most physically transparent way and with the simplest analytic structure, it is convenient to introduce a partition of the radiation phase space in sectors by using a set of functions, $\mathcal{W}_{ij}$ (where i, j run over the $n+1$ partons), inspired by the FKS scheme [1]. We require these functions to select the minimum number of IR singularities in each sector and to recover the entire phase space once we have summed over i and j. Moreover, the sum over all the sectors sharing a singular configuration has to reduce to unity, such that the analytic integration of the counterterm does not involve the (in case) complicated expression of $\mathcal{W}_{ij}$. We are able to define the local counterterm K_{ij} by collecting the leading singular contributions of the real matrix element projected in the sector ij

$$K_{ij} = [\mathbf{S}_i + \mathbf{C}_{ij} - \mathbf{S}_i\mathbf{C}_{ij}] R\, \mathcal{W}_{ij} \tag{4}$$

where the action of the soft $\mathbf{S}_i$, the collinear $\mathbf{C}_{ij}$ and the nested $\mathbf{S}_i\mathbf{C}_{ij}$ limit on R factorises a universal kernel and a Born-like matrix element according to the following expressions [13]

$$\mathbf{S}_i R(\{k\}) \propto \sum_{c,d} \frac{k_c \cdot k_d}{k_i \cdot k_c\, k_i \cdot k_d}\, B_{cd}(\{k\}_{\not i})\,, \qquad \mathbf{C}_{ij} R(\{k\}) \propto \frac{P^{\mu\nu}_{ij}}{k_i \cdot k_j}\, B_{\mu\nu}(\{k\}_{\not i \not j}, k_i + k_j)\,, \tag{5}$$

$$\mathbf{S}_i\mathbf{C}_{ij} R(\{k\}) \propto \frac{k_j \cdot k_r}{k_i \cdot k_j\, k_i \cdot k_r}\, B(\{k\}_{\not i})\,.$$

In the soft limit, B_{lm} is the color-connected Born-level squared matrix element and $\{k\}_{\not{i}}$ is the set of the initial $n+1$ final-state momenta $\{k\}$ with k_i removed. The collinear limit features the spitting kernel $P_{ij}^{\mu\nu}$ and the spin-correlated Born-level squared matrix element $B_{\mu\nu}$. The latter depends on the momenta set $(\{k\}_{\not{i}\not{j}}, k_i + k_j)$, obtained from $\{k\}$ by removing k_i and k_j and including their sum $k_i + k_j$. The two kinematics sets, $\{k\}_{\not{i}}$ and $(\{k\}_{\not{i}\not{j}}, k_i + k_j)$, do not satisfy the momentum conservation and the on-shell mass condition away from the exact $\mathbf{S}_i$ and $\mathbf{C}_{ij}$ limits. For this reason, a momentum mapping is a necessary ingredient to provide a counterterm suited for a proper subtraction algorithm. By taking advantage of the freedom in choosing the mapping, we introduce a Catani-Seymour final state mapping [3], different for each sector and for each term contributing to the soft kernel. After the mapping procedure, identified with a bar over the relevant operators, and the sum over sectors we obtain the complete counterterm

$$\overline{K} = \sum_{i,j\neq i} \overline{K}_{ij} = \sum_i \overline{\mathbf{S}}_i R + \sum_{i,j>i} \overline{\mathbf{C}}_{ij}(1 - \overline{\mathbf{S}}_i - \overline{\mathbf{S}}_j)R \equiv \overline{K}^{(s)} + \overline{K}^{(c)} - \overline{K}^{(sc)} \,. \tag{6}$$

To maximally simplify the integration of $\overline{K}$, we choose to parametrise the phase space according to the relevant kinematic mapping of each contribution, following the Catani-Seymour scheme. This way, the local counterterm has a minimal structure and can be analytically integrated [14].

4. Factorisation

The identification of the counterterms can be also derived from general considerations based on the factorisation properties of virtual QCD amplitudes [15]. The aim of this procedure is to improve our understanding of the kernels structure prior to any complete subtraction algorithm, *i.e.* no sector partition and no kinematic mapping are implemented. In massless QCD, a generic n-parton scattering amplitude splits according to the factorisation formula [16]

$$\mathcal{A}_n\left(\frac{p_i}{\mu}\right) = \prod_{i=1}^{n}\left[\frac{\mathcal{J}_i((p_i\cdot n_i)^2/(n_i^2\mu^2))}{\mathcal{J}_{i,\mathrm{E}}((\beta_i\cdot n_i)^2/n_i^2)}\right] \mathcal{S}_n(\beta_i\cdot\beta_j)\,\mathcal{H}_n\left(\frac{p_i\cdot p_j}{\mu^2}, \frac{(p_i\cdot n_i)^2}{n_i^2\mu^2}\right) . \tag{7}$$

Here n_i^μ is an auxiliary vector that allows for the full factorisation of the collinear content of $\mathcal{A}_n$ and enforces its gauge invariance, and β_i^μ is a four-velocity vector obtained by extracting the hard scale from the initial momenta p_i^μ, namely $p_i^\mu = Q\beta_i^\mu$ with $\beta_i^2 = 0$. The soft content of the amplitude is encoded in the soft function $\mathcal{S}_n$, a color operator acting on the finite hard reminder $\mathcal{H}_n$, which is the only process-dependent object in Eq.7. The ratio $\mathcal{J}_i/\mathcal{J}_{i,\mathrm{E}}$ embeds all and only the pure hard-collinear singularities of the process, such that the double counting of the soft-collinear configurations is automatically avoided. The functions appearing in the factorisation formula are universal, gauge invariant and defined at all orders in perturbation theory *via* matrix elements of field operators and semi-infinite Wilson lines [17]. The soft function is defined as the correlator of n Wilson lines oriented along the classical trajectories of the n hard partons, while the jet function $\mathcal{J}_i$ is a color singlet operator, depending on the nature of the emitting parton,

$$\mathcal{S}_n(\beta_i\cdot\beta_j) = \langle 0|\prod_{k=1}^{n}\Phi_{\beta_k}(\infty,0)\,|0\rangle\;, \qquad \bar{u}_s(p)\mathcal{J}_q\left(\frac{(p_i\cdot n_i)^2}{n_i^2\mu^2}\right) \equiv \langle p,s|\,\bar{\psi}(0)\Phi_n(0,\infty)\,|0\rangle\;. \tag{8}$$

Finally, the eikonal jet function $\mathcal{J}_{i,\mathrm{E}}$ is the soft approximation of the collinear function, featuring a Wilson line instead of the fermion field. In order to mimic the radiative counterpart of the virtual amplitude (see the second term of the r.h.s in Eq.1) and model its IR behaviour, we need to extend the virtual definition of $\mathcal{S}_n, \mathcal{J}_i$ and $\mathcal{J}_{i,\mathrm{E}}$ to generic real-radiation functions, involving the emission of m extra gluons. To obtain such generalisations, we decide to add real particles in the final states and define the *eikonal form factor* as

$$\mathcal{S}_{n,m}(\{k_m\};\beta_i) \equiv \langle k_1,\lambda_1\ldots k_m,\lambda_m|\prod_{i=1}^{n}\Phi_{\beta_i}(\infty,0)\,|0\rangle\;, \tag{9}$$

where we are assuming that this function, in the simplified case involving only soft divergences, regulates at all orders the factorisation of a generic radiative amplitude up to a finite remainder. At the cross section level, the *radiative soft function* reads

$$S_{n,m}(\{k_m\};\beta_i) \equiv \sum_{\{\lambda_i\}} \langle 0| \prod_i \Phi_{\beta_i}(0,\infty)\, |k_1,\lambda_1 \dots k_m,\lambda_m\rangle \,\langle k_1,\lambda_1 \dots k_m,\lambda_m| \prod_i \Phi_{\beta_i}(\infty,0)\,|0\rangle \;, \quad (10)$$

and naturally provides a finite quantity after summing over all the possible real gluons and integrating over their phase space. The resulting fully inclusive object is finite order by order in perturbation theory, and links virtual and radiative soft functions:

$$\sum_{m=0}^{\infty} \int d\Phi_m\, S_{n,m}(\{k_m\};\beta_i) = \langle 0| \prod_i \Phi_{\beta_i}(0,\infty) \prod_i \Phi_{\beta_i}(\infty,0)\,|0\rangle \;. \quad (11)$$

The first non trivial perturbative order in the coupling constant gives the relation

$$S^{(1)}_{n,0}(\beta_i) + \int d\Phi_1\, S^{(0)}_{n,1}(k,\beta_i) = \text{finite}\,, \quad (12)$$

where it is now straightforward to recognise in the second term a candidate for the soft counterterm. Indeed, $S^{(0)}_{n,1}$ reproduces exactly the expression of the tree-level eikonal current known from the literature [13] and it is in perfect agreement with the explicit expression of $K^{(s)}$, introduced in our subtraction scheme in the previous section. With a similar procedure we define the radiative jet function at cross-section level as

$$J_{q,m}(\{k_m\};p,n) \equiv \int d^dx\, e^{il\cdot x} \sum_{\{\lambda_i\}} \langle 0|\, \Phi_n(\infty,x)\psi(x)\, |p,s;k_j,\lambda_j\rangle \,\langle p,s;k_j,\lambda_j|\, \overline{\psi}(0)\Phi_n(0,\infty)\,|0\rangle \;, \quad (13)$$

where $l^\mu = p_i^\mu + \sum_i^m k_i^\mu$ is the total momentum flowing in the final state. Also in this case, a completeness relation for the final states sets a link between a pure virtual function and its radiative counterpart. Specifically, starting from the all-orders relation

$$\sum_{m=0}^{\infty} \int d\Phi_{m+1}\, J_{q,m}(k;l,p,n) = \text{Disc}\left[\int d^dx e^{il\cdot x}\, \langle 0|\, \Phi_n(\infty,x)\psi(x)\bar{\psi}(0)\Phi_n(0,\infty)\,|0\rangle\right], \quad (14)$$

where the r.h.s. is finite since it is fully inclusive in the final state, we obtain at the lowest perturbative order that

$$J^{(1)}_{i,0}(l,p,n) + \int d\Phi_1\, J^{(0)}_{i,1}(k;l,p,n) = \text{finite}\,. \quad (15)$$

In analogy to the soft case, we recognise in the second term of Eq.15 a candidate collinear counterterm. However, in order to compare the integrand function with the counterterm $K^{(c)}$ in Eq.6, it is still necessary to impose the collinearity between the hard quark and the emitted gluon. For this reason, we express the real radiation and the emitting particle momenta through a Sudakov parametrisation introducing a transverse momentum $k_\perp$ orthogonal to the collinear direction, and then we take the leading behaviour of $J^{(0)}_{i,1}$ for $k_\perp \to 0$. For a quark-induced jet this procedure yields the DGLAP splitting kernel for the branching $q \to qg$ [18]. The eikonal jet contribution completes the list of counterterms at NLO

$$\begin{aligned} K^{(s)}_{n+1} &= \mathcal{H}^{(0)\dagger}_n(p_i)\, S^{(0)}_{n,1}\, \mathcal{H}^{(0)}_n(p_i) \quad (16)\\ K^{(c)}_{n+1} &= \sum_i \mathcal{A}^{(0)\dagger}_n(p_1 \dots p_{i-1},l,p_{i+1}\dots p_n)\, J^{(0)}_{i,1}(k_i;l,p_i,n_i)\, \mathcal{A}^{(0)}_n(p_1 \dots p_{i-1},l,p_{i+1}\dots p_n)\\ K^{(sc)}_{n+1} &= \sum_i \mathcal{A}^{(0)\dagger}_n(p_1 \dots p_{i-1},l,p_{i+1}\dots p_n)\, J^{(0)}_{i,\mathrm{E},1}(k_i;l,p_i,n_i)\, \mathcal{A}^{(0)}_n(p_1 \dots p_{i-1},l,p_{i+1}\dots p_n)\;. \end{aligned}$$

At this point it is straightforward to map the terms above with the last expression appearing in Eq.6. As discussed at the beginning of this section, no mapping is implemented in Eq.16.

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5. Generalisation to NNLO

Both approaches are generalisable at NNLO and they both point out interesting properties of the subtraction procedure at higher perturbative orders. The expression of a generic distribution receives now contributions from double-virtual VV_n, real-virtual RV_{n+1} and double real RR_{n+2} configurations

$$\frac{d\sigma^{\rm NNLO}}{dX} = \lim_{d\to 4} \left\{ \int d\Phi_n \, VV_n \, \delta_n + \int d\Phi_{n+1} \, RV_{n+1} \, \delta_{n+1} + \int d\Phi_{n+2} \, RR_{n+2} \, \delta_{n+2} \right\}. \tag{17}$$

As a consequence, the pattern of cancellations among counterterms becomes more involved. In particular, to subtract all the unresolved configurations of RR we introduce $K^{(\mathbf{1})}$, that encodes the single unresolved contributions, and $(K^{(\mathbf{12})} + K^{(\mathbf{2})})$ to cure the double unresolved configurations. In the latter, $K^{(\mathbf{12})}$ collects all the limits where one parton becomes unresolved with a faster rate with respect to the other one, and $K^{(\mathbf{2})}$ contains all the "homogeneous" double-unresolved singularities. Moreover, we introduce $K^{(\mathbf{RV})}$ to treat the real-virtual divergences coming from the regions of phase space where the radiated parton becomes unresolved. Denoting the corresponding integrated counterterms with

$$I^{(\mathbf{i})} = \int d\Phi_{{\rm rad},i} \, K^{(\mathbf{i})} \,, \quad I^{(\mathbf{12})} = \int d\Phi_{{\rm rad},1} \, K^{(\mathbf{12})} \,, \quad I^{(\mathbf{RV})} = \int d\Phi_{\rm rad} \, K^{(\mathbf{RV})} \,, \quad \mathbf{i} = 1, 2 \,, \tag{18}$$

the subtracted distribution reads

$$\begin{aligned} \frac{d\sigma^{\rm NNLO}}{dX} = & \int d\Phi_n \left[VV_n + I^{(\mathbf{2})} + I^{(\mathbf{RV})} \right] \delta_n \\ & + \int d\Phi_{n+1} \left[(RV_{n+1} + I^{(\mathbf{1})}) \, \delta_{n+1} - (K^{(\mathbf{RV})} - I^{(\mathbf{12})}) \, \delta_n \right] \\ & + \int d\Phi_{n+2} \left[RR_{n+2} \, \delta_{n+2} - K^{(\mathbf{1})} \delta_{n+1} - (K^{(\mathbf{12})} + K^{(\mathbf{2})}) \delta_n \right]. \end{aligned} \tag{19}$$

In the second line $I^{(\mathbf{1})}$ manifests the same poles in ϵ as RV, $I^{(\mathbf{12})}$ locally subtracts the explicit poles of $K^{(\mathbf{RV})}$ and $I^{(\mathbf{RV})} + I^{(\mathbf{2})}$ has the same $1/\epsilon$ poles as the double virtual. This way, the contribution of each line is finite in $d = 4$ and integrable numerically. In order to implement an efficient subtraction algorithm one has to adopt the same main ingredients introduced at NLO, taking into account the contribution of new singular limits. In particular, we have the configuration $\mathbf{S}_{ij}$ where partons ij are homogeneously soft, $\mathbf{C}_{ijk}$ with three partons become collinear, $\mathbf{C}_{ijkl}$ where the parton pairs (ij) and (kl) are independently collinear, and $\mathbf{SC}_{ijk}$ with a soft parton i and a collinear pair (jk). In addition, we implement sector functions $\mathcal{W}_{ijkl}$ with as many indices as the maximum number of partons that can become unresolved at the same time in a NNLO configuration. Functions $\mathcal{W}_{ijkl}$ obey sum rules similar to those discussed in Sec.1. Moreover, they factorise NLO sector functions in single-unresolved regimes, enabling $I^{(\mathbf{1})}$ to subtract sector by sector the poles of RV.

On the other hand, the counterterm structure can also be derived from the factorised expression of the virtual amplitude. The idea is to expand Eq.7 up to the fourth power in the coupling constant $g_{\rm s}$ and then organise the result in terms of cross-section-level virtual functions, which are related to radiative quantities through the completeness relations in Eq.11 and Eq.14. The radiative functions provide the desired counterterms according to different kinematic structures. The full list of counterterms can be found in Ref.[15]. To present an example of the matching between the subtraction algorithm and the factorisation approach at NNLO, we analyse the contribution to $K^{(\mathbf{2})}$ that involves $\mathbf{S}_{ij}$. In our scheme, after the sum over sector functions and kinematic remapping, identified with a bar, it reads

$$\overline{K}^{(\mathbf{2})}\big|_{\rm soft} = \sum_i \sum_{j>i} \overline{\mathbf{S}}_{ij} \, RR \,. \tag{20}$$

Instead, starting from the factorisation formula, we deduce the double soft contributions to the virtual correction

$$VV_n|_{\text{soft}} = \mathcal{H}_n^{(0)\,\dagger}\, S_{n,0}^{(2)}\, \mathcal{H}_n^{(0)} + (\mathcal{H}_n^{(0)\,\dagger}\, S_{n,0}^{(1)}\, \mathcal{H}_n^{(1)} + \text{h.c.})\,, \tag{21}$$

and then we exploit the following relation

$$S_{n,0}^{(2)}(\beta_i) + \int d\Phi_1\; S_{n,1}^{(1)}(k,\beta_i) + \int d\Phi_2\; S_{n,2}^{(0)}(k_1,k_2,\beta_i) = \text{finite}\,, \tag{22}$$

to identify the double real emission soft counterterm

$$K_{n+2}^{(\mathbf{2},\,2s)} = \mathcal{H}_n^{(0)\,\dagger}\, S_{n,2}^{(0)}\, \mathcal{H}_n^{(0)}\,. \tag{23}$$

As expected, $K_{n+2}^{(\mathbf{2},\,2s)}$ involves two soft gluons with the same "softness". Its analytical expression matches the result in Ref.[13] and is the analogous of Eq.20 in the unbarred kinematics.

6. Conclusions

We have presented an innovative and efficient subtraction method based on the sector partition of the radiative phase space and the analytical integration of a minimum set of counterterms, valid for any IR-safe observable up to NNLO. Moreover, we have derived the counterterms structure at NLO and at NNLO from the factorised expression of a generic massless QCD amplitude. Work is in progress to investigate the properties of the counterterms at higher perturbative order and to generalise the subtraction scheme to processes with hadrons in the initial state. In particular, by studying the properties of the factorised amplitude we hope to provide important information for the construction of an efficient subtraction algorithm at NNNLO. Although the study of a higher perturbative order means to consider a larger number of contributing configurations, we expect to reconstruct macro-structures similar to those introduced at NNLO, and possibly to point out a fully general pattern.

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