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Research Article

Studies of Three-Body Decay of B to $J/\psi\eta K$ and $B(B_s)$ to $\eta_c\pi K^*$

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We investigate the $B^0 \to J/\psi\eta K^0$ and $B^+ \to J/\psi\eta K^+$ decay by using the Dalitz plot analysis. As we know there are tree, penguin, emission, and emission-annihilation diagrams for these decay modes in the factorization approach. The transition matrix element is factorized into a $B \to \eta K$ form factor multiplied by J/ψ decay constant and also a $B \to K$ form factor multiplied by $J/\psi\eta$ decay constant. According to QCD factorization approach and using the Dalitz plot analysis, we calculate the branching ratios of the $B^0 \to J/\psi\eta K^0$ and $B^+ \to J/\psi\eta K^+$ three-body decay in view of the $\eta-\eta'$ mixing and obtain the value of the $(9.22^{+2.67}_{-1.47}) \times 10^{-5}$, while the experimental results of them are $(8 \pm 4) \times 10^{-5}$ and $(10.8 \pm 3.3) \times 10^{-5}$, respectively. In this research we also analyze the $B(B_s) \to \eta_c\pi K^*$ decay which is similar to the previous decay, but there is no experimental data for the last decay. Since for calculations of the $B(B_s) \to \eta_c\pi K^*$ decay we use assumptions of the $B \to J/\psi\eta K$ decay, we hope that if this decay will be measured by the LHCb in the future, the experimental results will be in agreement with our calculations.

1. Introduction

The three-body decay of $B^{0(+)} \to J/\psi\eta K^{0(+)}$ was originally measured by the BABAR [1] and BELLE [2] collaborations and later on tabulated by the Particle Data Group [3]. A long time ago, the $D \to K\pi\pi$ decay was studied with the Dalitz plot analysis [4]. According to this technique, we can find many articles such as [5–9] which do not assume that the mass of the K-meson is heavy in front of the pi-meson one. So the momentum of the K-meson is sizable against the momentum of the pi-mesons. In these kinds of decay, when three particles are light, given that the theoretical momentums of the output particles are not directly calculable, momentums and form factors are written in terms of the $s = (p_B - p_3)^2$ and $t = (p_B - p_1)^2$ [10, 11] and the Dalitz plot analysis should be used for calculation of the decay rate integral from $s_{\min}$, $t_{\min}$ to $s_{\max}$, $t_{\max}$. In our selected decay with the experimental values of $BR(B^0 \to J/\psi\eta K^0) = (8 \pm 4) \times 10^{-5}$ and $BR(B^+ \to J/\psi\eta K^+) = (10.8 \pm 3.3) \times 10^{-5}$ [3], we obtain $BR(B \to J/\psi\eta K) = (9.22^{+2.67}_{-1.47}) \times 10^{-5}$ by using the Dalitz plot analysis.

Note that, to implement the $\eta-\eta'$ mixing, we will use the two-mixing-angle formalism proposed in [12, 13], in which one has

$$\begin{aligned} |\eta\rangle &= \cos\theta_8 |\eta_8\rangle - \sin\theta_0 |\eta_0\rangle, \\ |\eta'\rangle &= \sin\theta_8 |\eta_8\rangle + \cos\theta_0 |\eta_0\rangle, \end{aligned} \tag{1}$$

where η_8 and η_0 are, respectively, the flavor SU(3)-octet and SU(3)-singlet components. In the quark basis they are given by

$$\begin{aligned} |\eta_8\rangle &= \frac{1}{\sqrt{6}} |u\bar{u} + d\bar{d} - 2s\bar{s}\rangle, \\ |\eta_0\rangle &= \frac{1}{\sqrt{3}} |u\bar{u} + d\bar{d} + s\bar{s}\rangle. \end{aligned} \tag{2}$$

The relations for the pseudoscalar decay constants in this mixing formalism involving the axial-vector currents A_μ^8 and A_μ^0 are

$$\langle 0|\, A_\mu^8 \,|\eta(p)\rangle = i f_\eta^8 p_\mu, \qquad \langle 0|\, A_\mu^8 \,|\eta'(p)\rangle = i f_{\eta'}^8 p_\mu,$$
$$\langle 0|\, A_\mu^0 \,|\eta(p)\rangle = i f_\eta^0 p_\mu, \qquad \langle 0|\, A_\mu^0 \,|\eta'(p)\rangle = i f_{\eta'}^0 p_\mu. \quad (3)$$

In order to get four unknown parameters (θ_8, θ_0, f_8, and f_0) allowed values one has to use as constraints the experimental decay widths of $(\eta, \eta') \rightarrow \gamma\gamma$ [3]:

$$\Gamma(\eta \longrightarrow \gamma\gamma) = (0.511 \pm 0.0026)\ \text{keV},$$
$$\Gamma(\eta' \longrightarrow \gamma\gamma) = (4.338 \pm 0.1592)\ \text{keV}. \quad (4)$$

On the other hand [14, 15]

$$\Gamma(\eta \longrightarrow \gamma\gamma) = \frac{\alpha^2 m_\eta^3}{96\pi^3}\left(\frac{c\theta_0/f_8 - 2\sqrt{2}s\theta_8/f_0}{c\theta_0 c\theta_8 + s\theta_0 s\theta_8}\right)^2,$$
$$\Gamma(\eta' \longrightarrow \gamma\gamma) = \frac{\alpha^2 m_{\eta'}^3}{96\pi^3}\left(\frac{s\theta_0/f_8 + 2\sqrt{2}c\theta_8/f_0}{c\theta_0 c\theta_8 + s\theta_0 s\theta_8}\right)^2. \quad (5)$$

If $f_{\eta'}^c$ is that large, the radiative decay $J/\psi \rightarrow \eta'\gamma$ may be dominated by a contribution where the $c\bar{c}$ pair runs from the J/ψ to the η' meson instead of being annihilated. On that supposition the width of that process can be calculated along the same lines as that one for the $J/\psi \rightarrow \eta\gamma$ decay. The ratio of the two decay widths reads [3]

$$R_{J/\psi} = \frac{\Gamma(J/\psi \to \eta'\gamma)}{\Gamma(J/\psi \to \eta\gamma)} = \left(\frac{m_{\eta'}^2\left(f_8 \sin\theta_8 + \sqrt{2}f_0\cos\theta_0\right)}{m_\eta^2\left(f_8\cos\theta_8 - \sqrt{2}f_0\sin\theta_0\right)}\right)^2 = 4.674 \pm 0.289. \quad (6)$$

The best-fit values of the $(\eta - \eta')$ mixing parameters yield

$$\theta_8 = (-22.2 \pm 1.8)^\circ,$$
$$\theta_0 = (-8.7 \pm 2.1)^\circ,$$
$$f_8 = (167.296 \pm 7.842)\ \text{MeV},$$
$$f_0 = (154.23 \pm 5.228)\ \text{MeV}, \quad (7)$$

which are used to calculate the decay rates in which η and/or η' are involved. In the B meson decay into states with an η, in the case of $B \rightarrow J/\psi\eta K$, since the η meson is described by the $u\bar{u}$, $d\bar{d}$, and $s\bar{s}$ combination, there are two different color-suppressed internal W-emission Feynman diagrams; the $B^0 \rightarrow J/\psi\eta K^0$ decay mode contains $s\bar{s}$ and $d\bar{d}$ pairs while the $B^+ \rightarrow J/\psi\eta K^+$ decay mode includes $s\bar{s}$ and $u\bar{u}$ pairs of the η components. In addition, there is a penguin diagram where the $d\bar{d}$ and $u\bar{u}$ pairs are considered for both decay modes. The diagrams in which J/ψ is emitted via three-gluon exchange are called "hairpin" diagrams so that $u\bar{u}$ pairs of the η components are used. Before giving the matrix elements for the $B \rightarrow J/\psi\eta K$ decay, we discuss the parametrization of the decay constants and form factors which appear in the factorized form of the hadronic matrix elements. In the tree and penguin levels, the K and η mesons are placed in the form factors and the J/ψ meson is placed in the decay constant [16] in which the vector meson's decay constant, such as J/ψ, is expressed in terms of the matrix element $\langle J/\psi|\bar{c}\gamma_\mu c|0\rangle = f_{J/\psi} m_{J/\psi} \epsilon_{J/\psi}$ [17]. We also have a $B \rightarrow K$ form factor multiplied by $J/\psi\eta$ decay constant in the emission diagram with an emission-annihilation level.

The present analysis contains nonfactorizable effects, whereas the hadronic physics governing the $B \rightarrow M_1$ transition and the formation of the emission particle M_2 is genuinely nonperturbative; nonfactorizable interactions connecting the two systems (hard-scattering kernels) are dominated by hard gluon exchange. The hard-scattering kernels are calculable in perturbation theory, which starts at tree level and, at higher order in α_s, contains nonfactorizable corrections from hard gluon exchange.

The $B^{0(+)} \rightarrow J/\psi\eta K^{0(+)}$ decay channels can also receive contributions through intermediate resonances $K_2^*(1430)$ and $K_3^*(1780)$, namely, $B \rightarrow K^*(\rightarrow K\eta)J/\psi$. Therefore, in order to get a reliable estimation on the branching fraction, it is important to have an estimate of the resonant contributions.

2. Amplitudes of the $B^0 \rightarrow J/\psi\eta K^0$ and $B^+ \rightarrow J/\psi\eta K^+$ Decay

2.1. The Dalitz Plot Analysis

2.1.1. Nonresonant Background. In the factorization approach, the Feynman diagrams for three-body $B^0 \rightarrow J/\psi\eta K^0$ and $B^+ \rightarrow J/\psi\eta K^+$ decay are shown in Figure 1; the η meson is produced from three $u\bar{u}$, $d\bar{d}$, and $s\bar{s}$ components; according to Figure 1 to draw the Feynman diagrams of the $B^0 \rightarrow J/\psi\eta K^0$ decay, two $d\bar{d}$ and $s\bar{s}$ components are used for tree level and just $d\bar{d}$ component is considered for penguin contribution. These topics are shown in (a), (c), and (e) panels. For $B^+ \rightarrow J/\psi\eta K^+$ decay, the $u\bar{u}$ and $s\bar{s}$ components can be used for tree and $u\bar{u}$ pairs for penguin contributions. The panels of (b), (d), and (f) show the mentioned content. Panels (g)–(k) show the emission and emission-annihilation diagrams in which J/ψ is emitted via three-gluon exchange, which are the so-called hairpin diagrams; as we can see, both decay modes have the same amplitudes. Under the factorization approach, the $B^+ \rightarrow J/\psi\eta K^+$ and $B^0 \rightarrow J/\psi\eta K^0$ decay amplitudes consist of three distinct factorizable terms: (i) the tree and penguin processes, $\langle B \rightarrow \eta K\rangle \times \langle 0 \rightarrow J/\psi\rangle$, (ii) the J/ψ meson emission process, $\langle B \rightarrow K\rangle \times \langle 0 \rightarrow J/\psi\eta\rangle$, and (iii) the emission-annihilation process, $\langle B \rightarrow 0\rangle \times \langle 0 \rightarrow J/\psi\eta K\rangle$, where $\langle A \rightarrow B\rangle$ denotes an $A \rightarrow B$ transition matrix element. Here $\langle B \rightarrow \eta K\rangle$ denotes two-meson transition matrix element. The leading nonfactorizable diagrams in

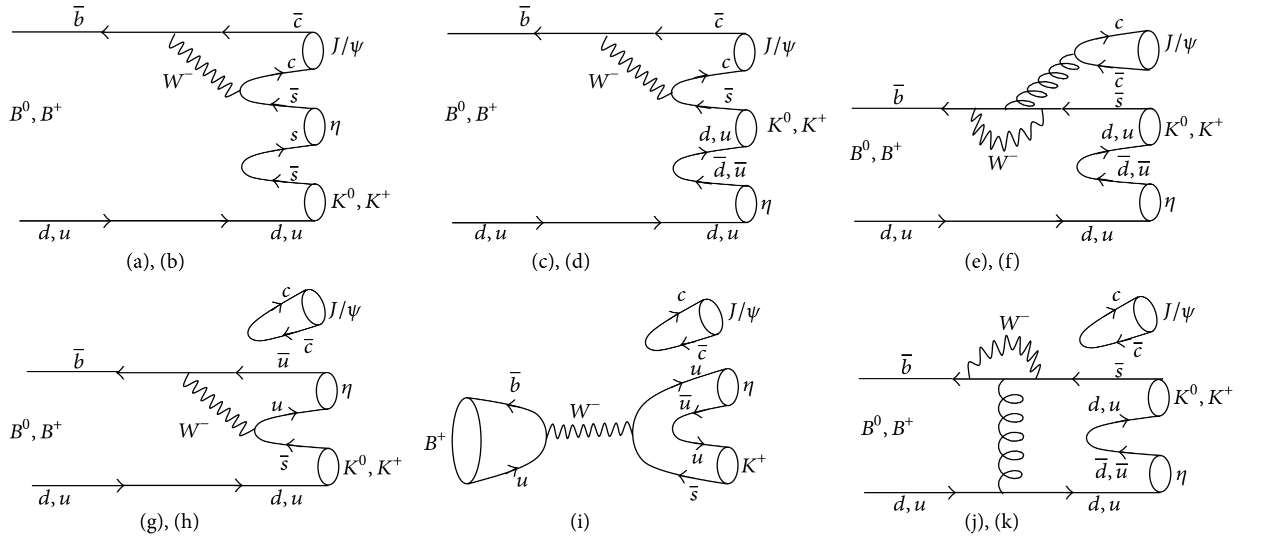


FIGURE 1: Quark diagrams illustrating the processes $B^0 \to J/\psi\eta K^0$ and $B^+ \to J/\psi\eta K^+$ decay. Panels (a)–(d) show the tree, (e)-(f) show the penguin, (g)-(h) show the emission, and (i)–(k) show the emission-annihilation transitions. The diagrams in which J/ψ is emitted via gluon exchange are called "hairpin" diagrams.

Figure 2 should be taken into account. To this, we employ the QCD factorization framework, which incorporates important theoretical aspects of QCD like color transparency, heavy quark limit, and hard scattering and allows us to calculate nonfactorizable contributions systematically.

The matrix elements of the $B \to J/\psi\eta K$ decay amplitude are given by

$$\begin{aligned}\left\langle \frac{J}{\psi}\eta K \left| H_{\text{eff}} \right| B \right\rangle &\propto \left\langle \frac{J}{\psi} \left| (c\bar{c})_{V-A} \right| 0 \right\rangle \left\langle \eta K \left| (s\bar{b})_{V-A} \right| B \right\rangle \\ &+ \left\langle \frac{J}{\psi}\eta \left| (s\bar{u})_{V-A} \right| 0 \right\rangle \left\langle K \left| (u\bar{b})_{V-A} \right| B \right\rangle \\ &+ \left\langle \frac{J}{\psi}\eta K \left| (s\bar{u})_{V-A} \right| 0 \right\rangle \left\langle 0 \left| (u\bar{b})_{V-A} \right| B \right\rangle .\end{aligned} \tag{8}$$

For the current-induced process, the two-meson transition matrix element $\langle \eta K | (s\bar{b})_{V-A} | B \rangle$ has the general expression as [10]

$$\begin{aligned}&\left\langle \eta(p_1) K(p_2) \left| (s\bar{b})_{V-A} \right| B \right\rangle \\ &\quad = ir(p_B - p_1 - p_2)_\mu + i\omega_+(p_2 + p_1)_\mu + i\omega_-(p_2 - p_1)_\mu ,\end{aligned} \tag{9}$$

and the decay constant is defined as [17]

$$\left\langle 0 \left| V_\mu \right| \frac{J}{\psi}(\epsilon_3, p_3) \right\rangle = f_{J/\psi} m_{J/\psi} \epsilon_3 . \tag{10}$$

The direct three-body decay of mesons in general receives two distinct contributions: one from the point-like weak transition and the other from the pole diagrams that involve three-point or four-point strong vertices [6]. So the r, ω_+, and ω_- form factors are computed from point-like and pole diagrams; we also need the strong coupling of $B^* B^{(*)}\eta$, $B^* B^{(*)} K$, and $BB\eta K$ vertices. These form factors are given by [10]

$$\begin{aligned} r &= \frac{f_B}{2 f_\eta f_K} - \frac{f_B}{f_\eta f_K} \frac{p_B \cdot (p_2 - p_1)}{(p_B - p_1 - p_2)^2 - m_B^2} \\ &+ \frac{2 g f_{B_s^*}}{f_\eta f_K} \sqrt{\frac{m_B}{m_{B_s^*}}} \frac{(p_B - p_1) \cdot p_1}{(p_B - p_1)^2 - m_{B_s^*}^2} \\ &- \frac{4 g^2 f_B}{f_\eta f_K} \frac{m_B m_{B_s^*}}{(p_B - p_1 - p_2)^2 - m_B^2} \\ &\times \frac{p_1 \cdot p_2 - p_1 \cdot (p_B - p_1)\, p_2 \cdot (p_B - p_1) / m_{B_s^*}^2}{(p_B - p_1)^2 - m_{B_s^*}^2}, \\ \omega_+ &= -\frac{g}{f_\eta f_K} \frac{f_{B_s^*} m_{B_s^*} \sqrt{m_B m_{B_s^*}}}{(p_B - p_1)^2 - m_{B_s^*}^2} \left[1 - \frac{(p_B - p_1) \cdot p_1}{m_{B_s^*}^2} \right] \\ &+ \frac{f_B}{2 f_\eta f_K}, \\ \omega_- &= \frac{g}{f_\eta f_K} \frac{f_{B_s^*} m_{B_s^*} \sqrt{m_B m_{B_s^*}}}{(p_B - p_1)^2 - m_{B_s^*}^2} \left[1 + \frac{(p_B - p_1) \cdot p_1}{m_{B_s^*}^2} \right]. \end{aligned} \tag{11}$$

The other two-body matrix element can be related to the η and J/ψ matrix element of the weak interaction current

$$\left\langle \eta(p_1) \frac{J}{\psi}(p_3) \left| (u\bar{u})_{V-A} \right| 0 \right\rangle = (p_3 - p_1)_\mu F^{J/\psi\eta}(q^2), \tag{12}$$

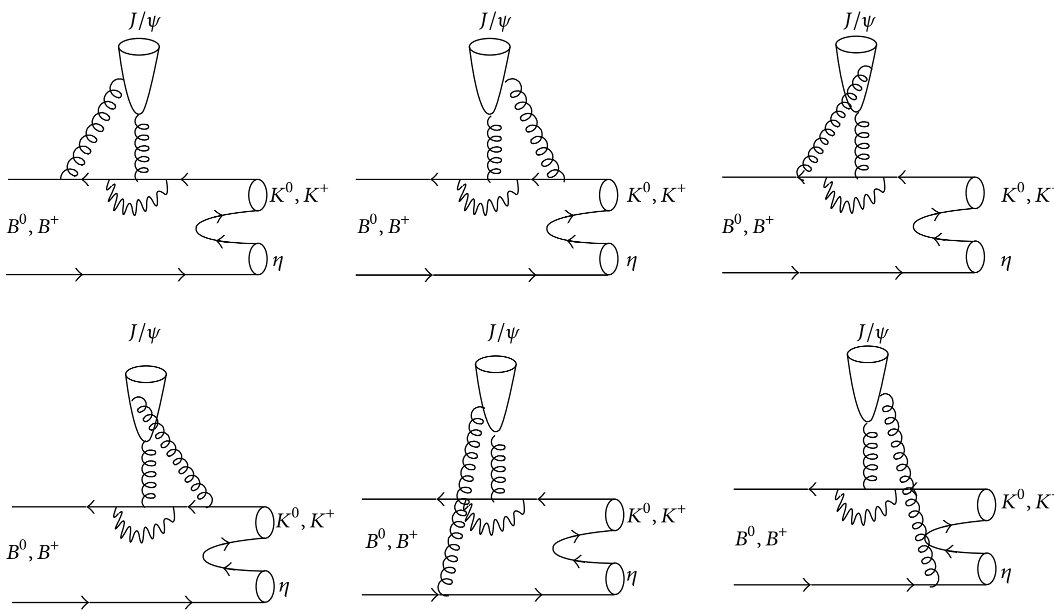


Figure 2: Nonfactorizable diagrams for $B^0 \rightarrow J/\psi\eta K^0$ and $B^+ \rightarrow J/\psi\eta K^+$ decay.

where $F^{J/\psi\eta}(q^2)$ is the J/ψ to η transition form factor and needs to be determined from experiment [2]. This transition occurs by two gluons where both of the gluons are off shell or by two-photon decay widths of the η meson in the $J/\psi \rightarrow \eta\gamma$ decay [19]. The contribution of gluonic wave function to the transition form factor $F_{g^* g^* - \eta}(Q_1^2, Q_2^2)$ has been tested in [20, 21], and the $B \rightarrow K$ form factor is defined as follows [17]:

$$\begin{aligned} &\left\langle K(p_2) \left| (u\bar{b})_{V-A} \right| B \right\rangle \\ &= \left((p_B + p_2)_\mu - \frac{m_B^2 - m_2^2}{q^2} q_\mu \right) F_1(q^2) \\ &+ \frac{m_B^2 - m_2^2}{q^2} q_\mu F_0(q^2), \end{aligned} \tag{13}$$

where $q_\mu = (p_B - p_2)_\mu$. The emission-annihilation matrix element is assumed to be [22]

$$\begin{aligned} &\left\langle \eta(p_1) K(p_2) \frac{J}{\psi}(p_3) \left| (s\bar{u})_{V-A} \right| 0 \right\rangle \\ &= \frac{2i}{f_K} \left(p_{2\mu} - \frac{p_B \cdot p_2}{p_B^2 - p_3^2} p_{B\mu} \right) F^{J/\psi\eta K}(q^2), \end{aligned} \tag{14}$$

and the form factor $F^{J/\psi\eta K}(q^2)$ is parameterized as

$$F^{J/\psi\eta K}(q^2) = \frac{1}{1 - q^2/\Lambda_\chi^2}, \tag{15}$$

where $\Lambda_\chi = 830$ Mev is the chiral-symmetry breaking scale. Then the matrix elements read

$$\begin{aligned} &\left\langle \eta(p_1) K(p_2) \frac{J}{\psi}(\epsilon_3, p_3) \left| H_{\text{eff}} \right| B \right\rangle \\ &\propto i f_{J/\psi} m_{J/\psi} (\epsilon_3 \cdot p_3 r + (\epsilon_3 \cdot p_2 + \epsilon_3 \cdot p_1)\omega_+ \\ &\qquad + (\epsilon_3 \cdot p_2 - \epsilon_3 \cdot p_1)\omega_-) \\ &+ \Bigg((p_3 \cdot p_B - p_1 \cdot p_B + p_3 \cdot p_2 - p_1 \cdot p_2) F_1^{BK}(q^2) \\ &- \frac{m_B^2 - m_2^2}{(p_1 + p_3)^2} (m_3^2 - m_1^2)\left(F_1^{BK}(q^2) - F_0^{BK}(q^2)\right) \Bigg) \\ &\times F^{J/\psi\eta} - \frac{2 f_B (p_2 \cdot p_B)}{f_K} \left(1 - \frac{m_B^2}{m_B^2 - m_3^2} \right) F^{J/\psi\eta K}, \end{aligned} \tag{16}$$

where under the Lorentz condition $\epsilon_3 \cdot p_3 = 0$. The J/ψ meson polarization vectors become

$$\begin{aligned} \epsilon_3^{(\lambda=0)} &= \frac{\left(|\vec{p}_3|, 0, 0, p_3^0\right)}{m_3}, \\ \epsilon_3^{(\lambda=\pm1)} &= \frac{\mp(0, 1, \pm i, 0)}{\sqrt{2}}. \end{aligned} \tag{17}$$

Consider the decay of B meson into three particles of masses m_1, m_2, and m_3. Denote their 4 momenta by p_B, p_1, p_2, and p_3, respectively. Energy-momentum conservation is expressed by

$$p_B = p_1 + p_2 + p_3. \tag{18}$$

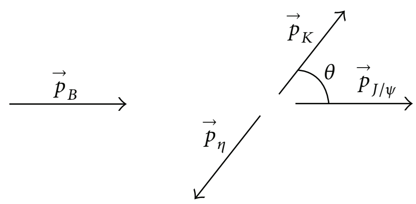


Figure 3: Definition of helicity angle θ, for the decay $B \to J/\psi\eta K$.

Define the following invariants:

$$\begin{aligned} s_{12} &= (p_1+p_2)^2 = (p_B-p_3)^2, \\ s_{13} &= (p_1+p_3)^2 = (p_B-p_2)^2, \\ s_{23} &= (p_2+p_3)^2 = (p_B-p_1)^2. \end{aligned} \tag{19}$$

The three invariants s_{12}, s_{13}, and s_{23} are not independent; it follows from their definitions together with 4-momentum conservation that

$$s_{12}+s_{13}+s_{23} = m_B^2+m_1^2+m_2^2+m_3^2. \tag{20}$$

We take $s_{12}=s$ and $s_{23}=t$, so we have $s_{13}=m_B^2+m_1^2+m_2^2+m_3^2-s-t$. In the center of mass of $\eta(p_1)$ and $K(p_2)$, according to Figure 3, we find

$$\begin{aligned} |\vec{p}_1| &= |\vec{p}_2| = \frac{1}{2}\sqrt{s-4m_1^2}, \\ p_1^0 &= p_2^0 = \frac{1}{2}\sqrt{s}, \\ |\vec{p}_3| &= p_3^3 = \frac{1}{2\sqrt{s}}\sqrt{(m_B^2-m_3^2-s)^2-4sm_3^2}, \\ p_3^0 &= \frac{1}{2\sqrt{s}}\left(m_B^2-m_3^2-s\right), \\ |\vec{\epsilon}_3| &= \frac{1}{2m_3\sqrt{s}}\left(m_B^2-m_3^2-s\right), \\ \epsilon_3^0 &= \frac{1}{2m_3\sqrt{s}}\sqrt{(m_B^2-m_3^2-s)^2-4sm_3^2}, \end{aligned} \tag{21}$$

and the cosine of the helicity angle θ between the direction of $\vec{p}_2$ and that of $\vec{p}_3$ reads

$$\cos\theta = \frac{1}{4|\vec{p}_2||\vec{p}_3|}\left(m_B^2+m_3^2+2m_2^2-s-2t\right). \tag{22}$$

With these definitions, we obtain multiplying of the 4-momentum conservation as

$$\begin{aligned} \epsilon_3\cdot(p_2+p_1) &= 2p_1^0\epsilon_3^0, \\ \epsilon_3\cdot(p_2-p_1) &= 2|\vec{\epsilon}_3||\vec{p}_1|\cos\theta. \end{aligned} \tag{23}$$

In this framework, nonfactorizable contributions (hard-scattering and vertex corrections) to $B \to K\eta J/\psi$ can be obtained by calculating the diagrams in Figure 2. Each of the diagrams in Figure 2 contains a leading-power contribution relevant to power-suppressed terms, which are not factorized in general. An important class of such power-suppressed effects is related to certain higher-twist meson distribution amplitudes. Fortunately, it turns out that ratios of the different hard-scattering contribution have very small uncertainties so we just put the vertex corrections in the calculation of the Wilson coefficients.

Now we can derive the nonresonant amplitude with $\eta-\eta'$ mixing angle θ_p as

$$\begin{aligned} &M_{\mathrm{NR}}\left(B\longrightarrow\eta(p_1)K(p_2)\frac{J}{\psi}(p_3)\right) \\ &= i\frac{G_F}{2\sqrt{2}}\left(\frac{1}{\sqrt{6}}\cos\theta_p-\frac{1}{\sqrt{3}}\sin\theta_p\right)^2 \\ &\times\Bigg[\left(a_2V_{cb}^*V_{cs}+a_3\lambda_p\right) \\ &\times\Bigg(\omega_+\sqrt{(m_B^2-m_3^2-s)^2-4sm_3^2} \\ &+\omega_-\left(m_B^2-m_3^2-s\right)\sqrt{1-\frac{4m_1^2}{s}}\cos\theta\Bigg)f_{J/\psi} \\ &-2ia_2V_{ub}^*V_{us}\Bigg((t-s)F_1^{BK} \\ &-\frac{m_B^2-m_2^2}{m_B^2+m_1^2+m_2^2+m_3^2-s-t} \\ &\times\left(F_1^{BK}-F_0^{BK}\right)\Bigg)F^{J/\psi\eta} \\ &+2i\left(a_1V_{ub}^*V_{us}+a_4\lambda_p\right)\frac{f_B}{f_K}\left(s+t-m_1^2-m_3^2\right) \\ &\times\left(1-\frac{m_B^2}{m_B^2-m_3^2}\right)F^{J/\psi\eta K}\Bigg]. \end{aligned} \tag{24}$$

The vertex corrections to the $B \to J/\psi\eta K$ decay, denoted as f_I in QCDF, have been calculated in the NDR scheme and can be adopted directly. Their effects can be combined into the Wilson coefficients associated with the factorizable contributions [18]:

$$\begin{aligned} a_1 &= c_1+\frac{c_2}{3}+\frac{\alpha_s}{4\pi}\frac{C_F}{3}c_2\left(-18+12\ln\frac{m_b}{\mu}+f_I\right), \\ a_2 &= c_2+\frac{c_1}{3}+\frac{\alpha_s}{4\pi}\frac{C_F}{3}c_1\left(-18+12\ln\frac{m_b}{\mu}+f_I\right), \\ a_3 &= c_3+\frac{c_4}{3}+\frac{\alpha_s}{4\pi}\frac{C_F}{3}c_4\left(-18+12\ln\frac{m_b}{\mu}+f_I\right), \end{aligned}$$

$$a_4 = c_4 + \frac{c_3}{3} + \frac{\alpha_s}{4\pi}\frac{C_F}{3}c_3\left(-18 + 12\ln\frac{m_b}{\mu} + f_I\right),$$
$$\lambda_p = \sum_{p=u,c} V_{pb}^* V_{ps}, \tag{25}$$

where

$$f_I = \frac{2\sqrt{6}}{f_{J/\psi}}\int dx\phi_{J/\psi}^L(x)\left[\frac{3(1-2x)}{1-x}\ln(x) - 3\pi i + 3\ln\left(1-r^2\right) + \frac{2r^2(1-x)}{1-r^2x}\right], \tag{26}$$

and $\phi_{J/\psi}^L(x)$ is the J/ψ meson asymptotic distribution amplitude which is given by [23]

$$\phi_{J/\psi}^L(x) = 9.58\frac{f_{J/\psi}}{2\sqrt{6}}x(1-x)\left[\frac{x(1-x)}{1-2.8x(1-x)}\right]^{0.7}. \tag{27}$$

2.1.2. Resonant Contributions. According to Figure 1, the decay channels of $B \to J/\psi\eta K$ can also receive contributions through intermediate resonances $K_2^*(1430)$ and $K_3^*(1780)$. Resonant effects are described in terms of the usual Breit-Wigner formalism

$$\langle \eta K |(\bar{s}b)_{V-A}| B\rangle^R = \sum_i \frac{g^{K_i^* \to K\eta}}{m_{K_i^*}^2 - \left(p_\eta + p_K\right)^2 - i m_{K_i^*}\Gamma^{K_i^* \to K\eta}} \times \sum_{\text{pol}} \epsilon_{K_i^*}\cdot\left(p_K - p_\eta\right)\langle K_i^* |(\bar{s}b)_{V-A}| B\rangle, \tag{28}$$

where

$$\left\langle K_i^*(p_1,\epsilon_1)\left|V_\mu - A_\mu\right| B(p_B)\right\rangle = -i\left(\epsilon_{K_i^*} - \frac{\varepsilon_{K_i^*}\cdot q}{q^2}q_\mu\right)\left(m_B + m_{K_i^*}\right)A_1^{BK_i^*} + i\left(\left(p_B + p_{K_i^*}\right)_\mu - \frac{m_B^2 - m_{K_i^*}^2}{q^2}q_\mu\right)\left(\epsilon_{K_i^*}\cdot q\right) \times \frac{A_2^{BK_i^*}}{m_B + m_{K_i^*}},$$
$$g^{K_i^* \to K\eta} = \sqrt{\frac{12\pi m_{K_i^*}^2 \Gamma^{K_i^* \to K\eta}}{2p_c^3}}, \tag{29}$$

where K_i^* denote $K_2^*(1430)$, $K_3^*(1780)$, and p_c is the c.m. momentum. In determining the coupling of $K_i^* \to K\eta$, we have used the partial widths $\Gamma^{K^{*0}(1430)\to K^0\eta} = 0.164$ Mev, $\Gamma^{K^{*+}(1430)\to K^+\eta} = 0.148$ Mev, and $\Gamma^{K_3^*(1780)\to K\eta} = 47.70$ Mev measured by PDG [3]. Then the decay amplitude through resonance intermediate reads

$$M_R\left(B \longrightarrow \frac{J}{\psi}\eta K\right) = -i\frac{G_F}{\sqrt{2}\sqrt{6}}m_{J/\psi}f_{J/\psi}\left(a_2 V_{cb}^* V_{cs} + a_3\lambda_p\right)\sum_i\left(2\vec{\epsilon}_{K_i^*}\cdot\vec{p}_K\right) \times \left[\left(\epsilon_{K_i^*}\cdot\epsilon_{J/\psi}\right)\left(m_B + m_{K_i^*}\right)A_1^{BK_i^*} - \left(\epsilon_{K_i^*}\cdot p_{J/\psi}\right) \times \left(\epsilon_{J/\psi}\cdot\left(p_B + p_{K_i^*}\right)\right)\frac{A_2^{BK_i^*}}{m_B + m_{K_i^*}}\right] \times \frac{g^{K_i^* \to K\eta}}{m_{K_i^*}^2 - s - i m_{K_i^*}\Gamma^{K_i^* \to K\eta}}. \tag{30}$$

Finally by using the full amplitude, the decay rate of $B \to J/\psi\eta K$ is then given by [22]

$$\Gamma\left(B \longrightarrow \frac{J}{\psi}\eta K\right) = \frac{1}{(2\pi)^3 32 m_B^3} \times \int_{s_{\min}}^{s_{\max}}\int_{t_{\min}}^{t_{\max}}\left|M_{\text{NR}}\left(B \longrightarrow \frac{J}{\psi}\eta K\right) + M_R\left(B \longrightarrow \frac{J}{\psi}\eta K\right)\right|^2 ds\,dt, \tag{31}$$

where

$$t_{\min,\max}(s) = m_1^2 + m_3^2 - \frac{1}{2s}\left(\left(m_B^2 - s - m_3^2\right)\left(s - m_2^2 + m_1^2\right) \mp \sqrt{\lambda\left(s, m_B^2, m_3^2\right)}\sqrt{\lambda\left(s, m_1^2, m_2^2\right)}\right),$$
$$s_{\min} = \left(m_1 + m_2\right)^2,$$
$$s_{\max} = \left(m_B - m_3\right)^2, \tag{32}$$

where $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + xz + yz)$.

3. Amplitudes of the $B \to \eta_c\pi K^*$ and $B_s \to \eta_c\pi K^*$ Decay

The Feynman diagrams for three-body decay of $B \to \eta_c\pi K^*$ and $B_s \to \eta_c\pi K^*$, in the factorization approach, are shown in Figure 4 and types of these decay modes can be obtained from the following options.

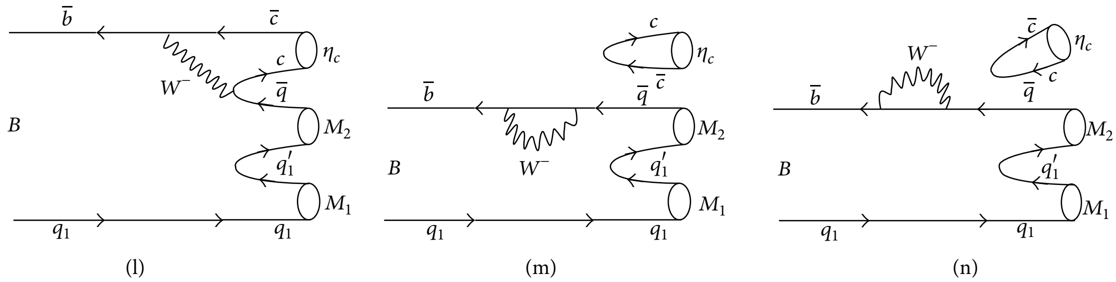


FIGURE 4: Quark diagrams illustrating the processes $B(B_s) \to \eta_c \pi K^*$ decay.

(1) For choices of $\bar{q} = \bar{s}$ and

(1-1) $q_1 = q_1' = d$, decay mode becomes $B^0 \to \eta_c \pi^0 K^{*0}$,

(1-2) $q_1 = d, q_1' = u$, decay mode becomes $B^0 \to \eta_c \pi^- K^{*+}$,

(1-3) $q_1 = q_1' = u$, decay mode becomes $B^+ \to \eta_c \pi^0 K^{*+}$,

(1-4) $q_1 = u, q_1' = d$, decay mode becomes $B^+ \to \eta_c \pi^+ K^{*0}$.

(2) For selection of $\bar{q} = \bar{d}$ and

(2-1) $q_1 = s, q_1' = u$, decay mode becomes $B_s^0 \to \eta_c \pi^+ K^{*-}$,

(2-2) $q_1 = s, q_1' = d$, decay mode becomes $B_s^0 \to \eta_c \pi^0 \bar{K}^{*0}$.

For this decay, according to Figure 2, the decay constant is defined as

$$\langle 0 | A_\mu | \eta_c(p) \rangle = i f_{\eta_c} p_{\eta_c}. \quad (33)$$

Then the nonresonant amplitude can be obtained by

$$\begin{aligned} & M(B \longrightarrow \pi(p_1) K^*(p_2) \eta_c(p_3)) \\ & = -\frac{G_F}{2\sqrt{2}} \Bigg[f_{\eta_c} \left(a_2 V_{cb}^* V_{cs} + a_3 \lambda_p \right) \\ & \quad \times \Big(2 r m_3^2 + \omega_+ \left(m_B^2 - s - m_3^2 \right) \\ & \quad + \omega_- \left(2t + s - m_B^2 - 2m_2^2 - m_3^2 \right) \Big) \\ & \quad + 2 a_4 \lambda_p \frac{f_B}{f_{K^*}} \left(s + t - m_1^2 - m_3^2 \right) \\ & \quad \times \left(1 - \frac{m_B^2}{m_B^2 - m_3^2} \right) F^{\eta_c \pi K} \Bigg]. \end{aligned} \quad (34)$$

In the B_s decay modes we useV_{cd} instead of V_{cs} (also within the λ_p). Note that when the final states contain π^0 meson, the decay amplitudes multiplied by $1/\sqrt{2}$. In addition, several intermediate resonant states involving $K_1(1270)$, $K_1(1400)$, $K^*(1410)$, $K_2^*(1430)$, $K^*(1680)$, and $K_3^*(1820)$ resonances are used in the calculations [3].

TABLE 1: The Wilson coefficients c_i in the NDR scheme at three different choices of the renormalization scale μ [18].

NLO	c_1	c_2	c_3	c_4
$\mu = m_b/2$	1.137	−0.295	0.021	−0.051
$\mu = m_b$	1.081	−0.190	0.014	−0.036
$\mu = 2m_b$	1.045	−0.113	0.009	−0.025

4. Numerical Results

The theoretical input parameters used in our analysis, together with their respective ranges of uncertainty, are summarized below.

The Wilson coefficients c_i have been calculated in different schemes. In this paper we will use consistently the naive dimensional regularization (NDR) scheme. The values of c_i at the scales $\mu = m_b/2$, $\mu = m_b$, and $\mu = 2m_b$ at the next to leading order (NLO) are shown in Table 1.

There is a potentially quite large error that could come from the uncertainty in the parameter g available on the form factors. This parameter is determined from the $D^* \to D\pi$ decay and we use [10]

$$g = 0.3 \pm 0.1. \quad (35)$$

For the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, we use the values of the Wolfenstein parameters and obtain

$$\begin{aligned} & |V_{ud}| = 0.97425 \pm 0.00022, \quad |V_{us}| = 0.2252 \pm 0.0009, \\ & |V_{ub}| = 0.00415 \pm 0.00049, \quad |V_{cd}| = 0.230 \pm 0.011, \\ & |V_{cs}| = 1.006 \pm 0.023, \quad |V_{cb}| = 0.0409 \pm 0.0011, \\ & |V_{td}| = 0.0084 \pm 0.0006, \quad |V_{ts}| = 0.0429 \pm 0.0026, \\ & |V_{tb}| = 0.89 \pm 0.07. \end{aligned} \quad (36)$$

Table 2: Branching ratios of $B \rightarrow J/\psi\eta K$ (in units of 10^{-5}) and $B(B_s) \rightarrow \eta_c\pi K^*$ (in units of 10^{-4}) decay by using the Dalitz plot analysis.

Mode	$\mu = m_b/2$	$\mu = m_b$	$\mu = 2m_b$	Exp. [3]
$B^0 \rightarrow J/\psi\eta K^0$	$2.42^{+0.70}_{-0.39}$	$9.22^{+2.67}_{-1.47}$	$17.73^{+4.64}_{-3.16}$	8 ± 4
$B^+ \rightarrow J/\psi\eta K^+$	$2.42^{+0.70}_{-0.39}$	$9.22^{+2.67}_{-1.47}$	$17.73^{+4.64}_{-3.16}$	10.8 ± 3.30
$B^0 \rightarrow \eta_c\pi^- K^{*+}$, $B^+ \rightarrow \eta_c\pi^+ K^{*0}$	$3.62^{+0.71}_{-0.41}$	$13.82^{+2.70}_{-1.56}$	$26.00^{+5.08}_{-2.93}$	—
$B^0 \rightarrow \eta_c\pi^0 K^{*0}$, $B^+ \rightarrow \eta_c\pi^0 K^{*+}$	$1.81^{+0.36}_{-0.21}$	$6.91^{+1.35}_{-0.78}$	$13.00^{+2.54}_{-1.47}$	—
$B_s^0 \rightarrow \eta_c\pi^+ K^{*-}$	$0.20^{+0.04}_{-0.02}$	$0.75^{+0.14}_{-0.09}$	$1.40^{+0.27}_{-0.16}$	—
$B_s^0 \rightarrow \eta_c\pi^0 K^{*0}$	$0.10^{+0.02}_{-0.01}$	$0.38^{+0.07}_{-0.05}$	$0.70^{+0.14}_{-0.08}$	—

The meson masses and decay constants needed in our calculations are taken as (in units of Mev) [3]

$$\begin{aligned}
&m_{B_s^*} = 5415.4^{+2.4}_{-2.1}, \qquad m_{B^0} = 5279.58 \pm 0.17,\\
&m_{B^\pm} = 5279.25 \pm 0.17, \qquad m_{J/\psi} = 3096.916 \pm 0.011,\\
&m_{\eta_c} = 2981.0 \pm 1.1, \qquad m_{K^*} = 891.66 \pm 0.26,\\
&m_{\eta} = 547.853 \pm 0.024, \qquad m_{K^0} = 497.614 \pm 0.024,\\
&m_{K^\pm} = 493.677 \pm 0.016, \qquad m_{\pi^0} = 134.9766 \pm 0.0006,\\
&m_{\pi^\pm} = 139.57018 \pm 0.00035,\\
&f_B = 176 \pm 42, \qquad f_{B_s^*} = 220,\\
&f_{J/\psi} = 418 \pm 9, \qquad f_{\eta_c} = 387 \pm 7,\\
&f_\eta = 131 \pm 7, \qquad f_K = 159.8 \pm 1.84,\\
&f_{K^*} = 217 \pm 5,
\end{aligned} \tag{37}$$

and the form factors at zero momentum transfer are taken as [17]

$$\begin{aligned}
&F_0^{BK} = F_1^{BK} = 0.35 \pm 0.05, \qquad A_1^{BK^*} = 0.35 \pm 0.07,\\
&A_2^{BK^*} = 0.34 \pm 0.06.
\end{aligned} \tag{38}$$

Using the parameters relevant to the $B \rightarrow J/\psi\eta K$ and $B(B_s) \rightarrow \eta_c\pi K^*$ decay, we calculate the branching ratios of this decay, which are shown in Table 2. Note that, as we mentioned before, both decay of $B^0 \rightarrow J/\psi\eta K^0$ and decay of $B^+ \rightarrow J/\psi\eta K^+$ have similar amplitudes. Since the masses of the B^0 and B^+ and also K^0 and K^+ mesons are very close to each other, both branching ratios are the same.

5. Conclusion

In this work, we have calculated the branching ratios of the $B^0 \rightarrow J/\psi\eta K^0$ and $B^+ \rightarrow J/\psi\eta K^+$ decay by using the Dalitz plot analysis. In this calculation we have used factorizable terms, nonfacorizable effects, and $\eta - \eta'$ mixing. According to QCD factorization approach, we have obtained $BR(B \rightarrow J/\psi\eta K) = (9.22^{+2.67}_{-1.47}) \times 10^{-5}$ while the experimental results of them are $(8 \pm 4) \times 10^{-5}$ and $(10.8 \pm 3.3) \times 10^{-5}$, respectively. The branching ratios obtained by applying the Dalitz plot analysis are compatible with the experimental results.

Moreover we have analyzed the $B(B_s) \rightarrow \eta_c\pi K^*$ decay which is similar to the $B \rightarrow J/\psi\eta K$ decay, but there are no experimental data for the last decay. Since for calculations of the $B(B_s) \rightarrow \eta_c\pi K^*$ decay we have used assumptions of the $B \rightarrow J/\psi\eta K$ decay, we hope that if this decay will be measured by the LHCb in the future, the experimental results will be in agreement with our calculations.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

References

[1] BABAR Collaboration, B. Aubert, R. Barate et al., "Evidence for the rare decay $B \rightarrow J/\psi\eta K$," http://arxiv.org/pdf/hep-ex/0307032.pdf?origin=publication_detail.

[2] Belle Collaboration and T. Iwashita, "Search for a J/psi eta resonance in B+- to J/psi eta K+- decay at Belle," http://arxiv.org/abs/1209.0886.

[3] J. Beringer and Particle Data Group, "Review of Particle Physics," *Physical Review D*, vol. 86, Article ID 010001, 2012.

[4] R. H. Dalitz, "On the analysis of τ-meson data and the nature of the τ-meson," *Philosophical Magazine*, vol. 44, no. 357, pp. 1068–1080, 1953.

[5] A. Furman, R. Kaminski, L. Lesniak, and P. Zenczykowski, "Final state interactions in $B^\pm \rightarrow K^+K^-K^\pm$ decays," *Physics Letters B*, vol. 699, no. 1-2, pp. 102–107, 2011.

[6] H. Y. Cheng, C. K. Chua, and A. Soni, "Charmless three-body decays of B mesons," *Physical Review D*, vol. 76, Article ID 094006, 2007.

[7] H. Y. Cheng, "Charmless 3-body B decays: resonant and nonresonant contributions," *International Journal of Modern Physics A*, vol. 23, article 3229, 2008.

[8] M. Gronau and J. L. Rosner, "Symmetry relations in charmless $B \rightarrow PPP$ decays," *Physical Review D*, vol. 72, Article ID 094031, 2005.

[9] H. Y. Cheng, C. K. Chua, and A. Soni, "CP-violating asymmetries in B^0 decays to $K^+K^-K^0_{S(L)}$ and $K^0_{S(L)}K^0_{S(L)}K^0_{S(L)}$," *Physical Review D*, vol. 72, Article ID 094003, 2005.

[10] B. Bajc, S. Fajfer, R. J. Oakes, T. N. Pham, and S. Prelovšek, "The CP violating asymmetry in $B^\pm \rightarrow M\overline{M}\pi^\pm$ decays," *Physics Letters B*, vol. 447, no. 3-4, pp. 313–320, 1999.

[11] S. Fajfer, R. J. Oakes, and T. N. Pham, "Penguin operators in nonresonant $B^- \rightarrow M\overline{M}\pi^-(M = \pi^-, K^-, K^0)$ decays," *Physical Review D*, vol. 60, Article ID 054029, 1999.

[12] H. Leutwyler, "On the *1/(N*-expansion) in chiral perturbation theory," *Nuclear Physics B*, vol. 64, no. 1–3, pp. 223–231, 1998.

[13] P. Herrera-Siklódy, J. I. Latorre, P. Pascual, and J. Taron, "$\eta^-\eta'$ mixing from $\cup(3)_L \otimes \cup(3)_R$ chiral perturbation theory," *Physics Letters B*, vol. 419, no. 1–4, pp. 326–332, 1998.

[14] R. Akhoury and J. M. Frère, "η, η' mixing and anomalies," *Physics Letters B*, vol. 220, no. 1-2, pp. 258–264, 1989.

[15] A. V. Kiselev and V. A. Petrov, "Two schemes of $\eta - \eta'$-mixing," *Zeitschrift für Physik C*, vol. 58, pp. 595–600, 1993.

[16] B. Mohammadi and H. Mehraban, "Nonresonant three-body decays of B to $J/\psi\pi\pi$," *Europhysics Letters*, vol. 100, no. 3, Article ID 31002, 2012.

[17] A. Ali, G. Kramer, and C. D. Lu, "Experimental tests of factorization in charmless nonleptonic two-body B decays," *Physical Review D*, vol. 58, Article ID 094009, 1998.

[18] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, "QCD factorization in $B \rightarrow \pi\pi$K, $\pi\pi$ decays and extraction of Wolfenstein parameters," *Nuclear Physics B*, vol. 606, no. 1-2, pp. 245–321, 2001.

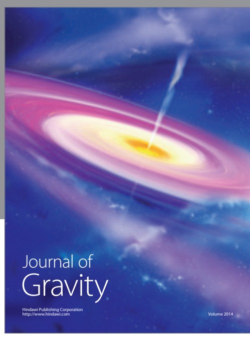
[19] T. Feldmann and P. Kroll, "Flavor symmetry breaking and mixing effects in the $\eta\gamma$ and $\eta'\gamma$ transition form factors," *The European Physical Journal C*, vol. 5, no. 2, pp. 327–335, 1998.

[20] M. Z. Yang and Y. D. Yang, "Revisiting charmless two-body B decays involving η' and η," *Nuclear Physics B*, vol. 609, no. 3, pp. 469–498, 2001.

[21] A. Ali and A. Y. Parkhomenko, "$\eta'\ g^* g^*$ vertex with arbitrary gluon virtualities in the perturbative QCD hard scattering approach ," *Physical Review D*, vol. 65, Article ID 074020, 2002.

[22] H. Y. Cheng and C. K. Yang, "Nonresonant three-body decays of D and B mesons," *Physical Review D*, vol. 66, Article ID 054015, 2002.

[23] P. V. Pobylitsa, "Exotic baryons and the large-N_c expansion," *Physical Review D*, vol. 69, no. 7, Article ID 074030, 2004.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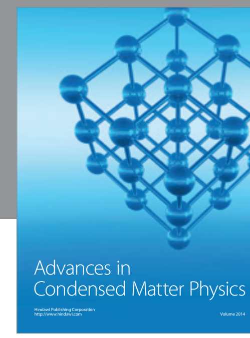
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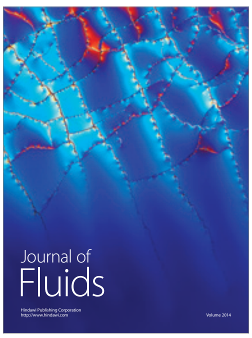
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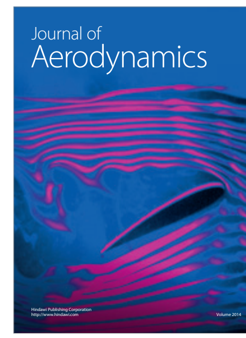
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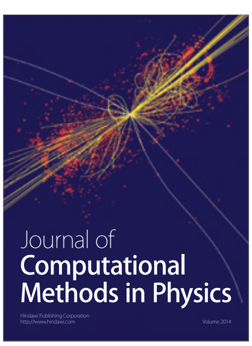
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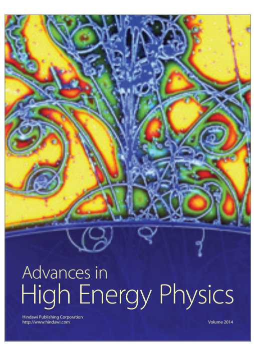
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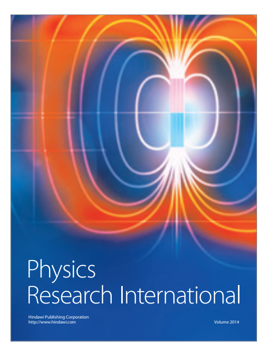
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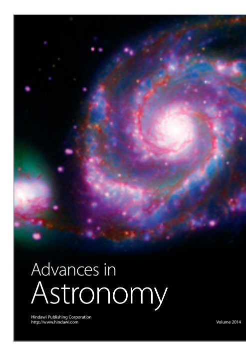
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